Application of TOPSIS in evaluating initial training aircraft under a fuzzy environment

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Abstract

This paper develops an evaluation approach based on the Technique for Order Performance by Similarity to Ideal Solution (TOPSIS), to help the Air Force Academy in Taiwan choose optimal initial training aircraft in a fuzzy environment where the vagueness and subjectivity are handled with linguistic terms parameterised by triangular fuzzy numbers. This study applies the fuzzy multi-criteria decision-making (MCDM) method to determine the importance weights of evaluation criteria and to synthesize the ratings of candidate aircraft. Aggregated the evaluators’ attitude toward preference; then TOPSIS is employed to obtain a crisp overall performance value for each alternative to make a final decision. This approach is demonstrated with a real case study involving 16 evaluation criteria, seven initial propeller-driven training aircraft assessed by 15 evaluators from the Taiwan Air Force Academy.

Keywords: Multiple criteria decision-making (MCMD); Initial training aircraft; Fuzzy sets theory; TOPSIS

1. Introduction

Airforce construction generally requires vast amounts of money and time, particularly for training combat pilots (Chiang, 1999). A combat pilot must be able to adapt experience to circumstances and make instantaneous judgments. The experience was accumulated in daily and routine training (Penrice, 2000). Pilots can be trained by either flight simulators or physical airplane flying (Bills, 1987). Therefore, the most efficient and effective training systems are needed. Combat aircraft manoeuvres consist of standard take-off, climbing, steep turns and slow flight. Although training beginner pilots directly in high-performance jet combat planes may be appropriate, it has economic difficulties and serious safety problems. Suitable special-purpose trainer aircraft are normally manufactured when new fighter planes are introduced. Propeller-driven training aircraft can be the stepping-stone to fly bigger, faster and more expensive jet combat planes. Initial training tasks have to encompass all basic flight programs (primary, basic, advanced and combat) to allow trainees to gather all required experience for high-level flight. To shorten the training cycle, improve efficiency and save expenses, beginners can be trained using propeller-driven training aircraft rather than jet combat planes. A good initial training aircraft is important for flight training success, so the performance of initial training aircraft is a significant factor. As stated by Cheng (1996, 1999), the performance evaluation and optimal selection of weapon systems have multi-level and multi-factor features, so such difficulties can be regarded as multiple criteria decision-making. Subjectivity, uncertainty and ambiguity usually exist in the weapon system evaluation process, and were not easily solved until the development of Zadeh’s fuzzy sets theory. Some research has started on the utilization of fuzzy sets theory in military applications. Cheng and Mon (1994) assessed weapon systems with AHP based on fuzzy scales. Cheng and Lin (2002) applied fuzzy decision theory to evaluate main battle tanks. Mon, Cheng, and Lin (1994) utilized fuzzy AHP

The main objectives of this study are to propose a systematic evaluation model to help the Air Force Academy in Taiwan select an optimal training aircraft among a set of available propeller-driven aircraft (T-34, PC-7, PC-9, PC-7 MK2, T-6A, KT-1 and T-27) under fuzzy multi-evaluator and multi-criteria environment, and help enhance understanding by this organization of the performance criteria of training aircraft that are important to the drillmasters and trainees. Evaluation within the aircraft selection process is mainly from the perspectives of pilot drillmasters and trainees. Operation, cruising, stalling, climbing, takeoff and landing performance have to be addressed simultaneously. Hence, this study utilizes a multi-criteria decision-making method to determine the importance weights of evaluation criteria, and TOPSIS to obtain the performance ratings of the feasible alternatives in linguistic terms parameterized with triangular fuzzy numbers. This approach is employed for four reasons: (a) TOPSIS logic is rational and understandable; (b) the computation processes are straightforward; (c) the concept permits the pursuit of best alternatives for each criterion depicted in a simple mathematical form, and (d) the importance weights are incorporated into the comparison procedures (Chu & Lin, 2002; Deng, Yeh, & Willis, 2000; Olson, 2004).

The remainder of this study is structured as follows. Section 2 briefly introduces fuzzy sets theory as utilized in multi-criteria decision-making processes. A hierarchical framework to evaluate initial training aircraft using TOPSIS under fuzzy environment is derived in Section 3. Section 4 presents an empirical initial propeller-driven training aircraft selection case elicited from the Air Force Academy in Taiwan. Conclusions and suggestions are also proposed in Section 5.

2. Fuzzy sets theory in multi-criteria decision-making

Natural language to express perception or judgement is always subjective, uncertain or vague. Such uncertainty and subjectivity have long been handled with probability and statistics (Dubois & Prade, 1997). Since words are less precise than numbers, the concept of a linguistic variable approximately characterizes phenomena that are too cumbersome or poorly defined to be described with conventional quantitative terms (Herrera & Herrera-Viedma, 2002). To resolve the vagueness, ambiguity and subjectivity of human judgment, fuzzy sets theory (Zadeh, 1965) was introduced to express the linguistic terms in decision-making (DM) process. Zadeh and Bellman (1970) were the first researchers to survey the decision-making problem using fuzzy sets, and initiated the fuzzy multi-criteria decision-making (FMCDM) methodology. FMCDM was developed to resolve the lack of precision in assigning importance weights of criteria and the ratings of alternatives regarding evaluation criteria (Chen & Klein, 1997). This approach helps decision-makers solve complex decision-making problems in a systematic, consistent and productive way (Carlsson & Fuller, 1996), and has been widely applied to tackle DM problems with multiple criteria and alternatives. Some studies have provided interesting results on multi-criteria decision-making with the help of fuzzy sets theory (Aouam, Chang, & Lee, 2003; Carlsson, 1982; Cheng et al., 1999; Fan, Hu, & Xiao, 2004; Fan, Ma, & Zhang, 2002; Li, 1999; Noci & Toletti, 2000; Ölcer & Odabasi, 2005; Roubens, 1997; Scott & Antonsson, 1998; Wang & Parkan, 2005). Applications on solving MCDM problems by fuzzy sets theory have been published in professional journals of diversified disciplines, such as transfer strategy selection in biotechnology (Chang & Chen, 1994), automotive industry (Alrock & Krause, 1994), electronic marketing strategies evaluation in the information service industry (Tang, Tseng, & Wang, 1999), tool steel material selection (Chen, 1997), election prediction (Royes & Bastos, 2001), enterprise intranet website evaluation (Tseng, Yang, Lin, & Chen, 2005), planning and design tender selection (Hsieh, Lu, & Tseng, 2004), nature resource management (Liu & Stewart, 2004), broadband service market evaluation (Tseung & Chiu, 2005), assessment of climate change (Bell, Hobbs, & Ellis, 2003), sustainable fishing development strategies evaluation (Biswas, 1995; Chen & Lee, 1999), distribution centre location selection (Chen, 2001), airline service quality evaluation (Tsaur, Chang, & Yen, 2002), selecting strategic alliances partners for liner shipping (Ding & Liang, 2005) and others (Choo, Schoner, & Wedley, 1999; Mikhailov, 2002; Tsoa & Chu, 2001; Tseng, Tzen, Chen, & Oricovic, 2002). In the following, for the purpose of reference, some important definitions and notations of fuzzy sets theory from (Chen, 1996; Cheng & Lin, 2002; Dou, Woldt, Bogardi, & Dahab, 1997; Lin, Duh, & Liu, 2002; Raj & Kumar, 1999) will be reviewed.

Let $X$ be the universe of discourse, $X = \{x_1, x_2, \ldots, x_n\}$. A fuzzy set $A$ of $X$ is a set of order pairs $\{(x_1, f_A^x(x_1)), (x_2, f_A^x(x_2)), \ldots, (x_n, f_A^x(x_n))\}$, where $f_A^x: X \rightarrow [0, 1]$, is the membership function of $A$, and $f_A^x(x_i)$ stands for the membership degree of $x_i$ in $A$.

**Definition 2.1.** When $X$ is continuous rather than a countable or finite set, the fuzzy set $A$ is denoted as: $A = \int_X f_A^x(x)dx / \mu_X$, where $x \in X$.

**Definition 2.2.** When $X$ is a countable or finite set, the fuzzy set $A$ is represented as: $A = \sum f_A^x(x_i) / \mu_X$, where $x_i \in X$.

**Definition 2.3.** A fuzzy set $\tilde{A}$ of the universe of discourse $X$ is normal when its membership function $f_A^x(x)$ satisfies: $\max_x f_A^x(x) = 1$. 
Definition 2.4. A fuzzy number is a fuzzy subset in the universe of discourse \( X \) that is not only convex but also normal.

Definition 2.5. The \( \alpha \)-cut \( \tilde{A}_\alpha \) and strong \( \alpha \)-cut \( \tilde{A}_{\alpha^+} \) of the fuzzy set \( \tilde{A} \) in the universe of discourse \( X \) is defined by

\[
\tilde{A}_\alpha = \{ x | f_\alpha(x) \geq \alpha, x \in X \}, \quad \text{where } \alpha \in [0, 1]
\]

(1)

\[
\tilde{A}_{\alpha^+} = \{ x | f_\alpha(x) > \alpha, x \in X \}, \quad \text{where } \alpha \in [0, 1]
\]

(2)

Definition 2.6. A fuzzy set \( \tilde{A} \) of the universe of discourse \( X \) is convex if and only if every \( \tilde{A}_\alpha \) is convex, that is \( \tilde{A}_\alpha \) is a close interval of \( \mathbb{R} \). It can be written as

\[
\tilde{A}_\alpha = [p^{(\alpha)}_1, p^{(\alpha)}_2], \quad \text{where } \alpha \in [0, 1]
\]

(3)

Definition 2.7. A triangular fuzzy number can be defined as a triplet \((a_1, a_2, a_3)\); the membership function of the fuzzy number \( \tilde{A} \) is defined as (see Fig. 1):

\[
f_{\tilde{A}}(x) = \begin{cases} 
0, & x < a_1, \\
(x - a_1)/(a_2 - a_1), & a_1 \leq x \leq a_2, \\
(a_3 - x)/(a_3 - a_2), & a_2 \leq x \leq a_3, \\
0, & x > a_3.
\end{cases}
\]

(4)

Let \( \tilde{A} \) and \( \tilde{B} \) be two triangular fuzzy numbers (TFN) parameterized by the triplet \((a_1, a_2, a_3)\) and \((b_1, b_2, b_3)\) respectively, then the operational laws of these two triangular fuzzy numbers are as follows:

\[
\tilde{A}(+)\tilde{B} = (a_1, a_2, a_3)(+) (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)
\]

(5)

\[
\tilde{A}(-)\tilde{B} = (a_1, a_2, a_3)(-) (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)
\]

(6)

\[
\tilde{A}(\times)\tilde{B} = (a_1, a_2, a_3)(\times) (b_1, b_2, b_3) = (a_1b_1, a_2b_2, a_3b_3)
\]

(7)

\[
\tilde{A}(\div)\tilde{B} = (a_1, a_2, a_3)(\div) (b_1, b_2, b_3) = \frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}
\]

(8)

According to the vertex method stated in Chen (2000), the distance between fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) is calculated as

\[
d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]}
\]

(9)

For example, Fig. 2 shows three fuzzy numbers \( \tilde{A} = (1.2, 3.5, 5.2) \), \( \tilde{B} = (2, 4.8, 7.3) \) and \( \tilde{C} = (5.3, 8.2, 9.8) \). Based on the above Eq. (11), the distance measurement can be computed as

\[
d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(1.2 - 2)^2 + (3.5 - 4.8)^2 + (5.2 - 7.3)^2]} = 1.4988
\]

(10)

\[
d(\tilde{A}, \tilde{C}) = \sqrt{\frac{1}{3}[(1.2 - 5.3)^2 + (3.5 - 8.2)^2 + (5.2 - 9.8)^2]} = 4.4743
\]

3. Framework for initial training aircraft evaluation

A hierarchical analysis structure (see Fig. 3) for tackling the problems of evaluating the initial training aircraft using TOPSIS in a fuzzy environment is constructed in this section. The content consists of three subsections, investigating the evaluation criteria, determining the importance
References


