A Physical Layer Scheme in Two-Way Relay Networks with Soft Network Coding

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Abstract—The previous schemes are introduced, including traditional network coding (NC) scheme, physical-layer network coding (PNC) scheme, soft network coding (SNC) scheme and pseudo physical-layer network coding (PPNC) scheme. As an improved scheme of PNC, PPNC does not need synchronization mechanism, however its BER performance gets worse due to the hard decision applied on the relay node. The new proposed pseudo physical-layer soft network coding (PPSNC) scheme adopts soft decision not hard decision compared with PPNC scheme. The computational complexity and BER performance of the proposed scheme are analyzed and compared with the previous schemes. Theoretical analysis shows that the proposed scheme has similar computational complexity with PPNC scheme, and has lower computational complexity than the other schemes. The simulation results show that the proposed scheme has better BER performance than PPNC scheme. At the same time, the performance of PPSNC in MIMO environment is also discussed. The simulation results show that PPSNC scheme in MIMO environment outperforms the previous schemes.

Index Terms—Network Coding; Physical-Layer Network Coding; Soft Network Coding; Two-Way Relay Networks

I. INTRODUCTION

The concept of network coding (NC) [1-5] is a major breakthrough in the field of communication. It combines route and coding, it has been an efficient technology to improve throughput, robustness, and security since the relay node not only store and forward but also process the data flow. NC was first applied in wired network [6-8]. For wireless network, its broadcast property makes the use of network coding more feasible, however, due to influence of channel fading, interference and noise, the application of network coding is difficult. Wireless two-way relay network is a typical wireless network. Recently, many researches on network coding in wireless two-way relay network draw much attention [9-13].

The system needs 4 time slots to exchange information based on traditional multi-hop network. Network coding is applied to two-way relay system for improving effectiveness, the system only need 3 time slots to exchange information [14]. So the system throughput can be increased about 33%. Ref. [15] proposed physical network coding (PNC). Ref. [16] proposed a simplified version of PNC called analog network coding (ANC). PNC and ANC are all two time slots scheme, even though the throughput is improved, PNC has synchronization problem and ANC amplifies and forwards the superimposed signal without denoising. Ref. [17] proposed soft network coding (SNC), in SNC, the system will save 50% computational complexity of channel decoding with a small degradation in BER performance compared with NC. Ref. [18] proposed a new physical layer network coding scheme called pseudo physical-layer network coding (PPNC), in PPNC, the system can save computational complexity compared with other schemes, but its BER performance is bad.

MIMO is widely used in the current communication systems, and it can increase the channel capacity [19]. To make the best use of PNC in current wireless system, the combination of PNC and MIMO can be considered. In [20], the authors study the PNC in MIMO environments, the demodulation based forwarding scheme was proposed for the MIMO TWRC. However, this scheme did not exploit the fact that relay don’t need to know the signals of each end nodes. In [21], a wireless relay network is analyzed where the source and the destination nodes are equipped with multiple antennas and each relay has a single antenna employing distributed space time code (DSTC).

This paper focuses on the bad BER performance of PPNC and proposes an improved scheme called pseudo physical-layer soft network coding (PPSNC) scheme. At the same time, this paper also discusses the combination of PPSNC and MIMO, and analyzes the BER performance of PPSNC in MIMO environment. The rest of this paper is organized as follows: Section II compares previous schemes, proposes PPSNC scheme, and discusses the performance of PPSNC in MIMO environment. Section III analyzes the performance and achievable rate. Section IV give the conclusion.

II. NETWORK CODING SCHEMES IN TWO-WAY RELAY SYSTEM

A. System Model

This paper focuses on the typical wireless two-way relay system model which contains two source nodes and one relay node, we assume that all nodes are half-duplex and equipped with single antenna, the system mode is as shown in Fig. 1. In this section, we introduce 3 time slots system model, the 2 time slots model can be seen in [15].

In Fig. 1, we assume there is no direct link between node A and B, which exchange information with the help
of node R. Signal transmission process is as follows: a). in
time slot 1, node A transmits $X_A$ to relay node R; b). in
time slot 2, node B transmits $X_B$ to relay node R; c). in
time slot 3, node R broadcast packet $X_R$ which is the
combination of $X_A$ and $X_B$. The destination node can
get the target information decoding from the received
signal with the help of self-information from the nodes.
The received signal at node R is:

$$
\begin{align*}
y_A &= h_{AR} \sqrt{P_A} x_A + n_1 \\
y_B &= h_{BR} \sqrt{P_B} x_B + n_2
\end{align*}
$$

In (1), $h_{iR}$ presents the channel gain between node i (A
or B) and node R, $P_A$, $P_B$, $P_R$ are the node A, node B and
node R transmitting power respectively. In this paper, we
assume $h_{R}=1$, $P_A=P_B=P$, $n_1$, $n_2$ present the noise at
time slot 1 and time slot 2 respectively, $n_1$, $n_2$ are
Additive White Gaussian noise (AWGN) with double-
sided power spectral density $N_0/2$. In time slot 3, the node
R performs network coding to improve the performance,
the signal processing procedure will be depicted in next
section.

![Wireless two-way relay system model](image)

**Figure 1. Wireless two-way relay system model**

**B. Previous Contributions and Proposed Scheme**

In this section, we introduce the existing network
coding schemes, and depict the new scheme—PPSNC.
All the schemes use Turbo channel coding denoted by $\Gamma$ and
BPSK modulation.

1). **NC scheme**: The signal processing procedure is
depicted in Fig. 2. Node R demodulates and decodes the
signals from node A and node B respectively and gets the
data packet $\hat{S}_A$ and $\hat{S}_B$, then combine $\hat{S}_A$ and $\hat{S}_B$:

$$
S_R = \hat{S}_A \oplus \hat{S}_B
$$

Finally, $S_R$ will be channel coded, BPSK modulated
and broadcasted to node A and node B in time slot 3.

2). **SNC scheme**: The signal processing procedure is
depicted in Fig. 3. Node R receives $Y_A$ and demodulates
it to get the hard/soft information $V_A$. Then it uses a
buffer to store $V_A$ until node R demodulates $Y_B$ and get
$V_B$, here we adopt hard decision; after that node R
combine $V_A$ and $V_B$ to get $V_R$:

$$
V_R = \frac{1 - V_A \ast V_B}{2} = \frac{1 - \hat{X}_A \ast \hat{X}_B}{2} = \frac{1 - (1 - 2D_A) \ast (1 - 2 \hat{D}_B)}{2} = \frac{1 - (1 - 2(D_A \oplus D_B))}{2}
$$

Finally the decoder decodes $V_R$ to get $S_B$:

$$
S_B = \Gamma^{-1}(V_R) = \Gamma^{-1}(\hat{D}_A \oplus \hat{D}_B) = \Gamma^{-1}(\hat{D}_A) \oplus \Gamma^{-1}(\hat{D}_B) (4)
$$

In (4), $\Gamma^{-1}$ denote turbo channel decoding process.
There are two channel decoding processes in NC scheme,
however, only one channel decoding process in SNC
scheme. So SNC scheme will save 50% channel decoding
computational complexity compared with NC.

3). **PNC scheme**: The signal processing procedure is
depicted in Fig. 4. The signals from two source nodes
combine naturally in the air. The received signal $y$ at
node R can be expressed as:

$$
y = \sqrt{P}(x_A + x_B) + n
$$

where $n$ is channel noise with double-sided power
spectral density $N_0/2$. Then node R makes a hard
decision and PNC mapping, and the decoder decodes $V_R$ to get $S_R$.
The hard decision result is:

$$
u_R = x_A + x_B = \begin{cases} -2 & s_A, s_B = 1,1 \\ 0 & s_A, s_B = 1,0,0,1 \\ 2 & s_A, s_B = 0,0 \end{cases}
$$

The PNC mapping result is:

$$
v_R = d_A + d_B = \begin{cases} 0 & u_R = \pm 2 \\ 1 & u_R = 0 \end{cases}
$$

Finally the output of channel decoder is:

$$
S_R = \Gamma^{-1}(V_R) = \Gamma^{-1}(\hat{D}_A \oplus \hat{D}_B) = \Gamma^{-1}(\hat{D}_A) \oplus \Gamma^{-1}(\hat{D}_B)
$$

We can note that the hard decision and PNC mapping
steps can be seen as a special BPSK demodulation
process and have an approximately computation
complexity with traditional BPSK demodulation. In PNC,
there are only one demodulation process and one channel
decoding process, so it can save approximately 50%
computation complexity of demodulation and channel decoding compared with NC scheme.

4). PPNC scheme: The signal processing procedure is depicted in Fig. 5. Node R receives $y_A$ and stores it to prepare for addition combination process until node R gets $y_B$.

$$y = y_A + y_B = \sqrt{P}(x_A + x_B) + n_1 + n_2$$  \hspace{1cm} (9)

See (9), we can note that it is similar to the received superimposed signal in PNC scheme. And the difference is just the doubled noise power. So we can use the same signal processing procedure as PNC scheme. In PPNC, the computational complexity of channel decoding and demodulation is less than NC and SNC. And compared with PNC, there is no synchronization problem.

5). PPSNC scheme: Compared with NC, SNC and PNC, PPNC scheme has lower computational complexity, but its BER performance gets worse. For that case, we propose an improved scheme called PPSNC. The signal processing procedure is depicted in Fig. 6. Node R receives two signals respectively and superimposes them to get y, this procedure is the same to PPNC and y in PPSNC equals to the superposed signal in (9). Node R uses mapping function in (10) to obtain the equivalence of $GF(2)$ summation of $d_A$ and $d_B$, the mapping function is called PPSNC mapping function.

$$v_R = -\frac{1}{2} \text{abs}(y_A + y_B) + 1$$  \hspace{1cm} (10)

See (10), abs () changes $y_A + y_B$ into absolute value. Then set $v_R$ to channel decoder. PPSNC use soft decision not hard decision, and the soft decision is an important part of Turbo channel decoder. This paper adopts Turbo code, the channel decoder uses Log-MAP algorithm to get the soft information, let $LLR()$ denotes the soft information.

$$LLR(s_k) = \ln \frac{p(s_k = 1)}{p(s_k = 0)} = L_s y_k^s + L_o(s_k) + \max_{s} (A_{x_s}(s) + 1/2 L_s x_k^s + B_k(s)) - \max_{s} (A_{x_s}(s) + 1/2 L_s x_k^s + B_k(s)) \tag{11}$$

where $s, y, x$ present information symbol, the received information and coded signal, the value of channel reliability is $L_s = 4aE_s / N_0$. $L_o(s_k)$ is prior information, $\max()$ is to get the maximized value, $A_{x_s}(s)$ and $B_k(s)$ present the forward recursive probability and backward recursive probability. Finally, the decoder makes a decision to the soft information $LLR(s_k)$.

$$s_k = \begin{cases} 1 & LLR(s_k) \geq 0 \\ 0 & LLR(s_k) < 0 \end{cases} \tag{12}$$

PPNC scheme adopts hard decision and that will lose the useful information. PPSNC scheme adopt soft decision to retain the soft information and restrain noise. At the same time, the computational complexity of PPSNC is lower than PPNC.

C. PPSNC in MIMO Environment

Although the BER performance of PPSNC is better than PPNC, its performance is not the best. In the next section, we will improve the PPSNC scheme, and propose a new scheme which is the combination of MIMO and PPSNC.

1) system model: As shown in Fig. 1, we assume a system with one antenna at each end node and two antennas at the relay node. We also assume that all nodes are half-duplex, there is no direct link between A and B, and node A and B exchange information with the help of node R. The received signal at node R is:
where $y_i$ denotes the received signal at the $i$-th antenna of the node $R$, $h_{ij}$ is the channel gain from node $j$ (A or B) to the $i$-th antenna of the node $R$, $P_A, P_B$ are the node A and node B transmitting power respectively, $n_i$ are Additive White Gaussian noise(AWGN) with double-sided power spectral density $N_0/2$. The equation (13) can be rewritten as followed form:

$$Y = HX + N$$

After receiving the signal $Y$, the relay node $R$ tries to get the network-coded signal of the signals from two source nodes.

2) **PPSNC in MIMO environment:** The system of MIMO-PPSNC scheme can be seen as a point-to-point 2-by-2 MIMO system, and the aim of the node $R$ is to exploit the network coded form of two end nodes’ signals. In traditional MIMO-NC scheme, the node is to get the signals from two exact source nodes and then transform them into network coded signal $s_A \oplus s_B$. However, the MIMO-PPSNC scheme does not need to know the exact signal of $s_A$ and $s_B$. It can directly make use of a mapping function to get the network coded signals. Next, we will describe the details of MIMO-PPSNC. Here we assume the channel matrix $H$ is a special one.

$$H = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$  (15)

In (15), $H$ is not a full-rank matrix and the relay cannot get $s_A$ and $s_B$ from the received signal.

In PPSNC sheme, the aim of node $R$ is to obtain $s_A \oplus s_B$ from $Y$ without first estimating $s_A$ and $s_B$. This procedure is different with MIMO-NC scheme. The signal $s_A + s_B$, which can be obtained directly from $Y$, it is a useful intermediate step as far as the estimation of $s_A \oplus s_B$ is concerned.

The signal processing procedure is depicted in Fig. 8. Node $R$ receives two signals respectively and superimposes them to get $Y$, the relay first combine the signals from the two receiving antennas.
where \( n' = \frac{n_1 + n_2 + n_3 + n_4}{2} \). After that, the node R maps y to \( x_1 \oplus x_2 \), and the mapping function is the same to PPSNC. The followed signal processing procedure is the same to PPSNC. Fig. 7 shows the signal processing procedure of MIMO-PPNC.

### III. PERFORMANCE ANALYSIS

#### A. Computation Complexity Analysis

We assume that the packets from node A and node B have the same length N. If we demodulate N-length packet based on BPSK demodulation, we need N times decision and N times mapping. Considering combination complexity, XOR computation in NC has similar complexity to addition computation in PPNC and PPSNC. In PNC, the signals combine in the air naturally, so it does not need artificial combination process. Seen (3), in SNC, the combination process contains two processes: multiplication combination and mapping. So it has higher complexity compared with XOR and addition computation. The decoder in these schemes is same. The PNC scheme always assumes symbol, carrier-frequency and carrier-phase synchronization, but it can not achieve this without synchronization mechanism, which will add additional complexity to PNC scheme. The main complexity parameters are listed in Table I according to different schemes.

We can summarize the computational complexity of PPSNC according to Table I. It is easy to see that PPSNC scheme has lower computational complexity than NC, SNC. Compared with PNC, PPSNC add a combination process which contains N times addition operation. PPSNC does not need synchronization mechanism and hard decision computation. Based on this point, PPSNC has lower computational complexity. The drawback of PPSNC is that PPSNC is a 3 time slots scheme which leads to a worse throughputs performance than PNC scheme. Seen (10), the mapping in PPSNC scheme has similar computational complexity to the special demodulation process (hard decision and mapping) in PNC scheme. So PPSNC and PPSNC can achieve the similar complexity.

#### B. Achievable Rates Analysis

We assume \( h_{AR} = h_{BR} = h \), \( P_A = P_B = P_R = P \), \( \sigma^2 = N_0 / 2 \), the sum-rate of wireless two-way relay system is

\[
R_{\text{sum}} = R_{\text{AR}} + R_{\text{BR}} = \min \{ R_{\text{AR}}, R_{\text{RB}} \} + \min \{ R_{\text{BA}}, R_{\text{RA}} \} \tag{17}
\]

where \( R_{\text{AR}} \), \( R_{\text{BR}} \) are the transmission rate of \( A \rightarrow R \rightarrow B \) and \( B \rightarrow R \rightarrow A \) respectively. \( R_{\text{AR}} \) and \( R_{\text{BR}} \) present transmission rate between source node and relay node.

(a) In traditional Multi-hop network every destination node can receive a new data packet every 4 time slots, the transmission rate of every each time slot is

\[
R_{\text{AR}} = R_{\text{BR}} = \frac{1}{4} \log_2(1 + \text{SNR}) \tag{18}
\]

The sum-rate of two-way relay system based on traditional multi-hop network is

\[
R_{\text{sum multi-hop}} = \frac{1}{2} \log_2(1 + \frac{2|h|^2P}{N_0}) \tag{19}
\]

(b) In NC, SNC, PPNC, PPSNC every destination node can receive a new data packet every 3 time slots, the transmission rate of every each time slot is

\[
R_{\text{AR}} = R_{\text{BR}} = \frac{1}{3} \log_2(1 + \text{SNR}) \tag{20}
\]

The sum-rates of two-way relay system based on NC, SNC, PPNC and PPSNC are

\[
R_{\text{sum NC}} = R_{\text{sum SNC}} = R_{\text{sum PPNC}} = R_{\text{sum PPSNC}} = \frac{2}{3} \log_2(1 + \frac{2|h|^2P}{N_0}) \tag{21}
\]

(c) In PNC, every destination node can receive a new data packet every 2 time slots, the transmission rate of every each time slot is

\[
R_{\text{AR}} = R_{\text{BR}} = \frac{1}{2} \log_2(1 + \text{SNR}) \tag{22}
\]

The sum-rate of two-way relay system based on PNC is

\[
R_{\text{sum PNC}} = \log_2(1 + \frac{2|h|^2P}{N_0}) \tag{23}
\]
Fig. 9 shows the sum-rate of each scheme, here we assume $h = 1$, the noise power is 10W. As we can see from Fig. 9, the sum-rate of PPSNC, the same as NC, SNC, and PPNC, is higher than traditional multi-hop network and lower than PNC.

C. Simulation Analysis

Based on the analysis of above network coding schemes, in this section, the simulation is done by MATLAB software. In TABLE II we list the simulation parameters.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Information packet size</td>
<td>100000 bits</td>
</tr>
<tr>
<td>Channel coding</td>
<td>Turbo code</td>
</tr>
<tr>
<td>Constraint length</td>
<td>3</td>
</tr>
<tr>
<td>Generator polynomial</td>
<td>[1 1 1 0 1]</td>
</tr>
<tr>
<td>Code rate</td>
<td>1/3</td>
</tr>
<tr>
<td>Modulation</td>
<td>BPSK</td>
</tr>
<tr>
<td>Channel model</td>
<td>Gaussian channel</td>
</tr>
</tbody>
</table>

As is shown in TABLE II, it assumes that the information packet size is 100000 bits. Turbo code is adopted with constraint length 3, generator polynomial [1 1 1 0 1], and code rate 1/3. BPSK modulation is used in every modulator module. All channels are AWGN channels. For simplicity, we assume $h = 1$.

As shown in Fig. 10, we plot the BER of NC scheme, SNC scheme, PNC scheme, PPNC scheme and PPSNC scheme. It can be seen that the SNR difference is about 3 dB between PPNC and PNC when BER is $10^{-5}$. The difference between PPNC and SNC is slightly less than 3 dB. At the same time we can see proposed PPSNC has a better BER performance than PPNC. The SNR difference is about 1 dB between PPNC and PPSNC when BER is $10^{-5}$. PPSNC also has lower computational complexity, so it is a better choice than other schemes when system focuses on low computational complexity and high power efficiency of relay nodes rather than BER performance. Although the BER of PPSNC is better than PPNC, its BER performance is worse than PNC, it needs still to be improved.

The Fig. 11 plots the BER performance of PPSNC and PPNC in MIMO environment. It shows the deference between MIMO-PPSNC, MIMO-PPNC, PPSNC and PPNC. We can see that MIMO-PPSNC scheme outperforms the other schemes. The SNR difference is about 3 dB between MIMO-PPSNC and PPSNC when BER is $10^{-5}$, and about 1dB between MIMO-PPSNC and MIMO-PPNC.

IV. CONCLUSION

Firstly this paper introduced the existing network coding schemes which are used in two-way relay wireless communication networks. The BER performance of PPNC based on hard decision is bad, for this case this paper proposes an improved scheme called PPSNC. PPSNC is based on soft decision, and it can reduce the bit error rate without increasing computational complexity. The SNR difference between PPNC and PPSNC is about 1 dB when BER is $10^{-5}$ through simulation analysis. In summary, PPSNC scheme is more suitable for the application when systems focus on low computational complexity and high power efficiency of intermediate nodes. At the same time, the performance of PPSNC in MIMO environment is also discussed, and MIMO-PPSNC scheme which is the combination of MIMO and PPSNC is proposed. Simulation result shows that the BER performance of MIMO-PPSNC is better than PPNC, PPSNC and MIMO-PPNC: MIMO-PPSNC outperforms PPSNC about 3 dB when BER is $10^{-5}$, and outperforms...
MIMO-PPNC about 1 dB when BER is $10^{-5}$. Meanwhile it has no synchronization problem compared with PNC. Due to 3 time slots, the achievable rate of MIMO-PPSNC is the same to MIMO-PPNC, PPNC, PPSNC.

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