Correlation-based Visual Odometry for Ground Vehicles

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Abstract

Reliable motion estimation is a key component for autonomous vehicles. We present a visual odometry method for ground vehicles using template matching. The method uses a downward facing camera perpendicular to the ground and estimates the motion of the vehicle by analyzing the image shift from frame to frame. Specifically, an image region (template) is selected and using correlation we find the corresponding image region in the next frame. We introduce the use of multi-template correlation matching and suggest template quality measures for estimating the suitability of a template for the purpose of correlation. Several aspects of the template choice are also presented. Through an extensive analysis we derive the expected theoretical error rate of our system and show its dependence on the template window size and image noise. We also show how a linear forward prediction filter can be used to limit the search area to significantly increase the computation

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performance. Using a single camera and assuming Ackerman-steering model, the method has been implemented successfully on a large industrial forklift and a 4×4 vehicle. Over 6 km of field trials from our industrial test site, an off-road area and an urban environment are presented illustrating the applicability of the method as an independent sensor for large vehicle motion estimation at practical velocities.

Keywords

Localization, Computer Vision, Field Robotics, Ground Robots

1 Introduction

The recent advancements in computer hardware are slowly making practical the use of cameras as a main sensor for localization. Still, vision-based localization methods for outdoor purposes are only at an evaluation stage. This was observed during the recent DARPA Urban Challenge events, where none of the finalists used vision as their main localization sensor (Burdick et al., 2007; Leonard et al., 2007; Urmson et al., 2006). Similarly, the Mars Exploration Rovers use vision as a secondary localization sensor after the inertial navigation system (INS) (Cheng et al., 2006). Vision-based localization is maturing and recent advancements indicate that it is only a matter of time before such systems are deployed outdoors in unstructured environments.

In our previous work (Nourani-Vatani et al., 2009, 2008) we introduced the concept of correlation-based visual odometry for car-like vehicles. Our visual odometry method uses a camera facing down (Figure 1(a)) and estimates the vehicle motion by analyzing the image shift. Figure 1(b) illustrates how the vehicle motion parameters can be extracted from the image shift in two consecutive frames. More precisely, an image region, also called a template image, is selected and using correlation the corresponding image region in the next frame is
found, making it possible to extract motion information.

In this paper, we propose a multi-template correlation method and a measure for estimating the quality of image templates. This Template Quality Measure (TQM) is used to determine the appropriateness of the selected image region for template matching. In a multi-template configuration, the TQM is used to select the best template image as the template candidate for the correlation. We show that a higher correlation in the template matching is possible using multi-template configurations, compared to a single-template configuration, resulting in better visual odometry on a wide variety of surfaces. This improved correlation is achieved with only a small added fraction of the correlation processing cost. We also show how a linear forward prediction finite impulse response (FIR) filter can be used to predict the next correlation location. This predictor is used initially to select a more appropriate location for the template windows and then to restrict the search area to achieve maximum speed up. Furthermore, through an extensive statistical analysis we derive the expected theoretical error rate of the system, showing its dependence on the template window size and signal noise. To show the achieved improvements, the new multi-template correlation matching method is compared with single-template methods.

The paper is structured as follows. In the following section we discuss the recent visual odometry works and contrast them to the proposed method. In Section 3 we explain how the vehicle’s pose is estimated from the displacement seen in the downward facing camera through a simplified motion model. In Section 4 we reiterate the use of template matching for estimation of image shifts, introduce the concepts of template quality measure and multi-template correlation, and describe the linear prediction filter. In Section 5 we derive a statistical analysis to determine the error rate of the proposed method. We present simulation results and several experimental field trials in Section 6, analyzing and illustrating the applicability and accuracy of the method. Finally, in Section 7 we discuss the findings and propose future directions for the research.
Figure 1: Illustration of the camera setup and the vehicle motion model.

2 Related Work

One approach to visual odometry is based on image feature tracking techniques. The idea is to find good features in one frame and the corresponding features in the next frames, calculating the perceived motion of these features and translating that to the motion of the camera. This method can be implemented using a single camera (Nister et al., 2005) but better results are often achieved with a stereo camera pair (Nister et al., 2005; Matthies et al., 2007). The advantage of using a stereo pair is that scale can be retrieved and more salient features—present in both cameras—increase the chance of tracking in subsequent frames. This method is often limited by its computational complexity and its requirement for a well-calibrated stereo pair. Johnson et al. (2008) have demonstrated significant speed-ups in such stereo-based systems.

A slightly different approach avoids the problems of finding and tracking robust features over multiple frames and instead looks at the change in brightness in the image between
two consecutive frames. The change in brightness results from the apparent motion in the image. This method, called optical flow (OF), is much simpler and computationally cheaper than the extraction and tracking of features but comes at the cost of less precision over time. Dense OF algorithms, such as the Horn-Schunck method (Horn and Schunck, 1981), calculate the velocity at each pixel by using global constraints. These methods completely avoid feature extraction but are less robust to noise. Sparse OF algorithms, such as the Lucas-Kanade approach (Lucas and Kanade, 1981), assume local smoothness. This provides more robustness to noise but offers a much sparser flow field. Due to this increased robustness, though, the sparse OF algorithms are preferable over dense OF algorithms for odometry usage.

One method of deriving odometry using OF was presented by Campbell et al. (2005). The authors mounted one camera on top of their robot and tilted it downwards to image more ground and less sky. They sub-divided their frame into three regions—ground, horizon, and sky—and used the OF calculated from the ground region to derive the robot’s translational motion and the OF from the sky region to estimate the rotation. Labrosse (2006) has shown how the vehicle heading can be estimated by comparing the shift in panoramic images. A different method, which can be used with either monocular or binocular vision, has been presented by Konolige and Agrawal (2007). In the monocular version, first the relative motion of sparse features between three consecutive frames are estimated. RANSAC is then employed to find a good motion hypothesis before using the 5-point method to hypothesize an essential matrix for the first and third frames. The best hypothesis is then optimized using sparse bundle adjustment.

Corke et al. (2004) compared a structure-from-motion (SFM) method with a sparse OF method using omni-directional vision. To achieve a more fair comparison of the two methods, the same extracted features, using Kanade-Lucas-Tomasi (KLT) feature tracker (Tomasi and Kanade, 1991), were used for both methods. OF methods only use the frame to frame displacement of the features, while the SFM method will take advantage and track the features over longer sequences. Their finding was that OF while being simpler is more
robust but SFM methods produce higher precision at the cost of higher computational needs. Furthermore, they also observed that the precision of the OF method also depends on the feature tracker used. Harris corner detector (Harris and Stephens, 1988) extracts more features compared to KLT, which results in better short term odometry but worse over the longer path. Recently, Scaramuzza et al. (2009) have shown that, by taking advantage of the limited vehicle motion kinematic of car-like vehicles and using a 1-point Random Sample Consensus (RANSAC) filtering, more reliable OF-based odometry is achievable.

Srinivasan (1994) presented an image-interpolation method to calculate the OF. The developed method generates displaced versions of the reference image and estimates the position of the moving image with respect to these reference images using interpolation. This technique is efficient and can also derive the rotation in the image but can only deal with very small displacements between the frames. Using this method, the ego motion of the robot was derived by pointing two cameras at the ceiling tracking changes in the light pattern.

The idea of deriving the vehicle’s ego-motion from the ceiling lights, applied by Srinivasan (1994), is interesting but is not always valid as ceiling heights are not constant. It is not valid in outdoor applications either. On the other hand, by flipping the camera such that it is pointing at the ground, it is fair to assume, using a ground vehicle, that the camera-ground distance is constant. Kelly (2000) used a downward looking camera located under the vehicle for mapping and localization in large scale indoor environments. Using image mosaicking a map of the area is generated and the current position is retrieved from the map via correlation. Most recently, Dille et al. (2009) have used a combination of a commercially available vision module—consisting of four optical mouse sensors—and an INS-GPS module to perform visual odometry on their lunar rover. The vision module is mounted under the vehicle facing down and reports the lateral motion velocities. The heading is calculated from the IMU yaw rate. They achieve fairly accurate odometry performance over hundreds of meters on different surfaces but report difficulties with textureless concrete surfaces.

As mentioned earlier, the proposed visual odometry set up consists of a downward looking
camera, perpendicular to the ground. The goal is to estimate the vehicle motion by analyzing the image information and its shift from frame to frame. Examples of images grabbed from such a set up are given in Figure 2 for different types of surfaces. When faced with very smooth and almost textureless surfaces, such as concrete and asphalt, feature trackers and sparse OF algorithms do not perform well (Campbell et al., 2005; Dille et al., 2009; Chhaniyara et al., 2008). Template matching using the normalized correlation coefficients, on the other hand, has proven reliable on all the surfaces we have tested: asphalt, concrete, gravel, and grass.

In the specific topic of planar surface tracking, several sophisticated algorithms have been
recently proposed (Zimmermann et al., 2009; Benhimane et al., 2007; Mei et al., 2006; Molton et al., 2004). However, a drawback of such gradient-based tracking methods that minimize cost functions, is that they require significant overlap between the patches from frame to frame. This limits the inter-frame displacement and consequently the velocity of the robot in our application. Template matching, in contrast, is able to handle larger displacements. In the experiments section we present comparisons between optical-flow and correlation-based visual odometry, illustrating that larger displacements can be achieved with the correlation option.

Like more recent works by Scaramuzza et al. (2009); Dille et al. (2009), we took advantage of knowledge of the vehicle’s motion constraints—in this case an Ackerman-like steering model (Borenstein et al., 1996)—to limit and simplify the motion model. This simplified motion model has two components; a pure forward/reverse translation and a pure rotation around the center of motion (CoM) on the rear wheel axle. See Fig. 1(b). These two vehicle motion parameters are directly derivable from the two camera motion parameters observed from the shift in the image.

Next we will show how the vehicle motion can be estimated from the displacement in the camera before proposing our method for estimating this translation in the camera.

3 Vehicle Motion Estimation

In order to calculate the odometry estimate, the pixel displacement observed by the camera must be translated into vehicle motion. In our approach we make the following assumptions:

1. the vehicle is car-like and its motion is constrained to Ackerman’s steering model. Therefore, its motion is comprised of a forward translation and a rotation around the center-of-motion of the rear axle.

2. there is no sideways translation so we ignore any sideways slippage.

3. when the displacement from frame to frame is very small, the vehicle motion can be
approximated as piece-wise straight motion.

This more simplified vehicle motion allows for easier odometric calculation.

The motion estimation is a four-step process that consists of (i) conversion from pixels to distance, (ii) coordinate system transformation, (iii) displacement observed by vehicle, and (iv) integration to global coordinate frame. Each of these steps are discussed in the following.

### 3.1 Conversion from Pixels to Distance

This first step translates the displacement from number of pixels to actual distance in meters. For this purpose the knowledge of camera intrinsic parameters and the camera position is required. A standard radial distortion model is used to calibrate and retrieve the intrinsic camera parameters (Zhang, 1999).

Equation (1) shows the conversion from pixels to meters.

\[
distance = \frac{h_{cam}}{f \cdot size_{pixel}} \cdot pixels
\]  

where \( h_{cam} \) is the camera-ground-distance, \( f \) is the camera focal-length and \( size_{pixel} \) is the size of each pixel on the chip. For a given setup \( h_{cam} \) is also a constant and we can define the camera constant: \( c_{cam} = \frac{1}{f \cdot size_{pixel}} \). Although in practice there is a variation in \( h_{cam} \) due to suspension oscillation as the vehicle moves, it is reasonable to assume this variation as zero-mean (ElMadany and Abduljabbar, 1989), such that differences are canceled out throughout the run. Pixel displacements, \( \Delta u \) and \( \Delta v \), will therefore result in physical camera displacement, \( \Delta c \):

\[
\Delta c = \begin{bmatrix}
\Delta x \\
\Delta y \\
0 \\
1
\end{bmatrix} = h_{cam} \cdot c_{cam} \cdot \begin{bmatrix}
\Delta u \\
\Delta v \\
0 \\
1
\end{bmatrix}
\]  

(2)
3.2 Conversion from Camera Coordinate Frame to Bumper Coordinate Frame

A coordinate frame transformation is necessary to convert from the camera coordinate frame, \( \{C\} \), to the vehicle bumper coordinate frame, \( \{B\} \):

\[
\Delta_B = T \times \Delta_C \tag{3}
\]

In our case the camera is facing downwards with the camera \( X \)-axis pointing to the right and the \( Z \)-axis pointing towards the ground. The vehicle coordinate frame is defined with the \( X \)-axis pointing forwards and the \( Z \)-axis pointing upwards. Using Euler angles, \( T \) is calculated by first rotating \( \theta = -\pi/2 \) around the \( Z \)-axis and then rotating \( \phi = \pi \) around the new \( X \)-axis:

\[
T = \begin{bmatrix}
\cos(\theta) & -\sin(\theta) & 0 & 0 \\
\sin(\theta) & \cos(\theta) & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \times \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(\phi) & -\sin(\phi) & 0 \\
0 & \sin(\phi) & \cos(\phi) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
T = \begin{bmatrix}
\cos(\theta) & -\sin(\theta) \cos(\phi) & \sin(\theta) \sin(\phi) & 0 \\
\sin(\theta) & \cos(\theta) \cos(\phi) & -\cos(\theta) \sin(\phi) & 0 \\
0 & \sin(\phi) & \cos(\phi) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
T = \begin{bmatrix}
\cos(\theta) & \sin(\theta) & 0 & 0 \\
\sin(\theta) & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \tag{4}
\]

At this stage, the camera misalignment with the vehicle axis, \( \alpha \), is taken into consideration by setting \( \theta = \alpha - \pi/2 \). A perfectly aligned camera corresponds to \( \alpha = 0 \). In this case, the camera has the simple transformation matrix:

\[
T = \begin{bmatrix}
0 & -1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \tag{5}
\]
3.3 Displacement Observed by Vehicle

When dealing with an Ackerman-steered vehicle, it is possible to take advantage of the motion restrictions described earlier and simplify the calculation of the vehicle displacement. The displacement of the vehicle around the center-of-motion (Figure 1(b)) then becomes:

\[
\Delta x = x_B \tag{6}
\]
\[
\Delta \theta = \text{atan2}(y_B, x_{\text{cam}}) \tag{7}
\]

where \(x_B\) and \(y_B\) are the x and y components of the displacement in the vehicle bumper frame, and \(x_{\text{cam}}\) is the displacement of the camera along the x-axis of the vehicle, i.e. the bumper-to-center-of-motion distance. Here we set \(\Delta y = 0\) as we assume no sideways motion.

Furthermore, because the observed rotation between two frames is much smaller than the turning radius of the vehicles, we assume translation only in the image field.

The fact that the observed displacement \(\Delta u\) is much smaller than the displacement of the camera along the vehicle x-axis, \(x_{\text{cam}}\), leads to a simplified version of (2)-(7) since \(\text{atan2}(y_B, x_{\text{cam}}) \approx y_B/x_{\text{cam}}\) when \(y_B << x_{\text{cam}}\):

\[
\begin{bmatrix}
\Delta x \\
\Delta \theta
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & \frac{1}{x_{\text{cam}}}
\end{bmatrix} \cdot \begin{bmatrix}
x_B \\
y_B
\end{bmatrix} \tag{8}
\]
\[
= \begin{bmatrix}
1 & 0 \\
0 & \frac{1}{x_{\text{cam}}}
\end{bmatrix} \cdot \begin{bmatrix}
cos(\theta) & \sin(\theta) \\
\sin(\theta) & 0
\end{bmatrix} \cdot \begin{bmatrix}
\Delta u \\
\Delta v
\end{bmatrix} \cdot h_{\text{cam}} \cdot c_{\text{cam}} \tag{9}
\]
\[
= \begin{bmatrix}
1 & 0 \\
0 & \frac{1}{x_{\text{cam}}}
\end{bmatrix} \cdot \begin{bmatrix}
\Delta u \cdot \cos(\theta) + \Delta v \cdot \sin(\theta) \\
\Delta u \cdot \sin(\theta)
\end{bmatrix} \cdot h_{\text{cam}} \cdot c_{\text{cam}} \tag{10}
\]

If we assume near perfect alignment of the camera with the vehicle motion axis, \(\alpha \sim 0 \rightarrow \theta = -\pi/2\), we can further reduce this

\[
\begin{bmatrix}
\Delta x \\
\Delta \theta
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & \frac{1}{x_{\text{cam}}}
\end{bmatrix} \cdot \begin{bmatrix}
-\Delta v \\
-\Delta u
\end{bmatrix} \cdot h_{\text{cam}} \cdot c_{\text{cam}} \tag{11}
\]
\[
= \begin{bmatrix}
-\Delta v \\
-\Delta u
\end{bmatrix} \cdot h_{\text{cam}} \cdot c_{\text{cam}} \tag{12}
\]
From this equation we observe that the forward/reverse displacement is a function of the vertical displacement in the image while the estimation of the rotation is a function of the horizontal displacement in the image over the camera-wheel-axis distance. Due to this, a poor estimation of $x_{cam}$ results in systematic errors in $\Delta \theta$. However, as $x_{cam}$ is increased, this systematic error is decreased. For a standard car-like vehicle, mounting the camera on the front bumper yields a ratio such that the error is not significant.

3.4 Increment Vehicle Displacement

The final step is to increment the 3D robot pose, $\mathbf{X}$, described with a $3 \times 1$ position vector, $\mathbf{t} = [x, y, z]^\top$, and a $4 \times 1$ quaternion rotation vector, $\mathbf{Q} = [q_1, q_2, q_3, q_4]^\top$, where the scalar component is the first element. The robot pose is calculated as:

$$
\mathbf{t}_n = \mathbf{t}_p + \mathbf{R}_p \cdot \mathbf{t}_i
$$

$$
\mathbf{Q}_n = \begin{bmatrix}
q_{1p} \cdot q_{1i} - q_{2p} \cdot q_{1i} - q_{3p} \cdot q_{3i} - q_{4p} \cdot q_{4i} \\
q_{1p} \cdot q_{2i} + q_{1i} \cdot q_{2p} + q_{3p} \cdot q_{4i} - q_{4p} \cdot q_{3i} \\
q_{1p} \cdot q_{3i} + q_{1i} \cdot q_{4p} + q_{2p} \cdot q_{4i} - q_{2p} \cdot q_{4i} \\
q_{1p} \cdot q_{4i} + q_{1i} \cdot q_{3p} + q_{2p} \cdot q_{3i} - q_{3p} \cdot q_{2i}
\end{bmatrix}
$$

where subscripts $n$, $p$, and $i$ denote the new, previous, and incremental poses, respectively.

The incremental pose is derived from the previously estimated motion displacement, (6)-(7) (or their simplified version in (12)), as:

$$
\mathbf{t}_i = [\Delta x, 0, 0]^\top
$$

$$
\mathbf{Q}_i = [\cos(\Delta \theta/2), 0, 0, \sin(\Delta \theta/2)]^\top
$$

$\mathbf{R}$ is the $3 \times 3$ rotation matrix derived from the quaternion matrix $\mathbf{Q}_p$:

$$
\mathbf{R} = \begin{bmatrix}
1 - 2q_{3p}^2 - 2q_{4p}^2 & 2q_{2p}q_{3p} - 2q_{1p}q_{4p} & 2q_{2p}q_{4p} + 2q_{1p}q_{3p} \\
2q_{2p}q_{3p} + 2q_{1p}q_{4p} & 1 - 2q_{2p}^2 - 2q_{4p}^2 & 2q_{3p}q_{4p} - 2q_{1p}q_{2p} \\
2q_{2p}q_{4p} - 2q_{1p}q_{3p} & 2q_{3p}q_{4p} + 2q_{1p}q_{2p} & 1 - 2q_{2p}^2 - 2q_{3p}^2
\end{bmatrix}
$$
3.5 Expansion to 3D

It is straightforward to expand the odometry calculations to 3D. To do so, the vehicle’s pitch and roll need to be estimated. It is not possible to estimate these parameters from the downward looking camera. Here we show how the pitch and roll derived from an on-board INS can be incorporated to get a full 3D odometry calculation. The estimate displacement by the downward facing camera is still done in the plane. However, instead of considering this plane to be horizontal, the roll and pitch information is used to rotate the plane before incrementing the estimated planar motion.

To incorporate the INS measurements we expand the previously explained four step calculation by adding one more step. In the new step (which is performed between steps 3 and 4 as previously defined), the current robot pitch and roll are read from the INS and are set in $Q_p$ in (14) before incrementing the vehicle displacement. Equation (18) shows the calculation of the rotation quaternion using the roll and pitch readings.

$$Q_p = \begin{bmatrix}
q_{1p} \\
q_{2p} \\
q_{3p} \\
q_{4p}
\end{bmatrix}
= \begin{bmatrix}
cRh \cdot cPh \cdot cYh + sRh \cdot sPh \cdot sYh \\
sRh \cdot cPh \cdot cYh - cRh \cdot sPh \cdot sYh \\
cRh \cdot sPh \cdot cYh + sRh \cdot cPh \cdot sYh \\
cRh \cdot cPh \cdot sYh - sRh \cdot sPh \cdot cYh
\end{bmatrix} \quad (18)$$

where $cRh = \cos(roll/2)$, $sRh = \sin(roll/2)$, $cPh = \cos(pitch/2)$, $sPh = \sin(pitch/2)$, etc.

Here, the yaw is not read from the INS, but is kept from the odometry calculation. If the yaw estimate from the INS is precise it can be used to ensure that the visually estimated rotation is correct.

In the next section we describe a method for robustly estimating the displacement in the camera, $\Delta u$ and $\Delta v$. 
Figure 3: The correlation processing time depends on the image size and template size.

4 Correlation-based Shift Detection

Displacement between shifted image frames can be estimated using correlation (Horn, 1986). The correlation can be applied to the entire frame or to a small area of one frame that can be found in a larger frame. The latter method is also called template matching (Gonzalez and Woods, 2002).

As discussed in Section 3, our motion estimate assumes piecewise linear motion. If a vehicle follows a different motion model from the one used in this paper and sharp turns (relative to the vehicle speed and camera frame rate) are expected, alternative models for rotation invariant template matching should be considered.

The processing time of the correlation matching method depends on the size of the image. This is one important design parameter for real-time (> 30 Hz) on-board odometry estimation as can be seen from Figure 3.

Another important parameter is the correlation template window. The choice of the template, its location, and size are of vital importance to the performance of the overall system. Figure 4, for example, illustrates the effect of different template window sizes on the correlation. The correlation score is in the interval [0 : 1], where 1 corresponds to a perfect match. A smaller template window can contain less information, resulting in local mini-
Figure 4: The correlation similarity score on concrete surface. The legend shows the average correlation over the 250 frames for 64, 128 and 256 pixel template windows.

A larger template window can be more unique but at the cost of smaller displacement and consequently lower vehicle velocity. Moreover, larger templates can be more expensive computationally, as plotted in Figure 3.

One potential shortcoming of template matching is its deficiency in dealing with foreshortening. It is expected that less accurate or incorrect results can be produced on uneven surfaces where two consecutive frames have different perspectives. Experimental results from off-road field trials have shown that this is not a limitation of the system (Nourani-Vatani et al., 2008).

In the following sub-sections we will explain in detail the most important parameters for the image shift displacement estimation using correlation which makes this approach suitable for real-time high speed estimation of the vehicle motion.

### 4.1 Restricted Search and Template Location

A prior estimate of the position of the correlation match in the next frame can be used to restrict the correlation search area. This is a very desirable outcome because of two important factors. Firstly, as was shown in Figure 3 the correlation processing time increases
exponentially with the size of the search area. Limiting the search area can therefore speed
up the processing time considerably. Finally, what is less obvious, is that the probability of
false matches due to local minima in the image also decreases with the size of the search
area. This is illustrated later in the experimental section.

To have a prior estimate of the position of the correlation match in the next frame, a forward
linear prediction (FIR) filter (Manolakis et al., 2000) is used. This filter predicts the current
value of a real-valued time series based on the past samples, minimizing the prediction error
in a least squares sense. We do not use a Kalman filter because the measurement error
cannot be approximated as Gaussian, as errors in the correlation position usually occur as
large jumps from the ground-truth.

On initialization, the system has no prior knowledge about the motion of the vehicle. The
obvious choice of template location is the center of the frame. The match to the template
will be in the center of the next frame if the vehicle is still. Once the vehicle starts moving,
the match will occur away from the center of the frame. To allow for more shift (and
higher velocities) we choose the template location in the opposite direction to the predicted
location according to the filter. Let \( x \) represent an image region (template) extracted from
a full image \( a \), this can be expressed as:

\[
\begin{align*}
   u_{x,k} &= w_a/2 - w_x/2 + u_{x_{pf},k} \\
   v_{x,k} &= h_a/2 - h_x/2 + v_{x_{pf},k}
\end{align*}
\]

where \( k \) is the frame number, \( u_{x,k} \) and \( v_{x,k} \) define the template location in the horizontal and
vertical directions, respectively, and \( u_{x_{pf},k} \) and \( v_{x_{pf},k} \) are the output of the prediction filter.
\( w_x \) and \( h_x \) are the width and height of the template window, \( w_a \) and \( h_a \) are the full image
width and height.

This simple method takes into consideration the vehicle motion, pushing the window in the
same direction as the vehicle motion, creating more space for displacement detection and
higher velocity accordingly. The maximum vehicle velocity that can be estimated is

\[
v_{max} = (h_a - h_x) \cdot h_{cam} \cdot c_{cam} \cdot f_r
\]
where $h_{\text{cam}}$ is the camera mount height, $c_{\text{cam}} = \frac{1}{f \cdot \text{size}_{\text{pixel}}}$ is the camera constant derived from the camera focal length ($f$) and the chip pixel size ($\text{size}_{\text{pixel}}$), and $f_r$ is the camera frame rate in frames per second (fps). Figure 5 shows the correspondence between template mask size, camera frame rate, and maximum allowable vehicle velocity. Correspondingly, (21) indicates the minimum overlap necessary between consecutive frames. If the overlap is less than $h_a - h_x$, the displacement estimation fails due to the lack of correspondence between the frames.

In a 640 $\times$ 480 image, a template size of 128 $\times$ 128 pixels provides an average template matching quality of 0.77 on concrete flooring (Figure 4), while still allowing for a 352 pixels image shift. To achieve high velocities of more than 50 km/h (or 14 m/s), using a camera mount height of 0.60 m above ground, a frame rate of more than 40 fps is necessary according to (21). If smaller velocities are expected, larger template windows can be used to achieve better template matching.
4.2 Multi-template Configurations

The performance of the template matching depends on the information available in the template image. For example, in a plain white image the template matching will fail because all regions in the image have the same pixel value. To increase the probability of having a higher correlation, we propose the use of multiple templates.

Many different template location configurations are applicable. Figure 6 shows four possible configurations when using a single, two, three, and five template images. The more templates we choose, the more likely it is that a good candidate is found. The limiting factors on the number of templates analyzed are processing time restrictions and image displacement restrictions.

With processing times of 20 – 50 ms to perform a single template match (Figure 3), it is not feasible to perform several correlations for real-time usage. The idea is to analyze multiple template windows using a template quality measure, find the most suitable, and use that template only to perform template matching. By analyzing larger regions, the probability of having a higher correlation is increased.
4.3 Template Quality Measure

The template quality measure (TQM) is used to evaluate the appropriateness of each template window for correlation. We will show in Section 5 that the quality of the correlation depends on the noise level as well as on the luminance variability of the template. On average, image templates with more contrast and variability yield better correlation performance than very smooth templates, which are less robust to noise. For this reason we investigate four metrics to measure variability:

- **Auto-correlation** gives a measure on the amount of randomness (or uncorrelatedness) of the signal, taking into account the neighboring structure of the pixels’ luminance.

- **Variance** is used to measure gray-level contrast and can also be used to establish descriptors of relative non-smoothness.

- **Median absolute difference** (MAD) also gives a measure of statistical dispersion of data, but it is more robust towards outliers than the variance as it is based on the median rather than the mean (Hoaglin et al., 1983).

- **Entropy** gives a measure of uncertainty taking into account probability of occurrence of each gray level (Gonzalez and Woods, 2002).

The metrics are used to calculate the TQM in a multi-template configuration. Only the template with the highest TQM is selected for template matching, and the resulting correlation score is observed. Figure 7(a) shows the performance of the correlation quality for the template chosen by the different TQMs. In this experiment, the TQM was used to select “the best” template for correlation among five template candidates. These results indicate that all the metrics are efficient in estimating the best template candidate. The auto-correlation, variance, MAD, and entropy presented similar performances with respect to the correlation quality. However, the auto-correlation is impractical as its computation time is significantly longer (approximately 1000 times slower than the other metrics). In contrast, the entropy presented the best performance time (Figure 7(b)). For this reason and considering that the
Figure 7: a) The correlation similarity score on concrete surface where the template window candidate is selected using auto-correlation, variance, MAD and entropy as the TQM. The average correlation over 250 frames is shown in the legend. b) The quality measure processing time for the variance, MAD and entropy for different template window sizes. In contrast, the average template matching for a $640 \times 480$ pixel image is 50 ms. The times presented here are relative to the OpenCV implementation running on a Core2 Duo 2GHz 2GB notebook. The auto-correlation implementation is part of the C++ Template Image Library and yields processing times more than 1000 times slower.

entropy is a well-known measure of image variability we use the entropy as the TQM in our final experiments presented in Section 6.

One important requirement regarding the TQM is that its processing time has to be much smaller than the processing time of the template matching. The processing time of the entropy is approximately $1.0e^{-4}$ s, which is only a very small fraction of the template matching calculation time of $20 - 50e^{-3}$ s. It is therefore possible to evaluate several template windows without significantly affecting the overall processing time of the algorithm.
4.4 Optimization through Fast Fourier Transform

To speed up the correlation process, we carry out the correlation in the frequency domain by making use of the correlation theorem (Kuglin and Hines, 1975). This theorem states that

$$a \ast x \leftrightarrow F(a)F(x)$$

(22)

where \( \ast \) represents the correlation operation and \( F(\cdot) \) represents the Fast Fourier Transform (FFT).

Depending on the ratio between the size of the image and the size of the template, template matching in the frequency domain via the FFT can be much faster than the spatial counterpart. With respect to computational complexity, correlation in the spatial domain should be performed only if \( x \) is much smaller than \( a \) (Gonzalez and Woods, 2002). As will be shown later, to achieve a practical odometer, a template size in the order of 100 \( \times \) 100 pixels (for a full frame of size 640 \( \times \) 480) is suggested, making the frequency domain implementation more efficient.

4.5 Image Shift

The image shift, \( \Delta u \) and \( \Delta v \), is finally calculated as:

$$\Delta u_k = u_{x_k} - u_{c_k}$$

(23)

$$\Delta v_k = v_{x_k} - v_{c_k}$$

(24)

where \( u_{c_k} \) and \( v_{c_k} \) are the reported correlation location, \( u_{x_k} \) and \( v_{x_k} \) are the location of the template image and \( k \) is the frame number. This image shift is then used to estimate the vehicle motion as explained in Section 3. The results from (23) and (24) are inserted in (2) to estimate the camera displacement and consequently the vehicle displacement.

The pseudo-algorithm in Algorithm 1 summarizes all the steps used in the estimation of the vehicle displacement from the image shift and Figure 8 shows an example of a template match between two consecutive frames. Here the image size is \((w_a \times h_a) = (640 \times 480)\) pixels and the template size is \(w_x = h_x = 128\) pixels in a three-template configuration. According to the
output of the FIR prediction filter, the next displacement is going to be \((\Delta u_{x^{pf,k}}, \Delta v_{x^{pf,k}}) = (1, 48)\) pixels. Hence, the center of the template configuration is shifted to the left 0.5 pixels and up 24 pixels to accommodate for the forward and slightly left motion of the vehicle. According to the TQM, the right template is selected (blue box in Figure 8(a)) among the three templates (yellow boxes) for correlation. The estimated location of the correlation becomes \((u_{x_k}, v_{x_k}) = (321, 274)\) (green box in Figure 8(b)) according to the output of the prediction filter and the location of the template. Using the knowledge from the predictor the search area is restricted to 178 pixels \((w_x + 50\) pixels) still accommodating for sudden changes (magenta box). The actual correlation occurred at \((321, 278)\) (red box), which is only 4 pixels away from the prediction.

Algorithm 1 Pseudo-algorithm summarizing the steps in the estimation of the vehicle displacement from the image shift.

- Get ‘current image’
- Load ‘previous image’
- Use prediction to estimate location of the next match in the ‘current image’ (Section 4.1)
- Extract templates in ‘previous image’ according to the configuration (Section 4.2)
- Select best template according to TQM (Section 4.3)
- Perform template matching on ‘current image’ using the selected template (Section 4.4)
- Calculate the image shift (Section 4.5)
- Increment the pose by using the estimated image shift (Section 3)

5 Theoretical Error Analysis

Given the proposed approach for visual odometry, in this section we present a statistical analysis to determine the theoretical error rate in the template matching.

5.1 Statistical Assumptions Used in the Analysis

For the images considered in this application, employing a Gaussian model to represent their statistical distribution is a reasonable approximation. The histograms of the images shown
Figure 8: Template matching using correlation. a) The yellow boxes show the location of the template candidates. The blue box corresponds to the selected template with highest TQM. b) The predicted location of the correlation is shown by the green box. This prediction is used to restrict the search area (magenta box). In this example, the resulting match (red box) is only four pixel away from the prediction.

In Figures 2(a) to 2(d) are given in Figures 9(a) to 9(d), respectively. These histograms illustrate that it is reasonable to assume a Gaussian model for the images. In addition, due to the unstructured characteristic of the ground, we approximate the pixels to be spatially uncorrelated. This assumption is supported by the plot in Figure 10, which shows a strong peak (at lag 0,0) for the two-dimensional auto-correlation function of the image in Figure 2(a).

The Gaussian and uncorrelated models are used in the error rate analysis in Sections 5.3 and 5.4. As mentioned, these are of course approximations and can be seen as a relaxation of the real statistical characteristics of the signals such that a theoretical analysis can be performed. Some images of course contain elements that will cause the Gaussian assumption to fail (for example, the dark line in Figure 8(b)). In addition, the presented error analysis is valid for constrained rotation and assuming that no motion blur is present. In this sense, the main goal of this analysis is not to output an absolute number for the error rate, but to give an understanding of approximately how much each variable affects the system performance.
Figure 9: Histograms for different types of surfaces.
Figure 10: 2D auto-correlation plot of a typical image (asphalt) from the experiments. The strong peak shows that it is reasonable to approximate the distribution of the pixels as uncorrelated for analysis purposes.

5.2 Definitions

Let $x_k$ represent an image area (template) cropped from a full image $a_k$ corresponding to the $k^{th}$ frame. Consider that for frame $k$, $a_k$ is a modified version of $a_{k-1}$ which has been translated (due to vehicle motion) and distorted with additive white Gaussian noise (AWGN). As discussed in Section 4, the task is to identify the area $x_k$ in frame $k$ that is most likely to correspond to $x_{k-1}$. As a likelihood measure, we employ the linear correlation, as linear correlation is the optimal detection metric when the noise can be modeled by AWGN (Barni and Bartolini, 2004).

With the assumptions above, we may write

$$x_k = x_{k-1} + z_k$$  (25)

where $z$ represents the noise. In our interpretation, the noise $z$ corresponds to the small differences that are observed when looking at the same ground area in two consecutive frames (which are displayed in different parts of the image when the vehicle is in motion).
So this includes very small changes in the observation angle and very small illumination changes, for example, apart from the sensor-noise.

Let us define the linear correlation between vectors \( f \) and \( g \) as

\[
C = \frac{1}{N} \sum_{i=1}^{N} f(i)g(i)
\]

where \( f(i) \) and \( g(i) \) correspond to the \( i^{th} \) element in \( f \) and \( g \), respectively. \( N \) corresponds to the number of elements in the vectors.

Ideally, in every new frame \( a_k \), the correlation detection should have its maximum at the point where \( x_k \) corresponds to \( x_{k-1} \). Let us define the result of the linear correlation in the correct position as

\[
C_c = \frac{1}{N} \sum_{i=1}^{N} x_{k-1}(i)[x_k(i) + z_k(i)]
\]

where \( x_k(i) \) corresponds to the \( i^{th} \) element of \( x_k \), considering a lexicographic representation of the images.

The definition of an error is: the correlation is higher in a position other than the correct corresponding region. Let us define the result of the linear correlation in an incorrect position as

\[
C_w = \frac{1}{N} \sum_{i=1}^{N} x_{k-1}(i)[s_k(i) + z_k(i)]
\]

where \( s_k(i) \) corresponds to the \( i^{th} \) element of \( s_k \), which represents an image block in a “wrong” position.

In this case, the template matching operation will yield an “erroneous” output when there is a value for \( s_k : C_w(s_k) > C_c \). In order to determine the system’s theoretical error rate, we need to determine the error probability \( \Pr(C_w > C_c) \). Given the Gaussian assumptions for \( x_k \) (argued in Section 5.1 and justified by results shown later) and \( z_k \), \( C_w \) and \( C_c \) follow a normal distribution because the correlation operation is linear. Therefore, \( C_w \sim \mathcal{N}(\mu_{C_w}, \sigma_{C_w}^2) \) and \( C_c \sim \mathcal{N}(\mu_{C_c}, \sigma_{C_c}^2) \). In this case, \( \Pr(C_w > C_c) \) can be determined from \( \mu_{C_c}, \sigma_{C_c}^2, \mu_{C_w} \) and \( \sigma_{C_w}^2 \).
5.3 Case 1: Determine $\mu_{C_c}$ and $\sigma^2_{C_c}$

The expected value of $C_c$ is given by

$$
\mu_{C_c} = E\{C_c\} = E \left\{ \frac{1}{N} \sum_{i=1}^{N} x_{k-1}(i)x_k(i) \right\} = E \left\{ \frac{1}{N} \sum_{i=1}^{N} x_{k-1}(i)[x_{k-1}(i) + z_k(i)] \right\} \tag{29}
$$

Without loss of generality, for notation simplicity, we remove the indexes $k - 1$ and $k$ in the following.

$$
\mu_{C_c} = E \left\{ \frac{1}{N} \sum_{i=1}^{N} x(i)[x(i) + z(i)] \right\} = \frac{1}{N} \sum_{i=1}^{N} E\{x^2(i)\} + E\{x(i)z(i)\} \tag{30}
$$

Recalling that $x \sim \mathcal{N}(\mu_x, \sigma^2_x)$, $z \sim \mathcal{N}(0, \sigma^2_z)$ and $\sigma^2_x = E\{x^2\} - \mu^2_x$, Equation (30) can be written as

$$
\mu_{C_c} = \frac{1}{N} \sum_{i=1}^{N} (\sigma^2_x + \mu^2_x) + E\{x(i)\} E\{z(i)\}_{=0} = \sigma^2_x + \mu^2_x \tag{31}
$$

The variance of $C_c$ is given by

$$
\sigma^2_{C_c} = E\{C^2_c\} - \mu^2_{C_c} = E \left\{ \left[ \frac{1}{N} \sum_{i=1}^{N} x^2(i) + x(i)z(i) \right]^2 \right\} \tag{32}
$$

In Appendix 8 we show that $\sigma^2_{C_c}$ can be written as

$$
\sigma^2_{C_c} = \frac{2\sigma^4_x + 4\sigma^2_x\mu^2_x + \sigma^2_x\sigma^2_z + \mu^2_x\sigma^2_z}{N} \tag{33}
$$
5.4 Case 2: Find $\mu_{C_w}$ and $\sigma^2_{C_w}$

When the correlation is performed on a “wrong” position we have

$$C_w = \frac{1}{N} \sum_{i=1}^{N} x(i)[s(i) + z(i)]$$  \hspace{1cm} (34)

The expected value $\mu_{C_w}$ is

$$\mu_{C_w} = \mathbb{E}\left\{ \frac{1}{N} \sum_{i=1}^{N} x(i)[s(i) + z(i)] \right\}$$

$$= \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}\{x(i)s(i)\} + \mathbb{E}\{x(i)z(i)\}$$  \hspace{1cm} (35)

Using the same statistical assumptions employed in Section 5.3, (35) becomes

$$\mu_{C_w} = \frac{1}{N} \sum_{i=1}^{N} \mathbb{E}\{x(i)\} \mathbb{E}\{s(i)\} + \mathbb{E}\{x(i)\} \mathbb{E}\{z(i)\} = 0$$

$$= \mu_x \mu_s$$  \hspace{1cm} (36)

and $\sigma^2_{C_w}$ is

$$\sigma^2_{C_w} = \frac{\sigma_x^2 \sigma_s^2 + \mu_x^2 \sigma_s^2 + \mu_x^2 \sigma_s^2 + \sigma_z^2 \sigma_z^2 + \mu_z^2 \sigma_z^2}{N}$$  \hspace{1cm} (37)

5.5 Merging $C_c$ and $C_w$

The probability of error $p_e$ that a “wrong” position presents a higher detection than a “correct” position is given by

$$p_e = \Pr(C_w > C_c)$$  \hspace{1cm} (38)

In Appendix 9 we show that $p_e$ may be expressed as

$$p_e = 1 - \frac{1}{2} \text{erfc} \left( \frac{-\mu_R}{\sqrt{2} \sigma_R} \right)$$  \hspace{1cm} (39)

where $R = C_c - C_w$ and \text{erfc}(x) = $\frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$.

Equation (39) represents the error probability for one single position in the image. The final error rate must take into account $S$ detections, where $S$ is the number of pixels in the full
image a. Following a Bernoulli trial framework (Papoulis and Pillai, 2001), the probability of $s$ successes (where the probability of a success is $p_e$) in $S$ trials is given by:

$$\binom{S}{s} p_e^s (1 - p_e)^{S-s}$$  \hspace{1cm} (40)

Therefore, the error rate for the system is given by:

$$P_e = 1 - \binom{S}{s} p_e^s (1 - p_e)^{S-s}$$

$$= 1 - \binom{S}{0} p_e^0 (1 - p_e)^{S-0}$$

$$= 1 - (1 - p_e)^S$$ \hspace{1cm} (41)

Equation (41) depends on the parameters $\mu_x$, $\sigma_x$, $\mu_s$, $\sigma_s$, $\sigma_z$, and $N$. This can be observed by inserting the results from Sections 5.3 and 5.4 into (39). The experiments in Section 6 illustrate the validity of the analysis and its applicability.

As shown in Equation (21), the maximum allowable drive velocity depends on the size of the template window, the available camera frame rate, and the mount height. The camera mount height and the camera frame rate are fixed for any given setup, such that only the template window size can be varied. From Equation (41) it is observed that the theoretical error rate is related to the size of the template window (blue line in Figure 11), as $p_e$ depends on (33) and (37). This implies that, by reducing the template window size to achieve higher velocities, the error rate of the correlation match is increased (Vanderbrug and Rosenfeld, 1977). This figure also shows that for an error rate of less than 1.0%, for example, the corresponding template window size needs to be larger than 220 pixels. From Figure 5 we see that this corresponds to a maximum vehicle velocity of 1.75 m/s at 15 fps.

6 Experiments

This section is divided in two parts. The first part presents results comparing different template sizes, number of templates, and their performance on different ground surfaces. Moreover, the validity of the derived theoretical error rate is also illustrated. The second
part evaluates the visual odometry system using different platforms in various outdoor environments and demonstrates its applicability as a robust odometry module.

In the plots presented the number of templates used and their corresponding sizes are represented by \(<\text{number}>T<\text{size}>\) in the legends. For example, a three-template configuration of size 128 pixels is written as 3T128.

6.1 Part 1: Analysis of the Template Image

6.1.1 Theoretical Error Rate

In this section we validate the result for the error rate, presented in Section 5. The noise levels were set to \(\sigma_z^2 = 100\) and \(\sigma_x^2 = \sigma^2_s = 2000\). \(\sigma_x^2\) and \(\sigma^2_s\) are set to 2000 because this corresponds approximately to the average variance of typical images used in the experiments (considering the range 0-255). The variance \(\sigma_z^2\) is set to 100 because we are not modeling \(z\) as sensor-noise only. In our interpretation in the theoretical analysis, \(z\) also corresponds to the small differences that are observed when looking at the same ground area in two consecutive frames (which are displayed in different parts of the image when the vehicle is in motion). So this includes very small changes in the observation angle and very small non-uniform illumination changes, for example, apart from the sensor-noise itself.

The curve in Figure 11 shows a plot of the result in Equation (41) for a variable number of pixels in the template (varying from 100 to 300) in an image with dimension 640 \(\times\) 480 pixels. The circles illustrate the template matching simulation results. For this simulation we use signals with the same statistical characteristics as assumed in the analysis. The excellent correspondence between the circles and the line plot indicates the validity of the theoretical analysis.

The crosses in Figure 11 correspond to the error rates using real images obtained in the trial runs. In this case we can see a discrepancy between the theoretical error rate and the error rate from the experimental results. This difference is expected due to a relaxation in
Figure 11: Theoretical error rate for the system with $\sigma_z^2 = 100$ and $\sigma_x^2 = \sigma_s^2 = 2000$.

the statistical assumptions used in the error analysis. Despite the difference, the analysis still yield a reasonable correspondence between theory and practice. We argue that, more significantly than the actual numerical values, this analysis provides an understanding of how much each of the modifiable parameters affect the performance of the system.

6.1.2 Importance of the Template Choice

It is important to evaluate whether it is possible to achieve improved correlation due to informed template choice. In Section 4.2 possible template location configurations were introduced, as illustrated in Figure 6. Figure 12 compares the single-template configuration against the three and five-template configurations. As expected, the template matching quality increases with the number of templates. This is shown as a relative correlation score $> 1.0$. More importantly, there are situations where more templates results in more than 20% higher correlation. This reduces the probability of a template mismatch.

6.1.3 Correlation Quality on Different Surfaces

In this section, we compare the performance of the template quality measure and the benefit of using it on different surfaces. The tests are performed on concrete, asphalt, and grass surfaces (see Figure 2). Figure 13 shows the correlation score ratio between four-template
and single-template configurations. Two main conclusions can be drawn from these results. Firstly, statistically the template matching quality is better when using more templates on all surfaces (the mean score is above 1.0). In contrast, the improvement when using multiple-template configuration is reduced on more feature-rich surfaces. In this case, the figure shows that the improvement when using 4 templates as opposed to 1 template is higher for the asphalt (1.028) than for the grass (1.002), for example.

6.2 Part 2: Odometry Performance

This section compares the odometry performance for the single and multi-template configurations. Some of the experiments using the single-template configuration only were originally presented in previous work (Nourani-Vatani et al., 2009, 2008).

6.2.1 Analyzing Odometry Performance

All forms of odometry suffer from unbounded error. Furthermore, the systematic velocity error in odometry modules increases with the distance from origin but decreases as we loop around and head back towards the origin (Kelly, 2004). This implies that driving in a loop and measuring the overall global error by calculating the Euclidean distance to the origin is a
Figure 13: The correlation score ratio between three and single-template configurations on a) concrete, b) asphalt, and c) grass surfaces. The mean and standard deviation of the correlation score ratios are shown in the legends.

A technically weak measure of performance for odometry modules. A more appropriate method is used by Johnson et al. (2008). They first align a short part of the odometry path with their ground truth GPS path using a least squares method. Then the Euclidean distance error of their odometry system is measured 100 m down the path. The start position is shifted slightly and the procedure is repeated for 100 m sequences until the end of the driven path. By looking at the median and standard deviation of the calculated errors, a much better understanding is gained from the performance of the odometry module.

We follow this approach to present the performance of our odometry module with two additions. Firstly, we conduct the test not only for one interval but for several intervals; 10 m, 20 m, 50 m, and 100 m. This provides a better indication on how the 3D Euclidean distance error grows with the distance traversed. Secondly, when ground truth heading is available, we also calculate the heading error, which is often a weakness of odometry modules. For all the error calculations, the first 1 m of the ground truth path is aligned with the odometry path using a least squares method. A distance $\delta$ is traversed on the ground truth path and the odometry path corresponding to this, using the time stamp as guide, is extracted. Then
the 3D Euclidean distance error between the two end points of these paths are calculated. Finally, the paths are shifted—we have chosen $\delta/10$—and the process repeated.

We present the result using boxplots (Johnson, 1994). The boxplots show the median, the 25%, and the 75% quartiles. The whiskers extend from each end of the box to the adjacent values within 1.5 times the interquartile range. Outliers are data with values beyond the ends of the whiskers and not displayed in the plots.

### 6.2.2 System Calibration

After setting up the system and mounting the sensors, a calibration run is necessary due to the uncertainties in the mounting and sensor locations. In this step we try to determine three parameters, namely the camera height over ground, the camera displacement from the vehicle's center-of-motion, and the camera misalignment with the vehicle axis, given in (10) by $h_{\text{cam}}$, $x_{\text{cam}}$, and $\theta$, respectively. If an INS is mounted, its offsets are also determined. This is done by driving a figure-eight and logging the sensor data. The system parameters are derived by comparing the visual odometry path with a ground-truth path through a least-square fit. This approach is currently done manually but only needs to be done once for any installation.

### 6.2.3 Presentation of the Field Trials

The field trials are carried out on a Toyota Prado 4×4 and on an HMC, shown in Figure 14. The software is implemented and run on a Macbook Santa Rosa 2 GHz 2 GB laptop running Linux Ubuntu 9.04. In particular, the template matching is done using OpenCV’s implementation using `cvMatchTemplate` which internally calls `cvDFT` to perform the correlation in frequency domain. We call this function using the normalized correlation method, `CV_TM_CCORR_NORMED` to achieve more lighting invariance and to get the result in the normalized interval.

In experiments I and II the camera used is a simple Unibrain Fire-i 1394 camera running at
Figure 14: Test vehicles used for compilation of the results consist of the 20 Tonne HMC forklift and a standard Toyota Prado 4×4 (the sensor setup shown corresponds to that used in Experiment II).

30 Hz with a resolution of 640 × 480\(^1\). In experiment III a Basler scA1400-30fc\(^2\) camera is used, where the image resolution is cropped to 480 × 640 to be able to log at a maximum frame rate of 43 fps. The lens used is a Tamron 23FM65 2/3” Manual Iris, with f-stop set to 4 and a shutter speed of \(\frac{1}{2000}\) s.

**Experiment I:** The first experiment is carried out in an industrial setting using the HMC. The path traversed is approximately 778 m long. As ground truth a high precision beacon-based global laser localizer is used (Roberts et al., 2007). The global error of the ground truth system is up to 0.12 m. The results for the HMC run, including the HMC wheel odometer, are shown in Figures 15-16 and summarized in Table 1. This run was performed in hard and dry concrete and asphalt surfaces, and no wheel slippage was observed. In the legend, the number of templates used and their corresponding sizes are represented by \(<\text{number of templates}>T<\text{template size}>\). Looking at the results, we observe an improvement when using 220 × 220 pixel templates compared to 128 × 128 pixel templates. This verifies our assumption (Figure 4) that larger template windows result in more accurate correlation. Similarly, there is also a reduction in the standard deviation when using multiple templates compared to a single template. This corresponds to a more stable and

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\(^1\)Unibrain Fire-i Cameras: [http://www.unibrain.com/products/visioning/fire_i_dc.htm](http://www.unibrain.com/products/visioning/fire_i_dc.htm)

Table 1: Summary of setup and results for Experiment I.

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<td>4.31 m/10.44°</td>
<td>17.43 m/20.26°</td>
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</table>

robust odometry. In comparison, the wheel odometer currently employed on the HMC after a 100 m traversal has an estimated position and heading error of 17 m and 21°, respectively, which is considerably larger than the visual odometer.

The velocity results in Figure 15(b) shows that the visual odometors produce a similar velocity estimate as the wheel odometer. The difference in velocity estimate 150 s into the run corresponds to a shadow failure where the odometer starts tracking its own shadow with relative velocity of zero.

**Experiment II:** For the second experiment the Toyota Prado is used on an off-road hilly track with a combination of gravel and grass surfaces. This path is approximately 440 m. A MicroStrain 3DM-GX1 inertial measurement unit mounted on the bumper of the vehicle behind the camera is used to estimate the vehicle roll and pitch. The 3D vehicle pose is derived by fusing the motion in the plane estimated by the visual odometer with the vehicle roll and pitch from the INS (Section 3.5). An RTK-GPS system with an estimated accuracy of 0.02 m is used as ground truth. The RTK-GPS unit does not return a heading value. Hence, only the 3D Euclidean distance error is calculated for this run. The result from this experiment is presented in Figure 17 and summarize in Table 2. Figure 18 indicates that the three-template configuration is performing similar or slightly worse than the naive single-template configuration. This seems to contrast with the fact that the template matching performance is better for multi-template configuration as explained in Section 6.1.3. Further
Figure 15: Experiment I: Comparison of visual odometry performance using different template configurations with the laser localizer and the wheel odometer. a) 2D plot of the trajectories. The start location is at the origin (0, 0), the paths are ~750 m long, and end at (1.8, −18.2). b) The comparison of the velocity estimates of the different approaches.
Table 2: Summary of setup and results for Experiment II.

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<td>7.04m</td>
</tr>
</tbody>
</table>

investigation shows that during a part of the traversal there is some cast shadow from the vehicle visible in the side of the frame. This shadow is picked up by the three-template configuration (as it covers an area twice as wide) but not by the single-template version. It is clear that, the algorithm is performing correctly and choosing the region in the frame with the highest contrast. Unfortunately for this sequence, this is shadow.

**Experiment III:** The third experiment covers a distance of $\sim 4.75$ km through an urban neighborhood in Brisbane, Australia. The mount height of the camera is 0.42 m and the focal length of the lens is 1021 pixels. From (21), using a template size of 96 pixels will allow for a maximum vehicle velocity of 9.7 m/s or equivalently approximately 35 km/h.
Figure 17: Experiment II: Comparison of the odometry performances using single-template (solid line) and three-template (dashed line) configuration on the hilly off-road track compared with the RTK-GPS (tick line). Path have lengths of $\sim 413$ m and starts at $(0, 0, 0)$.

(a) 3D plot. Note that the $z$-axis has been scaled differently to show the height calculation more clearly.  
(b) 2D overhead plot.

Figure 18: Experiment II: Boxplot of the error growth as a function of distance traversed.
Table 3: Processing times in milliseconds for a setup using a 480 × 640 pixels image with template size of 92 × 92 pixels in a three-template configuration using the entropy as TQM and with a restricted search area of 130 pixels.

<table>
<thead>
<tr>
<th>Name</th>
<th>Number</th>
<th>Size</th>
<th>10m</th>
<th>20m</th>
<th>50m</th>
<th>100m</th>
</tr>
</thead>
<tbody>
<tr>
<td>3T92</td>
<td>3</td>
<td>92</td>
<td>0.25 m/0.30°</td>
<td>0.60 m/0.58°</td>
<td>3.06 m/1.44°</td>
<td>7.44 m/2.94°</td>
</tr>
<tr>
<td>3T92 R</td>
<td>3</td>
<td>92</td>
<td>0.23 m/0.27°</td>
<td>0.53 m/0.49°</td>
<td>2.36 m/1.16°</td>
<td>6.73 m/2.17°</td>
</tr>
</tbody>
</table>

Table 4: Summary of setup and results for Experiment III.

As ground truth a laser SLAM system is employed (Bosse and Zlot, 2008). The distance and heading accuracy of this system is 0.6% and 0.01%, respectively, of the traveled path. The visual odometer uses a horizontally distributed three-template configuration (Figure 6(c)) with a size of 92 pixels. To ensure the odometer can operate in real-time, the search area is restricted to 130 pixels around the prediction point. This brings down the average correlation time to 2.5 ms. Table 3 shows the processing times for each of the steps of the visual odometer. With current implementation a maximum frame rate of ∼ 200 Hz is achievable.

For the sake of comparison, the result of estimating the odometer using an unrestricted search is also included. Note that this unrestricted approach cannot execute in real-time as the correlation takes ∼ 50 ms (Figure 3). Figure 19 shows the map of the traversed path and presents a visual comparison between the ground truth laser SLAM path and the visual odometry paths. The distance and heading error estimates are presented in Figure 20 and summarized in Table 4. For shorter segments the error is below 3% but grows to 6% over 100 m segments. The heading error remains well under 5° even for distances over 100 m.

One important result is the significantly smaller error for the restricted approach. This shows the accuracy of the predictor and demonstrates that in addition to a great speedup, false
Figure 19: Experiment III: a) Map of the traversed path. Start is shown with a green flag and end with a red flag. The path length is \( \sim 4743 \) m. b) Visual comparison of the three-template odometry paths (red and blue lines) compared to the laser SLAM ground truth path (black line). The paths start at \((0, 0)\) and end at \((1724, -1566)\).

Figure 20: Experiment III: Boxplots of the error growth over \([10\text{m}, 20\text{m}, 50\text{m}, 100\text{m}]\) segment lengths using a three-template configuration with a restricted (solid line) or unrestricted search (dashed line). a) Euclidean distance error. b) Heading error.
matches can be reduced by restricting the search area. This is illustrated in Figure 21 where the estimated displacements, $\Delta u$ and $\Delta v$, are plotted for both the restricted and unrestricted search implementations. Without the restricted search area, at high velocities the correlation can lose track and find false matches anywhere in the frame. By restricting the search area, not only do we speed up the matching but also limit false positive matches due to local minima in the image. The errors between frames [6200-6400] in Figure 21(a) result in the large error in the odometry estimate at (450,-430) in Figure 19(b) for the unrestricted path (red line).

Probably the most important usage of an odometer is its velocity estimate. Figure 22 presents the comparison of the velocity estimates between the laser odometer, which is used for SLAM, and the visual odometer. We observe that the visual odometer tracks the laser odometer accurately and can be used for closed-loop control.

### 6.3 Comparison with Optical Flow

In this section we compare the correlation-based odometer with the optical flow-based alternative. Figure 23 plots the results of the template-based correlation method and an
Figure 22: Experiment III: The figure shows a) the velocity estimate by the laser odometer and by the visual odometer and b) the velocity error.

pyramidal Lucas-Kanade algorithm (Bouguet et al., 1999) for tracking large displacements on different surfaces. On grass and gravel, the input features for the optical flow algorithm are extracted by the Harris corners (Harris and Stephens, 1988). On asphalt and concrete the Harris corner detector is not able to produce reliable features, hence, Canny edges (Canny, 1987) are extracted and features are sampled on these edges. The displacement is plotted as the median (to remove outliers) of the magnitude of the optical flow vector lengths. On grass and gravel, which are more feature-rich surfaces, the pyramidal Lucas-Kanade can only track displacements up to $\sim 15$ pixels robustly. This number is even lower on concrete and asphalt flooring. On the contrary, the template-based correlation algorithm has no such limitation and can track patches as long as they are visible.

Considering dense optical flow, we also perform experiments on our dataset using an inverse compositional implementation of the Lucas-Kanade algorithm (Baker and Matthews, 2004). The results indicated that, for a template of size $200 \times 200$, on average more than 90% overlap between the template region in two consecutive frames was necessary for convergence. This corresponds to a maximum velocity of approximately $0.23 \text{ m/s}$. One reason that contributes to this large overlap demand is that the whole image is textured to some extent and that the template corresponds to a subset of the image. Hence, unless the initial condition is
Figure 23: Estimation of displacement on different surfaces using template-based correlation (solid line) and pyramidal Lucas-Kanade optical flow algorithm (dashed line). a) Asphalt, b) Concrete, c) Grass, and d) Gravel. We can observe that the optical flow can only track changes up to \( \sim 15 \) pixels on these surfaces and at these high velocities whereas the correlation easily tracks these displacements, for the same camera setup.
very close to the true solution, Lucas-Kanade falls in local minima in these type of images, diverging from the true solution.

6.4 Practical Aspects

In this section we discuss practical aspects of shadows and motion blur, which can affect the system performance.

6.4.1 Shadows

As observed in Experiments I and II, shadows can be of concern for vision-based motion estimation. In the case of a downward looking camera, there are different sources of shadows:

**Shadows from static objects** can increase the dynamic range of the image to more than what can be captured by the camera. In these scenarios it is not possible to capture the detail in both the shadow and the highlight region.

**Shadows from dynamic objects** have the same negative effect as shadows from static objects but in addition they are moving and can cause erroneous displacement estimation if the shadow edge falls inside the template window.

**Shadows from the vehicle**, are moving with a relative velocity close to zero. If these shadows fall inside the template window, they can cause the template matching to estimate zero displacement.

In all shadow/sun scenarios the lighting difference, i.e. the contrast, between the shade and sun can exceed the dynamic range of standard digital cameras of five f-stops. The camera exposure can be adjusted for either shadow or bright regions using masking (Nourani-Vatani and Roberts, 2007) or similar approaches. This approach only helps if the velocity is very low or the shadow/sun separation is at the edge of the image. At higher velocities the template window falls in the sunny part, whereas the corresponding match in the next frame is in the shade, or vice versa. In this case, the correlation is likely to fail as either the bright areas
can be overexposed or the dark areas underexposed.

If the image contrast is below the dynamic range of the camera, shadow removal algorithms can be employed. Another alternative is to only perform shadow edge detection and ensure that neither the template nor the corresponding match fall on this edge.

A solution which we have found to work most robustly and which is computationally free is to mount the camera under the front bumper which is a less exposed location. In addition we employ a screen which ensures that the camera always operates in the shade. Such a mechanical solution, together with on-board lighting, can ensure that the system operates robustly day and night.

6.4.2 Motion Blur

Another important aspect when traversing at high velocities is motion blur. The best way to deal with it without using motion blur compensation algorithms is to ensure that the camera shutter speed is kept sufficiently high. However, the camera lens must also be set to a medium to small iris opening (above f3.5) to have enough depth-of-field to ensure a sharp image. This results in a quick drop in exposure as the shutter speed is increased.

As reported in Experiment III, we use a shutter time of $1/2000$ s, where we drive at velocities above 25 km/h. Even though the experiment is carried out in bright lighting condition, it is not possible to decrease the shutter time any further with the iris set at f4.0, as the image exposure becomes too low. To achieve even higher velocities, on-board lighting can be employed so faster shutter speeds and higher f-stops can be used.

7 Discussion

We have presented a full framework for vision-based odometry for ground vehicles. The method uses a single downward facing camera to estimate the motion by analyzing the image shift from frame to frame. The image shift is estimated by selecting a template in one
frame and finding the corresponding match in the next frame using linear correlation.

We propose the use of several template candidates to achieve higher correlation. Among these candidates, one is selected based on a quality measure. For the quality measure, we investigated the use of auto-correlation, variance, median absolute deviation, and entropy of the template image. The results, in terms of computation time and correlation performance, indicate that the entropy is the most appropriate metric among these.

The performance of the system is affected by several different variables, such as template size and image noise. To better understand how each of these variables effect the system, we present a statistical analysis that determines the theoretical detection error rate for template matching as a function of these variables. We compared many different template configurations and evaluated the performance of the template matching in several environments, such as concrete, asphalt, gravel, and grass surfaces. As expected, the experiments have shown that an increase in the number of templates improves the correlation quality. For optimal performance, for a given computational budget, as many templates as possible should be evaluated according to the TQM. Similarly, large templates also result in better correlation performance. This, however, comes at the expense of reduced allowed displacement and vehicle velocity.

We have also demonstrated how a forward linear filter can be used to predict the location of the future correlation location. This prediction is used to shift the template to allow for larger displacements and higher velocities. Furthermore, the prediction is used to limit the search area. This brings along two improvements; Firstly, the correlation can be performed with a twenty-fold speed up. Secondly, the probability of false positive matches is reduced.

We have presented odometry results from various outdoor field trials using two different vehicles. The 3D Euclidean distance odometry error is \( \sim 5\% \) of the distance traversed and the average heading error is \( 0.05^\circ/\text{m} \). The reported errors makes this module suitable as input for a global localization system, such as SLAM. As an example, the wheel-based odometry on the HMC increases exponentially and after 100 m traversal has an average
distance and heading error of 17 m and 22°, respectively. Still, despite these larger errors (compared to our visual odometer), the wheel odometer has been successfully applied as input to the global localizer used in this vehicle (Roberts et al., 2007).

The presented results include maximum vehicle velocities of $\sim 8 \text{ m/s}$. This is sufficient for most autonomous operation of large vehicles in industrial settings but can be a limitation in other scenarios. This limitation is due to the maximum frame rate of the camera. Operating at vehicle velocities of 50 km/h will require a camera frame rate of more than 70 fps assuming a mount height of 0.4 m. In such scenarios, a Lucas-Kanade (or other gradient-based) tracker, would be impractical. This is due to the necessity of significant overlap between the template images from frame to frame for convergence, which severely limits the vehicle velocity.

The intended usage for this odometry module is on large scale car-like vehicles in outdoor environments. One important assumption of the presented method is that the vehicle employs Ackerman steering model. Although cars do not follow a pure Ackerman steering model, this assumption still yields good results in our application. For other steering models, such as that employed by skid steered vehicles, where the vehicle has more than 2 degrees of freedom, the vehicle motion cannot be estimated using a single camera. Bonarini et al. (2004) and Lee (2004) present different configurations in which two optical mice can be used to derive the motion of any ground vehicle moving in the plane. These configurations are directly applicable to our odometry system.

In outdoor scenarios the lighting is uncontrolled and may affect the system performance. In particular, shadows causes our correlation method to fail. To overcome this, possible solutions include the usage of a shadow detection/removal algorithm or mounting the camera at a more lighting invariant position, such as under the vehicle. For this reason, development of a measure for the system to self-estimate its performance is important as the template quality measure and the correlation performance score alone cannot be used to determine the performance of the odometry module.

The next research step is to integrate this visual odometry module with other localization
systems on the vehicles and apply it as an aiding sensor for higher level simultaneous localization and matching modules.

8 Appendix 1

This appendix derives the result presented in Equation (33). From Equation (32), the variance of $C_c$ is given by

$$
\sigma^2_{C_c} = \mathbb{E}\left\{C_c^2\right\} - \mu^2_{C_c}
$$

$$
= \mathbb{E}\left\{\left[\frac{1}{N} \sum_{i=1}^{N} x^2(i) + x(i)z(i)\right]^2\right\}
$$

$$
= \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \mathbb{E}\left\{\left[x^2(i) + x(i)z(i)\right]\left[x^2(j) + x(j)z(j)\right]\right\} - \mu^2_{C_c}
$$

In order to work with zero-mean random variables, let $p(i) = x(i) - \mu_x$. Therefore,

$$
\sigma^2_{C_c} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \mathbb{E}\left\{\left[(p(i) + \mu_x)^2 + (p(i) + \mu_x)z(i)\right]\left[(p(j) + \mu_x)^2 + (p(j) + \mu_x)z(j)\right]\right\} - \mu^2_{C_c}
$$

$$
= \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \mathbb{E}\left\{p^2(i)p^2(j)\right\} + \mathbb{E}\left\{p^2(i)p(j)\mu_x\right\} + \mathbb{E}\left\{p^2(i)\mu^2_x\right\}
$$

$$
+ \mathbb{E}\left\{p(i)\mu_xp^2(j)\right\} + \mathbb{E}\left\{4p(i)\mu_xp(j)\mu_x\right\}
$$

$$
+ \mathbb{E}\left\{[p^2(i) + p(i)\mu_x]z(i)[p^2(j) + p(j)\mu_x]z(j)\right\}
$$

$$
+ \mathbb{E}\left\{\mu_xp^3(j)\right\} + \mathbb{E}\left\{\mu^2_xp(i)\right\} + \mathbb{E}\left\{\mu^2_xp^2(j)\right\} + \mathbb{E}\left\{\mu^4_x\right\}
$$

$$
+ \mathbb{E}\left\{\mu_x[p(i) + \mu_x][p(j) + \mu_x]z(i)z(j)\right\}
$$

$$
- \mu^2_{C_c}
$$

The first term in the equation above can be solved by applying the Gaussian moments factorization property for real valued random variables, which states that (Manolakis et al., 2000):

$$
\mathbb{E}\{x^2(m)x^2(n)\} = \mathbb{E}\{x^2(m)\}\mathbb{E}\{x^2(n)\} + 2\mathbb{E}\{x(m)x(n)\}^2
$$

(44)
Equation (43) can then be written as:

\[
\sigma^2_{C_c} = \sigma_x^4 + \frac{1}{N} 2 \sigma_x^4 + \sigma_x^4 \mu_x^2 + \frac{1}{N} 4 \sigma_x^2 \mu_x^2 + \frac{1}{N} \sigma_x^2 \sigma_x^2 + \sigma_x^2 + \mu_x^4 + \frac{1}{N} \mu_x^2 \sigma_x^2 - \left( \frac{\mu_x^2}{(\sigma_x^2 + \mu_x^2)^2} \right)
\]

Finally, the variance \( \sigma^2_{C_c} \) can be expressed as

\[
\sigma^2_{C_c} = 2 \sigma_x^4 + 4 \sigma_x^2 \mu_x^2 + \sigma_x^2 \sigma_x^2 + \mu_x^2 \sigma_x^2
\]

which is the result given in Equation (33).

9 Appendix 2

This appendix derives the result presented in Equation (39). From (38), the probability of error \( p_e \) that a “wrong” position presents a higher detection than a “correct” position is given by

\[
p_e = \Pr(C_w > C_c)
= \Pr(C_w - C_c > 0)
= \Pr(C_c - C_w < 0)
\]

Let \( R = C_c - C_w \). The expected value \( \mu_R \) of \( R \) is

\[
\mu_R = \mu_{C_c} - \mu_{C_w}
= \sigma_x^2 + \mu_x^2 - \mu_x \mu_s
\]

The variance of the sum/difference of correlated random variables \( a \) and \( b \) is given by (Papoulis and Pillai, 2001):

\[
\text{var}(a \pm b) = \text{var}(a) + \text{var}(b) \pm 2 \text{cov}(a, b).
\]

Therefore, the variance \( \sigma^2_R \) of \( R \) is

\[
\sigma^2_R = \sigma^2_{C_c} + \sigma^2_{C_w} - 2 \text{cov}(C_c, C_w)
\]

The first two terms (I and II) are simply the results in Equations (33) and (37). The third term is given by

\[
\text{cov}(C_c, C_w) = \mathbb{E}\{[C_c - \mu_{C_c}][C_w - \mu_{C_w}]\}
= \frac{[\sigma_x^2 + \mu_x^2] \sigma_z^2}{N} + \left[ \sigma_x \mu_x + \frac{2 \sigma_x \mu_x}{N} + \mu_x^3 \right] \mu_s - \left[ \sigma_x^2 + \mu_x^2 \right] \mu_x \mu_s
\]
Finally $\sigma^2_R$ can be written as

$$\sigma^2_R = \frac{2\sigma^4_x + 4\sigma^2_x\mu^2_x + \sigma^2_x\sigma^2_z + \mu^2_x\sigma^2_z + \sigma^2_x\sigma^2_s + \mu^2_x\sigma^2_s + \sigma^2_x\sigma^2_s + \mu^2_x\sigma^2_s}{N}$$

$$- 2 \left\{ \left[ \frac{\sigma^2_x + \mu^2_x}{N} \right] \sigma^2_z + \left[ \frac{2\sigma_x\mu_x}{N} + \mu^3_x \right] \mu_s \left[ \sigma^2_x + \mu^2_x \right] \mu_x \right\}$$

(50)

Since $p_e = \Pr(R < 0)$

$$p_e = \int_{-\infty}^{0} \frac{1}{\sigma_R \sqrt{2\pi}} \exp \left( -\frac{(t - \mu_R)^2}{2\sigma^2_R} \right) dt$$

$$= \int_{-\infty}^{-\mu_R} \frac{1}{\sigma_R \sqrt{2\pi}} \exp \left( -\frac{t^2}{2\sigma^2_R} \right) dt$$

$$= \int_{\mu_R}^{\infty} \frac{1}{\sigma_R \sqrt{2\pi}} \exp \left( -\frac{t^2}{2\sigma^2_R} \right) dt$$

$$= 1 - \frac{1}{2} \text{erfc} \left( \frac{-\mu_R}{\sqrt{2\sigma^2_R}} \right)$$

(51)

where $\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt$. Equation (51) corresponds to the result presented in Equation (39).

10 Index to Multimedia Extensions

A video is available as Supporting Information in the online version of the article. The video illustrates the template matching candidates selection, the prediction, and the corresponding area found in the next frame, according to the correlation metric. In parallel, a plot with the estimated positions and velocities is also presented.

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References


