Decomposition of the Complete Hypergraph into Delta-Systems II

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I. INTRODUCTION

In this paper, we characterize the values of $n$ for which $K_n^c$ can be decomposed into $\Delta(1, 3, c)$’s, in the cases $c = 4, 5,$ and $6$. The necessary conditions (1.1) and (1.2) of [2] take here the following form:

If $K_n^4 \to \Delta(1, 3, 4)$ then $n \geq 9$ and either $n$ is even or $n = 1, 9, \text{ or } 17 \pmod{24}$.

If $K_n^5 \to \Delta(1, 3, 5)$ then $n \geq 11$ and $n = 0, 1, \text{ or } 2 \pmod{5}$.

If $K_n^6 \to \Delta(1, 3, 6)$ then $n \geq 18$ and $n = 0, 1, 2, 9, 10, 18, 20, 28, \text{ or } 29 \pmod{36}$.

We shall prove that those conditions are also sufficient except for $n = 9$, $c = 4$.

THEOREM 1.1. $K_n^4 \to \Delta(1, 3, 4)$ iff $n \geq 10$ and either $n$ is even or $n = 1, 9, \text{ or } 17 \pmod{24}$.

Proof. This result will follow from Propositions 3.1 to 3.4.

THEOREM 1.2. $K_n^5 \to \Delta(1, 3, 5)$ iff $n \geq 11$ and $n = 0, 1, \text{ or } 2 \pmod{5}$.

Proof. This result will follow from Propositions 4.1 to 4.6.

THEOREM 1.3. $K_n^6 \to \Delta(1, 3, 6)$ iff $n \geq 18$ and $n = 0, 1, 2, 9, 10, 18, 20, 28, \text{ or } 29 \pmod{36}$.

Proof. This result will follow from Propositions 5.1 to 5.7. However, we also shall give some new general results which will prove useful in those particular cases.