Transmission of Risk in a Supply Chain

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ABSTRACT

We develop an equilibrium model of price dynamics and the transmission of shocks in a supply chain. Starting with exogenous factors for the net supply of the upstream input and the demand for the downstream output, we determine the equilibrium price process for the input, the output, and the spread between input and output prices. We specify and calibrate our model for the case of crude oil and refined products that include gasoline and heating oil, and present comparative statics. As we show, the relative volatilities of oil and gasoline, as well as their correlation depends on the volatility and correlations of supply and demand shocks, elasticities, the convexity of the production function, and the competitiveness of the gasoline markets. In our analysis of a two stage supply chain that transforms oil to fuel and fuel to air travel we find that airlines may be able to hedge more effectively using crude oil rather than jet fuel futures.

JEL Classification: G13, G32, G43

Introduction

In its simplest form, a supply chain consists of three components: an upstream primary input, a downstream output product, and a capital asset that turns the input into the output. One can give numerous examples: crude oil is transformed into refined products at a refinery; electricity is generated with various fuels; bauxite ore is transformed into aluminum in aluminum smelters; tomatoes are converted into paste; and milk is converted to cheese. In each case, raw materials are processed into a final commodity. More complicated, multi-stage chains are also common: once crude oil is transformed into refined products at a refinery, refined products are transformed into travel through automobiles, airplanes, and ships.

In this paper, we present an equilibrium model that examines how various characteristics of the input and output markets influence how commodity price risk is transmitted through the supply chain. The specific characteristics we consider include the extent to which uncertainty is generated by shocks to the supply of the input versus demand for the output, the elasticities of the supply and demand functions, operational characteristics, such as the convexity of the production function that transforms the input into the output, and the competitiveness of the output market.

Our primary interest is on the time series pattern of input and output prices and the spread between those prices. In particular, we are interested in the relative volatilities of those prices, how the prices covary, and the relation between the price patterns of input and output prices and the spread between the prices. These price patterns can have important implications for both the valuation and the hedging of the capital asset that transforms the input into the output. In particular, we are interested in understanding the extent to which the producer of a downstream product, like gasoline, can hedge profits by hedging the costs of its inputs. In an extension we also examine hedging in a two stage supply chain that transforms crude oil to fuel and fuel to air travel.

Our analysis indicates that price patterns are very different depending on whether uncertainty is generated mainly by supply rather than demand shocks. In a competitive market characterized by supply, but not demand shocks, input prices are more volatile than output prices and the spread between input and output prices are negatively correlated with the input price. In this case, a producer that wants to minimize the variance of its profits, and which cannot hedge the output price, will partially hedge the price of its input needs. The extent to which the producer buys the input forward is determined by the elasticity of the demand for the output good and the convexity of the production function. The variance minimizing hedge ratio is also determined by the competitiveness of the output market. Specifically, a variance minimizing oligopolist will hedge more than its perfectly competitive counterparts.

In contrast to the case where uncertainty is generated mainly from supply shocks, output prices are more volatile than input prices in a market where uncertainty is generated by demand shocks. In this case, the spread between input and output prices is positively correlated with the input price, indicating that the producer might perversely hedge its input costs by selling, rather than buying, its inputs forward. The extent to which it does this is determined by the convexity of the production function, the elasticity of the supply function as well as the competitiveness of the market.

To illustrate and gauge the magnitude of these effects we apply our model to the U.S. oil refinery industry, where the input is crude oil and the outputs include refined products like gasoline, jet fuel, and heating oil.[1] This industry is of intrinsic interest, because of its importance in the overall economy, and there is readily available data for the input (crude

[1]Beyond gasoline and distillates; i.e., diesel fuel, jet fuel and heating oil, the refinery industry also produces residual products, such as gas oil, lubricants, and asphalt. However, gasoline is by far the most important product for US refineries. Gasoline yield; i.e., the ratio of gasoline output to total refinery output, ranges between 42% and 48%, and due to higher gasoline prices, its share in refinery revenue is around 60%.

oil), the output (gasoline and other refinery products), and the crack spread,[2] and operational characteristics of gasoline refineries.

Although our model is quite simple, it can be calibrated to roughly match the price patterns of oil and refined products during our 1990 to 2012 sample period. In particular, we show that our calibrated model is consistent with three empirical observations that we document. The first is that the operating margins in the refining business, the crack spread, has widened during this time period. The second is that the volatility of the crack spread has increased, and the third is that the correlation between the crack spread and oil prices has gone from being negative to being positive.

The refining industry provides a good laboratory for exploring the relation between input and output prices since both gasoline and oil are traded on organized exchanges and their prices are readily observable. However, because refiners can hedge both inputs and outputs, the problem of trying to hedge profits with forward purchases of just inputs is not particularly relevant in this case. Given this, we extend our model to we consider a two-stage supply chain where only the input can be hedged. In particular, we provide a preliminary analysis of airlines, who often hedge with either forward purchases of jet fuel or alternatively derivatives on crude oil.

The literature on the transmission of risk in the supply chain starts surprising early: in the fourth century BCE, in the first book of Politics, Part XI, Aristotle discusses the effect of a shock to input supply on the profit of a refining industry with a fixed short-term capacity. He describes how Thales of Miletus correctly forecast a good year for olive production in the island of Chios, and how, based on this forecast, secured the use of all the local olive presses. The subsequent realization of a good harvest resulted in an increased demand for the limited number of olive presses, and allowed Thales to rent out the olive presses for a large profit.

[2]The term "crack spread" refers to the difference between the price of certain derivatives of crude oil — mostly gasoline and heating oil — and the crude oil used to produce these refined products. Synthetic contracts traded in the New York Mercantile Exchange — NYMEX — and other commodity exchanges use a 3-2-1 ratio, meaning that the value of the contract is the value of the sum of two units of gasoline and one unit of heating oil, net three units of crude oil.

This anecdote can be used to illustrate much of the intuition of our model: given the limited capacity of olive presses, an increase in the supply of olives results in olive presses operating at full capacity, increasing the spread between the price of olives and the price of olive oil, thereby increasing the rental rate on olive presses. Aristotle attributes Thales' large profit to his monopoly position, which would of course contribute to such profits. But as our model illustrates, a good olive harvest would result in large profits for the olive press industry even when the olive press market is perfectly competitive.

In a more recent paper Hirshleifer (1988) proposes a framework, similar to the one we consider, to study the equilibrium between growers, processors, and speculators in a market with an exogenous demand shock. Beyond the difference in focus, our framework differs in that we consider both supply and demand shocks, both linear and non-linear production functions for the output, and also a case of a two-stage supply chain. Bessembinder and Lemmon (2002), and Aid, Campi, Huu, and Touzi (2009), among others, build structural models for the electricity sector. Chesnes (2009) looks into the refinery industry but has a different focus on investment and maintenance issues. Our paper adds to the literature by considering a detailed model, which we calibrate using historical prices. We also pay special attention to the level and volatility of spreads between the input and output prices.

The paper is organized as follows. Section I describes the base case model for the supply chain between crude oil and refined products and provides comparative statics in the cases of only supply or only demand shocks. Section II provides two propositions on comparative statics when either supply, or demand shocks, but not both, are present. Section IV extends the base case model to a market structure with an oligopoly that engages in Cournot competition in the refining market. Section III calibrates the base case model to data between 1990 and 2012, and Section V provides comparative statics computed using the model for the cases of the competitive and oligopolistic markets. Section VI discusses an extension to a two-stage supply chain. and Section VII concludes.

I. Base Case Model

A. Input: Crude Oil

We assume that the input commodity — crude oil — is supplied in a competitive market, implying that price equals the marginal cost of production. We employ a reduced form model of oil prices where the elasticity of price with respect to gasoline consumption is assumed to be constant:

$$P_C(t) = X_s(t)Q^{\gamma_s}(t) \tag{1}$$

where $P_C(t)$ is the price of crude oil at time t, $X_s(t)$ is the value of a stochastic factor at time t whose inverse represents net supply of oil to the United States, $Q(t)$ is the quantity of crude oil used to produce domestic gasoline at time t, and $\gamma_s \geq 0$ captures the elasticity of oil price with respect to gasoline consumption. It should be noted that X_s captures both demand shocks outside the United States — e.g. increased consumption in China and India, which limit the supply that is available to the United States — and supply shocks — e.g., a revolution in Iran, or technical improvements in deep water drilling.

We model the dynamics of the stochastic factor X_s as a mean-reverting process.

$$dX_s(t) = \mu_s(\overline{X_s} - X_s(t))dt + \sigma_s dW_s \tag{2}$$

where the mean reversion rate μ_s, the long term level $\overline{X_s}$, and the volatility σ_s are assumed to be constant. Given this specification, the supply factor is normally distributed for each time, t, and given by

$$\begin{aligned}
X_s(t) &= e^{-\mu_s t}X_s(0) + \overline{X_s}\left(1 - e^{-\mu_s t}\right) + \sigma_s e^{-\mu_s t}\int_0^t e^{\mu_s u}dW_s(u) \\
E(X_s(t)) &= e^{-\mu_s t}X_s(0) + \overline{X_s}\left(1 - e^{-\mu_s t}\right) \\
\text{var}(X_s(t)) &= \frac{\sigma_s^2}{2\mu_s}\left(1 - e^{-2\mu_s t}\right)
\end{aligned} \tag{3}$$

B. Output: Gasoline

The price of gasoline is determined by a demand factor X_d, and the aggregate supply of gasoline on the market Q. The stochastic demand factor X_d depends on long-term variables like the fuel-efficiency of the stock of vehicles in use and short-term shocks to income, weather, and seasonal factors.

The demand for gasoline is assumed to have a constant elasticity with respect to price, implying an inverse demand function given by

$$P_G(t) = S_d(t)X_d(t)Q^{-\gamma_d}(t) \tag{4}$$

where S_d is a deterministic seasonality factor, and $\gamma_d \geq 0$ is the elasticity parameter.

We assume that the deseasonalized demand factor X_d follows a mean-reverting process with shocks that are correlated with the shocks in the net supply of crude oil.

$$X_d(t) = \mu_d(\overline{X_d} - X_d(t))dt + \sigma_d dW_d \tag{5}$$

where μ_d is the speed of mean reversion, $\overline{X_d}$ the long term level of the deasonalized demand factor, and σ_d is the volatility of the shocks to gasoline demand. The correlation between the shocks to gasoline demand and crude oil supply, ρ, is assumed to be constant

$$dW_d dW_s = \rho dt \tag{6}$$

Given this specification, the demand factor is normally distributed at each time, t, and given by

$$
\begin{aligned}
X_d(t) &= e^{-\mu_d t} X_d(0) + \overline{X_d}\left(1 - e^{-\mu_d t}\right) + \sigma_d e^{-\mu_d t} \int_0^t e^{\mu_d s} dW_G(s) \\
E(X_d(t) &= e^{-\mu_d t} X_d(0) + \overline{X_d}\left(1 - e^{-\mu_d t}\right) \\
\operatorname{var}(X_d(t)) &= \frac{\sigma_d^2}{2\mu_d}\left(1 - e^{-2\mu_d t}\right)
\end{aligned} \tag{7}
$$

The covariance of the demand and supply factors is given by

$$
\operatorname{covar}(X_s(t), X_d(t)) = \rho \frac{\sigma_s \sigma_d}{\mu_d + \mu_s}\left(1 - e^{(\mu_s + \mu_d)t}\right) \tag{8}
$$

Depending on the magnitude of the supply shocks and the correlation between demand in the United States and in other parts of the world, the supply and demand factors can be either positively or negatively correlated.

C. Capital Asset: Refinery Industry

We assume the United States refined products market is self-contained; i.e., no imports or exports, and is supplied by a refinery industry that converts crude oil into refined products that include gasoline, heating oil and jet fuel.[3] To keep our analysis simple we will refer to gasoline as the sole output and consider a relatively short time period where refinery capacity is fixed. To characterize the production function we consider the following function that describes the total cost of gasoline production:

$$
\mathrm{TC}(Q) = F_C + P_C(Q, X_s)Q + P_{I,G}Q + \phi(Q) \tag{9}
$$

[3] In fact, the refining industry is becoming increasingly global, and in the next draft we will be modeling a global refining industry.

where total costs include a fixed cost component, F_C, and a variable cost component that includes:

- the cost of crude oil $P_C(Q, X_s)Q$;
- the cost of inputs other than crude oil, such as energy (especially natural gas), and chemicals used in the process of turning crude oil into gasoline $P_{I,G}Q$; and,
- other refining costs, e.g. maintenance and labor, modeled as $\phi(Q)$, which is continuous, three times differentiable, increasing, and convex, $\phi''(Q) > 0$. In our calibration we will use the function $\phi(Q) = \varphi Q^{\alpha}$.

D. Market Structure

We first consider the case of a perfectly competitive market where each participant is small and takes prices as given when making production choices. In such a market, the refiners produce until the marginal cost of gasoline production equals the price of gasoline

$$P_G = P_{I,G} + P_C + \phi'(Q^*) \tag{10}$$

which implies that, in equilibrium, the optimal production amount, Q^* is given by

$$S_d X_d Q^{*-\gamma_d} = X_s Q^{*\gamma_s} + P_{I,G} + \phi'(Q^*) \tag{11}$$

II. Comparative Statics — Theory

Given the structure of our model we are able to determine how different sources of risk affect different commodities depending on their position within the supply chain. To illustrate this point we start with the following propositions that describe the sensitivities of oil and gasoline prices to supply and demand shocks. The proofs of the propositions are provided in the Appendix.

Proposition 1. 1) Crude oil prices are more sensitive to supply shocks than are gasoline prices, $\frac{dP_C}{dX_s} \geq \frac{dP_G}{dX_s}$

2) The sensitivity of gasoline prices to supply shocks decreases as the elasticity of demand increases, $\frac{dP_G}{dX_s} \downarrow \gamma_d \uparrow$

3) If the third derivative of the production function, ϕ''', is sufficiently small, the sensitivity of gasoline prices to supply shocks decreases as the convexity of the production function increases, $\frac{dP_G}{dX_s} \downarrow \phi'' \uparrow$

Proposition 2. 1) Crude oil prices are less sensitive to demand shocks than are gasoline prices, $\frac{dP_C}{dX_d} \leq \frac{dP_G}{dX_d}$

2) The sensitivity of gasoline prices to demand shocks decreases as the elasticity of supply increases, $\frac{dP_G}{dX_d} \downarrow \gamma_s \uparrow$

3) If the third derivative of the production function, ϕ''', is sufficiently small, the sensitivity of gasoline prices to demand shocks increases as the convexity of the production function increases, $\frac{dP_G}{dX_d} \uparrow \phi'' \uparrow$

Propositions 1 and 2 have direct implications about the relation between oil and gasoline price volatility. For example, an implication of the first proposition is that increases in the volatility of supply shocks increase the volatility of oil prices more than gasoline prices. Indeed, if uncertainty comes solely from supply shocks, oil prices will be more volatile than gasoline prices. The magnitude of this difference in sensitivity to supply shocks is reduced as the production function becomes more convex and the elasticity of demand increases. Similarly, an implication of the second proposition is that increases in the volatility of demand shocks has a greater effect on the volatility of gasoline prices than oil prices. In this case, if uncertainty comes solely from demand shocks, gasoline prices will be more volatile than oil prices. Again, this difference depends on the convexity of the production function, and in this case, the elasticity of supply rather than demand. Finally, one can extend this proposition to examine the relation between changes in oil and gasoline prices and the crack spread. Specifically, as we summarize in the following proposition, the crack spread can be either positively or negatively correlated with oil and gasoline prices depending on the magnitude of the volatilities of supply and demand shocks.

Proposition 3. When we have only supply shocks the crack spread is negatively correlated with both crude oil and gasoline prices. On the other hand, when we have only demand shocks the crack spread is positively correlated with both crude oil and gasoline prices.

As Proposition 3 illustrates, the hedge ratio between the profits of a manufacturing facility, like a refinery, and the input price can be very different depending on whether uncertainty comes primarily from supply or demand shocks. When there are both supply and demand shocks the correlation between oil prices and the crack spread can be either positive or negative depending on the volatility of the shocks, and oil prices can be either more or less volatile than gasoline prices. In this case, the crack spread is no longer perfectly correlated with either crude oil or gasoline, since there are two sources of uncertainty. Of course, with two sources of uncertainty one can still hedge perfectly with both oil and gasoline futures.

III. Calibration

In this section we calibrate the model, with the goal of roughly matching the relevant moments of observed oil and gasoline prices, which we obtain from the Energy Information Administration (EIA) website. Specifically, we examine data from January 1990 until May 2012, on crude oil prices, US gasoline prices, and the amount of gasoline supplied every month.[4]

To estimate the model parameters we employ the method of simulated moments, which pick parameters to match as closely as possible the moments implied by our model with em-

[4]The amount supplied includes the amount produced, as well as monthly imports and exports, and changes in the amount of gasoline stored.

pirical moments observed in the data. Specifically, we match the level of crude oil and gasoline prices, the standard deviation of changes in crude oil prices; the standard deviation of changes in gasoline prices; the standard deviation of changes in the crack spread; the correlation between changes in crude oil and gasoline price changes; the correlation between changes in the crude oil and crack spread prices; and, the correlation between gasoline and crack spread price changes. We separately estimate parameters over four time windows that include 1990-1995, 1996-2000, 2001-2005, 2006-May 2012. We use the same elasticity parameters and production function for all four windows but allow the means, volatilities, and correlation between the demand and supply factors to vary; i.e., for each time window we match eight moments by varying the values of five parameters.[5]

For each time window, the calibration algorithm is described below:

Step 1: We guess initial values for the mean, standard deviation, and correlation of the supply and demand factors,

Step 2: Given our guesses, we generate scenarios for changes in the values of the demand factor and supply factors,

Step 3: For each scenario, we calculate the amount of gasoline produced, the change in the price of crude oil, and the change in the price of gasoline,

Step 4: We calculate the mean, standard deviation, and correlation of the supply and demand factors that minimize the sum of the percentage difference between the model-generated values and the empirically-observed values. If the differences between the model and empirical values have not improved by more than a certain percentage over the guess in step 1, we stop and report the model parameters; otherwise we change the guess for the model parameters to the one found by the optimization and repeat steps 1-4.

The common values for the elasticity parameters and the production function are given in Table I.

[5] We choose the values for the common parameters; i.e., the elasticity parameters and the parameters of the production function, to best match the moments for the 2006-2012 window.

Table I
Common Parameters

Parameter	Value
Elasticity of supply γ_s	0.19
Elasticity of demand γ_d	5.06
Production function convexity α	8.66
Coefficient φ per million barrels	4.7×10^{-8}
Marginal refining cost: $P_{I,G}$	\$4.41

Tables II, III, IV, V present the results of the calibration for each time window. The tables report the model and empirical moments for the price of crude oil, gasoline, crack spread, and the quantity of gasoline produced. In addition, the tables report the beta in the regression of changes in the price of the crack spread with respect to changes in the price of crude oil, the skewness of the crack spread, and the skewness of the aggregate profit for the entire refining industry (reported and available only from the model).

Overall, the model fits quite well. With the exception of the standard deviation of the quantity of gasoline consumed — in reality, the quantity consumed if much less volatile than the quantity suggested by our model — the calibrated model parameters are all within a few percent of their empirical counterparts. We are particularly interested in the correlation between the crack spread and oil prices, since this correlation provides insights about the extent to which hedging input prices effectively hedges a firm's profits. As the Tables reveal, the calibrated model generates correlations that are quite close to the empirically estimated correlations in each of the five year periods. This is somewhat surprising because the correlation changes quite a bit over this time period, from being slightly negative in the first time period to being very positive in the last time period.

Table II
Model and Empirical Moments — 1990-1995

Parameter	Model	Empirical
mean(P_C)	\$18.32	\$18.32
mean(P_G)	\$25.03	\$25.03
mean($P_G - P_C$)	\$6.71	\$6.71
Std. Dev. (ΔP_C)	\$1.83	\$1.83
Std. Dev. (ΔP_G)	\$2.44	\$2.44
Std. Dev. ($\Delta(P_G - P_C)$)	\$1.75	\$1.75
Corr($\Delta P_G, \Delta P_C$)	69.8%	69.8%
Corr($\Delta P_C, \Delta(P_G - P_C)$)	-7.6%	-7.6%
Corr($\Delta P_G, \Delta(P_G - P_C)$)	66.1%	66.0%
mean(Q)	7.33	7.35
Std. Dev. (Q)	0.86	0.30
beta(Δcrack, Δcrude)	-0.07	-0.07
skew(crack)	0.89	1.08
skew(profit)	1.05	

Table III
Model and Empirical Moments — 1996-2000

Parameter	Model	Empirical
mean(P_C)	\$17.37	\$17.52
mean(P_G)	\$23.94	\$23.08
mean($P_G - P_C$)	\$6.57	\$5.56
Std. Dev. (ΔP_C)	\$1.10	\$1.10
Std. Dev. (ΔP_G)	\$2.12	\$2.10
Std. Dev. ($\Delta(P_G - P_C)$)	\$1.53	\$1.53
Corr($\Delta P_G, \Delta P_C$)	72.1%	71.3%
Corr($\Delta P_C, \Delta(P_G - P_C)$)	27.7%	25.8%
Corr($\Delta P_G, \Delta(P_G - P_C)$)	86.5%	86.2%
mean(Q)	7.33	8.07
Std. Dev. (Q)	0.76	0.34
beta(Δcrack, Δcrude)	0.38	0.36
skew(crack)	0.79	1.14
skew(profit)	0.92	

Table IV
Model and Empirical Moments — 2001-2005

Parameter	Model	Empirical
mean(P_C)	\$28.32	\$28.32
mean(P_G)	\$37.10	\$37.10
mean($P_G - P_C$)	\$8.78	\$8.78
Std. Dev. (ΔP_C)	\$2.22	\$2.22
Std. Dev. (ΔP_G)	\$4.17	\$4.17
Std. Dev. ($\Delta(P_G - P_C)$)	\$2.86	\$2.86
Corr($\Delta P_G, \Delta P_C$)	76.4%	76.4%
Corr($\Delta P_C, \Delta(P_G - P_C)$)	34.0%	34.0%
Corr($\Delta P_G, \Delta(P_G - P_C)$)	86.6%	86.6%
mean(Q)	8.08	8.80
Std. Dev. (Q)	0.75	0.27
beta(Δcrack, Δcrude)	0.43	0.44
skew(crack)	0.83	0.93
skew(profit)	0.96	

Table V
Model and Empirical Moments — 2006-May 2012

Parameter	Model	Empirical
mean(P_C)	\$74.11	\$74.11
mean(P_G)	\$87.98	\$87.98
mean($P_G - P_C$)	\$13.87	\$13.87
Std. Dev. (ΔP_C)	\$6.38	\$6.38
Std. Dev. (ΔP_G)	\$9.06	\$9.06
Std. Dev. ($\Delta(P_G - P_C)$)	\$5.24	\$5.24
Corr($\Delta P_G, \Delta P_C$)	82.5%	82.5%
Corr($\Delta P_C, \Delta(P_G - P_C)$)	20.8%	20.8%
Corr($\Delta P_G, \Delta(P_G - P_C)$)	72.5%	72.5%
mean(Q)	9.00	9.04
Std. Dev. (Q)	0.70	0.23
beta(Δcrack, Δcrude)	0.17	0.17
skew(crack)	0.61	0.65
skew(profit)	0.71	

Table VI
Supply and Demand Factors

Parameter	90-95	96-00	01-05	06-12
Mean(Supply)	2.52	2.47	2.94	3.88
Supply Std.	0.11	0.07	0.08	0.09
Mean(Demand)	13.24	13.20	14.14	15.56
Demand Std.	0.71	0.65	0.61	0.51
Correlation	-20%	0%	18%	15%
Supply Std. %	4.33%	2.61%	2.65%	2.27%
Demand Std. %	5.37%	4.91%	4.28%	3.25%

To understand the changes in the correlation between the crack spread and oil prices we report, in Table VI, the calibrated model parameters for each of our four time periods. As can be seen from the table, the major change is the implied correlation between the demand and supply factor shocks, which is initially negative but becomes positive over time.[6] This change reflects the growing importance of demand shocks in China and other emerging markets — recall, we are modeling a positive shock to demand in China as being equivalent to a negative shock to the supply of oil available to U.S. consumers. Hence, a positive global shock to demand would imply an increase in demand in the United States and, at the same time, less supply available to the United States. Hence, as the demand from outside the United States becomes larger, and/or more correlated with the demand in the United States, we will see this correlation increase. As our model implies, this increase in the correlation between the supply and the demand factors leads to an increase in the correlation between crude oil prices and the crack spread.

[6]We note that since our supply factor is inversely related to supply, a positive correlation between our supply and demand factors corresponds to a negative correlation between the levels of supply and demand.

IV. Symmetric Oligopoly — Cournot Competition

The previous section presents calibrated parameters for our model that assumes a perfectly competitive refinery market. In reality, given the relatively small number of firms that own and operate refineries, and the inelasticity of the demand for refined products, we expect the players in this market to have at least some market power.[7] To explore the implications of market power in the refinery market we model the market as a game with N identical competitors who engage in Cournot competition. In Cournot competition each participant considers the actions of all other participants as given and then optimizes his expected profit, taking into account the impact of his actions on the market prices of gasoline and crude oil.

Given that all the participants are identical, in the symmetric equilibrium the same amount of gasoline is produced in each refinery. If the amount of gasoline produced by the i^{th} participant is given by Q_i, while the aggregate amount of gasoline produced by all the other participants is given by Q_{-i}, then the total cost for each participant is given by:

$$\text{Total cost}_i(Q_i) = \frac{1}{N}F_C + P_C Q_i + P_{I,G} Q_i + \frac{1}{N}\phi(NQ_i) \tag{12}$$

The prices of gasoline and crude oil are given by

$$\begin{aligned} P_C &= X_s(Q_i + Q_{-i})^{\gamma_s} \\ P_G &= S_d X_d (Q_i + Q_{-i})^{-\gamma_d} \end{aligned} \tag{13}$$

[7] While some major producers of gasoline are vertically integrated, several companies are involved exclusively in the refining business. Gilbert and Hastings (2005) show that oil companies that own refineries may use strategic pricing of crude oil at regional level to influence the input cost of their rivals. Du and Hayes (2008) estimate the Herfindahl-Hirschman Index (HHI) for five "Petroleum Administration for Defense Districts" (PADDs) and conclude that, except for the East Coast, the HHI corresponds to the case of competitive markets. Even on the East Coast, it can not be concluded that gasoline market is not competitive because this region imports a lot of gasoline from other regions. Oladunjoye (2008) also reports similar results for the areas of New York, the Gulf Coast, and Los Angeles.

Each participant maximizes his profit

$$\max_{Q_i} P_G Q_i - \text{Total cost}_i(Q_i) \tag{14}$$

assuming that the quantity produced by the other participants stays constant. Given the symmetry between the participants, the aggregate amount of gasoline produced is equal to NQ_i, which is the solution to the following equation

$$\begin{aligned} \frac{\partial}{\partial Q_i}\left(P_G Q_i - \text{Total cost}_i(Q_i)\right)|_{Q_i+Q_{-i}=NQ_i} = 0 \\ (1+\frac{\gamma_s}{N})P_C + P_{I,G} + \phi'(NQ_i) = \left(1-\frac{\gamma_d}{N}\right)P_G \end{aligned} \tag{15}$$

V. Comparative Statics — Numerical

Table VII presents empirical and model generated moments for the base case parameters, as well as six comparative statics. The first five comparative statics examine the effect of 10% increases in the elasticity of demand, the elasticity of supply, the convexity of the refinery production function, the standard deviation of the demand factor increases and the standard deviation of the supply factor. The last comparative static explores how changes in the level of demand affect prices, spreads and correlations. The change in the level of demand is chosen to correspond to an average of 10% increase in production of gasoline, or, equivalently, a 10% increase in the utilization rate.

From the table we note that an increase in the elasticity of demand decreases the price of gasoline from \$87.98 to \$78.42 and cuts the crack spread approximately in half, from \$13.87 to \$6.81. It also reduces the standard deviation of gasoline prices by 50% and the standard deviation of the crack spread from \$5.24 to \$1.35. In addition, it increases the correlation of changes in crude oil and gasoline prices from 82% to 98% and reduces the correlation of

Table VII
Moment Comparative Statics — Competitive

This table presents model generated moments for the competitive market structure.

	Base case	Empirical	γ_d	γ_s	α	σ_d	σ_s	Demand
mean(P_C)	74.11	74.11	71.61	77.19	73.30	74.11	74.17	75.52
mean(P_G)	87.98	87.98	78.42	90.65	118.61	88.27	88.04	99.58
mean($P_G - P_C$)	13.87	13.87	6.81	13.47	45.30	14.17	13.87	24.06
Std. Dev. (ΔP_C)	6.38	6.38	6.16	6.66	6.30	6.42	6.99	6.50
Std. Dev. (ΔP_G)	9.06	9.06	6.56	9.22	21.39	9.69	9.45	12.89
Std. Dev. ($\Delta(P_G - P_C)$)	5.24	5.24	1.35	5.04	19.25	5.88	5.22	9.90
Corr($\Delta P_G, \Delta P_C$)	0.82	0.82	0.98	0.85	0.47	0.81	0.84	0.66
Corr($\Delta P_C, \Delta(P_G - P_C)$)	0.21	0.21	0.20	0.22	0.19	0.24	0.18	0.20
Corr($\Delta P_G, \Delta(P_G - P_C)$)	0.72	0.72	0.39	0.71	0.96	0.77	0.68	0.87
mean(Q)	9.00	9.04	7.53	8.95	8.49	9.00	9.00	9.93
Std. Dev. (Q)	0.70	0.23	0.59	0.70	0.50	0.77	0.70	0.70

changes in the gasoline prices and the crack spread from 72% to 39%. In contrast, an increase in the elasticity of supply has only a small effect.

An increase in the convexity of the production function generates a large increase in the level of gasoline prices from \$87.98 to \$118.61, and generates an increase in the crack spread from \$13.87 to \$45.30. It also results in an increase in the standard deviation of gasoline prices from \$9.06 to \$21.39 and the standard deviation of the crack spread from \$5.24 to \$19.25. The correlation between crude oil and gasoline price changes decreases from 82% to 47% with an increase in convexity and the correlation between changes in gasoline prices and changes in the crack spread increases from 72% to 96%.

Consistent with our theoretical comparative statics, an increase in the standard deviation of demand shocks leads to a relatively larger increase in the standard deviation of changes in gasoline prices, from \$9.06 to \$9.69, while the standard deviation of changes of crude oil prices increases only modestly, from \$6.38 to \$6.42. Similarly, an increase in the standard deviation of supply shocks leads to a relatively larger increase in the standard deviation of changes in crude oil prices. A 10% increase in the standard deviation of supply shocks generates an increase in the standard deviation of changes in oil prices from \$6.38 to \$6.99, but increases the standard deviation of changes in gasoline prices much less, from \$9.06 to \$9.45.

If we increase the level of demand enough to increase the production of gasoline by 10%, the price of gasoline increases from \$87.98 to \$99.58, and the crack spread increases from \$13.87 to \$24.06, and the monthly standard deviation of changes in the crack spread increases from \$5.24 to \$9.90. The intuition behind these increases is that when the production of gasoline is higher, because of the convexity of the production function the marginal cost of production is higher, leading to a larger crack spread, as well as a more volatile crack spread. All these changes are consistent with the observed changes in crack spreads since 1990 and indicate that the increase in the level of the crack spread and its volatility can be explained in terms of an increase in the level of demand.

Similar to the calibration for the case of a competitive market, we have tried to calibrate our model under the assumption of Cournot competition. However, as we illustrate in Table VIII, the estimated moments are substantially different from empirical estimates even when we increase the number of competitors to as high as 30. In particular, we cannot come close to matching the magnitude of the crack spread in the Cournot model. This failure suggests that even though there are relatively few participants in the refining market, they appear to compete much more aggressively than would be suggested by the Cournot model, suggesting that some form of a Bertrand model for oligopoly may be more appropriate. The challenge is illustrated in first two columns of Table VIII, which presents model generated moments for the base case parameters calibrated to the case of perfect competition, with the additional assumption that there are 30 identical market participants that operate as oligopolists. We note that, compared to the case of perfect competition, the level of the crack spread doubles, from \$13.87 to \$29.28, mostly due to an increase of the price of gasoline from \$87.98 to \$102.95.

It should be noted that the correlation between the crack spread and crude oil price changes is substantially greater in an oligopolistic market — from 21% in the competitive market

Table VIII
Moment Comparative Statics — Oligopoly

This table presents model generated moments for the oligopolistic market with 30 participants.

	Base case	Empirical	γ_d	γ_s	α	σ_d	σ_s
mean(P_C)	73.67	74.11	71.14	76.68	72.97	73.67	73.73
mean(P_G)	102.95	87.98	95.03	106.17	133.60	103.25	103.01
mean($P_G - P_C$)	29.28	13.87	23.88	29.49	60.63	29.58	29.29
Std. Dev. (ΔP_C)	6.34	6.38	6.12	6.62	6.27	6.38	6.95
Std. Dev. (ΔP_G)	10.04	9.06	7.87	10.26	22.54	10.66	10.55
Std. Dev. ($\Delta(P_G - P_C)$)	5.53	5.24	2.07	5.38	20.10	6.20	5.53
Corr($\Delta P_G, \Delta P_C$)	0.87	0.82	0.99	0.88	0.51	0.85	0.88
Corr($\Delta P_C, \Delta(P_G - P_C)$)	0.43	0.21	0.79	0.46	0.26	0.44	0.42
Corr($\Delta P_G, \Delta(P_G - P_C)$)	0.82	0.72	0.88	0.82	0.96	0.84	0.80
mean(Q)	8.73	9.04	7.27	8.67	8.29	8.73	8.73
Std. Dev. (Q)	0.70	0.23	0.57	0.69	0.51	0.76	0.70

model to 43% in the oligopolistic market — indicating that there is a greater incentive to hedge input costs in less competitive markets.[8]

Comparing the moments between the competitive and oligopolistic markets for the various comparative statics, we also note that when the elasticity of demand increases, the correlations between the price changes of crude oil and the crack spread and gasoline and the crack spread increase from 20% and 39% in the competitive market, to 79% and 88% in the oligopolistic market. A similar increase occurs when the elasticity of demand rises, with the correlation of crude oil price changes and crack spread changes increasing from 22% to 46% and the correlation of gasoline price changes and crack spread changes increasing from 71% to 82% between the competitive and oligopolistic markets respectively.

[8] We have found that the greater correlation between changes in the crack spread and changes in the price of crude oil is a feature of our model that holds for all of the parameters we have examined. While we have been unable to prove the generality of this result for our model, we can prove the result in a model with linear inverse demand and supply functions and a quadratic production function.

VI. Two Stage Supply Chain

We now consider the case of a two-stage supply chain. Specifically, we consider air travel, which uses gasoline — actually uses jet fuel — which is in turn refined from crude oil. To study the transmission of risk along the supply chain we make the simplifying assumption that changes in the logarithm of the demand for gasoline are perfectly correlated with changes in the logarithm of the demand for air travel.

Similar to our model in the single stage supply chain we assume that the price of crude oil is exogenously determined by a supply factor, X_s, and the amount of gasoline produced, Q_G

$$P_C = X_s Q_G^{\gamma_s} \tag{16}$$

The price of a unit of gasoline is determined by a demand factor X_G, which is correlated with the supply of crude oil with correlation ρ, and the amount of gasoline produced Q_G. The inverse demand function is given by

$$P_G = S_d X_d Q_G^{-\gamma_G} \equiv X_G Q_G^{-\gamma_G} \tag{17}$$

The price of a unit of air travel, which we define as the passenger miles that can be generated by one barrel of crude oil, is determined by a demand factor X_A, which is a multiple of the demand for gasoline, and the amount of air travel supplied, Q_A

$$\begin{aligned} \Delta \ln X_A &= c \Delta \ln X_G \\ P_A &= X_A Q_A^{-\gamma_A} \end{aligned} \tag{18}$$

The total cost for producing Q_G units of gasoline and Q_A units of air travel is given by

$$\begin{aligned} \text{Total cost}_{\text{gas}}(Q_G) &= F_G + P_C Q_G + P_{I,G} Q_G + \phi_G(Q_G) \\ \text{Total cost}_{\text{air}}(Q_A) &= F_A + P_G Q_A + P_{I,A} Q_A + \phi_A(Q_A) \end{aligned} \tag{19}$$

Assuming a competitive market structure for both the refinery and the airline industries, the amounts of gasoline and air travel produced are given by equating the marginal cost of producing a unit of gasoline to the price of gasoline and the marginal cost of producing a unit of air travel to the price of air travel

$$\begin{aligned} P_G &= P_C + P_{I,G} + \phi'_G(Q_G) \\ P_A &= P_G + P_{I,A} + \phi'_A(Q_A) \end{aligned} \tag{20}$$

A rough calibration of the two-stage supply chain is based on the following stylized facts:

a) U.S. airlines utilize approximately 800 thousand barrels of fuel each day;

b) fuel cost accounts for approximately 35% of the operating cost of an airline;

c) the elasticity of demand for air travel is approximately 0.5, as reported in Jung and Fujii (1976).

Given these stylized facts, we choose parameters such that the price of air travel per unit of fuel is approximately triple the price of fuel. We do this by setting the unit input cost, $P_{I,A}$ equal to \$110, the unit fixed cost equal to \$55, and the other costs at \$55, and the standard deviation of monthly changes of the price of air travel per unit of fuel equal to approximately 3%. In addition, we assume that air travel demand is perfectly correlated with gasoline demand, and that the convexity of the air travel production function is equal to 2.0. Under these assumptions, Table IX, presents the model moments for the base case parameters from Table I and for the demand and supply factor values corresponding to the 2006-2012 period, given in Table V.

Table IX
Two Stage Supply Chain Moments

This table presents model moments for a two stage supply chain from crude oil to gasoline to air travel. The base case parameters are given in Table I. In addition, the convexity of the production function for air travel is equal to 2.

Moment	Value
mean(P_a)	288.47
Std. Dev. (ΔP_a)	8.61
mean($\Delta(P_a - P_g)$)	200.49
Std. Dev. ($\Delta(P_G - P_a)$)	3.68
mean(Q_a)	0.82
Std. Dev. (Q_a)	0.03
corr($\Delta P_a, \Delta P_c$)	0.54
corr($\Delta P_a, \Delta P_g$)	0.91
corr($\Delta P_a, \Delta(P_g - P_c)$)	0.92
corr($\Delta(P_a - P_g), \Delta P_c$)	-0.76
corr($\Delta(P_a - P_g), \Delta P_g$)	-0.32
corr($\Delta(P_a - P_g), \Delta P_a$)	0.09
corr($\Delta(P_a - P_g), \Delta(P_g - P_c)$)	0.38

Of the moments reported in Table IX we are particularly interested in the correlations between the change in the airline operating margins, $\Delta(P_a - P_g)$, and the changes in the price of gasoline, ΔP_g, crude oil, ΔP_c, and the operating margins of a refinery, $\Delta(P_g - P_c)$. These correlations can be used as a measure of the effectiveness of a hedging policy for airline profits using forward contracts for crude oil or gasoline, or, in the case of the correlation between the air travel spread and the crack spread, using a refinery as a hedge for airline profits.[9] These correlations reveal that, for the calibrated parameter values, crude oil provides a better hedge against changes in operating margins than gasoline, and that refinery profits are positively correlated with airline profits.

Since gasoline, and not crude oil, is the input fuel for an airline, it is perhaps surprising that crude oil provides the better hedge for airline profits. Forward purchases of gasoline provide a less effective hedge because demand shocks, that increase gasoline prices, also increase airline revenues. As we showed in the last section, demand shocks also affect oil prices, but not by as much. However, oil prices are affected more than gasoline prices by supply shocks, and price increases that are due to supply shocks cannot be passed on to air travellers as much as demand shocks. Hence, when supply shocks are relatively important, as in our calibrated example, crude oil is the better hedge.

We further investigate the impact of our assumption on the standard deviation of the change in the price of a unit of air travel in Table X by calibrating the parameters of our model to a standard deviation of 2%, rather than the 3% of Table IX. We note that the correlations between the changes in airline operating margins and the change in the price of crude oil and gasoline all increase, indicating that hedging using forward contracts become more efficient. In addition, for these parameter values we find that the correlation between the change in the operating margins of an airline and the operating margins of a refinery are negatively correlated (-40% correlation), indicating that an airline may decide to own a refinery as a hedge.

[9] Reuters reports that "Delta Air Lines took the keys to the Trainer, Pennsylvania, refinery from Philips 66 on Friday (June 22nd, 2012), becoming the first air carrier to wade into fuel production in a bid to bring down costs." Our results suggest that, beyond controlling the cost of fuel, owning a refinery may reduce the volatility of earnings of Delta Air Lines.

Table X
Two Stage Supply Chain — Comparative Statics

This table presents model moments for a two stage supply chain from crude oil to gasoline to air travel. The parameters are the same as the ones used in Table X, other than the multiplied c that connects the logarithm of the changes in the demand for gasoline to changes in the logarithm in the demand of air travel. It is calibrated such that the standard deviation of the monthly change of the price of a unit of air travel is approximately 2% of the average price of a unit of air travel.

Moment	Value
mean(P_a)	288.35
Std. Dev. (ΔP_a)	5.79
mean($\Delta(P_a - P_g)$)	200.37
Std. Dev. ($\Delta(P_G - P_a)$)	3.85
mean(Q_a)	0.82
Std. Dev. (Q_a)	0.03
corr($\Delta P_a, \Delta P_c$)	0.64
corr($\Delta P_a, \Delta P_g$)	0.96
corr($\Delta P_a, \Delta(P_g - P_c)$)	0.87
corr($\Delta(P_a - P_g), \Delta P_c$)	-0.97
corr($\Delta(P_a - P_g), \Delta P_g$)	-0.91
corr($\Delta(P_a - P_g), \Delta P_a$)	-0.75
corr($\Delta(P_a - P_g), \Delta(P_g - P_c)$)	-0.39

The intuition behind these results is that when the demand for air travel is less sensitive to economy-wide demand shocks than the demand for gasoline, then aggregate demand shocks can have an opposite effect on refineries and airlines. The intuition is that a strong demand shock can have a large effect on gasoline prices, making refineries very profitable, but if airlines are not able to pass those costs on to their customers, airlines become less profitable.

VII. Conclusions

This paper presents an equilibrium model that allows us to study the process generating the prices of commodities along a supply chain. In our application to the refinery industry we found that the model worked reasonably well assuming perfect competition. However, even though there are a limited number of competitors in the refining industry, our model with perfect competition fits the data much better than our model with Cournot competition, suggesting that the market participants compete more aggressively than would be suggested by the Cournot model. Our preliminary analysis of a two-stage supply chain that we apply to the airline industry also generates interesting results about the efficacy of hedging airline profits with oil and jet fuel.

Finally, it should be noted that although our focus has been on hedging strategies, our approach can be applied to the valuation of capital assets, like refineries, which transform downstream to upstream commodities. As we show, the profits generated in our model can be quite skewed, and the risks associated with the profits change from period to period depending on the level of demand. In future research we plan to consider the extent to which traditional valuation heuristics may systematically misprice capital assets in this setting.

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A. Proof of Propositions

We will prove Proposition 1. The proof of Proposition 2 is similar.

First we derive the sensitivity of the optimal quantity of gasoline produced with respect to the supply and demand factors, X_s and X_d.

$$
\begin{aligned}
\frac{d}{dX_s}: \quad & -S_d\gamma_d X_d Q^{*-\gamma_d-1}\frac{dQ^*}{dX_s} = Q^{*\gamma_s} + \gamma_s X_s Q^{*\gamma_s-1}\frac{dQ^*}{dX_s} + \phi''(Q^*)\frac{dQ^*}{dX_s} \\
& \Rightarrow \frac{dQ^*}{dX_s}\left(-S_d\gamma_d X_d Q^{*-\gamma_d-1} - \gamma_s X_s Q^{*\gamma_s-1} - \phi''(Q^*)\right) = Q^{*\gamma_s} \\
\frac{d}{dX_d}: \quad & S_d Q^{*-\gamma_d} - S_d\gamma_d X_d Q^{*-\gamma_d-1}\frac{dQ^*}{dX_d} = \gamma_s X_s Q^{*\gamma_s-1}\frac{dQ^*}{dX_d} + \phi''(Q^*)\frac{dQ^*}{dX_d} \\
& \Rightarrow \frac{dQ^*}{dX_d}\left(-S_d\gamma_d X_d Q^{*-\gamma_d-1} - \gamma_s X_s Q^{*\gamma_s-1} - \phi''(Q^*)\right) = -S_d Q^{*-\gamma_d}
\end{aligned}
\tag{21}
$$

From the equations above we conclude that changes in the supply factor and the demand factor have opposite effects on the optimal output quantity of gasoline produced. Specifically, the optimal quantity of gasoline produced decreases with positive shocks to the supply factor, $dQ^*/dX_s < 0$, and increases with positive shocks to the demand factor, $dQ^*/dX_d > 0$. Since the supply factor corresponds inversely to the net supply of crude oil, while the demand factor corresponds directly to demand, we have that as either supply or demand increase, the optimal quantity of gasoline produced increases.

A. Proof of part 1

Proof. We have that

$$\frac{dP_G}{dX_s} = \frac{dP_C}{dX_s} + \phi''(Q^*)\frac{dQ^*}{dX_s} \tag{22}$$

Since $\phi''(Q) \geq 0$, and $dQ^*/dX_s < 0$, we have that

$$\frac{dP_C}{dX_s} \geq \frac{dP_G}{dX_s} \tag{23}$$

In addition, we can calculate the sensitivity of gasoline prices with respect to shocks in the supply factor

$$\frac{dP_G}{dX_s} = -\gamma_d S_d X_d Q^{*-\gamma_d-1} \frac{dQ^*}{dX_s} > 0 \tag{24}$$

Collecting our results, we have the following for the sensitivities of the gasoline price, the crude oil price, and the crack spread, with respect to the supply factor

$$\begin{aligned} \frac{dP_G}{dX_s} &> 0 \\ \frac{dP_C}{dX_s} > \frac{dP_G}{dX_s} &> 0 \\ \frac{d(P_G - P_C)}{dX_s} &< 0 \end{aligned} \tag{25}$$

From the results above, we have that, with positive supply shocks — which are inversely related to the supply factor, X_s — the price of crude oil changes more than the price of gasoline, that both the price of crude oil and the price of gasoline decrease, and that the crack spread increases. These results also imply that, with supply shocks only, the price of crude and the price of gasoline are positively correlated with each other and negatively correlated with the crack spread, proving part 1 of Proposition 3. □

B. Proof of part 2

Proof. We first show that when the elasticity of demand increases, the amount of gasoline produced decreases. We have, from Equation 11

$$\begin{aligned} &\frac{d}{d\gamma_d} : S_d X_d Q^{*-\gamma_d} = X_s Q^{*\gamma_s} + P_{I,G} + \phi'(Q^*) \\ &\quad \Rightarrow S_d X_d Q^{*-\gamma_d} \left(-\ln Q^* - \frac{\gamma_d}{Q^*} \frac{dQ^*}{d\gamma_d} \right) = \gamma_s X_s Q^{*\gamma_s - 1} \frac{dQ^*}{d\gamma_d} + \phi''(Q^*) \frac{dQ^*}{d\gamma_d} \\ &\quad \Rightarrow \left(\gamma_d S_d X_d Q^{*-\gamma_d-1} + \gamma_s X_s Q^{*\gamma_s-1} + \phi''(Q^*) \right) \frac{dQ^*}{d\gamma_d} = -S_d X_d Q^{*-\gamma_d} \ln Q^* \\ &\quad \Rightarrow \frac{dQ^*}{d\gamma_d} < 0 \end{aligned} \tag{26}$$

Collecting the information from Equations 24, 21, we have that

$$\begin{aligned}
\frac{dP_G}{dX_s} &= -\gamma_d S_d X_d Q^{*-\gamma_d-1}\frac{dQ^*}{dX_s} > 0 \\
&= \frac{\gamma_d S_d X_d Q^{*\gamma_s-\gamma_d-1}}{\gamma_s S_d X_d Q^{-\gamma_d-1}+\gamma_s X_s Q^{\gamma_s-1}+\phi''} \\
&= \frac{Q^{\gamma_s}}{1+\frac{\gamma_s X_s Q^{\gamma_s-1}+\phi''}{\gamma_d S_d X_d Q^{-\gamma_d-1}}}
\end{aligned} \tag{27}$$

Since the optimal quantity, Q^*, decreases when the elasticity of demand, γ_d, increases, we have also that Q_s^{γ} decreases, and $\gamma_d Q^{-\gamma_d}$ increases, meaning that, if changes in the convexity of the production function, ϕ'' are small enough, as the elasticity of demand, γ_d increases, the sensitivity of gasoline/refined product prices with respect to supply shocks increase dP_G/dX_s also increases.

□

C. Proof of part 3

Proof. Collecting the information from Equations 24, 21, we have that

$$\begin{aligned}
\frac{dP_G}{dX_s} &= -\gamma_d S_d X_d Q^{*-\gamma_d-1}\frac{dQ^*}{dX_s} \\
&= \frac{\gamma_d S_d X_d Q^{*\gamma_s-\gamma_d-1}}{\gamma_s S_d X_d Q^{-\gamma_d-1}+\gamma_s X_s Q^{\gamma_s-1}+\phi''}
\end{aligned} \tag{28}$$

In addition, from Equation 11 we have that

$$\begin{aligned}
\frac{d}{d\phi''} :& S_d X_d Q^{*-\gamma_d} = X_s Q^{*\gamma_s} + P_{I,G} + \phi'(Q^*) \\
&\Rightarrow -\gamma_d S_d X_d Q^{*-\gamma_d-1}\frac{dQ^*}{d\phi''} = \gamma_s X_s Q^{*\gamma_s-1}\frac{dQ^*}{d\phi''} + \phi''\frac{dQ^*}{d\phi''} \\
&\Rightarrow \frac{dQ^*}{d\phi''}\left(\gamma_d S_d X_d Q^{*-\gamma_d-1} + \gamma_s X_s Q^{*\gamma_s-1} + \phi''\right) = 0
\end{aligned} \tag{29}$$

i.e., the optimal amount of gasoline produced does not depend on the convexity of the production function (as long as everything else remains the same).

Collecting this information, we conclude that, as the convexity increases, $\phi'' \uparrow$, the sensitivity of gasoline/refinded product prices with respect to supply shocks decreases $dP_G/dX_s \downarrow$.

To prove the second part of the proposition, we note that the optimal quantity of gasoline produced does not depend on the convexity of the production function. We deduce that the crude oil price and the gasoline price are also independent of the convexity of the production function. From Equation 22 we then conclude that, when the convexity of the production function increases, for the same shock in supply we have a larger shock in the crack spread.

□