A Deadlock Prevention Policy for Flexible Manufacturing Systems Using Siphons

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Abstract

In this paper, we present a new deadlock prevention algorithm for the class Petri nets. A new class of net that is extended from S\textsuperscript{3}PR, called ES\textsuperscript{3}PR where deadlocks are related to unmarked siphons. This method is an iterative approach by adding two kinds of control places called ordinary control place and weighted control place to the original model to prevent siphons from being unmarked. We have obtained the relation of the algorithm and the liveness and reversibility of the controlled net. Finally, a flexible manufacturing example is presented for illustrating the method.

1. Introduction

Petri nets (PNs) have been recognized as one of the most powerful tools for modeling FMS [12]. The increasing interest in PN’s is stimulated by their analysis of the modeled systems. However, several fundamental problems remain open. One of them is the prevention of deadlocks. In essence, the deadlock prevention problem is an important issue in manufacturing systems because of two reasons. First, the existence of deadlocks causes a certain degree of disruption of production, and thus may significantly increase the production cost. Second, the computation of deadlocks is in general an exponential-time problem.

Some deadlock prevention avoidance schemes for controlling an FMS have been proposed in prior work [1] [3-4][6-8]. Many of them adopted PN models as a formalism to describe FMS’s and to develop deadlock avoidance policies. In [1][5-6], the deadlock avoidance problem is solved using the concept of siphons. In particular, Ezpeleta \textit{et al.} [6] proposed a deadlock prevention control policy that is implemented by means of the addition to the initial PN model of some new elements such that the final model is live. That is, new places are added to the net imposing restrictions that prevent the presence of unmarked siphons (direct cause of deadlocks). Unfortunately, all output arcs of the new control places are added to source transitions of the resultant net if all places representing resources (we shall call them resource places) in the original net are removed. Since the source transitions denote the entry points of the processing of raw parts, this policy is rather conservative. In [3], the deadlock detection and avoidance is proposed using a graph-theoretic approach. The approach is simple because it only looks ahead one step. Unfortunately, the approach considers only single resources in a state. In [8], the authors use an algebraic polynomial kernel for analysis and deadlock avoidance for augmented marked graphs. A main drawback of the prevention method is that the algorithm directly controls source transitions. Other developments involving deadlock prevention avoidance were reported in [4][6][11].

In [2], the authors have developed a fast deadlock detection approach based on mixed integer programming (MIP) for structurally bounded nets whose deadlocks tied to unmarked siphons. Since no explicit enumeration of siphons is required, this formulation opens a new avenue for checking deadlock-freeness of large systems. Its computational efficiency is relatively insensitive to the initial marking and expected to be more efficient than classical state enumeration methods [9]. The MIP method is able to find a maximal siphon unmarked at a reachable marking. Based on this, we can formalize an algorithm that can efficiently obtain a minimal siphon from the result of the MIP method.

In [5], we have proposed a deadlock prevention algorithm for S\textsuperscript{3}PR modeled manufacturing systems. The algorithm is an iterative approach based on Chu and Xie’s MIP method. At each iteration, the MIP technique is used to find an unmarked maximal siphon. Next, an unmarked minimal siphon is obtained from the maximal siphon. The algorithm consists of two main stages: one stage is called siphons control, the other stage is called augmented siphons control. This latter stage assures that there are no new siphons generated. We have shown the...
relation of the algorithm and the liveness and reversibility of the controlled net. Comparing with prior work, this approach provides better performance since control is not solely added to the entry point of jobs.

In this paper, we present a new deadlock prevention algorithm for a new class of nets that are extended from S'PR called ES'PR where deadlocks are related to unmarked siphons. This method is an iterative approach by adding two kinds of control places to the original model to prevent siphons from being unmarked. One is added a control place, called ordinary control place, to an original net G with its related arcs such that the controlled net is still an ordinary Petri net; the other is added a weighted control place, called weighted control place, to an original net G with its related arcs such that the controlled net become to a generalized Petri net. We have obtained the relation of the algorithm and the liveness and reversibility of the controlled net. Finally, a flexible manufacturing example is presented for illustrating the method.

The rest of the paper is organized as follows: Section 2 presents basic definitions and properties of Petri nets that are related to this paper. Section 3 presents our deadlock avoidance algorithm. Section 4 gives an example. Conclusions are presented in section 5.

2. Basic Definitions and Properties

Consider a generalized PN \( G = (P, T, F, W, M_0) \) where \( P \) is the set of places, \( T \) is the set of transitions, \( F \subseteq (P \times T) \cup (T \times P) \) is the set of directed arc, \( W: F \rightarrow \text{IN}[0] \) gives the weights of arcs, and \( M_0: P \rightarrow \text{IN} \) is the initial marking, where \( \text{IN} \) is the set of nonnegative integers. A net such that \( W: F \rightarrow \{1\} \) is called an ordinary net, and can be denoted as \( G = (P, T, F, M_0) \). The set of input (resp. output) transitions of a place \( p \) is denoted by \( \bullet p \) (resp. \( p \bullet \)). Similarly the set of input (resp. output) places of a transition \( t \) is denoted by \( \bullet t \) (resp. \( t \bullet \)). For any subset of places \( S \), \( S \bullet \) (resp. \( S \bullet \)) denotes the set of transitions with at least one output (resp. input) place belonging to \( S \). A transition \( t \) is enabled and can fire under a marking \( M \) iff \( M(p) \geq 1, \forall p \in \bullet t \). A transition \( t \) is said to be live if for any \( M \in R(M_0) \), there exists a sequence of transitions fireable from \( M \) that contains \( t \). A PN is said to be live if all the transitions are live. A PN is said to be deadlock-free if at least one transition is enable at every reachable marking. A place \( p \) is said to be bounded if there exists a constant \( K \) such that \( M(p) \leq K \) for all \( M \in R(M_0) \). A PN is said to be bounded if all the places are bounded. It is said to be structurally bounded if it is bounded whatever the initial marking is. A PN is said to be reversible if, for any \( M \in R(M_0) \), \( M_0 \) is reachable from \( M \). A subset of places \( S \) is called a siphon if \( S \subseteq S \bullet \), i.e., any input transition of \( S \) is also an output transition of \( S \).

It is called a trap if \( S \subseteq S \bullet \). A siphon is minimal if it does not contain any other siphons [10].

Our deadlock prevention method uses the MIP deadlock detection approach proposed by Chu and Xie [2] since the MIP approach can deal with large systems within reasonable computation time according to their experimental study. The MIP approach is briefly summarized as follows: Given a siphon \( S \), two binary indicators are introduced:

\[
\begin{align*}
    v_p &= 1 [p \notin S] \quad \text{and} \quad z_t = 1 [t \notin S \bullet] \quad (1)
\end{align*}
\]

It is obvious that any \( p \) with \( v_p = 1 \) and any \( t \) with \( z_t = 1 \) will not be included in a siphon. Since \( S \) is a siphon, \( v_p = 0 \Rightarrow z_t = 0, \forall t \in p \bullet \) and \( z_t = 1 \Rightarrow v_p = 1, \forall p \in t \bullet \) which lead to

\[
\begin{align*}
    z_t &\geq \sum_{p \in S \bullet} v_p - |S| + 1, \forall t \in T \quad (2) \\
    v_p &\geq z_t, \forall (t, p) \in F \quad (3) \\
    v_p, z_t &\in \{0, 1\}. \quad (4)
\end{align*}
\]

For a structurally bounded net, we have

\[
\begin{align*}
    v_p &\geq M(p)/SB(p), \forall p \in P \quad (5)
\end{align*}
\]

where the structural bound \( SB(p) \) is defined as: \( SB(p) = \max \{ M(p) | M = M_0 + CY, M \geq 0, Y \geq 0 \} \). Therefore, the maximal siphon unmarked at a given marking \( M \) can be determined by the following MIP problem and there exist siphons unmarked at \( M \) iff \( G_{MIP}^M < |P| \):

\[
\begin{align*}
    G(M) = \text{Minimize} \sum_{p \in P} v_p
\end{align*}
\]

such that constraints (1)-(4) are satisfied and

\[
\begin{align*}
    M = M_0 + CY, \quad M \geq 0, Y \geq 0 \quad (6)
\end{align*}
\]

Several important properties are presented below.

**Property 1**: The system (2)-(4) is properly speaking an integer linear system.

**Property 2**: Let \( S \) be a minimal siphon and let \( p \) and \( q \) be two places such that \( p \bullet \subseteq q \bullet \) then \( q \in S \Rightarrow p \notin S \).

**Property 3**: Let \( S \) be a minimal siphon. Then for any \( p \in S \), there exists \( t \in p \bullet \) such that \( t \cap S = \{p\} \).

**Property 4**: Let \( S \) be a minimal siphon. Then the subnet induced by \( S \) and \( S \bullet \) is strongly connected.

The following properties (see [2]) show the importance of siphons and traps in the detection of deadlocks.

**Property 5**: A siphon free of tokens at a marking remains token-free whatever the transition firings. A trap marked by a marking remains marked. For any marking such that no transition is enabled, the set of unmarked places forms a siphon.
Property 6: A Petri net is deadlock-free if no minimal siphon eventually becomes unmarked.

3. The Deadlock Prevention Algorithm
The deadlock prevention algorithm proposed in this paper is targeted at systems modeled by a class of ordinary PNs called ES3PR. An ES3PR is defined as follows:

Definition 1: An ES3PR net \( G \) is a net that results from adding a set \( R \) of initially marked places (resource places) to a set of process nets such that:

(i) Each resource place \( r \) is associated with a set of operation places, \( OP(r) \). This implies that these operation places require resource \( r \).

(ii) For each input transition \( t \) of some \( p \in OP(r) \), there exists an arc from \( r \) to \( t \) if \( t \cap IOP(r) = \phi \).

(iii) For each output transition \( t \) of some \( p \in OP(r) \), there exists an arc from \( t \) to \( r \) if \( t \cdot IOP(r) = \phi \).

ES3PRs properly include S3PRs and partially overlap with RCN merged nets. Fig. 1 helps us to understand an ES3PR net.

As described above, our deadlock avoidance algorithm is an iterative approach. At each iteration, the proposed method adds a control place to the original net \( G \) with its output arcs to the sink transitions of a minimal siphon, or to the source transitions of the resultant net \( G^* \) if all resource places are removed.

Based on the discussions of the above section, this section presents and illustrates our deadlock avoidance algorithm. Since the algorithm uses the MIP approach described in Section 2 to obtain maximal siphons unmarked at some markings. We also use an algorithm [5] to obtain unmarked minimal siphons from the maximal siphons.

For each minimal siphon \( S \) that can become unmarked, two cases are possible.

When an ordinary control place (OC) is added on an ordinary net, the resultant net is still an ordinary net (i.e. depicted in Fig. 2(a)); but when a weighted control place (WC) is added to an ordinary net, the resultant net becomes a generalized net (i.e. depicted in Fig. 2(b)).

As mentioned previously, our method consists of two kinds of control places: We add an OC to a net \( G \) with its output arcs to the sink transitions of a minimal siphon. However, we add a WC control place to a net \( G \) with its output arcs to the source transitions of the net. The principles and theorems are depicted as follows.

Fig. 1: An ES3PR net.

Definition 2:

(i) An operation place \( p \not\in S \) is called stealing place if \( p \in OP(r) \) for some resource place \( r \in S \).

(ii) Let \( O(S) \) be the set of stealing place of \( S \).

(iii) Let \( N_p = \sum_{r \in S \cap R} I(p \in OP(r)) = \) Number of resource tokens of \( S \) stolen by a token in place \( p \).

Case I: \( N_p = 1, \forall p \in O(S) \)
To prevent \( S \) from becoming unmarked, an OC is added as follows:

Definition 3: For each siphon \( S \) such that \( N_p = 1, \forall p \in O(S) \), a new place \( OC \) is added such that:

(i) \( M_0(OC) = \sum_{r \in S \cap R} M_0(r) - 1 \),

(ii) For any input transition \( t \) of a place in \( O(S) \), there exists an arc from \( OC \) to \( t \) if \( t \cap I(S) = \phi \).

(iii) For any output transition \( t \) of a place in \( O(S) \), there exists an arc from \( t \) to \( OC \) if \( t \cdot I(S) = \phi \).

Theorem 1: The net with the place \( OC \) is also an ES3PR net where \( OC \) can be considered as a resource place with \( OP(OC) = O(S) \).
Case II: \( N_p > 1, \forall p \in O(S) \)

To prevent \( S \) from becoming unmarked, a WC is added as follows:

**Definition 4:** \( W_p = \max_{p \in L_p} N_p \) where \( L_p \) is the set of operation place in all paths from \( p \) to the sink transitions of \( G' \) with obvious extension \( N_p = 0 \) for all \( p \notin O(S) \).

**Definition 5:** \( W_t = \max_{p \in L_t} N_p \) where \( L_t \) is the set of operation place in all paths from \( p \) to the sink transitions of \( G' \) with obvious extension \( N_p = 0 \) for all \( p \notin O(S) \).

Note that \( W_t = W_p \), for the unique operation place \( p \in t^* \).

**Definition 6:** For each siphon \( S \) such that \( N_p > 1, \forall p \in O(S) \), a new place WC is added such that:

(i) \( M_0(WC) = \sum_{r \in S, r \in R} M_0(r) - 1 \),

(ii) For all source transition \( t \) of \( G' \) such that \( W_t > 0 \), add an arc \((WC, t)\) of weight \( W_t \),

(iii) For any output transition \( t \) that is not a source transition \( G' \), let \( p \) be its input operation place. Add an arc \((t, WC)\) of weight \( W_p - W_t \) if \( W_p > W_t \).

**Theorem 2:** The net obtained by adding WC is potentially live.

**Theorem 3:** The net obtained by adding WC is reversible iff no siphon \( S \) without WC cannot become unmarked.

As previously mentioned, the net has several features as follows:

(i) There may be more than one siphon that can become unmarked.

(ii) Adding OC may create a new unmarked siphon.

(iii) Adding WC will not create new unmarked siphons.

From the above features, they imply that an iterative process is needed to eliminate all existing/new unmarked siphons. As a result, our goal is to eliminate any unmarked siphons at each iteration. More detail steps are shown as follows:

**Algorithm Deadlock_Prevention**

**Step 1:**

Starting from an ES3PR net \( G^0 \) an unmarked siphon \( S_1 \), is determined and a control place \( C_1 \) is added.

Let \( (G^0 + C_1) \) denote the new net.

**Step 2:**

**Case I:** \( C_i \) is an OC.

(i) Then, the new net \( (G^0 + C_i) \) is an ES3PR net.

(ii) The next step determines an unmarked siphon \( S_2 \) of the net \( (G^0 + C_i) \).

(iii) The related control place \( C_2 \) is added to obtain net \( (G^0 + C_i + C_2) \).

**Case II:** \( C_i \) is an OW.

(i) Then, the new net \( (G^0 + C_i) \) is a generalized net.

(ii) According to the Theorem 15, the next step determines a siphon \( S_2 \) in net \( G^0 \) that can become unmarked in \( (G^0 + C_i) \). Note that the WC with its arcs added on the net cannot generate any new siphons.
(iii) The related control place $C_2$ is added to obtain net $(G^0 + C_1 + C_2)$.

For convenience to explain our policy, in the next section, we will show an example.

4. Example

Fig. 1 shows the net model of a flexible production system where two types of processes execute concurrently and share a set of common resources. The system net is an ES^3PR.

In Fig. 3, we obtain an unmarked maximal siphon \{p_3, p_5, p_6, p_7, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}\} of the net. From the maximal siphon, an unmarked minimal siphon \{p_5, p_6, p_7, p_{10}, p_{11}, p_{14}, p_{15}\} by which the sub-net generated is shown in Fig.3. Hence, we can point out that $t_2$, $t_5$ and $t_{13}$ are sink transitions, \{p_3, p_4, p_9\} $\in \mathcal{O}(S)$, and $M_\delta(p_{17}) = M_\delta(p_{14}) + M_\delta(p_{15}) - 1 = 2$.

Note that $p_{17}$ is considered a resource place in the algorithm. Similarity, we obtain an unmarked maximal siphon \{p_3, p_4, p_5, p_6, p_7, p_{10}, p_{12}, p_{13}, p_{14}, p_{17}\} of the net. From the maximal siphon, an unmarked minimal siphon \{p_3, p_4, p_5, p_6, p_{12}, p_{13}, p_{14}\} is obtained. Hence, we can point out that $t_1$ and $t_{12}$ are sink transitions, \{p_2, p_{10}, p_{11}\} $\in \mathcal{O}(S)$, and $M_\delta(p_{18}) = M_\delta(p_{13}) + M_\delta(p_{14}) - 1 = 1$. In the next iteration, by the same way, we obtain an unmarked maximal siphon \{p_3, p_5, p_6, p_7, p_{10}, p_{11}, p_{12}, p_{13}, p_{15}, p_{17}, p_{18}\} from the maximal siphon, an unmarked minimal siphon \{p_3, p_4, p_{13}, p_{17}, p_{18}\} by which the sub-net generated is shown in Fig.4. Hence, we can point out that $t_2$ and $t_{13}$ are sink transitions and source transition as well. This situation is different from the former iteration. Since $N_{p_2} = 2$, in this case, we add a weighted control place to the net. Specifically, we obtain $W_{t_1} = 2$ and $W_{t_{13}} = 1$, and the weight of arc $(p_{19}, t_1)$ is equal to 2 and the weight of $(p_{19}, t_{13})$ is equal to 1. It is worth to note that $p_{19}$ (WC) cannot generate a new siphon (Theorem 3).

Due to that $p_{19}$ cannot be considered a resource place and generates a new siphon, in fact, add a restriction to the MIP method. The objective is to eliminate some unreachable states. The restriction is as follows:

$$M(3) + M(4) + M(12) + M(13) + M(17) + M(18) \leq 3$$

As a result, in the new iteration, we can obtain another minimal siphon \{p_5, p_6, p_7, p_{10}, p_{11}, p_{15}, p_{17}, p_{18}\} and the controlled sub-net is depicted in Fig. 5.

Finally, we can obtain another minimal siphon \{p_3, p_4, p_{10}, p_{11}, p_{17}, p_{18}\} and the controlled sub-net is depicted in Fig. 6.

In summary, we add three OCs control place and two WC s control place to the original net. After adding those control places, the resultant net is live and reversible.

5. Conclusions

This paper presents a deadlock prevention method for a class of FMS where deadlocks are caused by unmarked siphons in their PN models. The FMSs are modeled using ES^3PRs, which are extended from S^3PR nets.
The proposed deadlock control policy can be implemented by adding two kinds of control places to the original model. The algorithm main object is to decide which one kind of control place is suitable. More specific, the method used two kinds of control places to add on the original net $G$. We have obtained the relation of the algorithm and the liveness and reversibility of the controlled net. Finally, a flexible manufacturing example is presented for illustrating the method.

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