Design, Modeling, and Verification of High-Performance AC–DC Current Shunts From Inexpensive Components
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Abstract—A new type of ac current shunt for ac–dc current transfer is presented for which the ac–dc current differences at frequencies from 10 to 100 kHz are calculated with uncertainties smaller than ±9 µA/A in the current range of 30 mA–10 A. This is an independent realization of the ac–dc current-transfer standards in addition to the step-up method used at most National Metrology Institutes. The construction, modeling, and experimental verification of the shunts are described.

Index Terms—AC current, ac–dc difference, analog design, current measurement, current shunts.

I. INTRODUCTION

THERMAL converters are the most accurate standards for the transfer of ac to the equivalent dc quantities. Commercially available single-junction thermal converters are mostly used as thermal current converters (TCCs), and uncertainties down to a few microamperes per ampere in the ac–dc current-transfer differences have been obtained in the milliamperes and kilohertz range [1]. Multijunction thermal converters are used to obtain the lowest uncertainties, but these are not readily available commercially [2]. Thin-film or planar multijunction thermal converters (PMJTCs) have been developed during the last 20 years [3]–[5]. They have very low ac–dc current-transfer differences in the milliamperes/audio frequency range and are used at most national measurement institutes (NMIs) but are not readily available commercially.

To extend the current range of up to 20 A, usually, commercially available [6], [7] or self-designed shunt resistors in parallel with the converters are used [8]–[11]. These have ac–dc current differences of less than 10 µA/A at audio frequencies but up to several hundred microamperes per ampere for frequencies from 20 to 100 kHz. The ac–dc transfer differences of these current converters for higher currents are measured in a step-up procedure, starting with the known PMJTC at 10 mA. They are not calculable. The high differences limit the minimum attainable uncertainties for ac currents in this frequency range.

This situation, together with the ongoing interlaboratory comparison CCCEM-K12, motivated Justervesenet (JV) to develop new shunts with very low and calculable ac–dc current-transfer differences in the frequency range of 10 Hz–100 kHz.

II. DESIGN CONSIDERATIONS

At JV, the standards used for the ac–dc current transfer are of the PMJTC type [3], [4]. These have 90-Ω heaters with a rated current of 10 mA and have to be shunted by appropriately rated resistors for higher currents.

We set out with the aim to develop a shunt set calculable with an uncertainty of less than 20 µA/A for the range 10 Hz to 100 kHz at 30 mA to 10 A, with low ac–dc current-transfer differences and current-level dependencies. The resistance values of the shunts should be such that they were well suited for use with PMJTCs.

Using a simplified model with stray inductances in series with the resistive element and stray capacitances in parallel, initial calculations indicated that for the lowest current ranges of up to 300 mA, small capacitances and, for higher currents, small inductances are important.

Stray magnetic fields outside the shunt itself should be kept as low as possible to avoid magnetic coupling to the closely connected TCC and other parts of the measurement setup. A coaxial or at least a good approximation to a coaxial structure would be the best solution. Coaxial shunts also have better frequency behavior.

A structure with a large surface area for good heat dissipation is needed for currents above 1 A to achieve a low temperature rise in the resistive element.

The shunts should also be easy and cheap to manufacture, without the need for special tools such as bonding machines, etc.

The design requirements could then be summarized as follows:

1) small capacitance for the low current ranges;
2) small inductance for the high current ranges;
3) coaxial or near coaxial structure;
4) good heat-conduction path from the resistive element to the surface of the housing;
5) large surface area for efficient heat dissipation;
6) low component cost;
7) easy to manufacture with simple tools;
8) influence of the skin effect as small as possible.

III. PRACTICAL REALIZATION OF THE SHUNTS

To keep the capacitances small, low-permittivity insulation materials such as air or polytetrafluorethylene (PTFE) can be used. These materials give the additional benefit of low
dielectric losses for high frequencies. For the higher currents, when the capacitance and dielectric losses are not as important, fiberglass-epoxy printed circuit-board material (FR-4) is good enough and is usually an order of magnitude cheaper than PTFE.

The requirement for low inductances for the higher currents suggested that resistors intended for surface mounting could be used. Due to their small size, these resistors have very low inductances but limited power-handling capability. Consequently, it would be necessary to use many resistors in parallel, which would also reduce the resulting effective inductance. To keep all stray inductances low, all forward and return current paths in the shunt structure should enclose a small area. A compact design was thus needed.

To obtain minimum influence from the skin effect, a material with small copper-layer thicknesses compared to the skin depth for all frequencies should be used. At 100 kHz, the skin depth in copper is approximately 200 µm. This indicates that a circuit-board material with 35-µm copper thickness should be satisfactory, but there should still be enough effective copper cross-sectional area to carry the total current in the shunt.

To be able to use the shunts in a setup for the ac–dc current transfer with the desired low uncertainties, the shunts need to have temperature coefficients of the resistance smaller than 10 µΩ/Ω per K. Surface-mount resistors of the cylindrical metal-electrode face-bonding (MELF) type are cheap and easily available with temperature coefficients below 10 µΩ/Ω per K. Unfortunately, these resistors are ferromagnetic, which increases the inductance and its frequency dependency. This effect must then be taken into account in the mathematical model. Measurements indicate that the inductance is approximately inversely proportional to the square root of frequency.

The surface-mount resistors were mounted in holes through the surface of the double-sided circuit board and soldered to the copper layers on both sides. MELF resistors are cylindrical in shape and fit very well into holes with the proper diameter. The circuit board with the resistors was then mounted in a structure, as shown in Fig. 1. This way, quite good heat dissipation from the resistors to both the copper layer and the insulating material can be obtained, and the input-current paths enclose areas that are quite small. The output voltage is measured at the center of the rectangle of resistors on the right side in Fig. 1 in such a way that the enclosed area for the output current is even smaller. The magnetic coupling between input and output becomes very small due to the small loop areas. The basic idea of the structure is an improvement of the design published by Hammond and Budovsky [12], [13]. A photograph of the output (voltage) side of a 1-A shunt is shown in Fig. 2.

The requirements for the shunts and the number of resistors needed gave three different practical sizes of the circuit boards: 50 × 50 mm for the range of 30–300 mA, 100 × 100 mm for the range of 1 A, and 160 × 160 mm for the range of 3–10 A. For the current range of 5–10 A, so many resistors were needed for power dissipation and heat distribution that they had to be mounted in two separate boards or “decks.” The deck to the left in Fig. 1 was used as the second deck. This slightly changed design made it necessary to modify the model developed in

![Fig. 1. Basic structure of the shunts.](image1)

![Fig. 2. A 1-A shunt, as shown from the output (voltage) end.](image2)

the following section for the calculations for the shunts at this current level. This is to take into account the different placement of the extra resistors and the resulting effects.

The given dimensions for the various shunts are the optimum for each current level.

IV. MODELING THE AC–DC TRANSFER BEHAVIOR

Fig. 3 shows a general circuit model of the shunt using lumped circuit elements. The model includes approximations to the parasitic reactive and resistive components from the chosen shunt geometry. Component values are measured and/or calculated from the geometry and material properties.

In Fig. 3, \( R \) represents the shunt resistance, \( L_s \) is the equivalent series inductance of the shunt resistor, and \( L_p \) and \( C_2 \) are the series inductance and parallel capacitance, of the output port of the shunt, respectively. \( R_{PAC} \) is the effective parallel resistance caused by dielectric losses. It was measured with a commercial impedance meter. The loss is a function of the material in the plates and is highly frequency-dependent. \( L_f \) and \( C_1 \) are the series inductance and parallel capacitance, as seen from the current input. \( R_{IAC} \) and \( R_{IDC} \) represent the
resistive losses in the structure for ac (including skin effect) and dc, respectively. \( R_T \) represents the input resistance of the thermal voltage converter (TVC) used to measure the output voltage across the shunt.

The ac–dc difference \( \delta \) of the shunt is defined as

\[
\delta = \frac{I_{AC} - I_{DC}}{I_{DC}}
\]

where \( I_{AC} \) is the effective value of the ac current producing the same output as the mean of the reversed dc current \( I_{DC} \).

The ac current can be expressed in terms of the circuit parameters as

\[
I_{AC} = \frac{Z_2 \cdot Z_5}{Z_1 \cdot Z_4 \cdot Z_6} \cdot U_{TAC}
\]

where \( U_{TAC} \) is the normalized ac voltage across the TVC heater

\[
Z_1 = \frac{R_T \cdot \frac{1}{j\omega C_2}}{R_T + \frac{1}{j\omega C_2}}
\]

\[
Z_2 = \frac{R_T \cdot \frac{1}{j\omega C_2} + j\omega L_p}{R_T + \frac{1}{j\omega C_2}}
\]

\[
Z_4 = \frac{Z_2 \cdot \frac{R_{RAC}}{R_T + R_{RAC}} + j\omega L_s}{Z_2 + \frac{R_{RAC}}{R_T + R_{RAC}} + j\omega L_s}
\]

\[
Z_5 = Z_4 + R_{TAC} + j\omega L_I
\]

\[
Z_6 = \frac{(Z_4 + R_{TAC} + j\omega L_I) \cdot \frac{1}{j\omega C_1}}{Z_4 + R_{TAC} + j\omega L_I + \frac{1}{j\omega C_1}}
\]

\[
I_{DC} = \frac{U_{TDC}}{R_T + \frac{1}{R_T + R}}
\]

Because \( I_{AC} \) in (2) is a complex number and thus contains a phase information, the absolute value of the ac current has to be used. The ac–dc difference then can be expressed as

\[
\delta = \frac{|I_{AC}| - I_{DC}}{I_{DC}}
\]

with \( U_{TAC} = U_{TDC} \). This means that we compare the ac and dc conductance and look for the ac–dc conductance differences of the shunt.

The aforementioned equations were analyzed and solved with a commercial mathematical software package [14]. The full analytical solution for the ac–dc current difference is very complex and thus impractical to be given here. The calculated numerical values of the ac–dc differences for all shunts from 30 mA to 10 A as a function of frequency are shown in Fig. 4.

V. TESTING AND VERIFICATION

The shunts were tested, using PMJTCs of the Physikalisch-Technische Bundesanstalt (Germany) (PTB) type [3] to measure the voltage on the output, in a digital bridge setup for comparing the ac–dc current differences. The setup has been described earlier [15], [16] and is similar to the system also described by Rydler [17].

A complete step-up from 10 mA to 10 A was made. In the step-up, the unknown shunt TVC for the next higher current is calibrated at the current level of the known converter and then used at its rated current. At 10 mA, the shunt/PMJTC was calibrated directly against the primary reference for the ac–dc current difference, a 3-D multijunction thermal converter of the PTB type described by Klonz [2]. The other steps were 30 mA, 100 mA, 300 mA, 1 A, 3 A, 5 A, and 10 A. A relatively small number of shunts are needed because of the large dynamic range and high signal-to-noise ratio of the PMJTCs.

The measured ac–dc current differences at all current levels for eight frequencies from 10 Hz to 100 kHz are shown in Fig. 5. The step-up results have been corrected for the known ac–dc differences of the PMJTCs. In the frequency range of 10–55 Hz, the measured results differ from the model predictions. This can be attributed to the low-frequency power dependency of the ac–dc transfer differences of the PMJTCs.

To test the repeatability in the production of the shunts, three shunts at all current levels have been produced. The measured
Fig. 6. Measured spread (deviation from average value) in ac–dc current differences for three shunts at 100 mA as a function of frequency.

Fig. 7. Measured spread (deviation from average value) in ac–dc current differences for three shunts at 5 A as a function of frequency.

Fig. 8. Average values of three measured shunts at the current levels of 30 mA, 100 mA, and 5 A with uncertainties \((k = 2)\), which are compared to model calculations.

ac–dc current differences for 100 mA and 5 A are shown in Figs. 6 and 7. The maximum difference in the ac–dc current differences between the sets is smaller than 5 \(\mu\)A/A at 30 mA and 1 \(\mu\)A/A at 100 mA to 5 A. This indicates that the design and production is quite repeatable.

Average values for the ac–dc differences with uncertainties \((k = 2)\) for three shunt sets at three current levels are shown together with corresponding calculated data in Fig. 8. The uncertainties for the experimental data have been calculated using the Guide to the Expression of Uncertainty in Measurements (GUM) method.

In Tables I and II, the uncertainty budgets at 300 mA and 5 A, which are calculated from the uncertainties in the input parameters using the model and the GUM method, are given. Uncertainty calculations should ideally be done analytically. The complex structure of the formula for the ac–dc differences makes the formulas for partial derivatives too large to be handled by the commercial mathematics software. To calculate the uncertainty contributions, each parameter value in the ac–dc difference formulas (1)–(9) has been varied separately with their uncertainties. The respective changes in the calculated ac–dc differences provide approximations to the uncertainty contribution from each parameter.

VI. RESULTS

The experimental results agree very well with the model at all current levels and frequencies above about 100 Hz. The deviations for frequencies below 100 Hz can most probably be explained by the power level dependency of the PMJTCs [4]. For the lowest current step (10 to 30 mA), one PMJTC is also running in the voltage mode (with 30-mA shunt), and the other, which has no shunt, is running in the current mode in the setup for comparing the ac–dc current differences. PMJTCs exhibit different ac–dc differences in the current and voltage modes for low frequencies, as described by Klonz [18] and Laiz [19]. To investigate this discrepancy, measurements to estimate the power dependency of the PMJTCs in the voltage mode in the 10–30-mA step were made for 10 Hz, 20 Hz, 55 Hz, and 1 kHz for currents from 3.2 to 10 mA. By using the experimental values for 10 mA, the experimental data in Fig. 8 can be corrected for power dependency in the lowest step. The corrected data are shown with the model data in Fig. 9.

The corrected experimental data show even better agreement with the model. This indicates that the shunts themselves have a level dependency of smaller than 1.5 \(\mu\)A/A in the ac–dc current differences for 30% to 100% of range current step and frequencies from 10 to 100 Hz. From 100 Hz to 100 kHz, the level dependency is negligible.

To further verify the properties and stability of the shunts and the model, one 300-mA shunt was shipped to SP Technical Research Institute of Sweden (SP), the Swedish NMI, and two complete sets (30 mA–10 A) to PTB, the German NMI. The measured ac–dc current differences from SP and PTB, with corresponding results from JV, are shown in Fig. 10 [20], [21], together with the calculated data. The results verify the very low ac–dc current differences of the new shunts and, thus, the modeling as well.

VII. CONCLUSION

A new type of current shunts has been developed for which the ac–dc current differences at frequencies from 10 Hz to 100 kHz can be calculated with uncertainties smaller than \(\pm 9 \mu\)A/A in the current range of 30 mA–10 A. The level dependency of the ac–dc differences of the shunts should further be investigated, but preliminary results indicate that it is less than \(\pm 1.5 \mu\)A/A for 30% to 100% of current range for all frequencies from 10 Hz to 100 kHz in the current range of 10 mA–10 A.

An independent realization of the ac current scale in the whole frequency/current range described is thus possible, given the agreement of the model with the measured results.
TABLE I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measured/estimated nominal value</th>
<th>Estimated Relative Uncertainty</th>
<th>Uncertainty Contribution to ( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>38 pF</td>
<td>±20 %</td>
<td>0.028 ( \mu )A</td>
</tr>
<tr>
<td>( L_1 )</td>
<td>5 nH</td>
<td>±15 %</td>
<td>0.011 ( \mu )A</td>
</tr>
<tr>
<td>( L_3 )</td>
<td>10 nH</td>
<td>±15 %</td>
<td>0.79 ( \mu )A</td>
</tr>
<tr>
<td>( R_{AC} )</td>
<td>4.9 M( \Omega )</td>
<td>+80 % to -60 %</td>
<td>1.17 ( \mu )A</td>
</tr>
<tr>
<td>( L_a )</td>
<td>0.5 nH</td>
<td>±15 %</td>
<td>0 ( \mu )A</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>44 pF</td>
<td>±20 %</td>
<td>0.034 ( \mu )A</td>
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<tr>
<td><strong>Total estimated uncertainty</strong> (k=2)</td>
<td><strong>2.8 ( \mu )A</strong></td>
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**TABLE II**

<table>
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<tr>
<th>Parameter</th>
<th>Measured/estimated nominal value</th>
<th>Estimated Relative Uncertainty</th>
<th>Uncertainty Contribution to ( \delta )</th>
</tr>
</thead>
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<tr>
<td>( C_1 )</td>
<td>700 pF</td>
<td>±20 %</td>
<td>0.087 ( \mu )A</td>
</tr>
<tr>
<td>( L_1 )</td>
<td>0.15 nH</td>
<td>±15 %</td>
<td>0.006 ( \mu )A</td>
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<tr>
<td>( L_3 )</td>
<td>1.4 nH</td>
<td>±15 %</td>
<td>4.13 ( \mu )A</td>
</tr>
<tr>
<td>( R_{AC} )</td>
<td>0.16 M( \Omega )</td>
<td>+80 % to -60 %</td>
<td>1.23 ( \mu )A</td>
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<tr>
<td>( L_a )</td>
<td>1 nH</td>
<td>±15 %</td>
<td>0.045 ( \mu )A</td>
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<tr>
<td>( C_2 )</td>
<td>760 pF</td>
<td>±20 %</td>
<td>0.15 ( \mu )A</td>
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<td><strong>Total estimated uncertainty</strong> (k=2)</td>
<td><strong>8.6 ( \mu )A</strong></td>
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**REFERENCES**


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