Modeling and Control of Magnetic Levitation System via Fuzzy Logic Controller

Kashif Ishaque¹, Yasir Saleem², S.S Abdullah¹, M. Amjad¹, Munaf Rashid¹ and Suhail Kazi¹
¹Universiti Teknologi Malaysia.
Johor Bahru
MALAYSIA.

²University of Engineering & Technology Lahore.
Lahore
PAKISTAN.
kashif@fkegraduate.utm.my, yasir@uet.edu.pk, shahrum@ic.utm.my

Abstract

Fuzzy logic controller (FLC) is an attractive alternative to existing classical or modern controllers for designing the challenging Non-linear control systems. It does not require any system modeling or complex mathematical equations governing the relationship between inputs and outputs. Fuzzy rules are very easy to learn and use, even by non-experts. It typically takes only a few rules to describe systems that may require several lines of conventional software code, which reduces the design complexity. By considering these advantages, this paper presents the design and analysis of a FLC controller for the magnetic levitation system. Additionally, a classical PID controller is also designed to compare the performance of both types of controllers. Results reveal that FLC found to give better transient and steady state results compare to the classical PID.

1. Introduction

Magnetic suspension systems, as a kind of contact-free and wear-free suspension, play an important role in precision motion control and have been studied and applied in many research fields, such as mechanical engineering, automatic control and biomedical engineering [1]-[5]. In order to achieve an overall better performance, trade-offs have to be taken among precision, rising time, overshoot and stable-state deviation in different magnetic suspension systems. Growing attention has been paid to its optimization and control in past several decades and the control algorithms have been developed from typical linear control strategies to intelligent control methods [6]-[14].

Although PID control is a proficient technique for the handling of non-linear systems but modeling these systems is often troublesome and sometimes impossible using the laws of physics. Therefore, using a classical controller is not suitable for nonlinear control application [15]. Alternatively, Fuzzy Logic Control are useful when the processes are too complex for analysis by conventional quantitative techniques or when the available sources of information are interpreted qualitatively, inexact, or uncertainly [16]. It does not require any system modeling or complex mathematical equations governing the relationship between inputs and outputs. Fuzzy rules are very easy to learn and use, even by non-experts. It typically takes only a few rules to describe systems that may require several lines of conventional software code, which reduces the design complexity [17]. PID control requires the model of the system for the determination of the parameters of PID controller using control theory and finally the development of an algorithm for the controller. Whereas in case of fuzzy logic the system behavior is characterized using human knowledge which directly leads to the design of control algorithm on the basis of fuzzy rules. These rules are in terms of the relationship of inputs to their corresponding outputs, and precisely determine the controller parameters. Any adjustment or debugging only requires modification in these fuzzy rules instead of the redesigning the controller. Hence control technique based on fuzzy logic not only simplifies the design, but also reduces the monotonous task of solving complex mathematical equations for nonlinear systems. As a result, fuzzy logic controller delivers a better performance in cases where the conventional controller does not cope well with the non-linearity of a process under control [18]. In fuzzy control we focus on gaining an intuitive understanding of how to best control the process, then we load this information directly into the fuzzy controller.

This paper describes an implementation of FLC. The design procedure utilizes MATLAB® Fuzzy Logic toolbox and is implemented using SIMULINK® version 7.1. One of the great advantages of the Fuzzy Logic toolbox is the ability to take fuzzy systems directly into SIMULINK® and test them out in a simulation environment [19]. Although the authors have no experience of controlling the ball and beam system before, a somewhat better controller is designed based on the simple fuzzy rules. The triangular membership functions are used, and the centroid method is used for...
defuzzification. A classical PID controller has been also
designed in MATLAB® for the system studied here,
which will be used as a comparison to the FLC designed.
The remainder of the paper is organized as follows. In
Section 2, we have presented a mathematical modeling of
the magnetic levitation system. Section 3 discusses a
design scheme of FLC controller In Section 4, results are
discussed and a comparison has been made between PID
and FLC based on simulation results. Finally in section 5,
we have concluded all the discussion presented in our
paper.

2. Modeling of Magnetic levitation system
The magnetic levitation system is a magnetic ball
suspension system which is used to levitate a steel ball on
air by the electromagnetic force generated by an
electromagnet.

Fig. 1: Magnetic Levitation system

Figure 1 shows the magnetic ball suspension system
consists of an electromagnet, a ball rest, a ball position
sensor, and a steel ball. The magnetic ball suspension
system can be categorized into two systems: a mechanical
system and an electrical system. The ball position in the
mechanical system can be controlled by adjusting the
current through the electromagnet where the current
through the electromagnet in the electrical system can be
controlled by applying controlled voltage across the
electromagnet terminals.

From Ampere’s circuit law and faraday’s inductive
law, the magnitude of the force \( f(x,i) \) exerted across an
air gap \( h \) by an electromagnet through which current \( i \)
flows can be described as:

\[
f(h,i) = -\frac{i^2}{2} \frac{dL(h)}{dh}
\]  

The total inductance \( L \) is a function of the distance
and given by

\[
L(h) = L_1 + \frac{L_0 H_0}{h}
\]  

Where \( L_i \) is the inductance of the electromagnetic
(coil) in the absence of the levitated object, \( L_o \) is the
additional inductance contributed by its presence, and \( X_o \)
is the equilibrium position. The parameters are
determined by the geometry and construction of the
electromagnet, and can be determined experimentally.
Substituting equation (2) into (1) yields

\[
f = \frac{L_0 X_o}{2} \left( \frac{i}{h} \right)^2 = \beta \left( \frac{i}{h} \right)^2
\]  

\[
\beta = \frac{L_0 H_0}{2}
\]  

\[
f = f_0 + \left( \frac{\partial f}{\partial L_0} \right) \Delta L_0 + \left( \frac{\partial f}{\partial H_0} \right) \Delta H_0 + \left( \frac{\partial f}{\partial i} \right) \Delta i + \left( \frac{\partial f}{\partial h} \right) \Delta h
\]  

Eliminating higher order terms give

\[
f = f_0 + \left( \frac{\partial f}{\partial i} \right) \Delta i + \left( \frac{\partial f}{\partial h} \right) \Delta h
\]  

Evaluating equation (6) using (4) and (5) yields

\[
f = \beta \left( \frac{I_o}{H_0} \right) + \left( \frac{2 \beta I_o}{H_0^2} \right) \Delta i - \left( \frac{2 \beta I_o^2}{H_0^3} \right) \Delta h
\]  

Where, \( I_o \) is the equilibrium value. At equilibrium,
the weight of the object is suspended by the
electromagnet force, \( f_i \). The force required to maintain
equilibrium, \( f_i \), is

\[
f_i = f - f_0
\]  

Combining equations (7) and (8) gives

\[
f_i = \left( \frac{2 \beta I_o}{H_0^2} \right) \Delta i - \left( \frac{2 \beta I_o^2}{H_0^3} \right) \Delta h
\]  

The voltage equation of the electromagnetic coil is
given in equation 1.

\[
V = iR + L(h) \frac{di}{dt}
\]  

Assuming the suspended object remains close to its
equilibrium position, \( h=h_0 \), and therefore

\[
L(h) = \frac{L_0 H_0}{h}
\]  

Also assuming that \( L_i >> L_o \) equation (10) can be
simplified as

\[
V = iR + L_i \frac{di}{dt}
\]  

The principal equation for the suspended object
comes by applying Newton’s second law of motion. For
this one degree of freedom system, a force balance taken at the centre of gravity of the object yields

\[ M \frac{d^2 h}{dt^2} = -f_1 \quad (13) \]

The sensor can be modeled as a gain element,

\[ V_s = K_s h \quad (14) \]

Where \( V_s \) is the sensor output voltage and \( K_s \) is an experimental gain between the object’s position and the output voltage.

\[-M \frac{d^2 h}{dt^2} = \left( \frac{2\beta I_0}{H_0^2} \right) i - \left( \frac{2\beta I_0^2}{H_0^3} \right) h \quad (15)\]

\[-M \frac{d^2 h}{dt^2} = K_i i - K_2 h \quad (16)\]

Where, \( K_1 = \frac{2\beta I_0}{H_0^2} \) and \( K_2 = \frac{2\beta I_0^2}{H_0^3} \)

The Laplace transform of above equation obtained as:

\[ (Ms^2 - K_2) H(s) = -K_i I(s) \quad (17) \]

The Laplace transform of equation (28) is

\[ I(s) = \frac{V(s)}{L_1 s + R} \quad (18) \]

The overall transfer function of the Maglev system is obtained as:

\[ G(s) = \frac{V_i(s)}{V(s)} = \frac{-K_s K_1}{s^2 + \frac{R}{L_1} \left( s^2 - \frac{K_2}{M} \right)} \quad (19) \]

Table 1 summarizes the variables and parameters use in this problem. Here the problem is to maintain the ball at its operating point (position) of 0.03 meters from the coil.

Table 1. Parameters of the magnetic Levitation system.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_0 )</td>
<td>equilibrium height of ball (m)</td>
<td>0.03</td>
</tr>
<tr>
<td>( M )</td>
<td>Mass of ball bearing (kg)</td>
<td>0.225</td>
</tr>
<tr>
<td>( R )</td>
<td>Resistance (( \Omega ))</td>
<td>2.48</td>
</tr>
<tr>
<td>( L_1 )</td>
<td>Inductor (H)</td>
<td>0.18</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Constant related to magnetic force (Nm²/Å²)</td>
<td>( 7.93 \times 10^5 )</td>
</tr>
<tr>
<td>( I_0 )</td>
<td>Equilibrium current of the coil (A)</td>
<td>5</td>
</tr>
<tr>
<td>( K_s )</td>
<td>Sensor gain factor (V/m)</td>
<td>200</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>constant (N/A)</td>
<td>0.882</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>constant (N/m)</td>
<td>147</td>
</tr>
</tbody>
</table>

3. Design Methodology of FLC

For the fuzzification process, the triangular membership functions are used for both input and output with the universe of discourse as follows:

- \( e = [-1,1] \)
- \( \Delta e = [-1,1] \)
- \( V = [-2, +2] \)

These values were obtained by observing the corresponding values of \( e, \Delta e \) and control input \( u \) in the conventional system using the classical PID controller. Figures 3 and 4 show the membership functions used for the input and output variables.
### 3.1 Inference mechanism

The Inference Mechanism provides the mechanism for invoking or referring to the rule base such that the appropriate rules are fired. Using trial and error approach, the best inference mechanism to use in this case seems to be the min-max method.

### 3.2 Rule base

The rule base used in the design is given in Table 1. The rule base follows closely the rules that were suggested in [16].

### 3.3 Defuzzification

The defuzzification technique used was also found using trial and error. The defuzzification technique in this case that gave the least integral square error was the Centre of Gravity approach.

### 3.4 Scaling factor

Here the controller was tuned using input and output scaling factors to improve the performance of the system. The output scaling factor was needed to ensure that the control input $u$ to the ball and beam is enough to move the beam accordingly in order to maintain its position. The scaling factors that were used are listed in Table 2.

### Table 3 Scaling Factors

<table>
<thead>
<tr>
<th>Scaling factors</th>
<th>Value used</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e$</td>
<td>1</td>
</tr>
<tr>
<td>$\Delta e$</td>
<td>0.05</td>
</tr>
<tr>
<td>$u$</td>
<td>2.5</td>
</tr>
</tbody>
</table>

### 4. Results and Discussion

Fig. 5 and 6 show the coil current and control input (voltage) for both types of controllers. The result shows that the controlled electromagnet current can stabilize the disturbances that otherwise, would cause the ball to either fall or attach itself to the electromagnet. In the steady state of Fig. 6, the electromagnet current with very few vibrations and the suspended position with very small errors coincide with the designed balance conditions: 5 A of electromagnet current producing a magnetic force which just counteracts the 0.225 kg weight of the ball suspended 0.03 m under the electromagnet. Fig. 7 shows the control surface of the FLC. It can be noted that, FLC has a linear control surface. This is due to the equal widths of membership function for input and output [20]–[21].

From the simulation results shown in Fig. 7, it can be observed that the fuzzy controller has better transient response than the classical controller. The overshoot of the FLC controller is 6% compared to 18% in the classical case. Furthermore, FLC has a faster transient response; it reaches to steady state in 0.3 second to that of 1 sec in PID. In comparison to the steady state value, both controllers satisfactorily attain the steady state value of 0.03 meter.
5. Conclusion

An attempt to control the position of a steel ball in a magnetic levitation system using fuzzy logic has been proposed. From the simulation results, it has been shown that the fuzzy controller can stabilize the system efficiently. Also, the performance during the transient period of the fuzzy system is better in the sense that less overshoot was obtained. Moreover, the fuzzy controller provides a zero steady state error.

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References


