Recent Advances in Harmony Search

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1. Introduction

The harmony search (HS) is a music-inspired evolutionary algorithm, mimicking the improvisation process of music players (Geem et al., 2001). The HS is simple in concept, few in parameters, and easy in implementation, with theoretical background of stochastic derivative (Geem, 2007a). The algorithm was originally developed for discrete optimization and later expanded for continuous optimization (Lee & Geem, 2005).
The following pseudo code describes how the HS algorithm works:

```
procedure HS
  // initialize
  initiate parameters
  initialize the harmony memory

  //main loop
  while (not_termination)
    for I = 1 to number of decision variables (N) do
      R1 = uniform random number between 0 and 1
      if (R1 < P_HMCR)  (memory consideration)
        X[I] will be randomly chosen from harmony memory
        R2 = uniform random number
        if (R2 < P_PAR)  (pitch adjustment)
          X[I] = X[I] ± Δ
        end if
      else  (random selection)
        X[I] = X ∈ Φ  (Φ = Value Set)
      end if
    end do

  // evaluate the fitness of each vector
  fitness_X = evaluate_fitness(X)

  // update harmony memory
  update_memory(X, fitness_X) % if applicable

  end while
end procedure
```


Source: Advances in Evolutionary Algorithms, Book edited by: Witold Kosiński, ISBN 978-953-7619-11-4, pp. 468, November 2008, I-Tech Education and Publishing, Vienna, Austria

Ensemble harmony search (EHS) is another variant of the HS where ensemble consideration is added to the original algorithm structure (Geem, 2006a). The new operation considers the relationship among decision variables. The EHS could overcome the drawback of genetic algorithm's building block theory which does not work well if less-correlated variables locate closely in a chromosome.

Mahdavi et al. (2007) proposed an improved harmony search (IHS), in which dynamic parameter adjusting is used in improvisation step. As the search progresses, P_{PAR} is increased linearly while adjusting amount is decreased exponentially. This modification improves the local exploitation capability of the HS algorithm.

Recently, Omran & Mahdavi (2007) proposed a new variant of harmony search, called the global-best harmony search (GHS), in which the concepts from swarm intelligence are borrowed to enhance the performance of HS such that the new harmony can mimic the best harmony in the harmony memory (HM).

The HS algorithm has been successfully applied to various artificial intelligence and engineering problems including music composition (Geem & Choi, 2007), Sudoku puzzle solving (Geem, 2007b), structural design (Lee & Geem, 2004; Saka, 2007), ecological conservation (Geem & Williams, 2008), aquifer parameter identification (Ayvaz, 2007), soil slip determination (Cheng et al., 2008), offshore structure mooring (Ryu et al., 2007), power economic dispatch (Vasebi et al., 2007), pipeline network design (Geem, 2006b), and dam operation (Geem, 2007c).

The goal of this chapter is to review various recent applications of the HS algorithm, helping other researchers to draw a big picture of the HS ability and to apply it to their own problems.

2. Recent applications

2.1 Music composition

The HS algorithm composed music pieces (Geem & Choi, 2007). When HS was applied to the organum (an early form of polyphonic music) composition, it was able to successfully compose harmony lines based on original Gregorian chant lines.

Gregorian chant is a monophonic religious song in the middle ages, and organum is an early form of harmonized music which accompanies the Gregorian chant melody. HS generates the harmony line (vox organalis) to accompany the original Gregorian chant (vox principalis).

The organum has the following composing rules: the harmony line progresses in parallel; for the parallel motion, the interval of perfect fourth is preferred; and, in order to distinguish the vox principalis from vox organalis, the former should always be located above the latter.

The above-mentioned rules were formulated as a optimization problem. Then, HS solved the problem, obtaining aesthetically pleasing organum as shown in Figure 1.

Figure 1 shows a Gregorian chant "Rex caeli Domine" and its organum composed by HS. The upper line in the figure is the Gregorian chant melody and the lower line is the organum line.

Fig. 1. Organum Composed by HS algorithm

2.2 Sudoku puzzle solving

HS was applied to a Sudoku puzzle (Geem, 2007b), which is formulated as an optimization problem with number-uniqueness penalties.

Sudoku means "singular number" in Japanese, and consists of 9 × 9 grid and 3 × 3 blocks for all the 81 cells. Each puzzle starts with some cells that already have numbers as shown in Figure 2 (the numbers in white cells are originally given). The goal of the puzzle is to find numbers for the remaining cells with three rules: (1) Each horizontal row should contain the numbers 1 - 9, without repeating any; (2) Each vertical column should contain the numbers 1 - 9, without repeating any; and (3) Each 3 × 3 block should contain the numbers 1 - 9, without repeating any.

2	5	4	3	1	6	8	9	7
7	6	3	9	8	5	1	2	4
1	9	8	4	2	7	6	5	3
9	8	1	7	5	3	2	4	6
6	3	2	8	4	9	7	1	5
5	4	7	2	6	1	9	3	8
4	7	5	6	9	2	3	8	1
3	1	9	5	7	8	4	6	2
8	2	6	1	3	4	5	7	9

Fig. 2. Sudoku Puzzle Solved by HS algorithm

The HS model found the optimal solution without any violation of three rules after 285 function evaluations as shown in Figure 2.

2.3 Structural design

Structural design involves in decision making about cross sectional dimensions of the members that constitute the structure and sometimes the geometry and topology of the structure itself. In the design of a steel frame, the decision making process necessitates selecting W or any other type of steel sections from practically available set of steel section tables for the members of the frame such that the response of the frame to external loads is within the limitations described in the steel design codes. It is not very difficult to imagine that one can come up with large number of different combinations selected from the available steel section set which may satisfy these requirements. However, the designer is interested in finding the combination which not only satisfies design code limitations but also minimizes the material weight or the overall cost. This is the optimal design. HS method is quite effective in finding the optimum solution of such combinatorial optimization problems. In this section the HS algorithm is applied to determine the solution of optimum design of grillage system, optimum geometry design of a steel dome and the optimum design of reinforced concrete continuous beam.

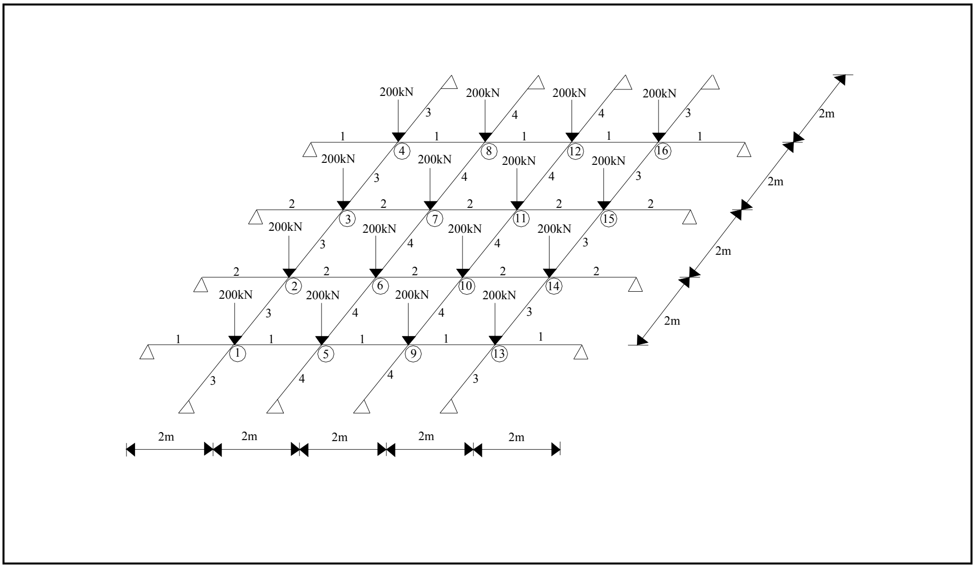

Fig. 3. 40-Member Grillage System

Optimum Design of 40-Member Grillage System: The grillage system shown in Figure 3 has 40 members which are collected in four groups such that the outer and inner longitudinal beams are considered to belong to groups 1 and 2 while the outer and inner transverse beams are taken as groups 3 and 4 respectively. This system is originally designed using HS (Erdal, 2007). The displacement and stress constraints are considered in the formulation of this design problem. The external loading that the grillage system is subjected to also shown in the figure. Under this loading it is required that the vertical displacements of joints 6, 7, 10 and 11 should not exceed 25mm. Furthermore it is the condition of the design criteria that nowhere in the longitudinal and transverse beams the bending stress should exceed the allowable bending stress of 250MPa. The 272 W-sections starting from W100X19.3 to W1100X499 are selected from LRFD-AISC (Manual of Steel Construction) as an available discrete design set for the optimum design procedure to select from. The task of the optimum design algorithm is to decide the appropriate W sections from this list for longitudinal and transverse beams of the grillage system such that the displacement and stress constraints described above are satisfied while the weight of the grillage system is the minimum. The solution of this problem is obtained by using HS as well as genetic algorithm (GA). The GA algorithm utilized in the solution of this design problem is a simple genetic algorithm where the initial population size is taken as 50 and two-point crossover is used to swap the genetic information between mating parents. While GA obtained the optimum solution after 40,000 structural analyses (function evaluations), HS required only 10,000 structural analyses to reach the optimum result. The optimum design (minimum weight = 7,075.84 kg) obtained by the HS method is 14% lighter than the one (8,087.91kg) determined by the GA in this particular design problem.

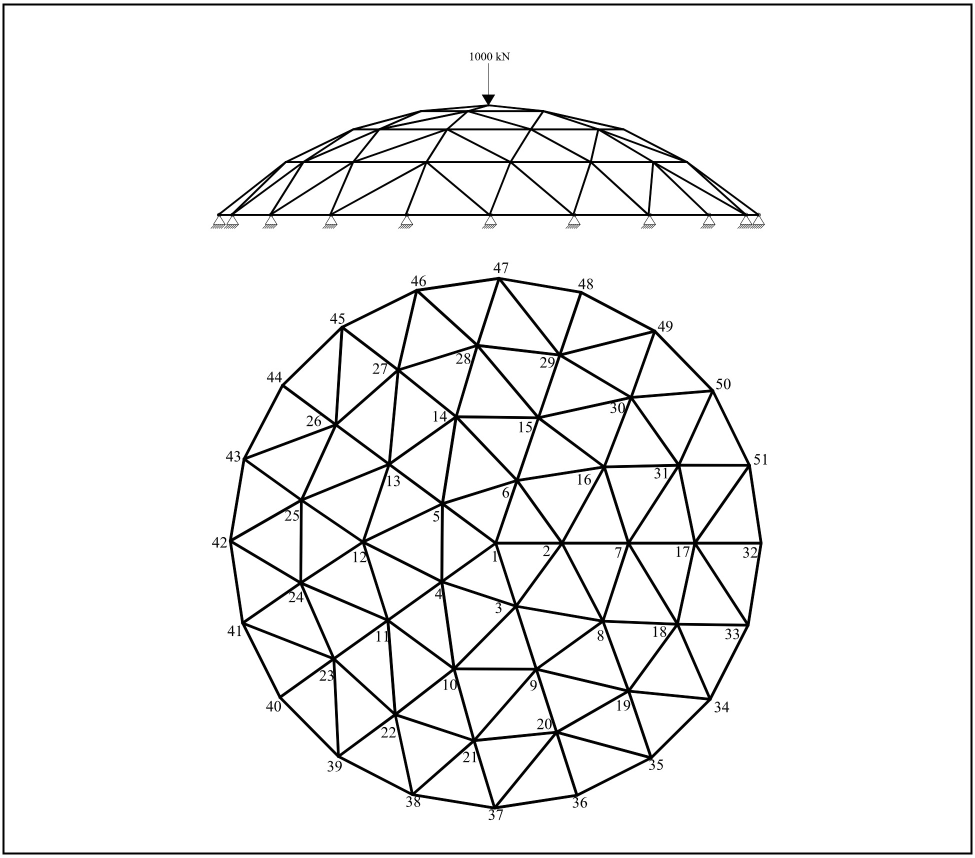

Fig. 4. Geodesic dome

Optimum Geometry Design of Geodesic Domes: Domes are economical structures in terms of materials that are used to cover large areas such as exhibition halls and stadiums where they provide a completely unobstructed inner space. Domes are given different names depending upon the way their surface is formed. Geodesic dome shown in Figure 4 is a typical example of a braced dome which is widely used in the construction of exhibition halls all over the world. A geodesic dome is comprised of a complex network of triangles that form a roughly spherical surface. Generally the area that is to be covered by the dome is provided by a client and the structural designer is required to come up with dimensions of pipe sections that are usually adopted for the dome members and also specify the height of the crown.

The design problem considered here is to determine the optimum height and circular steel hollow section designations for the geodesic dome that is suppose to cover the circular area of 20m as shown in Figure 4. The modulus of elasticity of the material is taken as 205kN/mm^2. The grade of steel adopted is grade 43. The dome is considered to be subjected to equipment loading of 1000kN at its crown. The formulation of the design problem and the construction of these constraints are explained in detail by Saka (2007). The solution of the design problem is obtained by HS. There are altogether 32 values for the HS algorithm to choose from.

It is apparent from Figure 4 that there are 3 rings in the dome. This number can also be treated as design variable. However for the simplicity here it is not taken as design variable. Two design problems are considered. In the first one all the members are decided to be made out of the same pipe section which means all the members are belong to the same group. In this case HS obtains the optimum height of the dome as 1.75m and PIP886 is adopted for the dome members. The minimum weight for this dome is 3750.6kg. It is noticed that while the displacements of the restricted joints are much smaller than their upper limits the strength ratios of some members are at their upper bound. This indicates that in the optimum design problem the strength constraints were dominant. Later, it is decided that those members between each ring are to be made one group and the members on each ring are another group. For example, if grouping is carried out such a way that the diagonal members between the crown and the first ring are group 1, the members on the first ring are group 2, the members between ring 1 and 2 are group3 and the group number of members on the ring 2 is 4 and so forth, then the total number of groups in the dome becomes twice the number of rings in the dome. In this case HS method determines the optimum height of the crown as 2m while the sectional designations for six groups of the dome members were PIP1143, PIP603.6, PIP483.2, PIP423.2 and PIP213.2. The minimum weight of the dome was 1244.42kg. Once more it is noticed that the strength constraints were dominant in the design problem

Optimum Design of Reinforced Concrete Continuous Beams: In the formulation of the optimum design problem of reinforced concrete continuous beams, design variables are selected as the width and height of beams and the reinforcement areas of longitudinal bars. These longitudinal bars are tensile reinforcements at each mid-span and supports and the shear reinforcement bar diameters for each beam. The general description of the design variables for four span continuous beams is given in Figure 5. The objective function is the total cost of the continuous beams which consists of cost of concrete, formwork and reinforcement steel. The design constraints consist of the ultimate strength requirements in bending and shear and minimum and maximum percentage of tensile and shear reinforcements. The details of these constraints are given by Akin (2007). The optimum design determined by the HS algorithm has the minimum cost of $11,406 while GA obtained $11,836.

Three different structural design problems are considered to demonstrate the robustness and effectiveness of the HS algorithm. The first problem is a size optimization problem where the HS method has selected optimum W sectional designations for longitudinal and transverse beams of grillage systems out of 272 discrete set of W steel sections. The solution obtained by HS is better than the one determined by simple genetic algorithm. The second design example is optimum geometry design of a geodesic dome where the HS algorithm has also effectively determined the optimum height of the crown as well as the optimum pipe designations for the dome members. Finally in the third design example, it is shown that HS can be successfully employed to determine the optimum cross sectional dimensions for beams as well as required reinforcement diameters and their total number in the design of reinforced concrete continuous beams.

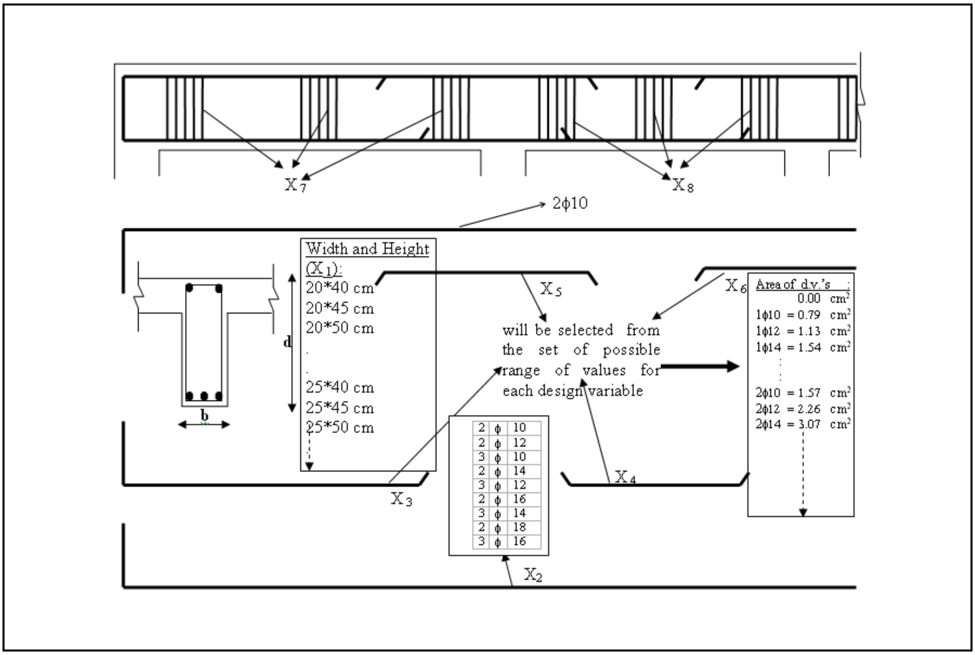

Fig. 5. Design Variables for Four Span Symmetrical Reinforced Concrete Continuous Beam

2.4 Ecological conservation

In today's industrialized life, to conserve ecosystem and its species becomes very important. In order to achieve the goal, quantitative techniques have been so far developed and utilized for the problem. HS was also applied to a natural reserve selection problem for preserving species and their habitats (Geem & Williams, 2008). The problem was formulated as an optimization problem (maximal covering species problem) to maximize the number of species protected within the reserve system given a specified number of sites that can be selected (ReVelle et al., 2002). The HS model developed for this problem was tested with real-world problem in the state of Oregon, USA, which consists of 426 species and 441 candidate sites as shown in Figure 6.

Harmony Search was applied to 24 cases, each involving a different limit on number of parcels that could be selected. HS found 15 global optimum solutions and 9 near-optimal solutions. When compared with simulated annealing (SA), the HS algorithm found better solutions than those of SA in 14 cases while the former found worse solution only once (Csuti et al., 1997).

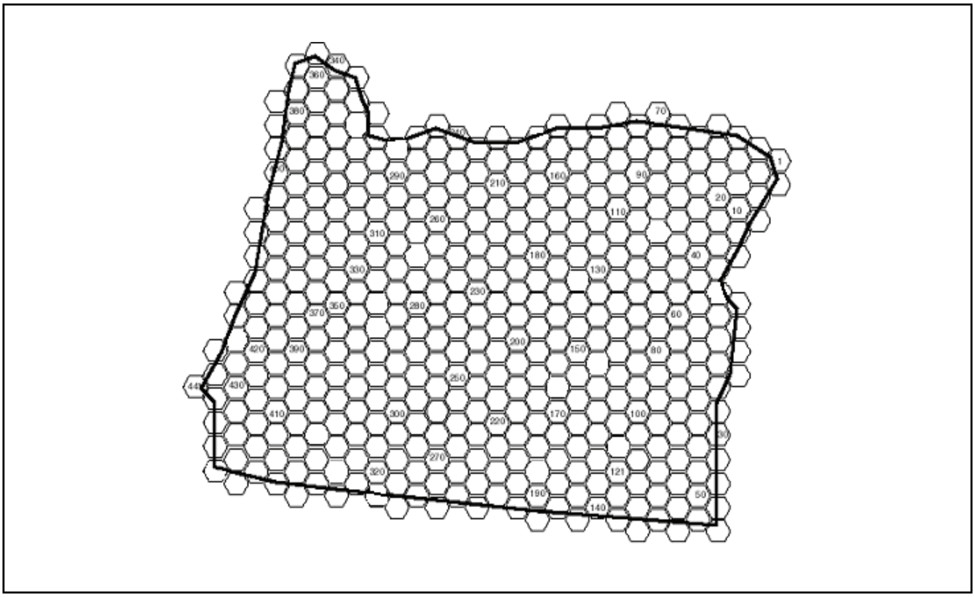

Fig. 6. Hex Map of Oregon

Another advantage of the HS algorithm is that it gives many alternative solutions because it handles multiple solutions as a time. For example, the HS found 25 alternative solutions for the case of 24 selected sites.

2.5 Aquifer parameter identification

Mathematical simulation models are widely used in the management of aquifer systems. These models require the spatial distributions of some hydrologic and hydro-geologic parameters for the solution process. However, aquifers are heterogeneous geological structures and usually distribution of their parameters is unknown. Thus, the determination of both aquifer parameters and their corresponding parameter structures based on field observations becomes an important step. The main goal of this study is to propose an S/O approach for simultaneously identification of transmissivity values and associated zone structures of a heterogeneous aquifer system. In the simulation model, the governing equation of groundwater flow is numerically solved using a block-centered finite difference solution scheme. The zone structure identification problem is solved through fuzzy c-means clustering (FCM) algorithm, and the HS algorithm is used as an optimization model to determine the optimum locations of cluster centroids and the associated transmissivity values within each zone (Ayvaz , 2007).

The main reason for applying FCM and HS to the groundwater inverse problem is to determine the zone structure and associated transmissivity values within each zone. The parameter zone structure of the aquifer is initiated using random cluster centroids and random transmissivity values are assigned to each cluster. Cluster centroids and transmissivity values are then optimized using HS by minimizing the residual error (RE) between the simulated and observed hydraulic heads at several observation wells.

The performance of the proposed S/O approach is tested on a hypothetical example. Figure 7 (Left) shows the plain view of two-dimensional confined aquifer.

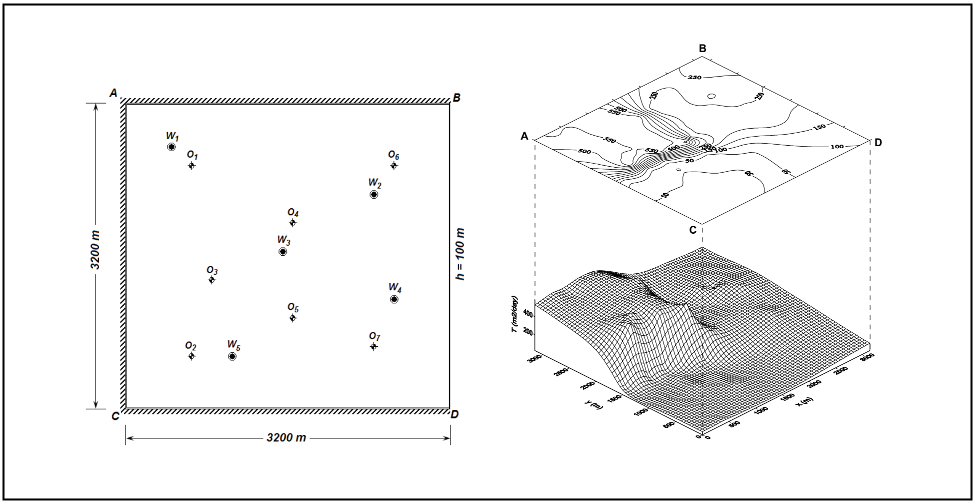

Fig. 7. (Left) Plain View of Confined Aquifer and (Right) True Transmissivity Field

As can be seen in Figure 7 (Left), the boundary conditions of the aquifer are 100 m constant head in the BD side and the no-flow in the other sides. The storage coefficient of the aquifer is the 0.0002. There are five pumping wells having the pumping rates of 4,000 cmd for Wells 1 to 4 and 2,000 cmd for Well 5. All the pumping wells are continuously operated for 10 days. There are seven observation wells and head observations are collected at the end of each day. The Gaussian noise of zero mean and 0.1 m standard deviation is added to the head observations. The true transmissivity field of the aquifer is shown in Figure 7 (Right). The main goal is to determine the best zonation pattern to satisfy the true transmissivity field. For the optimization process, five cases with different algorithm parameters are taken into account. Maximum number of improvisations (iteration) is set as 50,000 and the search process ends when the RE value remains unchanged through 1,000 improvisations. Note that, for comparison, the number of zones is fixed as 4 and the bounds of transmissivity values are set as 20 ~ 600 smd.

HS obtained the minimum RE (2.33) after 29,370 of function evaluations. Note that, GA (Tsai et al., 2003) solved the same problem, obtaining RE of 2.62 after 40,000 function evaluations. Although there are some differences, the identified transmissivity structures well capture the true transmissivity field.

2.6 Soil slip determination

Soil slopes are general in civil engineering and their stability assessment is of great importance to engineers. Up to now, limit equilibrium method is widely used by engineers and researchers for slope stability analysis. By using limit equilibrium method, a value F_s, also named the factor of safety can be estimated without the knowledge of the initial stress conditions and a problem can be defined and solved within a relatively short time. Limit equilibrium method is a statically indeterminate problem and different assumptions on the

internal forces distributions are adopted for different methods of analyses. At present, the famous method proposed by Morgenstern and Price (1965) is used to give the factor of safety for specified slip surface.
The minimum factor of safety of a slope and the corresponding critical failure surface are critical for the proper design of slope stabilization measures. The HS algorithm is employed to locate the critical failure surface in slope stability analysis. The generation of slip surfaces is as follows.
Consider the Cartesian system of reference Oxy as shown in Figure 8.

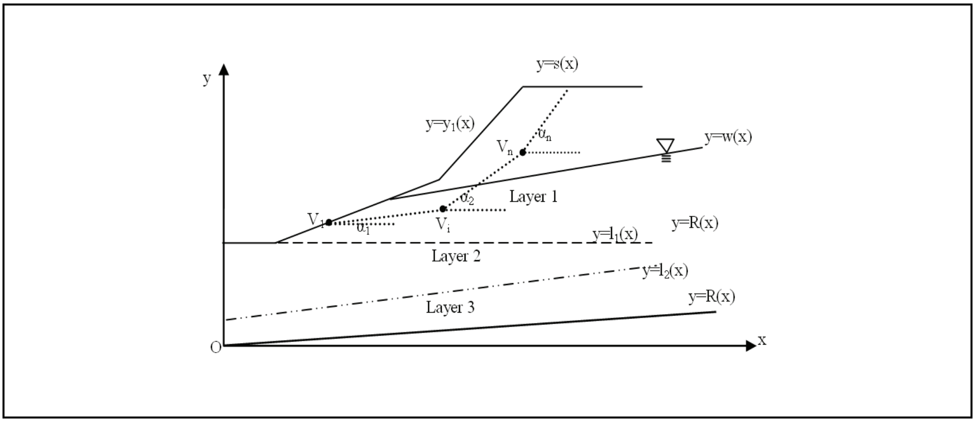

Fig. 8. Slip Surface and the Cross Section of a Slope

Function $y = y_1(x)$ describes the ground profile while the water table is represented by $y = w(x)$. The bed rock surface is represented by the function $y = R(x)$ and function $y = l_i(x)$ can be introduced to represent boundary between different soils. The trial failure surface is described by using the function $y = s(x)$.

To obtain the values of F_s requires the failure soil mass to be divided into n vertical slices and the slip surface is represented by n+1 vertices. Each slice can be identified by two adjacent vertices. Generally speaking, the potential slip surfaces are concave upward (kinematically acceptable requirement) with only few exceptions. The concave upward requirement can be formulated as follows:

$$\alpha_1 \le \alpha_2 \le \ldots \le \alpha_n \tag{1}$$

where α_i is the base inclination of slice i as shown in Figure 8. Every slip surface can be mathematically identified by the control variable vector $\mathbf{X}$ as follows:

$$\mathbf{X} = [x_1, y_1, x_2, y_2, \ldots, x_n, y_n, x_{n+1}, y_{n+1}]^T \tag{2}$$

The vector $\mathbf{X}$ is analogous to the harmony in music, and the HS algorithm can be performed to determine the critical slip surface with the minimum factor of safety.

The example is the one proposed by Zolfaghari (2005), where a slope in layered soil is analyzed using the GA and the Morgenstern and Price method. The number of slices n used in this study is assumed to be 20, 25, and 30. While GA (Zolfaghari, 2005) found minimum safety factor of 1.24, HS found 1.20 with 30 slices.

2.7 Mooring design of offshore floating structures

The mooring design of offshore platforms requires relatively significant amount of design cycles since a desired solution must satisfy the complex design constraints and be economically competitive. The complexity of these mooring design constraints may result from coupling between platform motion and mooring/riser system, maximum offset constraint of the riser system, multiple numbers of design parameters defining anchor leg system components, and uniqueness of site-dependent environmental conditions including water depth, wave/current/wind condition, seabed condition, etc. When the optimal cost is sought for this complex mooring design, the design process becomes even more complex.

Mooring design is to find an appropriate stiffness which is stiff enough and soft enough at the same time since the mooring system needs to satisfy mainly two design constraints: (1) required maximum horizontal offset and (2) reduction of extreme forces acting on the platform caused by interactions between environmental forces and platform responses. To reduce the trial and error effort in mooring design, Fylling (1997) addresses an application of mooring optimization of deepwater mooring systems. A nonlinear optimization program with frequency-domain analysis of mooring systems was presented, and the results showed that the suggested optimization could be a powerful tool for concept development and finding a feasible solution (Fylling, 1997). Fylling and Kleiven (2000) presented the simultaneous optimization of mooring lines and risers.

A single point mooring of a Floating, Production, Storage, and Offloading (FPSO) system was adopted for a case study. Deepwater and ultra-deepwater application of FPSOs becomes more attractive since they have advantages in early production and relatively big storage capacity compared to other types of offshore platforms. As we target for deeper water oil/gas fields, more technical challenges are confronted. For instance, prediction of deepwater oil offloading buoy motion becomes more difficult (Duggal and Ryu, 2005; Ryu, et al., 2006). Technical challenges due to deepwater and ultra-deepwater oil fields and project execution challenges due to the fast track schedule become a trend in FPSO projects. This deeper water and fast track trend naturally suggests a way of fast finding of a site and requirement specific feasible mooring design.

This section addresses a HS-based mooring optimization determining the length and diameter of each mooring component. In this design, only three design constraints were applied: (1) maximum platform offset, (2) factor of safety (FS) for intact case top tension, and (3) no uplift of the bottom chain. The objective function is the total cost of mooring system.

A total of 2,000 iterations were performed to find optimal mooring designs. Figure 9 presents the search history of optimal mooring cost as a function of iteration, and Figure 10 shows one final solution the HS algorithm found.

A mooring optimization design tool using the HS algorithm and a frequency domain global analysis tool was proposed to minimize the cost of the mooring system. This proposed cost-optimal mooring design tool successfully finds feasible mooring systems. A case study on a permanent turret mooring system for an FPSO in deepwater was conducted. The results show that the objective function (i.e. mooring system cost) converges well and HS provides several feasible mooring systems. In conclusion, a new HS-based mooring optimization tool, has a potential for fast finding the cost-optimal mooring system.

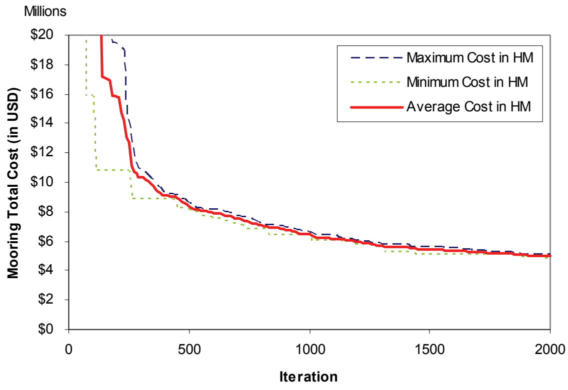

Fig. 9. Max, Min, and Mean Costs in Harmony Memory

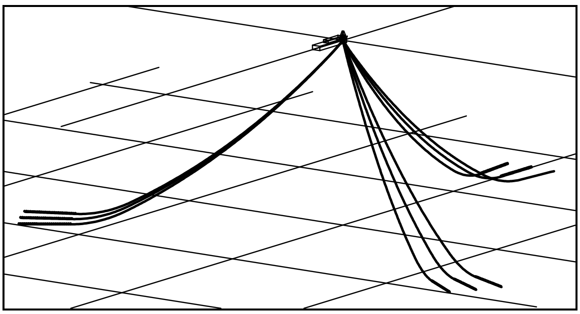

Fig. 10. Mooring Configurations

2.8 Heat & power generation

The conversion of primary fossil fuels, such as coal and gas, to electricity is a relatively inefficient process. Even the most modern combined cycle plants can only achieve efficiencies of between 50–60%. Most of the energy that is wasted in this conversion process is released to the environment as waste heat. The principle of combined heat and power (CHP), also known as cogeneration, is to recover and make beneficial use of this heat, significantly raising the overall efficiency of the conversion process. The best CHP schemes can achieve fuel conversion efficiencies of the order of 90%. In order to obtain the optimal utilization of CHP units, economic dispatch must be applied. The primary objective of

economic dispatch is to minimize the total cost of generation while honoring the operational constraints of the available generation resources. Complication arises if one or more units produce both electricity and heat. In this case, both of heat and power demands must be met concurrently. This section will show the application of the HS algorithm to solve the CHPED problem.
Figure 11 shows the heat-power Feasible Operation Region (*FOR*) of a combined cycle cogeneration unit. The feasible operation region is enclosed by the boundary curve ABCDEF.

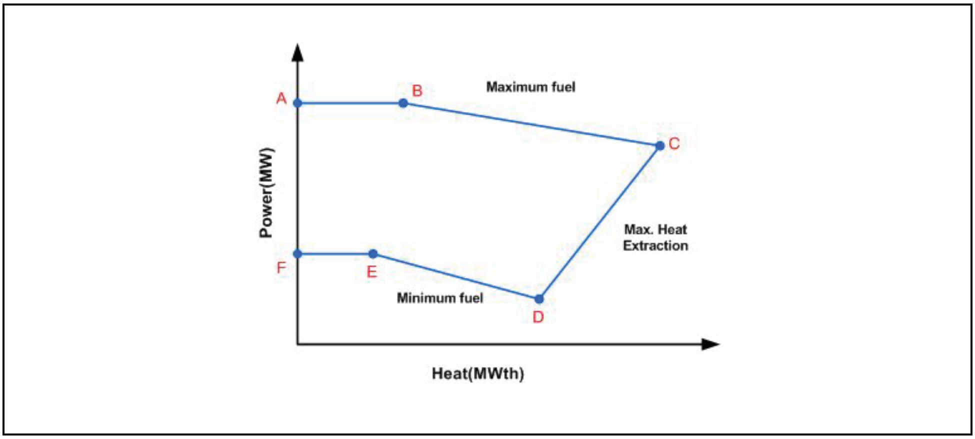

Fig. 11. Feasible Operation Region for a Cogeneration Unit

An example which is taken from the literature is used to show the validity and effectiveness of the HS algorithm. This example has been previously solved using a variety of other techniques (both evolutionary and traditional mathematical programming methods) after originally proposed by Guo et al. (1996). The problem consists of a conventional power unit, two cogeneration units and a heat-only unit. The objective is to find the minimum overall cost of units subject to constraints on heat and power production and demands.
After 25,000 function evaluations, the best solution is obtained with corresponding function value equal to $9257.07 (Vasebi et al., 2007). No constraints are active for this solution. The best solution of this problem obtained using the HS algorithm is compared with solutions reported by other researchers, showing that the result of HS is the same as the best known solution in the literature: $9257.07 by Lagrangian Relaxation (Guo et al., 1996); $9267.20 by GA (Song & Xuan, 1998); $9452.20 by ant colony search algorithm (Song et al., 1999); $9257.07 by improved GA (Su & Chiang, 2004).
Comparison between the results obtained by the HS method and those generated with other (evolutionary and mathematical programming) techniques reported in the literature clearly demonstrate that the HS method is practical and valid for CHPED applications.

3. Conclusions

This study reviews recent applications of the music-inspired HS algorithm, such as music composition, Sudoku puzzle solving, structural design, ecological conservation, aquifer parameter identification, soil slip determination, offshore structure mooring, and power economic dispatch.

As observed in most applications, the HS algorithm possesses a potential for obtaining good solutions in various optimization problems. Thus, the authors expect to see more successful applications in other scientific and engineering fields in near future. Also, theoretical progress in finding better solutions is expected.

4. Acknowledgements

The first author would like to thank many people who have helped in development of the HS algorithm, including Joel Donahue, Jack Galuardi, Teresa Giral, Kang Seok Lee, Audrey Leslie, Chung-Li Tseng, and Ronald Wiles.

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