Repeatable, extended Jacobian inverse kinematics algorithm for mobile manipulators

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Abstract An inverse kinematics algorithm for a robotic system is called repeatable, if it transforms closed paths in the task space into closed paths in the configuration space. In this paper we assume the endogenous configuration space approach and define a repeatable inverse kinematics algorithm for mobile manipulators, patterned on the extended Jacobian scheme. An examination of a dynamic system associated with this algorithm gives new insight into the mechanism of repeatability. As an illustration of the general design strategy an exemplary extended Jacobian algorithm is applied to a kinematic car-type platform carrying the POLYCRANK, 7 DOF on-board manipulator.

Key words Mobile manipulator, endogenous configuration space, inverse kinematics algorithm, extended Jacobian, repeatability

1 Introduction

In this paper by a mobile manipulator we shall mean a robotic device comprised of a nonholonomic mobile platform and a holonomic manipulator fixed to the platform. An increasing interest in mobile manipulators results both from their significant applications as well
as from a research challenge posed by their modeling, motion planning, and navigation. A comprehensive review of literature concerned with mobile manipulators has been made in [1]. Three representative more recent works that need to be mentioned in this context are: [2], where certain manipulability measures have been applied to the motion planning problem, [3], where an optimization theory based inverse kinematics algorithm for mobile manipulators has been derived, and [4] presenting a new adaptive control algorithm for task space path following, that takes into account the mobile manipulator’s dynamics.

The kinematics of a stationary or a mobile manipulator may be treated as a map from a configuration space into a task space. An inverse kinematics algorithm defines a map from the task space into the configuration space, inverse to the kinematics. The algorithm is referred to as repeatable, if closed paths in the task space are transformed into closed paths in the configuration space. Suppose that the inverse kinematics algorithm is processing a sequence of tasks in such a way that the solution of the previous task serves as an initial point of the algorithm for the next task. The property of repeatability implies that whenever a certain task repeats in the sequence, its solution each time will be the same, irrespective of the initial point. Repeatability has a very appealing geometric interpretation resulting from an observation that, outside singular configurations of the kinematics, the configuration space can be given the structure of a fiber bundle over the task space, whose fibers coincide with self-motion manifolds. Given a point in the task space, and a closed path beginning and ending at this point, an inverse kinematics algorithm lifts the path to the configuration space in such a way that images of the end points of the path belong to the fiber lying over this point. In this way, to every closed path in the task space the algorithm assigns a diffeomorphism of the fiber. Repeatability of the algorithm means that this collection of diffeomorphisms shrinks to the identity map. Equivalently, repeatability means that the connection defined by the inverse kinematics algorithm has trivial holonomy group.

There exists a rather complete collection of results concerned with re-
peatability of inverse kinematics algorithms for stationary manipulators. In [5] it has been observed that the Jacobian pseudoinverse algorithm is not repeatable. Transparent geometric conditions for repeatability have been provided in [6], and proved later in [7]. A repeatable inverse kinematics algorithm, known as the extended Jacobian algorithm, has been derived in [8] and developed further in [9,10].

A concept of repeatability of inverse kinematics algorithms for mobile manipulators has been introduced in [11], by exploiting the endogenous configuration space approach [1]. Also in [11] differential geometric necessary and sufficient conditions for repeatability have been provided, in the form of involutivity of a distribution associated with the algorithm. The integral manifolds of this distribution are invariant with respect to the algorithm’s dynamics, and form a transverse foliation of the endogenous configuration space, so that every fiber intersects the integral manifold in exactly one point. Using these conditions it may be shown that the Jacobian pseudoinverse and the adjoint Jacobian inverse kinematics algorithms for mobile manipulators are not repeatable. Since the repeatability concept in mobile manipulators bears a geometric resemblance to that for stationary manipulators, one may expect that a repeatable inverse kinematics algorithm for mobile manipulators can also be patterned upon the extended Jacobian algorithm construction. Indeed, this may be accomplished by an appropriate augmentation of the mobile manipulator kinematics in such a way that the associated distribution is annihilated by the differential of the augmenting kinematics map, making its level sets the integral manifolds of this distribution.

The main result of this paper consists in providing a design procedure of the extended Jacobian repeatable inverse kinematics algorithm for mobile manipulators, and in demonstrating that the repeatability of the extended Jacobian algorithm can be explained by examining its associated dynamic system evolving on an invariant manifold. Although the design procedure has been made formally identical to the construction for stationary manipulators presented in [8,9], two its aspects are novel and specific to mobile manipulators. First, the extended Jacobian construction is carried out in an infinite dimen-
sional configuration space, so the mobile manipulator could be regarded as an infinitely redundant system. Second, this construction remains valid when the mobile manipulator is underactuated in the sense that the number of degrees of freedom of the on-board manipulator is smaller than the dimensionality of the task space. In a limit case our construction applies even in the case when a mobile platform does not carry on board any manipulator. The dynamic system viewpoint assumed throughout this paper prompts the following explanation of repeatability. Given a sequence of inverse kinematic tasks to be solved by the extended Jacobian algorithm, a choice of the initial point for the first task fixes an invariant manifold on which the algorithm operates. It turns out that the dynamic system associated with the algorithm depends only on the task, and has an asymptotically stable equilibrium point corresponding to the zero task space error. For this reason, the configuration returned by the algorithm depends on the task, not on the initial configuration, and is unique due to transversality of the invariant manifold to the self-motions manifolds.

The composition of this paper is the following. In section 2 we briefly resume the concept of the extended Jacobian inverse kinematics algorithm for stationary manipulators. A repeatable inverse kinematics algorithm for mobile manipulators is derived in section 3. Section 4 develops a specific repeatable inverse kinematics algorithm and examines its associated dynamic system. Section 5 presents an example. The paper is concluded with section 6.

2 Stationary manipulator

Suppose that a coordinate representation of kinematics of a stationary manipulator takes the form of a map

\[
k: \mathbb{R}^p \rightarrow \mathbb{R}^r, \quad y = k(x),
\]

from the configuration space into the task space. We assume that \( p > r \), and let \( J(x) = \frac{dk}{dx}(x) \) stand for the analytic Jacobian of the manipulator. After setting \( s = p - r \) we introduce an augmenting kine-
matics map

\[ h : R^p \rightarrow R^s, \quad \tilde{y} = h(x), \]  \hspace{1cm} (2)

and the extended kinematics map

\[ l = (h, k) : R^p \rightarrow R^p, \quad \tilde{y} = (h(x), k(x)). \]  \hspace{1cm} (3)

The extended analytic Jacobian \( \tilde{J}(x) = \begin{bmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial k}{\partial x} \end{bmatrix}(x) \) is a square \( p \times p \) matrix. Outside the set \( S = \{ x \in R^p | \det \tilde{J}(x) = 0 \} \) of singular configurations, for any \( \eta \in R^r \), we define a right inverse of \( J(x) \)

\[ J^{e\#}(x)\eta = \tilde{J}^{-1}(x) \begin{pmatrix} 0 \\ \eta \end{pmatrix}. \]  \hspace{1cm} (4)

By definition, the inverse \( J^{e\#}(x) \) has the following properties

\[ \frac{\partial h}{\partial x}(x)J^{e\#}(x) = 0 \quad \text{and} \quad J(x)J^{e\#}(x) = I_r, \]  \hspace{1cm} (5)

of which the former asserts that the distribution spanned by columns of \( J^{e\#}(x) \) is annihilated by the differentials \( dh_i(x), i = 1, \ldots, s \), and the latter justifies calling \( J^{e\#}(x) \) a right inverse of the analytic Jacobian. Given a desirable task space point \( y_d \in R^r \), we denote the task space error by \( e(t) = k(x(t)) - y_d \), and define by means of (4) a dynamic system

\[ \dot{x} = -\gamma J^{e\#}(x)e \]  \hspace{1cm} (6)

for a certain \( \gamma > 0 \). Now, it is easily seen that along trajectories of (6)

\[ \dot{e} = J(x)\dot{x} = -\gamma J(x)J^{e\#}(x)e = -\gamma e, \]

i.e. the error vanishes exponentially, and \( x_d = \lim_{t \to +\infty} x(t) \) solves the inverse kinematics problem. Since by the first property of (5), the associated distribution of the algorithm (6) is involutive, the algorithm is repeatable [7].
3 Mobile manipulator

The derivation made in the previous section can be carried over to the case of mobile manipulators. Given a mobile manipulator composed of a nonholonomic mobile platform and a holonomic on-board manipulator, a control system representation of its kinematics takes the form of a driftless control system with outputs

$$\begin{align*}
\dot{q} &= G(q)u = \sum_{i=1}^{m} g_i(q)u_i, \\
y &= k(q,x),
\end{align*}$$

(7)

where $q \in R^n$ denotes the platform posture, $x \in R^p$ - the joint position of the manipulator, and $y \in R^r$ - the vector of task space coordinates. Admissible control actions $(u(\cdot), x)$ in the system (7) constitute the endogenous configuration space $X = L^2_m[0,T] \times R^p$ [1]. To every $(u(\cdot), x) \in X$ there correspond a platform trajectory $q(t) = \varphi_{q_0,t}(u(\cdot))$, and a task space trajectory $y(t) = k(q(t), x)$. The instantaneous kinematics of the mobile manipulator are a map $K_{q_0,T} : X \rightarrow R^r$ from the endogenous configuration space into the task space, defined as

$$K_{q_0,T}(u(\cdot), x) = y(T) = k(\varphi_{q_0,T}(u(\cdot)), x).$$

(8)

The kinematics determine reachable at $T$ end effector positions and orientations of the mobile manipulator subject to the control $(u(\cdot), x)$, provided that the platform starts from the initial posture $q_0$. The endogenous configuration space may be regarded as a Hilbert space with inner product

$$<(u_1(\cdot),x_1), (u_2(\cdot),x_2)> = \int_0^T u_1^T(t)u_2(t)dt + x_1^T x_2.$$  

(9)

Given an endogenous configuration $(u(\cdot), x) \in X$ we introduce the variational system associated with (7)

$$\begin{align*}
\dot{\xi} &= A(t)\xi + B(t)v, \\
\eta &= C(t,x)\xi + D(t,x)w,
\end{align*}$$

(10)
defined as the linear approximation to this system along \((u(t), x, q(t))\), whose matrices are computed in the standard way as follows

\[
A(t) = \frac{\partial}{\partial q} (G(q(t))u(t)), \quad B(t) = G(q(t)), \quad C(t, x) = \frac{\partial k}{\partial q} (q(t), x), \quad D(t, x) = \frac{\partial k}{\partial x} (q(t), x).
\]

Consequently, the analytic Jacobian

\[
J_{q_0,T}(u(\cdot), x)(v(\cdot), w) = C(T, x) \int_0^T \Phi(T, t)B(t)v(t)dt + D(T, x)w,
\]

of the mobile manipulator at the endogenous configuration \((u(\cdot), x)\) is identified with the output reachability map at \(T\) of the variational system (10) initialized at \(\xi_0 = 0\). The transition matrix \(\Phi(t, s)\) satisfies the usual evolution equation

\[
\frac{\partial}{\partial t} \Phi(t, s) = A(t)\Phi(t, s), \quad \Phi(s, s) = I_n.
\]

Now, we let \(p \geq r\) and set \(s = p - r\). By imitating the developments accomplished in section 2, we augment the kinematics (8) with a map

\[
H_{q_0,T} : X \longrightarrow L_m^2[0, T] \times R^i,
\]

consisting of two components

\[
H_{1q_0,T} : X \longrightarrow L_m^2[0, T], \quad H_{2q_0,T} : X \longrightarrow R^i,
\]

and define an extended kinematics map

\[
L_{q_0,T} = (H_{1q_0,T}, H_{2q_0,T}, K_{q_0,T}) : X \longrightarrow X.
\]

In the case of \(p < r\) we proceed by making an orthogonal decomposition \(L_m^2[0, T] \cong L_m^2[0, T] \oplus R^{r-p}\) that results in the representation \(X \cong L_m^2[0, T] \times R^r\), and then introduce one augmenting map \(H_{q_0,T} : X \longrightarrow L_m^2[0, T]\).

For the sake of similarity with stationary manipulators, in what follows we shall concentrate on the case when \(p \geq r\). The analytic Jaco-
bian \( J_{q_0,T}(u(\cdot),x) : X \rightarrow X \) of (14) can be computed as

\[
J_{q_0,T}(u(\cdot),x) = (DH_{1,q_0,T}(u(\cdot),x), DH_{2,q_0,T}(u(\cdot),x), J_{q_0,T}(u(\cdot),x)).
\]

As before, by

\[
S = \{(u(\cdot),x) \in X | \tilde{J}_{q_0,T}(u(\cdot),x) \text{ is not a linear isomorphism}\}
\]

we denote singularities of the extension (14). Outside the singular set the map \( \tilde{J}_{q_0,T}(u(\cdot),x) \) is invertible, and gives rise to a right inverse

\[
J_{q_0,T}^E(u(\cdot),x) : R^r \rightarrow X
\]

of the analytic Jacobian, called extended Jacobian inverse, defined as

\[
J_{q_0,T}^E(u(\cdot),x) \eta = J_{q_0,T}^{-1}(u(\cdot),x)(0(\cdot),0,\eta), \tag{16}
\]

for the zero function \( 0(\cdot) \in \tilde{L}_m^2[0,T], 0 \in R^{p-r} \), and \( \eta \in R^r \).

By definition, the inverse (16) has the following properties, cf (5),

\[
DH_{q_0,T}(u(\cdot),x)J_{q_0,T}^E(u(\cdot),x) = 0(\cdot), \quad J_{q_0,T}(u(\cdot),x)J_{q_0,T}^E(u(\cdot),x) = I_r. \tag{17}
\]

Now, for a desirable taskspace point \( y_d \in R^r \) we choose a smooth curve \( (u_\theta(\cdot),x(\theta)) \in X \), compute an error \( e(\theta) = K_{q_0,T}(u_\theta(\cdot),x(\theta)) - y_d \), and use the inverse (16) in order to define a dynamic system

\[
\frac{d}{d\theta}(u_\theta(t),x(\theta)) = -\gamma \left( J_{q_0,T}^E(u_\theta(\cdot),x(\theta))e(\theta) \right)(t), \tag{18}
\]

where \( \gamma > 0 \). This system defines the extended Jacobian inverse kinematics algorithm for mobile manipulators. After the differentiation of the error along a trajectory of (18) and suitable substitutions we obtain

\[
\frac{d}{d\theta}e(\theta) = J_{q_0,T}(u_\theta(\cdot),x(\theta)) \frac{d}{d\theta}(u_\theta(\cdot),x(\theta)) = -\gamma J_{q_0,T}(u_\theta(\cdot),x(\theta))J_{q_0,T}^E(u_\theta(\cdot),x(\theta))e(\theta) = -\gamma e(\theta),
\]

and conclude that the error vanishes exponentially, what means that the system (18) provides a solution to the inverse kinematics problem,
so that

\[ (u_d(t), x_d) = \lim_{\theta \to +\infty} (u_\theta(t), x(\theta)) \]

on condition that the limit exists.

The associated distribution of this inverse kinematics algorithm consists of vector fields \( J_{q_0,T}^E(u(\cdot), x) e_i, \) \( i = 1, 2, \ldots, r \), where \( e_i \) is the \( i \)th unit vector in \( \mathbb{R}^r \). By the first property of (17) this distribution is involutive, so the algorithm (18) is repeatable [11]. Moreover, by design of the algorithm, the augmenting map (13) remains constant along the solutions of (18).

4 A derivation for \( p \geq r \geq m \)

The result of the previous section might be considered as a design procedure of repeatable inverse kinematics algorithms for mobile manipulators, dependent on the choice of the augmenting map (13). Below we shall present a specific inverse \( J_{q_0,T}^E(u(\cdot), x) \), derived in agreement with this procedure. We consider a mobile manipulator with endogenous configuration space \( X = L^2_m[0, T] \times \mathbb{R}^p \), and the task space \( \mathbb{R}^s \). Let \( K_{q_0,T}(u(\cdot), x) \) denote the instantaneous kinematics, and \( J_{q_0,T}(u(\cdot), x) \) be the analytic Jacobian. Having set \( s = p - r \) we introduce a pair of augmenting maps

\[ H_{q_0,T}(u(\cdot), x) = (H_1_{q_0,T}(u(\cdot), x), H_2_{q_0,T}(u(\cdot), x)) \]  

such that \( H_1_{q_0,T}(u(\cdot), x) : X \rightarrow L^2_m[0, T] \) and \( H_2_{q_0,T}(u(\cdot), x) : X \rightarrow \mathbb{R}^r \). The extended kinematics map (14) takes the form

\[ L_{q_0,T}(u(\cdot), x) = (H_{q_0,T}(u(\cdot), x), K_{q_0,T}(u(\cdot), x)). \]  

The analytic Jacobian is equal to

\[ \tilde{J}_{q_0,T}(u(\cdot), x) = (DH_{q_0,T}(u(\cdot), x), J_{q_0,T}(u(\cdot), x)), \]
where the derivative decomposes into partial derivatives

\[
DH_{q_0,T}(u(\cdot),x)(v(\cdot),w) = (D_{u(\cdot)}H_{1,q_0,T}(u(\cdot),x)v(\cdot), D_xH_{1,q_0,T}(u(\cdot),x)w, \\
D_{u(\cdot)}H_{2,q_0,T}(u(\cdot),x)v(\cdot), D_xH_{2,q_0,T}(u(\cdot),x)w).
\]

The inverse (16) is defined as

\[
J_{q_0,T}^E(u(\cdot),x)\eta = J_{q_0,T}^{-1}(u(\cdot),x)(0(\cdot),0,\eta). \tag{22}
\]

Let us set \(J_{q_0,T}^E(u(\cdot),x)\eta = (v(\cdot),w)\), so \(J_{q_0,T}(u(\cdot),x)(v(\cdot),w) = (0(\cdot),0,\eta)\). This last identity is equivalent to

\[
\begin{align*}
D_{u(\cdot)}H_{1,q_0,T}(u(\cdot),x)v(\cdot) + D_xH_{1,q_0,T}(u(\cdot),x)w &= 0(\cdot) \\
D_{u(\cdot)}H_{2,q_0,T}(u(\cdot),x)v(\cdot) + D_xH_{2,q_0,T}(u(\cdot),x)w &= 0 \tag{23} \\
J_{q_0,T}(u(\cdot),x)(v(\cdot),w) &= \eta.
\end{align*}
\]

Suppose that the map \(D_{u(\cdot)}H_{1,q_0,T}(u(\cdot),x)\) is invertible for every \(x \in \mathbb{R}^p\). Then, from the first equality in (23) we compute

\[
v(\cdot) = - (D_{u(\cdot)}H_{1,q_0,T})^{-1}(u(\cdot),x)D_xH_{1,q_0,T}(u(\cdot),x)w. \tag{24}
\]

After a substitution of (24) into the second and the third equality in (23) we obtain

\[
P(u(\cdot),x)w = (0,\eta), \tag{25}
\]

where

\[
P(u(\cdot),x) =
\begin{bmatrix}
D_xH_{2,q_0,T} - D_{u(\cdot)}H_{2,q_0,T} \left( D_{u(\cdot)}H_{1,q_0,T} \right)^{-1} D_xH_{1,q_0,T} \\
D(T,x) - C(T,x) \int_0^T \Phi(T,t)B(t) \left( D_{u(\cdot)}H_{1,q_0,T} \right)^{-1} D_xH_{1,q_0,T}(u(\cdot),x)(t)dt
\end{bmatrix}.
\]

Assuming invertibility of the matrix \(P(u(\cdot),x)\), we are in a position to find \(w = P^{-1}(u(\cdot),x)(0,\eta)\) and compute the extended Jacobian in-
The above formula gives rise to the following form of the extended Jacobian inverse kinematics algorithm (18)

\begin{align}
\frac{du_0(t)}{d\theta} &= \gamma \left( D_{u(\cdot)H_1q_0,T}^{-1}(u_\theta(\cdot),x(\theta))D_xH_1q_0,T(u_\theta(\cdot),x(\theta))w(\theta)(t) \right) \\
\frac{dx(\theta)}{d\theta} &= -\gamma w(\theta)
\end{align}

(27)

where \( w(\theta) = P^{-1}(u_\theta(\cdot),x(\cdot))(0,e(\theta)). \)

Now, let us suppose for a while that \( p = r \), denote by \( E(u(\cdot),x) \) the lower block row of matrix \( P(u(\cdot),x) \) (the upper row will be absent), and look more closely at the inverse kinematics algorithm (27) solving a sequence of inverse kinematic tasks. Given an initial condition \((u_0(\cdot),x_0)\) of the algorithm, we deduce from the invariant manifold property that (now we have \( H_1q_0,T = H_{q_0,T} \))

\[ H_{q_0,T}(u_\theta(\cdot),x(\theta)) = H_{q_0,T}(u_0(\cdot),x_0). \]

(28)

This in turn yields that

\[ D_{u(\cdot)H_{q_0,T}}(u_\theta(\cdot),x(\theta))\frac{du_\theta(\cdot)}{d\theta} + D_xH_{q_0,T}(u_\theta(\cdot),x(\theta))\frac{dx(\theta)}{d\theta} = 0, \]

i.e.

\[ \frac{du_\theta(\cdot)}{d\theta} = -\left( D_{u(\cdot)H_{q_0,T}}^{-1}(u_\theta(\cdot),x(\theta))D_xH_{q_0,T}(u_\theta(\cdot),x(\theta)) \right)\frac{dx(\theta)}{d\theta}. \]

Thus, the operation of the extended Jacobian algorithm is determined solely by the dynamics of \( \frac{dx(\theta)}{d\theta} \). Furthermore, since \( D_{u(\cdot)H_{q_0,T}}(u(\cdot),x) \) is invertible, using the implicit function theorem, we can compute from (28) \( u_\theta(\cdot) = F(u_0(\cdot),x_0,x(\theta)) \). This being so, we conclude that in fact \( E(u_\theta(\cdot),x(\theta)) = E(u_0(\cdot),x_0,x(\theta)) \), and that the taskspace error

\[ J^E_{q_0,T}(u(\cdot),x)\eta = (v(\cdot),w) = \\
\left[ -\left(D_{u(\cdot)H_{q_0,T}}^{-1}(u(\cdot),x)D_xH_{q_0,T}(u(\cdot),x) \right) \right] P^{-1}(u(\cdot),x)(0,\eta). \]

(26)
\( e(\theta) = e_d(\theta) = K_{q_0,T}(u_0(\cdot), x_0, x(\theta)) - y_d \), the subscript \( d \) showing a dependence on the desirable taskspace point \( y_d \). Eventually, we get

\[
\frac{d x(\theta)}{d \theta} = -\gamma E^{-1}(u_0(\cdot), x_0, x(\theta))e_d(\theta),
\]  

(29)

whereas \( u_\theta(\cdot) = F(u_0(\cdot), x_0, x(\theta)) \). It easy to see that \( e_d(\theta) = 0 \) is the unique equilibrium point of the dynamic system (29). Now, let \((u_0(\cdot), x_0)\) and \((\bar{u}_0(\cdot), \bar{x}_0)\) denote a pair of initial points of the extended Jacobian algorithm such that

\[
H_{q_0,T}(u_0(\cdot), x_0) = H_{q_0,T}(\bar{u}_0(\cdot), \bar{x}_0) = H_{q_0,T}(u_\theta(\cdot), x(\theta)).
\]

Obviously, it holds that \( F(u_0(\cdot), x_0, x(\theta)) = F(\bar{u}_0(\cdot), \bar{x}_0, x(\theta)) \) so from both initial points the dynamics of the inverse kinematics algorithm are governed by the same dynamic system (29) that is locally asymptotically stable with equilibrium point corresponding to \( e_d(\theta) = 0 \). We conclude that given the taskspace location \( y_d \), the algorithm initialized anywhere on the invariant manifold will produce an endogenous configuration lying on this manifold and yielding \( y_d \). Uniqueness of this configuration results from repeatability. More details concerned with the issue of repeatability of extended Jacobian algorithms can be found in [12].

5 Example

For illustration of the inverse kinematics algorithm developed in section 4 we shall consider a mobile manipulator composed of the kinematic car mobile platform equipped with the POLYCRANK onboard manipulator. The POLYCRANK is a unique design of a fast 7 d.o.f. manipulator without joint limits and nearly diagonal inertia matrix, created at the Institute of Aeronautics and Applied Mechanics, Warsaw University of Technology [13,14]. Its kinematics expressed in Cartesian coordinates + ZXZ Euler angles are represented by the fol-
lowing control system with outputs

\[
\begin{align*}
\dot{q}_1 &= u_1 \cos q_3 \cos q_4, \\
\dot{q}_2 &= u_1 \sin q_3 \cos q_4, \\
\dot{q}_3 &= u_1 \sin q_4, \\
\dot{q}_4 &= u_2,
\end{align*}
\]
\[
y = (y_1, y_2, \ldots, y_6) = (q_1 + l_1 \cos (q_3 + x_1) + l_2 \cos x_2 + l_6 \cos x_3 + (l_4 \sin x_4 + l_5 \sin x_5) \sin x_3, q_2 + l_1 \sin (q_3 + x_1) + l_2 \sin x_2 + l_6 \sin x_3 - (l_4 \sin x_4 + l_5 \sin x_5) \cos x_3, l_3 + l_4 \cos x_4 + l_5 \cos x_5, x_3 + q_3, x_6, x_7).
\]
\]

We propose the following augmenting kinematics map (19)

\[
H_{q_0,T}(u(\cdot),x(t)) = \begin{pmatrix} u_1(t) \\ x_1^2 + \varepsilon \end{pmatrix}, \quad H_{q_0,T}(u(\cdot),x) = x_4 + x_5,
\]

for a positive constant \( \varepsilon \). After accomplishing necessary mathematical developments along the lines described in section 4, the following extended Jacobian inverse kinematics algorithm (27) for the mobile manipulator (30) has been obtained

\[
\begin{align*}
\frac{du(t)}{d\theta} &= 2 \text{diag} \left\{ \frac{x_1 u_1(t)}{x_1^2 + \varepsilon}, \frac{x_2 u_2(t)}{x_2^2 + \varepsilon} \right\} P_{72} \frac{dx(\theta)}{d\theta}, \\
\frac{dx(\theta)}{d\theta} &= -\gamma P^{-1}(u_\theta(\cdot),x(\theta))(0,e(\theta)),
\end{align*}
\]

where \( P_{72} \) denotes the projection matrix onto the first two coordinates of \( R^7 \), \( P(u(\cdot),x) = \begin{bmatrix} e_4^T + e_5^T \\ E(u(\cdot),x) \end{bmatrix} \), \( e_4, e_5 \) stand for suitable unit vectors in \( R^7 \), and

\[
E(u(\cdot),x) = D(T,x) + 2C(T,x) \int_0^T \Phi(T,t)B(t) \begin{bmatrix} x_1 u_1(t) \\ x_2 u_2(t) \end{bmatrix}, 0 \end{bmatrix} dt.
\]

Notice that since the augmenting map \( H_{q_0,T}(u(\cdot),x) = H_{q_0,T}(u_0(\cdot),x_0) \),
during the operation of the inverse kinematics algorithm (31) we have

\[ u_1(t) = \frac{x_1^2 + \varepsilon}{x_{01}^2 + \varepsilon} u_{01}(t), \quad u_2(t) = \frac{x_2^2 + \varepsilon}{x_{02}^2 + \varepsilon} u_{02}(t) \quad \text{and} \quad x_4 + x_5 = x_{04} + x_{05}. \]

This simplifies the algorithm considerably by reducing it merely to solving 6 equations out of the system

\[ \frac{d x(\theta)}{d \theta} = -\gamma P^{-1}(u_\theta(\cdot), x(\theta))(0, e(\theta)). \]

Using advantages of this last algorithm we have solved an exemplary inverse kinematics problem assuming the following data. The length of the car is set to 1. Geometric parameters of the POLYCRANK are equal to \( l_1 = 0.2975, \ l_2 = 0.18, \ l_3 = 1.552, \ l_4 = l_5 = 0.16, \ l_6 = 0.2562. \) Initial values of platform coordinates have been taken as \( q(0) = (-10, 1, 0, 0), \) initial joint positions of the onboard manipulator \( x(0) = (\pi/2, 0.1, \pi/2, 0.1, -0.1, \pi/2, \pi/2). \) The controls have been chosen in the form of truncated Fourier series containing constant terms and first order harmonics. Initial values of constant terms, respectively for \( u_1(t) \) and \( u_2(t), \) are 1 and \( -0.1, \) initial amplitudes of all harmonics are set to 0. The control time horizon \( T = 1. \) The algorithm’s parameters \( \gamma = 0.5, \ \varepsilon = 10^{-6}. \) A sample of results of computations is shown in figure 1.

6 Conclusion

Using the endogenous configuration space approach we have presented a method of constructing repeatable inverse kinematics algorithms for mobile manipulators. A pattern followed in the derivation has been the concept of extended Jacobian inverse kinematics algorithm for stationary manipulators. A basic tool of our method is an infinite dimensional augmenting kinematics map. The level sets of this map define invariant manifolds of the inverse kinematics algorithm. This construction we have made specific for the case when the number of degrees of freedom of onboard manipulator is greater than or equal to the taskspace dimension. It has been observed that having
Fig. 1. Solution of an exemplary inverse kinematics problem: desirable taskspace point \( y_d = (0, 0, 1.6, 0, 0, 0) \), number of iterations 236, final error 9.846638e-13.

restricted to the invariant manifold in the endogenous configuration space, and fixed a task, the algorithm will always deliver the same inverse kinematics solution, irrespective of its initial point. A general theory has been illustrated with the solution of an exemplary inverse kinematics problem for the kinematic car platform equipped with a 7 DOF on-board manipulator.

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