Abstract- This study investigates the problem of unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid past over a horizontal stretching sheet in the presence of uniform transverse magnetic field in the porous medium. Using the similarity transformed the governing time dependent boundary layer equations are transformed to ordinary differential equations, before being solved numerically by standard techniques. The solutions of non-linear differential equations are obtained for unsteady parameter, Magnetic parameter, Radiation parameter, Permeability parameter, Eckert number and Prandtl number on the velocity and temperature profile and discussed graphically.

Keywords- Viscous Dissipation; Thermal Radiation; Stretching Sheet; MHD; Unsteady Boundary Layer Flow; Porous Medium

I. INTRODUCTION

The study of boundary layer flow over a stretching sheet has gained considerable attention due to its applications in industrial manufacturing process, such as paper production, the aerodynamics extrusion of plastic sheet, glass blowing metal spinning, wire and fiber coating, chemical processing equipment etc. The production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets. The quality of final product depends on the rate of heat transfer at the stretching surface. Numerous researchers have investigated the heat transfer over stretching surface by considering the effect of magnetic field. In the pioneering work of Crane [1], the flow of Newtonian fluid over a linearly stretching surface was studied. Later, Laha et al. [2] studied Heat transfer characteristics of the flow of an incompressible viscous fluid over a stretching sheet. Andersson et al. [3] investigated using a similarity transformation the flow of a thin liquid film of a power-law fluid by unsteady stretching surface. Subsequently, the pioneering work of Crane [1] are extended by many authors to explore various aspects of the flow and heat transfer occurring in an infinite domain of the fluid surrounding the stretching surface [4-12]. The study of Unsteady magnetohydrodynamic boundary layer flow over a stretching surface with viscous dissipation and Joule heating have been investigated by Jat and Chaudhary [13], Shahy and Motsa [14] extended the work of El-Aziz [11] to include mass transfer. Ellahi and Riaz [15] investigated Analytical solution for MHD flow in a third grade fluid with variable viscosity. Analytic solution for MHD flow in an annulus was also studied by Ellahi et al. [16]. Singh et al. [17] studied the Effects of thermal radiation and magnetic field on unsteady stretching permeable sheet in presence of free stream velocity. Ellahi et al. [18] discussed Analysis of steady flows in viscous fluid with heat/mass transfer and slip effects. Numerical analysis of steady non-Newtonian flows with heat transfer analysis, MHD and nonlinear slip effects was investigated by Ellahi and Hameed [19]. Unsteady boundary layer flow past a stretching plate and heat transfer with variable thermal conductivity was studied by Misra et al. [20]. Recently, Verjavelu [21] investigated unsteady convective boundary layer flow of a viscous fluid at a vertical surface with variable fluid properties. Most Recently, Zeeshan and Ellahi [22] studied Series solutions for nonlinear partial differential equations with slip boundary conditions for non-Newtonian MHD fluid in porous space. However, radiation and magnetic effects on a stretching surface for time dependent moving viscous fluid in porous medium has been not considered so far in best of our knowledge. The study of MHD flow of an electrically conducting fluid caused by the deformation of the wall of a vessel containing fluid is of considerable interest in modern metallurgical and metal working process. The present paper is concern to investigate two-dimensional unsteady flow of a viscous incompressible fluid about stagnation point on permeable stretching sheet in the presence of time dependent free stream velocity with dissipation and radiation effect.

II. FORMULATION OF THE PROBLEM

Consider the unsteady laminar two-dimensional flow of a viscous incompressible electrically conducting fluid past a semi-infinite porous stretching sheet. Fluid is considered in the influence of transverse magnetic field of constant strength $B_0$ normal to the sheet. The magnetic Reynolds number is taken to be small and therefore the induced magnetic field is neglected. The Cartesian Reynolds coordinate system has its origin located at the leading edge of the sheet with positive x-axis extending along the sheet in the direction of flow, while y-axis is along normal to the surface of the sheet and is positive in the direction from the sheet to the fluid (fig.1). We assume that for time $t < 0$ the fluid and heat flows are steady. The unsteady fluid and heat flows start at $t = 0$, the sheet is being stretched with the velocity $U_w(x,t)$ along the x-axis, keeping the origin fixed and temperature $T_w(x,t)$ . The thermo-physical properties of the sheet and ambient fluid are assumed constant. Under the Boussinesq
and boundary layer approximations, the governing equations of the flow are:

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{\rho} u \]

(1)

\[ \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u + \frac{\mu}{\rho} \left( \frac{\partial u}{\partial y} \right)^2 + \sigma B_0^2 u^2 \]

(2)

\[ \rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \kappa \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \left( \frac{\partial u}{\partial y} \right)^2 + \sigma B_0^2 u^2 \]

(3)

Where \( u \) and \( v \) are the velocities in the x- and y-directions respectively, \( \rho \) is the density of the fluid, \( \mu \) is the dynamic viscosity, \( \nu = \frac{\mu}{\rho} \) is the kinematic viscosity, \( c_p \) is the specific heat at constant pressure, \( \kappa \) is thermal conductivity of the fluid under consideration, \( q_r \) is the radiative heat flux, \( T \) is the temperature.

The associated boundary conditions are:

\[ u(x,0) = U_w(x,t), \quad v(x,0) = 0, \]

\[ T(x,0) = T_w(x,t) \]

(4)

Following Shayeti and Motsa [14], the stretching velocity \( U_w(x,t) \) is assumed \( U_w = \frac{bx}{(1-\alpha t)} \) where both \( b \) and \( \alpha \) are positive constant. We have \( b \) as the initial stretching rate \( \frac{b}{(1-\alpha)} \) and it is increasing with time.

We assume the surface temperature \( T_w(x,t) \) of the stretching sheet to vary with the distance \( x \) along the sheet and time in the following form:

\[ T_w(x,t) = T_w + T_0 \left( \frac{bx^2}{\nu} \right)^{\frac{3}{2}} \]

(5)

where \( T_0 \) is a heating or cooling reference temperature. The radiative heat flux \( q_r \) is simplified by using Rosseland approximation (Rosseland [18]) as:

\[ q_r = -\frac{4\alpha \kappa T^4}{3\beta} \]

(6)

Where \( \alpha \) is the Stefan-Boltzmann constant and \( \beta \) is the Roseland mean absorption coefficient.

This approximation is valid at points optically far from the boundary surface and it is good for intensive absorption, which is for an optically thick boundary layer. It is assumed that the temperature difference with in the flow such that the term \( T^4 \) may be expressed as a linear function of temperature. Hence, expanding \( T^4 \) by Taylor series about \( T_w \) and neglecting higher-order terms gives:

\[ T^4 \approx 4T_w^3 - 3T_w^2 \]

(7)

Using equation (6) and (7), equation (3) reduces to:

\[ T = T_w + T_0 \left( \frac{bx^2}{2\nu} \right) (1-\alpha t)^{-\frac{3}{2}} \theta(\eta) \]

(8)

where

\[ \eta = \frac{b}{\sqrt{\nu(1-\alpha t)}} \]

(9)

The velocity components are then derived from the stream function expression and obtained as:

\[ u = \frac{bx}{(1-\alpha)} f(\eta), \quad v = -\frac{eb}{(1-\alpha)} f(\eta) \]

(10)

Then, the momentum and energy equations (2) and (8) are transformed to:
\[ f'' + f' - (f')^2 - \frac{A\eta}{2} f' - \left( A + M + \frac{1}{K} \right) f' = 0 \]  
\[ (3R + 4)\theta' + 3RPr \left\{ f\theta - 2f'\theta - \frac{A}{2} (3\theta + \eta\theta') + MEc (f')^2 + Ec (f')^2 \right\} = 0 \]

where
\[ \text{Re} = \frac{U_w x}{v} \]  
(Reynolds number)  

The corresponding boundary conditions are: 
\[ \eta = 0: \quad f = 0, \quad f' = 1, \quad \theta = 1 \]
\[ \eta \to \infty: \quad f' \to 0, \quad \theta \to 0 \]

Where prime (') denote the differentiation with respect to \( \eta \) and dimensionless parameters are:
\[ M = \frac{\sigma B^2}{\rho c} (1 - \alpha t) \]  
(Magnetic parameter)
\[ Ec = \frac{u^2}{C_p (T_w - T_\infty)} \]  
(Eckert number)
\[ Pr = \frac{\mu C_p}{\kappa} \]  
(Prandtl number)
\[ K = \frac{K_b}{\nu} (1 - \alpha t) \]  
(Permeability parameter)
\[ R = \frac{\beta\kappa}{4\alpha T_\infty^3} \]  
(Radiation parameter)
\[ A = \frac{\alpha}{b} \]  
(Unsteady parameter)

The physical quantities of interest are the skin-friction coefficient \( c_f \) and heat transfer rates i.e. the Nusselt number \( Nu \) are:
\[ c_f = \frac{\tau_w}{\rho U_w^2} = \frac{\mu}{\rho U_w^2} \left( \frac{\partial u}{\partial y} \right)_{y=0} \]
\[ \Rightarrow c_f = 2 \sqrt{\frac{v(1 - \alpha t)}{bx^2}} f'(0) = \frac{2}{\sqrt{\text{Re}}} f'(0) \]  
(16)

and
\[ Nu = -\frac{x}{T_w - T_\infty} \left( \frac{\partial \theta}{\partial y} \right)_{y=0} \]
\[ \Rightarrow Nu = -\sqrt{\frac{bx^2}{v(1 - \alpha t)}} \theta'(0) = -\sqrt{\text{Re}}\theta'(0) \]  
(17)

IV. RESULT AND DISCUSSION

The boundary layer equations of momentum and energy are solved numerically using Runge-Kutta forth order algorithm with a systematic guessing of \( f''(0) \) and \( \theta'(0) \) by the shooting technique until the boundary conditions at infinity are satisfied. The step size \( \Delta \eta = 0.001 \) is used while obtaining the numerical solution and accuracy up to the seventh decimal place i.e.1 x 10^{-7}, which is very sufficient for convergence. In this method, we choose suitable finite values of \( \eta \to \infty \), say \( \eta_\infty \), which depend on the values of the parameter used. The computations were done by a program which uses a symbolic and computational computer language Matlab. The computation through employed numerical scheme has been carried out for various values of the parameters such as Unsteadiness parameter A, Permeability parameter K, Magnetic parameter M, Radiation parameter R, Prandtl number Pr and Eckert number Ec. The velocity profile \( f'(\eta) \) for different values of the unsteady parameter A is shown in fig.2. It is observed that the velocity decreases with the increasing values of unsteady parameter A. It is interesting to note that the thickness of boundary decreases with increasing values of A. This is due to the fluid flow caused solely by the stretching sheet. The velocity profile \( f'(\eta) \) for different values of the magnetic parameter M is shown in fig.3. It is observed that velocity decreases with the increasing values of magnetic parameter M. As M increases, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow, and from fig.4, we observed that velocity increases for increasing values of permeability parameter K. The temperature profiles for different values of A, M, R, Pr and Ec are presented in figure 5 to figure 9. It is observed from the figures that the boundary conditions are satisfied asymptotically in all the cases, which supporting the accuracy of the numerical results obtained. From fig.6, we observed that temperature of the fluid is decreases with the increasing values of unsteady parameter A. Temperature at a point of surface decreases significantly with the increases of A i.e. rate of heat transfer increases with increasing values of A. Physically, it means that the temperature gradient at the surface increases as A increases, which imply the increases of heat transfer rate \( -\theta'(\eta) \) at the surface. From fig.5 and fig.8, we observed that temperature of the fluid is increases as magnetic parameter M and Eckert number Ec increases. It is observed from the fig.7 and fig.9, temperature of the fluid is decreases with the increasing values of Prandtl number Pr and Radiation parameter R, this is because of the
increase in Prandtl number Pr, indicates the increase of the fluid heat capacity or the decrease of the thermal diffusivity hence cause a diminution of the influence of the thermal expansion to the flow. This implies momentum boundary layer is thicker than thermal boundary layer.

ACKNOWLEDGEMENTS

- The authors wish to express their sincere appreciation to the learned referee for careful reading of the manuscript and valuable suggestions.
- This work has been carried out with the financial support of CSIR in the form of J.R.F awarded to one of the author (Gopi Chand).

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Fig. 2: Velocity profile against $\eta$ for various values of Permeability parameter $K$ for $A = 1.0$ and $M = 1.0$.

Fig. 3: Velocity profile against $\eta$ for various values of Magnetic parameter $M$ for $A = 1.0$ and $K = 0.5$.

Fig. 4: Velocity profile against $\eta$ for various values of Unsteady parameter $A$ for $M = 1.0$ and $K = 0.5$. 
Fig. 5: Temperature distribution against \( \eta \) for various values of Magnetic parameter \( M \) for \( Pr = 7.0, Ec = 0.5, R = 0.5, K = 0.5, A = 1.0 \).

Fig. 6: Temperature distribution against \( \eta \) for various values of Unsteady parameter \( A \) for \( Pr = 7.0, Ec = 0.5, R = 0.5, K = 0.5, M = 1.0 \).

Fig. 7: Temperature distribution against \( \eta \) for various values of Radiation parameter \( R \) for \( Pr = 7.0, Ec = 0.5, K = 0.5, M = 1.0, A = 1.0 \).
Fig. 8: Temperature distribution against $\eta$ for various values Eckert number Ec

for $Pr = 7.0$, $R = 0.5$, $K = 0.5$, $M = 1.0$, $A = 1.0$.

Fig. 9: Temperature distribution against $\eta$ for various values Prandtl number Pr

for $R = 0.5$, $Ec = 0.5$, $K = 0.5$, $M = 1.0$, $A = 1.0$. 