Non-equidistance Optimum Grey Model GM(1, 1) of Requirement Analysis of Urban Construction Land and Its Empirical Study

Bohong Zheng
School of Architecture and Art, Central South University
ChangSha, 410083, China

Lanxiang Luo *
School of Architecture and Art, Central South University
ChangSha, 410083, China
* Corresponding author: LanXiang LUO, e-mail: 545746678@qq.com

ABSTRACT

As the foundation of urban development, construction land is also the basic material security of regional urbanization. Besides, accurate forecast of the land demand for urban construction is also important for the planning compilation of construction land. Thus, a non-equidistance optimum model GM(1,1) was proposed using the minimum average relative error between restored value of the model and real value as objective function. This model was corresponding to the time response formula and reduction formula of non-equidistance model GM(1,1). With the help of mathematical software LINGO15.0, the global optimal solution was directly obtained. Furthermore, this model was suitable for both equidistance and non-equidistance model, with high accuracy and strong stability. Examples showed the model's practicality and reliability can offer decision basis to land management departments in land planning.

KEYWORDS: Requirement analysis of urban construction land; urban planning; GM(1,1) model; non-equidistance; mathematical software; grey system.

INTRODUCTION

Land resources are the basic resources and material wealth of human development. With the development of social economy, urban construction has accelerated its pace, with increasing demand of construction land. However, this also results in imbalance between supply and demand. As an important basis for land planning, demand forecast of urban construction land has made a significant contribution to land use and management. [1] An important part of the gray system theory is grey model. Since the gray system theory first proposed by Professor Deng Julong, gray model has been applied in many fields. [2] Especially the GM(1,1) model, which can be studied with "small sample" and "poor information", has been widely used for its simple operation and practical advantages. Gray system models mostly belong to equidistant sequence, while the raw data obtained in practice are non-equidistant sequences. Therefore, the establishment of non-equidistant series model has a practical and theoretical significance. To improve fitting and prediction accuracy of GM(1,1) model, numerous non-equidistant GM(1,1) models have been established in Reference [3-5] corresponding to
different structure methods of background value. Reference [6] proposed the accumulated generating operation in opposite direction, with the establishment of corresponding grey model GOM(1,1). Reference [7] put forward the accumulated generating operation in reciprocal number, based on which the grey model GRM(1,1) has been established. In Reference [8], the grey model GRM(1,1) has been improved with the establishment of improved grey model CGRM(1,1). However, the models established in References [7] and [8] were both equidistance model GRM(1,1). In reference [9], the step by step optimum model GM(1,1) was established by direct modeling method for the demand forecast of urban construction land. Besides, the non-equidistance grey model GRM(1,1) was proposed in Reference [10] for the requirement analysis of urban construction land. These models were all established using the residual between practical value and simulation value. However, the evaluation criterion of the model was the sum of squares of the relative error or absolute error between practical value and restored value. Therefore, there were some differences in modeling methods and evaluation standards. Namely, not all models met the conclusion that the minimum residual sum of squares achieved the highest precision. Based on this idea, the equidistance optimum grey model GM(1,1) was established in Reference [11]. In the work, the modeling method in Reference [11] was adopted. Besides, a non-equidistance optimum model GM(1,1) was proposed using the minimum average relative error between restored value of the model and real value as objective function. With the help of mathematical software LINGO15.0, the global optimal solution can be directly obtained. Except its high accuracy, this model also has a practical and theoretical significance. Examples of requirement analysis of urban construction land showed the model was of strong practicality and reliability.

**REQUIREMENT ANALYSIS OF URBAN CONSTRUCTION LAND: NON-EQUIDISTANCE OPTIMUM GREY MODEL GM(1, 1)**

**Non-equidistance Optimum Grey Model GM(1, 1)**

Definition 1: Supposing the sequence was \( X^{(0)} = [x^{(0)}(t_1), \ldots, x^{(0)}(t_m)] \), then \( X^{(0)} \) was the non-equidistance sequence if \( \Delta t_i = t_i - t_{i-1} \neq const \) and \( i = 2, \ldots, m \).

Definition 2: Supposing the sequence was \( X^{(1)} = [x^{(1)}(t_1), x^{(1)}(t_2), \ldots, x^{(1)}(t_m)] \), then non-equidistance sequence \( X^{(1)} \) was the first-order accumulated generation (1-RAG) if \( x^{(1)}(t_i) = x^{(0)}(t_i) \) and \( x^{(1)}(t_{k+1}) = x^{(1)}(t_k) + x^{(0)}(t_{k+1}) \cdot \Delta t_{k+1}, k = 1, \ldots, m-1 \).

To build the model, one accumulated generating operation was conducted on the original data, with the following new sequence.

\[
X^{(1)} = [x^{(1)}(t_1), x^{(1)}(t_2), \ldots, x^{(1)}(t_m)]
\]

where \( x^{(1)}(t_j) (j = 1, 2, \ldots, m) \) met the requirement of definition 2, namely:

\[
x^{(1)}(t_k) = \begin{cases} 
\sum_{j=1}^{k} x^{(1)}(t_j)(t_j - t_{j-1}) & (k = 2, \ldots, m) \\
0^{(0)}(t_1) & (k = 1) 
\end{cases}
\]
For $X^{(1)}$, after an accumulated generation, the established non-equidistance model GM(1,1) can be expressed as first-order grey differential equation \( \frac{dx^{(1)}}{dt} + ax^{(1)} = b \), where \( \zeta^{(1)} \) was the background value. Its albinism differential equation of GM(1,1) was as follows.

\[
\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{3}
\]

The discrete solution of \( \frac{dx^{(1)}}{dt} + ax^{(1)} = b \) was as follows.

\[
x^{(1)}(t_k) = \frac{b}{a} + (x^{(0)}(t_1) - \frac{b}{a})e^{-at_1-t_k}(k = 1,2,\ldots,m) \tag{4}
\]

After reduction, the fitted value of the original data reciprocal was as follows.

\[
\hat{x}^{(0)}(t_k) = \begin{cases} 
  x^{(0)}(t_1) & (k = 1) \\
  \frac{x^{(0)}(t_k) - x^{(0)}(t_{k-1})}{\Delta t_k} & (k = 2,3,\ldots,m)
\end{cases} \tag{5}
\]

The absolute error of fitting data was as follows.

\[
q(t_k) = \hat{x}^{(0)}(t_k) - x^{(0)}(t_k) \tag{6}
\]

In above GM(1,1) models, parameter \( a \) was the development coefficient and \( b \) the grey action. All these models were modeled using the residual between practical value and simulation value. Furthermore, different values of parameters \( a \) and \( b \) were identified by the least square method for different construction methods of background values, also with different errors.

### Nonlinear Programming Model with the Least Average Relative Error

Generally, the quality of GM(1,1) model was evaluated using the average relative error between simulation value and raw data sequence in final model. Therefore, it is feasible to establish a nonlinear programming model of average relative error.

\[
\min = \frac{1}{n} \sum_{k=2}^{n} |e(t_k)| \tag{7}
\]

s.t.

\[
a_{\min} \leq a \leq a_{\max}, b_{\min} \leq b \leq b_{\max}
\]

where

\[
e(t_k) = \frac{\hat{x}^{(0)}(t_k) - x^{(0)}(t_k)}{x^{(0)}(t_k)} \times 100
\]

When \( t_k = k(k = 1,2,\cdots,n) \), the non-equidistance model can be transformed into equidistance model. The design variable of model was \( x = [a,b] \).
Solving Optimization Model by LINGO

LINGO is a software package to solve mathematics programming issues under the environment of WIN. For its fast speed and high efficiency in analyzing mathematical programming problems, LINGO has been widely used in education, research and industry. LINGO is mainly used for solving linear programming, nonlinear programming, quadratic programming, integer programming and equations. Besides, it is also useful to find the roots of algebraic equations as well as solving linear and nonlinear equations. Meanwhile, it is a matrix generator which can provide a kind of language to solve optimal problems. With the help of LINGO, users can build thousands of constraints or objective-functions by only typing a line of characters. Therefore, it is important to master such language of model optimization for its simplification in solving large scale issues. LINGO consists of a modeling language and many common mathematical functions, which can be used for the modeling of mathematical programming models. The latest version of LINGO is LINGO15.0, including a series of versions. Although the solving scales are different, the kernel and directions of software are similar. The website of such software is http://www.lindo.com.

LINGO Model starts with the statement of “MODEL”, and ends with “END”. In addition to SETS, ENDSETS, DATA, ENDDATA, INIT, ENDINIT, MODEL, END, CALC and ENDCALC, all the statements end with a semicolon. In LINGO, numerous internal functions starting with “@” can be quoted during the establishment of optimum models and solution of equations. Generally, the variables in LINGO are assumed nonnegative. If the variable was negative, then “@FREE(Variable-name)” would be used to cancel the non-negative condition. The annotation starts with a symbol “!”.

For the optimized objective function, there must be a min or max; if the variable was an integer variable, there must be a statement "@gin(Variable-name)". The design of global optimization only requires setting the options at global optimization. Other directions and edition differences is in the help documentation of LINGO15.0. Using LINGO, users should note the following issues: (1) Trying to use real optimization model and reduce the number of integer constraints and variables; (2) Trying to use smooth optimization model rather than non-smooth function; (3) Trying to use linear optimization model, thus minimizing the number of nonlinear constraints and nonlinear variables; (4) Trying to make the upper and lower bounds of variables reasonable and provide the initial value of the variables; (5) Trying to set appropriate variable magnitude of the unit in model. With the update of software, more functions have been added to Lingo. Therefore, Lingo will prevail in the future rather than the Lindo (Linear, INteractive and Discrete Optimizer) without upgraded version. During the solution in LINGO, both the global optimization method and local optimization method are available. Generally, the local optimization method is used for fast solution, while global optimization method is for accurate solution. The model of Lingo is as follows:

```plaintext
model:
sets:
  k/1..8/:x0,x1,xr1,xr,tk,detatk;! The number 8 in this example can be modified when the data of x00 and tk vary;
endsets
data:
  x0=60 62 68 73.5 76.78 89.48 90.9 94.59; ! The data can be modified;
  tk=2001 2002 2003 2004 2005 2006 2007 2008; ! The data can be modified;
enddata
n=8;! The number 8 in this example can be modified when the data of x00 and tk vary.
@for (k(i)|i#GT#1:detatk=tk(i)-tk(i-1));
x1(1)=x0(1);
```
\begin{verbatim}
@for (k(i)|i#GT#1:x1=x1(i-1)+x0(i)*detatk(i));
min=100*@sum(k(i):@abs(x0(i)-xr(i))/x0(i)/n);
xr1(1)=x1(1);
@for (k(i)|i#GT#1:xr1=(xr1(1)-b/a)*@exp(-a*(tk(i)-tk(1)))+b/a);
xr(1)=xr1(1);
@for (k(i)|i#GT#1:xr=(xr1(i)-xr1(i-1))/detatk(i));
@bnd(-5,a,5);
@bnd(-3000,b,3000);
end
\end{verbatim}

EMPIRICAL RESEARCH

From 2001 to 2008, the built-up areas in the city center of Xinxiang are 60, 62, 68, 73.5, 76.78, 89.48, 90.9 and 94.59 square kilometer, respectively [10,12]. Then, the original data can be denoted as follows using the method in this work.

\[
X^{(0)} = [60, 62, 68, 73.5, 76.78, 89.48, 90.9, 94.59]
\]

\[
\]

After the establishment of model, the model parameters were: \(a=-0.07256307\), \(b=56.61996\), \(b/a=-780.2862\).

\[
\hat{x}^{(1)}(t_k) = 840.2862e^{0.07256307(t-2001)} - 780.2862
\]

The fitting value of original data was:

\[
\hat{x}^{(0)}(t_k) = [60.000, 63.240, 68.000, 73.118, 78.621, 84.538, 90.900, 97.741]
\]

Then, the average relative error of fitting data was 1.721%.

Using the accumulated generating operation in Reference [3], the average relative error of non-equidistance model was 1.8231%. However, the average value calculated by accumulated generating operation in reciprocal number was 1.7848% in Reference [10]. While modelling equidistance models, pretreatment of \(T=t-2000\) is required for \(t\). The method used in the work was direct calculation. If the original data of certain year missed, then direct modelling is also possible according to the method proposed in the work. Therefore, the method in the work is practical and scientific.

CONCLUSIONS

For the modelling and evaluation of GM model, there were some differences in modelling methods and evaluation standards. Namely, not all models met the conclusion that the minimum residual sum of squares achieved the highest precision. In this work, a non-equidistance optimum model GM(1,1) was proposed to simulate the land demand for urban construction. Such model was modelled using the minimum average relative error between restored value of the model and real value as objective function. With the general program LINGO15.0, the global optimal solution was directly obtained. Furthermore, this model was suitable for both equidistance and non-equidistance model, with high accuracy and strong stability. Examples showed the model's practicality and reliability can offer decision basis to land management departments in land planning. Therefore, this model was of practical and theoretical significance for application.
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REFERENCES


