Robust segmentation of cerebral arterial segments by a sequential Monte Carlo method: Particle filtering

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Abstract
In this paper a method to extract cerebral arterial segments from CT angiography (CTA) is proposed. The segmentation of cerebral arteries in CTA is a challenging task mainly due to bone contact and vein contamination. The proposed method considers a vessel segment as an ellipse travelling in three-dimensional (3D) space and segments it out by tracking the ellipse in spatial sequence. A particle filter is employed as the main framework for tracking and is equipped with adaptive properties to both bone contact and vein contamination. The proposed tracking method is evaluated by the experiments on both synthetic and actual data. A variety of vessels were synthesized to assess the sensitivity to the axis curvature change, obscure boundaries, and noise. The experimental results showed that the proposed method is also insensitive to parameter settings and requires less user intervention than the conventional vessel tracking methods, which proves its improved robustness.

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1. Introduction

Well-timed prognosis of various diseases on cerebral arteries such as a subarachnoid hemorrhage (SAH) necessitates cerebrovascular system to be segmented out. However, cerebral arteries build up a very complex structure and pass through the cranial bone structure. Therefore, they are more difficult to extract than any other vessel structures such as cardiovascular, pulmonary vascular or hepatic vascular system in a CTA volume.

There have been a number of literatures on vessel segmentation as surveyed in [1]. The most basic method is the combination of thresholding and region-growing [2]. However, it has limitations when applied to cerebral arteries in a CTA volume due to two complications; the overlapping Hounsfield unit (HU) value distributions of bone and vessels and the close contact between them. To deal with these problems, the proposed method takes into account anatomic priors that arteries are smoothly varying structures with elliptical cross-sections. The assumption of smooth variation along the axis suggests the tracking-based segmentation as a solution. Since tracking-based approaches apply local operators on a focus inside the vessel, they can be more robust than other approaches to detect objects or features in the whole image. Furthermore, tracking methods easily allow quantitative measurements of the vessel parameters. On the other hand, tracking needs initial conditions and requires re-initialization if it gets astray or makes a turnover.

Wink et al. [3] extracted the abdominal aorta based on tracking of the center line. The abdominal aorta is thick and...
mostly straight, so it is much simpler to extract than cerebral arteries. Therefore, when applied to cerebral arteries in the CTA, their work has limitations, especially when the boundaries are obscured by vein contamination.

Shim et al. [4] have partitioned a CTA volume into the lower and upper sub-volumes and applied a separate algorithm to each one, i.e., adaptive tracking method to the lower one and threshold-based region growing to the upper one. As a consequence, it has method-inconsistency in the whole volume and the problem of global thresholds in the upper sub-volume.

The work of Florin et al. [5] is the most recent and the most related to our work. They applied the particle filter (PF) to track an elliptical cross-section for segmentation of coronaries. Their measurement model used the assumption that coronary arteries are brighter than the background, which cannot be held true for cerebral arteries of the CTA because of bone structure. Furthermore, the distance between the prediction and the actual observation does not take into account the normal vector of the cross-section but only the intensity distribution and the mean difference, which may often disturb the tracking.

The proposed work also employs the PF which is the state-of-the-art in video tracking, and defines the system and measurement models adaptively to cerebral arteries. Especially in the update stage, the border points on the cross-section perpendicular to the axis are detected as the observations $z_k$. The thresholds for their detection are chosen properly according to the surroundings of the arteries. In the while, the weight associated with each particle is updated by the exponential sum of distances of the points in $z_k$ and the 3D ellipse represented by each particle. In this way, the proposed tracking method updates the normal vector of the cross-section at every discrete time and lowers the chance of getting astray or making a turnover.

2. Description of the method

First of all, some basic notions on particle filtering are briefly reviewed. The detailed description is available in [7]. The PF is a sequential Monte Carlo method to solve the non-linear Bayesian state estimation problem, which can be expressed by the pseudo code in Algorithm 1. For each particle $x_k^i$, the pdf $p(x_k^i|z_k)$ is obtained by the two stages of prediction and update. Then the PF estimates the current state as (4) and applies the resampling stage to the whole set of the particles.

Algorithm 1. The SIR PF at the discrete time $k$

- FOR $i = 1 : N_k$
  - Prediction : Draw $x_k^i \sim p(x_k^i|x_{k-1})$
  - Update : $q_k^i = q_k^i \cdot p(z_k^i|x_k^i)$
- END FOR
- Estimation:
  $$\hat{x}_k = \text{mean} \left( \sum_{i=1}^{N_k} q_k^i \delta(x_k - x_k^i) \right)$$
- Resampling : the same way as Algorithm 2 in [7]

2.1. The system model of the proposed method

At the current time $k$, the elliptical cross-section of a vessel segment can be represented by a $9 \times 1$ state vector which is divided into three $3 \times 1$ vectors as:

$$x_k = [c_{xk} \ c_{yk} \ c_{zk} \ n_{xk} \ n_{yk} \ n_{zk} \ a_k \ b_k \ \beta_k]^T$$

where $c_k = [c_{xk} \ c_{yk} \ c_{zk}]^T$ and $n_k = [n_{xk} \ n_{yk} \ n_{zk}]^T$ represent the 3D coordinates of the ellipse center and the unit vector perpendicular to the ellipse, respectively. $e_k = [a_k \ b_k \ \beta_k]^T$ specifies the elliptical shape with the two lengths of the semi-axes, and the rotation angle in the normal plane represented by $n_k$.

If the elliptical cross-section marches with a constant unit velocity, the state transition can be modelled as the addition of the unit normal vector to the center of the previous state. The transition is corrupted by the system noise $v_k$ which can be split into three independent noise vectors of $v_{c_k}$, $v_{n_k}$ and $v_{e_k}$. Then the system transition of (1) is represented by three $3 \times 1$ vector equations as:

$$c_k = c_{k-1} + n_{k-1} + v_{c_k},$$
$$n_k = f_{n_k}(n_{k-1}, v_{n_k}),$$
$$e_k = e_{k-1} + v_{e_k}.$$  

The transitions for $c_k$ and $e_k$ of (6) and (8) are straightforward. All the six components of the noises, $v_{c_k} = [v_{c_{xk}} \ v_{c_{yk}} \ v_{c_{zk}}]^T$ and $v_{e_k} = [v_{e_{xk}} \ v_{e_{yk}} \ v_{e_{zk}}]^T$, are assumed to be independent of each other and also independent of the time index $k$. Then, each of them is modelled as a Gaussian distribution with zero mean and constant standard deviation denoted by $\sigma_{c_{xk}}, \sigma_{c_{yk}}, \sigma_{c_{zk}}, \sigma_{c_{xk}}, \sigma_{c_{yk}}, \sigma_{c_{zk}}$, respectively.

For the transition of the normal vector $n_k$ corrupted by the random noise $v_{n_k}$ as (7), more sophisticated consideration is
needed. Let us consider the distribution of the random noise concentrated on the north pole (0, 0, 1) as illustrated in Fig. 1(a). Then, \( v_{n_b} \) is represented by the polar angle \( v_{\phi_b} \) and the azimuth \( v_{\theta_b} \) in Fig. 1(b) as

\[
v_{n_b} = \begin{bmatrix} \cos v_{\phi_b} \sin v_{\theta_b} \\ \sin v_{\phi_b} \sin v_{\theta_b} \\ \cos v_{\theta_b} \end{bmatrix}.
\]  

(9)

The rotational randomness of \( v_{n_b} \) is uniform through \([0, 2\pi]\) and the samples become sparser as they are farther away from the north pole, meaning

\[
v_{\phi_b} \sim U[0, 2\pi], \quad v_{\theta_b} \sim N(0, \sigma_b^2).
\]  

(10)

The unit normal vector \( n_k \) of (7) is randomized according to (10) around the previous normal vector \( n_{k-1} \) whose polar angle and azimuth are \( \phi_{k-1} \) and \( \theta_{k-1} \). Thus, \( n_k \) is obtained by rotating the random unit vector \( v_{n_b} \) of (9) around the y-axis by \( \phi_{k-1} \) and then rotating around the z-axis by \( \theta_{k-1} \). If these two rotations are represented by \( R_{z,\phi_{k-1}} \) and \( R_{y,\theta_{k-1}} \), then

\[
n_k = R_{z,\phi_{k-1}} R_{y,\theta_{k-1}} v_{n_b}.
\]  

(11)

2.2. The measurement model of the proposed method

The most dominant factor affecting the FF’s performance is getting trustworthy measurements. In this paper the measurement is set as the border points detected on the cross-section perpendicular to the central axis. This cross-section is determined directly from the CT volume data as follows.

As seen in Fig. 1(c), vessels are assumed to be cylinders, and the direction of the vessel axis and the area of its perpendicular cross-section are denoted by \( n_0 \) and \( S_0 \), respectively. Then, an arbitrary cross-section with the angle of \( \phi = 0 \) has its area \( S(\phi) = S_0 \cos \phi \) where \( 0 \leq \phi \leq \pi/2 \). Consequently the perpendicular cross-section is determined by minimizing \( S(\phi) \) which leads to \( \phi = 0 \). If the polar angle \( \phi \) and the azimuth \( \theta \) are sampled uniformly in the intervals \([0, \pi/2]\) and \([0, 2\pi]\) as \( \phi_i = (\pi/2)(i/N), i = 0, \ldots, N \) and \( \theta_i = 2\pi(j/M), j = 0, \ldots, M - 1 \), respectively, then the perpendicular cross-section is determined by having the minimum cross-sectional area over all the pairs of \( (\phi_i, \theta_j) \).

Next, from the candidate center, rays are cast along the equally sampled directions as shown in Fig. 2 and the radial gradient along each ray is used to detect the border point. In most cases cerebral arteries are surrounded by darker soft tissues and are named as NT (normal tissue) for short. Some border points lie between artery and bone and are named as BC (bone contact), while others lie between the artery and vein contamination (VC).

In Fig. 2(a) and (b), the border points of the types NT, BC, and VC are denoted by the cyan ‘Δ’s, the magenta ‘◦’s, and the yellow ‘◦’s, respectively. Each type has the corresponding threshold such as \( t_{nt} \), \( t_{bc} \), and \( t_{vc} \). An exemplary ray is also illustrated as the black line and the white one in Fig. 2(a) and (b), respectively. In Fig. 2(c) the radial gradient along the ray is plotted from the center outwards as the BC curve with ‘◦’ for the black ray in Fig. 2(a) and the VC curve with ‘◦’ for the white ray in Fig. 2(b). The NT curve with ‘Δ’ is for a ray when the artery is surrounded by darker normal tissues, thus the border point is detected as the first negative extremum below the threshold \( t_{nt} \). However, the artery often passes through bone structure which is generally denser than the enhanced arteries. Thus the positive threshold \( t_{bc} \) is applied to find the bone boundary. Once the bone boundary is detected, the border point should be moved inwards, since there is usually a gap between bone structure and the artery. The border point is moved inwards until the gradient is greater than a lower threshold \( t_{bc} \) with \( 0 < \delta < 1 \). Finally, when the artery is near to vein contamination, the border point is also located as the first negative extremum below \( t_{bc} \). \( t_{vc} \) is usually smaller than \( t_{nt} \) to account for the smoother intensity change due to vein contamination. The way to select a threshold among \( t_{nt} \), \( t_{bc} \), and \( t_{vc} \) is summarized in Algorithm 2. \( r \) is the radial index and \( R_{max} \) is the maximum of \( r \) at the brim of the cross-section. \( Grad(r) \) is the radial gradient at \( r \). The first FOR loop looks for the border points of the types NT and BC by applying \( t_{nt} \) and \( t_{bc} \) at every radial step. If there is no border detected, then \( t_{vc} \) is applied in the second FOR loop to find the border point of the type VC. The point at radius \( R_{max} \) is taken as an outlier, if the second FOR loop fails to find a border point.

\[
\text{Fig. 1} \quad \text{Randomized transition and determination of the unit normal vector: (a) random sampling concentrated on the north pole (0,0,1), (b) } v_{n_b} \text{ identified by the polar angle } v_{\phi_b} \text{ and the azimuth } v_{\theta_b}, \text{ and (c) determination of the normal vector by minimizing the cross-sectional area } S(\phi).
\]
Fig. 2 – Detection of the border point along a ray with \( N_z = 16 \): (a) and (b) The cases where the artery is adjacent to bone structure and to vein contamination, respectively. The cyan ‘△’s, the magenta ‘▽’s, and the yellow ‘<’ denote the border points of the types NT, BC, and VC, respectively. The green ‘//H17007’ is the improper detection. (c) The plots of the radial gradients along rays for the three types.

An outlier also exists when an inadequately threshold is used as the green ‘//H17007’ in Fig. 2(a). Although \( t_{nt} \) should have been used, the notable gradient between the vein and the normal tissue caused \( t_{nt} \) to be selected. The detected border points are assumed to be on an ellipse which can be uniquely defined by at least five points [12]. Then the border points except the above outliers are set as the actual measurement and are used to quantify the likelihood of each particle. Let \( N_z, N_o, \) and \( e'_{k-1} \) be the number of the rays, the number of the outliers, and the translation of the previous ellipse \( e_{k-1} \) by the previous normal vector \( n_{k-1} \), respectively. A border point is excluded as an outlier in the computation of the distance between each particle and the measurement, if it is one of the \( N_o \) farthest points from \( e'_{k-1} \). Then, the measurement \( z_k \) is a set of \( (N_z - N_o) \) 3D points on the normal plane as \( \{p_{kj}, i = 1, \ldots, (N_z - N_o)\} \) and it indicates the actual vessel boundary. The aim of \( z_k \) is to give a higher weight to the particle \( x_{ik} \) whose ellipse \( e_i \) is more similar to the measurement. The distance between \( z_k \) and \( x'_{ik} \) is defined as

\[
d(z_k, x'_{ik}) = \frac{1}{N_z - N_o} \sum_{j=1}^{N_z - N_o} d(p_{kj}, e'_j).
\]  

(12)

where \( d(p, e) \) denotes the 3D distance between a point \( p \) and the boundary of an ellipse \( e \). Then the updated weight is represented as

\[
q'_k \propto q'_{k-1} \frac{\exp(-Kd(z_k, x'_{ik}))}{\sum_{m=1}^{N_z} \exp(-Kd(z_k, x'_{im}))}.
\]  

(13)

where \( K \) is a normalizing constant depending on the relative range of the distance. Then the particles are resampled according to Algorithm 1.

Algorithm 2. Detection of a border point selecting among \( t_{nt} \), \( t_{bc} \) and \( t_{vc} \)

- FOR \( r = 1 : R_{max} \) % the first loop to use \( t_{nt} \) or \( t_{bc} \)
  - IF \( |\text{Grad}(r)| = r(r-1) \) \% the radial gradient
    - END FOR
  
  - ELSE IF \( |\text{Grad}(r)| < t_{nt} \)
    - THEN the negative extremum \( \Rightarrow \) NT type border
  - ELSE IF \( |\text{Grad}(r)| > t_{bc} \)
    - THEN the most inward point with \( |\text{Grad}(r)| > \delta_{bc} \Rightarrow \) BC type border
  
- END FOR

- FOR \( r = 1 : R_{max} \) % the second loop to use \( t_{vc} \)
IF \( \text{Grad}(r) < t_{vc} \) THEN the negative extremum \( \Rightarrow \) VC type border
\* END FOR

2.3. Checking end conditions and visualization of the vessel segment

Tracking of a vessel segment terminate if it gets astray out of the arteries. The number of tracking steps is also limited to prevent endless wandering. On the termination of the tracking the accumulated cross-sections are visualized by a volume rendering tool. The adjacent accumulated elliptical cross-sections are connected by triangular surface meshes of Delaunay triangulation [9] and then voxelized to a binary volume. The voxelization is performed using DDA (digital differential analyzer) algorithm [10] and the morphological operation of region filling. Fig. 3 shows the visualization results of an arterial segment composed of the left internal carotid artery (ICA) and the left middle cerebral artery (MCA). In spite of the varying diameter and the overlapping intensity distributions, the proposed tracking scheme has extracted the artery robustly and effectively.

2.4. Generation of synthetic vessel segments

The vessel synthesis model has a circular bar-like cross-section convolved with a Gaussian kernel as introduced in [11]. Thus, the intensity distribution of the cross-section is expressed as

\[
I_r(x, y) = f_r(x, y) \times G_{\sigma_p}(x, y),
\]  

(14)

where \( r \) is the radius of the vessel and \( G_{\sigma_p} \) is the Gaussian kernel with the standard deviation of \( \sigma_p \). The circular bar-like function \( f_r \) is defined by

\[
f_r(x, y) = \begin{cases} 
I_0 & \text{if } x^2 + y^2 \leq r^2 \\
0 & \text{otherwise}
\end{cases},
\]  

(15)

and is illustrated in Fig. 4(a) with \( I_0 = 300 \) and \( r = 7.3 \). Fig. 4(d) is the convolution of (a) with the Gaussian kernel of \( \sigma_p = 1.0 \).

Two kinds of vessel segments are generated to test the sensitivity of tracking with respect to the axis curvature and the obscure boundary. Fig. 4(b) shows a vessel segment with lower axis curvature compared to the one in Fig. 4(e). They have 2.0 and 4.5 wavelengths in the same distance, respectively. Next, the vessel segments in Fig. 4(c) and (f) are generated to evaluate the tracking performance in the low-contrast conditions. A vessel segment of sine-waveform and another straight segment are located in a relatively larger distance between their axes in Fig. 4(c) and in a smaller distance in Fig. 4(f). The smaller distance between the axes made their boundaries overlap with each other and simulates the low-contrast conditions of the boundary.

3. Experimental results and discussion

In this section we first discuss the robustness of the proposed algorithm with respect to the parameter variations. Subsequently, a variety of experiments are performed on a set of synthetic vessel segments and 15 actual clinical datasets.

3.1. Parameter settings

The sensitivity of the proposed tracking method with respect to the thresholds \( t_{nt}, t_{bc}, \) and \( t_{vc} \) needs to be analyzed, since the proposed method heavily depends on the correct detection of border points. To this end, 10 cross-sections were sampled
at each of the cervical portion, the petrous portion, and the cavernous portion of ICA’s of 7 different patients to the total sum of 30 planes. On these planes the contour of the arteries were delineated manually. On our experiment 480 (30 × 16) rays were tested with \( N_z = 16 \). At each ray the threshold type is selected and the border point is detected according to Algorithm 2. \( t_{nt} \) and \( t_{vc} \) were decremented from 0 to -100 and \( t_{bc} \) was incremented from 0 to 100. For each type the percentage of rays that actually terminate at the outlined position is plotted as the three curves of \( H_{nt}, H_{bc}, \) and \( H_{vc} \) in Fig. 5. The values on the \( x \)-axis are inverted to positive values for \( t_{nt} \) and \( t_{vc} \) so that the three curves may be plotted simultaneously. On the interval of \( \{ t_{nt} \mid -46 \leq t_{nt} \leq -26 \} \), the \( H_{nt} \) curve with ‘⋆’ is flat above 0.94 indicating that more than 94% of border points are correctly detected insensitively to \( t_{nt} \)-change. Similarly the \( H_{bc} \) curve with ‘○’ shows that \( t_{bc} \) consistently produces correct border points more than 92% if \( t_{bc} > 58 \). The threshold \( t_{bc} \) is sensitive compared to the others, but if \( -22 \leq t_{bc} \leq -15 \), the hit ratio \( H_{vc} \) is maintained to be 84.4% without regard to small change in \( t_{vc} \). As acquisition time of CT scan becomes shorter, vein contamination has become less important and appears only in the obsolete datasets leading to less affect of \( t_{vc} \). In our experiments, we set \( t_{nt} = -40, t_{bc} = -20, \) and \( t_{vc} = 60 \) and they produce 94.2% hit ratio with 480 rays.

### 3.2. Experiments on synthesized data

The synthetic vessel segments were used to test the sensitivity of the algorithms with respect to obscure boundaries, change in axis curvature, and noise. Each volume dataset has 128 × 128 × 128 voxels and the cross-section of the vessel segment has the intensity distribution of Fig. 4(d) and (14).

First, to simulate variations in the axis curvature, vessel segments having sine-waveforms with variant wavelengths were generated. The vessel segment in Fig. 4(b) has the lowest axis curvature with 2 wavelengths in the width of 128 voxels. The number of wavelengths in the same width is incremented from 2.0 to 4.5 by 0.1, and consequently 26 vessel segments were tested. Fig. 4(e) shows the vessel segment of the highest axis curvature of 4.5 wavelengths in 128 voxels. Then, the Gaussian noise with the standard deviation of \( \sigma_n = 0, 1, \ldots, 48 \) was added to each dataset, amounting to 1274 (26 × 49) datasets. As shown in Fig. 6(a), the proposed method indicated by the ‘●’ curve exhibits the best performance except for the

![Fig. 4](image-url) Generation of the synthetic vessel segments: (a) and (d) A circular bar-like cross-section with \( I_0 = 300 \) and \( r = 7.3 \) and its convolution with a Gaussian kernel of \( \sigma_p = 1.0 \), (b) and (e) a vessel segment with smooth change and sharp change in axis curvature. The boundary of the upper vessel segment is obscured by the lower one only a little in (c) and substantially in (f).

![Fig. 5](image-url) The percentage of rays that actually terminate at the manually outlined boundary is plotted for \( t_{nt}, t_{bc}, \) and \( t_{vc} \), respectively. The values for \( t_{nt} \) and \( t_{vc} \) are inverted to positive values for simultaneous display of three curves. The proposed method of border detection is robust to the threshold values if \( -46 \leq t_{nt} \leq -26, t_{bc} > 58, \) and \( -22 \leq t_{vc} \leq -15 \).
Fig. 6 – Experiments on synthetic vessel segments: (a) To test the sensitivity to the axis curvature, the sine-waveform segment varies the axis curvature by incrementing the number of wavelengths in 128 voxels from 2.0 to 4.5 by 0.1 as in Fig. 4(b) and (e). (b) To test the sensitivity to obscure boundaries, the boundary of the sine-waveform segment is interfered by the lower straight segment as illustrated in Fig. 4(c) and (f). The interference depends on the distance between the two axes which ranges from 2.5R to 3.5R.

Fig. 7 – A vessel segment tracked by the proposed method: (a) display of the every 10th step among 475 steps, (b) enlargement of the box in (a) representing the predicted state as the blue dashed ellipse, the three most weighted particles as the cyan dotted ones, and the weighted mean of all the particles as the red solid one. The 14 small black squares indicate the detected border points and compose the measurement $z_k$, while the two crosses represent the outliers.

3.3. Experiments on actual clinical data

The actual clinical dataset is composed of 15 CT studies of different patients. They have the spatial resolution in the $xy$-directions from 0.28 to 0.36 mm and in the $z$-direction from 0.3 to 0.5 mm. By a linear interpolation in the $z$-direction, every dataset is converted into an isotropic volume which has the same spatial resolution in all the $xyz$-directions.

The aim of this experiment is to track two vessel segments with two initial seeds located in the left and right internal vessel segments of higher curvature of more than four wavelengths. For these segments the method [3] of the ‘⋆’ curve shows the most successes. This is a wrong result caused by the fact that the high curvature waveforms are too tightly close to each other. It is supported by the rising number of successes of the method [3], while every method generally decreases in the success rate, as the axis curvature becomes sharper.

Another set of vessel segments were synthesized to test the sensitivity to obscure boundaries, i.e., the low-contrast conditions. Let $R$ to be 7.3. In Fig. 4(c) and (f), the sine-waveform segment of $r = R$ and $\sigma_p = 1.3$ is to be tracked and its boundary is interfered by the lower straight segment with $r = 2R$ and $\sigma_p = 4.5$. The distance between the two axes, $d$, is decremented by 0.1R from 3.5R (Fig. 4(c)) to 2.5R (Fig. 4(f)), making 11 volume datasets. For each dataset, the Gaussian noise was also added with $\sigma_n = 0, 1, \ldots, 48$. As illustrated in Fig. 6(b), the proposed method is the most robust and the tracking result of the sine-waveform segment is not affected by the lower straight segment. It appropriately deals with the outliers resulted from obscure boundaries, by making use of the accumulated statistics of many particles.
carotid arteries (ICA’s), respectively. Consequently the total number of vessel segments is 30. As an example, Fig. 7(a) shows one vessel segment tracked by the proposed method with 475 steps, where only every 10th step is displayed. Empirically we set $\sigma_c = \sigma_y = \sigma_z = 1.0$ and $\alpha = \pi/8$.

Each step consists of several ellipses which are enlarged in Fig. 7(b). The ellipse in the blue dashed line indicates the prior state mean ($p(x_k|x_{k-1})$) and the ones in the cyan dotted lines represent the three particles having the three largest weights. Since the number of particles $N_s$ is set to 500, there are 497 hidden ellipses of cyan dotted lines. Each particle is weighted

![Image](image_url)

Fig. 8 – Among the 30 vessel segments provided for the experiments, the three vessel segments in which the method of Shim et al. [4] failed and the proposed method succeeded. Failures of the method of Shim et al. [4], more specifically (a) at the petrous portion of the left ICA making a turnover, (c) at the petrous portion of the right ICA getting astray, (e) at the cavernous portion making a turnover. (b), (d), and (f) successes of the proposed method.
according to the exponential sum of distances using (12) and (13). The small black squares represent the 14 3D points composing the actual observation $z_k$ and the two crosses are the outliers, as we set $N_z = 16$ and $N_o = 2$. The ellipse in the red solid line is the updated state $x_k$ at time $k$ as the weighted mean of all the $N_z$ particles of (4). Fig. 7(a) illustrates that the proposed method does not get astray nor makes a turnover and it acceptably tracks the vessel segment. The time consumption is about 30 s by a system with Pentium IV 3.4 GHz and 2 GByte memory.

For comparison, the method of Wink et al. [3] was applied to the same vessel segment, but got astray eight times and made four turnovers. The method of Florin et al. [5] was also applied with only one Gaussian distribution in their appearance model. It may result from incomplete implementation, but the tracking did not work and got astray only after a few steps. It was caused by the normal vector which is unable to follow the axis curvature effectively. Their assumption of brighter arteries also limits its application to cerebral arteries. However, the proposed method is expected to be applicable to coronary arteries. Its improved robustness is due to the following properties:

- The normal vector at each cross-section is determined by minimizing the cross-sectional area over all the sampled pairs of $(\theta_i, \phi_j)$.
- The gradient threshold is modified when the artery is adjacent to veins.
- The outliers in the measurements at each normal plane are removed.

Next, more macroscopic comparison with the work of Shim et al. [4] is performed using the 30 vessel segments. Tracking is judged as a success when the whole ICA (from the seed to the supraclinoid portion) is extracted and tracking continues flowing into one of the main branches of the ICA, mostly the MCA. The method of Shim et al. [4] has recorded

Fig. 9 – Abnormal vessel formations: (a) at a bifurcation, the tracking progresses maintaining its direction. The other branch indicated by the arrow can be tracked using the new seed obtained by simple shape analysis [13], (b) at a stenosis, the proposed method continues tracking, and (c) at an aneurysm, the tracking gets astray. New tracking can be initiated in the same way as (a), (d) and (e) cerebral arterial tree structures extracted with only four seeds, showing not only the main arteries like ICA and VA but also a number of branches such as ACA, PCA, LLA, etc.
In this paper, a method to extract cerebral arterial segments from CTA was proposed. It tracked the centerline using an SIR (sampling importance resampling) particle filter. The measurement model defines the border points on the nominal cross-section as the observation. The border points are detected using the radial gradient along rays and the thresholds for detection are adaptively modified to bone contact and vein contamination. The outliers in the detection are discarded in the computation of the distance between each particle and the actual observation. The distance is measured as the exponential sum of distances of the border points in the measurement from the ellipse represented by each particle.

The sensitivity analysis to the thresholds $t_{nt}$, $t_{sc}$, and $t_{cc}$ confirmed that the proposed border detection method is very robust to these thresholds. Thirty vessel segments were also provided for the comparison of the proposed method to the method of Shim et al. [4] which also has acceptable tracking performance. The proposed method improved the success rate by virtue of the collective statistics of many particles, even in the cases of abrupt change in curvature and inaccurate normal vector computation. In addition, various synthetic vessel segments were generated for more systematic and quantitative evaluation. The proposed tracking method also showed more robustness than the conventional methods in the experiments on these synthetic data. However, the analysis of the clinical accuracy using the ground truth of the complete arteries of interest is left as another future work.

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