MIMO Radar and Communication Spectrum Sharing with Clutter Mitigation

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Outline

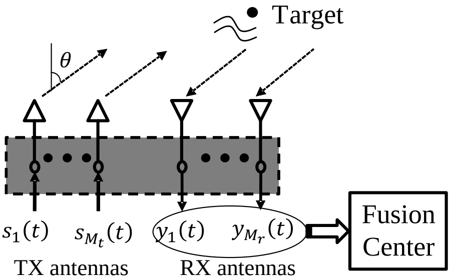
1. Introduction to MIMO Radars and Motivations
2. The Coexistence Signal Model
3. Spectrum Sharing with Clutter Mitigation
4. Simulation Results
5. Conclusions

Outline

Introduction to MIMO Radars

MIMO Radar:

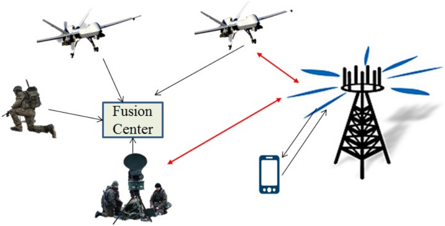
- independent waveforms, omnidirectional illumination
- high spatial resolution
- flexibility in waveform design

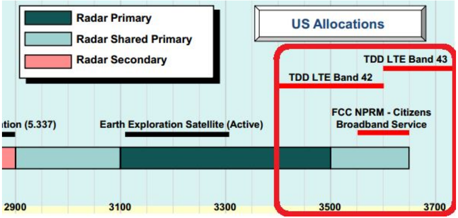


Motivation of Radar and Comm. Spectrum Sharing

[Lackpour et al, 11], [Sodagari et al, 12]

- Radar and communication systems may coexist and overlap in the spectrum.





- Existing spectrum sharing approaches basically include three categories.
  - Avoiding interference by large spatial separation.
  - Dynamic spectrum access based on spectrum sensing.
  - **Spatial multiplexing enabled by the multiple antennas at both the radar and communication systems.**

Existing MIMO based Spectrum Sharing Approaches

- **Spatial multiplexing enabled by the multiple antennas at both the radar and communication systems**
  - Projecting radar waveforms onto the interference channel null space [Sodagari et al, 12].
  - Spatial filtering to reject interference from the communication systems to the radar receiver [Deng et al, 13].

Existing approaches are non-cooperative.

Cooperative Spectrum Sharing

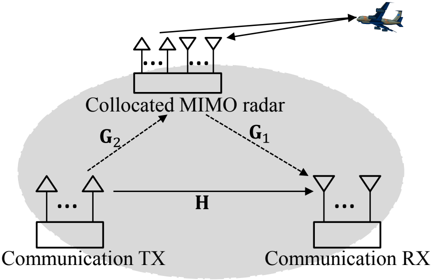
- What information should be shared and how? - feasibility
- What are the performance metrics? - heterogeneousness
- What is the overall objective? - fairness
- What algorithm should be used? - complexity

Outline

The Coexistence Signal Model

Consider a MIMO communication system which coexists with a MIMO-MC radar system as shown below. Assumptions:

- Flat fading channel, narrow band radar and comm. signals;
- Block fading: the channels remain constant for at least one PRI;
- The two systems are time-synchronized and have the same symbol rate;
- The two systems cooperate on channel estimation and feedback.



Received Signal by The MIMO Radar

The discrete time signal received by the radar for $l \in \mathbb{N}_{\tilde{L}}^{+}$ equals

$$\mathbf{y}_R(l) = \underbrace{\beta_0 \mathbf{v}_r(\theta_0)\mathbf{v}_t^T(\theta_0)\mathbf{P}\mathbf{s}(l-l_0)}_{\text{Target echoes}} + \underbrace{\mathbf{G}_2\mathbf{x}(l)e^{j\alpha_2(l)}}_{\text{Interference}} + \underbrace{\sum_{k=1}^{K}\beta_k\mathbf{v}_r(\theta_k)\mathbf{v}_t^T(\theta_k)\mathbf{P}\mathbf{s}(l-l_0)}_{\text{Clutter echoes}} + \underbrace{\mathbf{w}_R(l)}_{\text{Noise}},$$

where

- $M_{t,R}$ $M_{r,R}$, # of radar TX/RX antennas; $M_{t,C}$ $M_{r,C}$, # of comm. TX/RX antennas;
- L, length of the waveform; $\tilde{L}$, # of samples in one PRI; K, # of point clutters;
- $\mathbf{v}_t(\theta) \in \mathbb{C}^{M_{t,R}}$, $\mathbf{v}_r(\theta) \in \mathbb{C}^{M_{r,R}}$, TX/RX steering vectors;
- $\beta_k \sim \mathcal{CN}(0, \sigma_{\beta k}^2), \forall k \in \mathbb{N}_K$, target/clutter RCS,
- $\mathbf{P} \in \mathbb{C}^{M_{t,R} \times M_{t,R}}$, the transmit precoding matrix;
- $\mathbf{s}(l) \in \mathbb{C}^{M_{t,R}}$, l-th column of coded, orthonormal MIMO radar waveform;
- $\mathbf{G}_2 \in \mathbb{C}^{M_{r,R} \times M_{t,C}}$: the interference channel communication TX antennas → radar;
- $\mathbf{x}(l)$: the communication waveform.
- $e^{j\alpha_{2l}}$, the random phase offset between the MIMO radar and the comm. system. $\{\alpha_{2l}\}_{l=1}^{L}$ are distributed as $\mathcal{N}(0, \sigma_\alpha^2)$, where σ_α^2 is small [Razavi, 96].

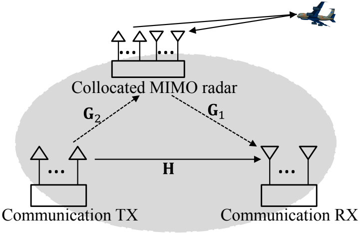
Received Signal by The Comm. System

The discrete time signal received by the comm. system equals

$$\mathbf{y}_C(l) = \underbrace{\mathbf{H}\mathbf{x}(l)}_{\text{Signal}} + \underbrace{\mathbf{G}_1\mathbf{P}\mathbf{s}(l)e^{j\alpha_1(l)}}_{\text{Interference}} + \underbrace{\mathbf{w}_C(l)}_{\text{Noise}}, \quad l \in \mathbb{N}^+_{\tilde{L}}, \tag{1}$$

where

- $\mathbf{H} \in \mathbb{C}^{M_{r,C} \times M_{t,C}}$: the communication channel;
- $\mathbf{G}_1 \in \mathbb{C}^{M_{r,C} \times M_{t,R}}$: the interference channel radar → communication RX antennas;
- $\mathbf{x}(l) \sim \mathcal{CN}(0, \mathbf{R}_x)$: the communication waveform.
- $e^{j\alpha_{1l}}$, the random phase offset between the MIMO radar and the comm. system.



Previous Work and Contribution In This Work

Method 1 [Li & Petropulu, ICASSP 2015]

- **Cooperation** on channel estimation and feedback.
- Directly subtract the radar interference based on **shared knowledge of radar waveform**. (Residual exists due to the random phase offset between radar and comm. systems.)
- Design $\mathbf{R}_{xl}$ to minimize interference to radar while achieving certain comm. rate
- Radar shares its waveform with the comm. system
- Precoding and clutter were not considered

Method 2 [Li & Petropulu, ICASSP 2016]

- **Cooperation** on channel estimation and feedback.
- Design $\mathbf{R}_{xl}$ and $\mathbf{P}$ to maximize radar SINR while achieving certain comm. rate
- Clutter was not considered

Main Contribution In This Work

- Spectrum sharing in the presence of point clutters
- An efficient algorithm based on SOCP

Outline

Cooperation & Knowledge Shared

- Cooperate on estimation and feedback of $\mathbf{G}_1$ & $\mathbf{G}_2$.
- Jointly design the $\mathbf{R}_x$ and $\mathbf{P}$.

Performance Metrics

- The Communication Rate
  - The covariance of interference plus noise in *two periods*:

$$\mathbf{R}_{\mathrm{Cin}l} = \begin{cases} \mathbf{G}_1\mathbf{\Phi}\mathbf{G}_1^H + \sigma_C^2\mathbf{I} & l \in \mathbb{N}_L^+ \\ \sigma_C^2\mathbf{I} & l \in \mathbb{N}_{\tilde{L}}^+ \setminus \mathbb{N}_L^+ \end{cases} \quad \text{where } \mathbf{\Phi} \triangleq \mathbf{P}\mathbf{P}^H/L \text{ is PSD.}$$

  - A lower bound on the *instaneous* information rate $\underline{C}(\mathbf{R}_x, \mathbf{\Phi}) \triangleq \log_2 \left| \mathbf{I} + \mathbf{R}_{\mathrm{Cin}l}^{-1}\mathbf{H}\mathbf{R}_x\mathbf{H}^H \right|$.
  - The average communication rate over $\tilde{L}$ symbols

$$\mathrm{C}_{\mathrm{avg}}(\mathbf{R}_x, \mathbf{\Phi}) \triangleq L/\tilde{L}\,\underline{C}(\mathbf{R}_x, \mathbf{\Phi}) + (1 - L/\tilde{L})\underline{C}(\mathbf{R}_x, \mathbf{0}), \tag{2}$$

- The Radar SINR
  - The clutter covariance matrix is *signal dependent* $\mathbf{R}_c = \sum_{k=1}^{K} \mathbf{C}_k\mathbf{\Phi}\mathbf{C}_k^H$ with $\mathbf{C}_k = \sigma_{\beta k}\mathbf{v}_r(\theta_k)\mathbf{v}_t^T(\theta_k)$.
  - The radar SINR:

$$\mathrm{SINR}(\mathbf{R}_x, \mathbf{\Phi}) = \mathrm{Tr}\left((\mathbf{R}_{\mathrm{Rin}} + \mathbf{R}_c)^{-1}\,\mathbf{D}_0\mathbf{\Phi}\mathbf{D}_0^H \right), \tag{3}$$

where $\mathbf{R}_{\mathrm{Rin}} \triangleq \mathbf{G}_2\mathbf{R}_x\mathbf{G}_2^H + \sigma_R^2\mathbf{I}$ and $\mathbf{D}_0 = \sigma_{\beta 0}\mathbf{v}_r(\theta_0)\mathbf{v}_t^T(\theta_0)$.

The Design Objective and Constraints

The Design Objective

- Maximizing the radar signal-to-interference-plus-noise ratio (SINR) $\text{SINR}(\mathbf{R}_x, \mathbf{\Phi})$

Design Constraints

- The power budget at the radar transmitter: $L\text{Tr}(\mathbf{\Phi}) \leq P_R$,
- The power budget at the communication transmitter: $\tilde{L}\text{Tr}(\mathbf{R}_x) \leq P_C$,
- The requirement on the average communication rate achieved during the $\tilde{L}$ symbol periods: $\text{C}_{\text{avg}}(\mathbf{R}_x, \mathbf{\Phi}) \geq C$.

$$(\mathbf{P}_1) \quad \max_{\mathbf{R}_x \succeq 0, \mathbf{\Phi} \succeq 0} \text{SINR}, \text{ s.t. } \text{C}_{\text{avg}}(\mathbf{R}_x, \mathbf{\Phi}) \geq C, \tag{4a}$$

$$\tilde{L}\text{Tr}\left(\mathbf{R}_x\right) \leq P_C, L\text{Tr}\left(\mathbf{\Phi}\right) \leq P_R. \tag{4b}$$

- The objective is a non-convex function of $\mathbf{\Phi}$. We propose to maximize a lower bound of the objective function

$$\text{SINR} \geq \frac{\sigma_{\beta 0}^2 M_{r,R}^2 \text{Tr}(\mathbf{\Phi}\mathbf{D}_t)}{\text{Tr}(\mathbf{\Phi}\mathbf{C}) + \text{Tr}(\mathbf{R}_x\mathbf{B}) + \sigma_R^2 M_{r,R}}, \tag{5}$$

where $\mathbf{D}_t \triangleq \mathbf{v}_t^*(\theta_0)\mathbf{v}_t^T(\theta_0)$, $\mathbf{C} \triangleq \sum_{k=1}^K \mathbf{C}_k^H \mathbf{v}_r(\theta_0)\mathbf{v}_r^H(\theta_0)\mathbf{C}_k$ and $\mathbf{B} \triangleq \mathbf{G}_2^H \mathbf{v}_r(\theta_0)\mathbf{v}_r^H(\theta_0)\mathbf{G}_2$.

The Approximate Optimization Problem

$$(\mathbf{P}_1') \ \max_{\mathbf{R}_x \succeq 0, \mathbf{\Phi} \succeq 0} \frac{\sigma_{\beta 0}^2 M_{r,R}^2 \mathrm{Tr}(\mathbf{\Phi D}_t)}{\mathrm{Tr}(\mathbf{\Phi C}) + \mathrm{Tr}(\mathbf{R}_x \mathbf{B}) + \sigma_R^2 M_{r,R}}, \quad \text{s.t. same constraints as} (\mathbf{P}_1). \tag{6}$$

Alternate optimization is applied to solve $(\mathbf{P}_1')$.

- The alternating iteration w.r.t. $\mathbf{R}_x$ with fixed $\mathbf{\Phi}$: convex, SDP

$$\min_{\mathbf{R}_x \succeq 0} \mathrm{Tr}(\mathbf{R}_x \mathbf{B}) \text{ s.t. } \mathrm{C}_{\text{avg}}(\mathbf{R}_x, \mathbf{\Phi}) \geq C, \tilde{L}\mathrm{Tr}(\mathbf{R}_x) \leq P_C. \tag{7}$$

- The alternating iteration w.r.t. $\mathbf{\Phi}$ with fixed $\mathbf{R}_x$: the constraint is non-convex, solve with the sequential convex programming

$$(\mathbf{P}_{\mathbf{\Phi}}) \ \max_{\mathbf{\Phi} \succeq 0} \frac{\mathrm{Tr}(\mathbf{\Phi D}_t)}{\mathrm{Tr}(\mathbf{\Phi C}) + \rho}, \text{ s.t. } \mathrm{Tr}(\mathbf{\Phi A}) \leq \tilde{C}/L, \mathrm{Tr}(\mathbf{\Phi}) \leq P_R/L. \tag{8}$$

where $\mathbf{A} \triangleq -\left(\frac{\partial C_{\text{avg}}(\mathbf{R}_x, \mathbf{\Phi})}{\partial \Re(\mathbf{\Phi})}\right)^T_{\mathbf{\Phi}=\bar{\mathbf{\Phi}}}$, the constant $\tilde{C}$ is introduced by the first order Taylor approximation of $C_{\text{avg}}(\mathbf{R}_x, \mathbf{\Phi})$, $\rho = \mathrm{Tr}(\mathbf{R}_x \mathbf{B}) + \sigma_R^2 M_{r,R}$, and $\bar{\mathbf{\Phi}}$ is updated as the solution of the previous repeated problem.

An Efficient SOCP Algorithm for ($\mathbf{P}_{\boldsymbol{\Phi}}$)

- ($\mathbf{P}_{\boldsymbol{\Phi}}$) could be formulated as a SDP via Charnes-Cooper Transformation.
- A more efficient SOCP algorithm is proposed based on the following

Proposition 2

Suppose ($\mathbf{P}_{\boldsymbol{\Phi}}$) is feasible. Then ($\mathbf{P}_{\boldsymbol{\Phi}}$) always has rank one solution.

Proof: Karush-Kuhn-Tucker conditions show that the optimal solution of ($\mathbf{P}_{\boldsymbol{\Phi}}$) must be rank one and unique.

Algorithm 1 The proposed algorithm for spectrum sharing with clutter mitigation ($\mathbf{P}_1'$).

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Input: $\mathbf{D}_0, \mathbf{C}_n, \mathbf{H}, \mathbf{G}_1, \mathbf{G}_2, P_{C/R}, C, \sigma^2_{C/R}, \delta_1$
Initialization: $\boldsymbol{\Phi} = \frac{P_R}{LM_{t,R}}\mathbf{I}$, $\mathbf{R}_x = \frac{P_C}{\tilde{L}M_{t,C}}\mathbf{I}$;
repeat
    Update $\mathbf{R}_x$ by solving (7) with fixed $\boldsymbol{\Phi}$;
    Update $\boldsymbol{\Phi}$ by solving a sequence of approximated problem ($\mathbf{P}_{\boldsymbol{\Phi}}$), which is in turn
    achieved by bisection search and SOCP solvers;
until $|\mathrm{SINR}^n - \mathrm{SINR}^{n-1}| < \delta_1$
Output: $\mathbf{R}_x, \mathbf{P} = \sqrt{L}(\boldsymbol{\Phi}^n)^{1/2}$
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Simulation Setup

- $M_{t,R} = M_{r,R} = 16, M_{t,C} = 8, M_{r,C} = 4.$ $\tilde{L} = 32,$ $L = 8,$ $\sigma_C^2 = \sigma_R^2 = 0.01.$
- One stationary target with RCS variance $\sigma_{\beta 0}^2 = 5 \times 10^{-5}$, and eight point clutters with identical RCS variances $\sigma_\beta^2 \rightarrow$ clutter to noise ratio (CNR) $10 \log \sigma_\beta^2 / \sigma_R^2$.
- θ_0 is randomly generated; clutter scatters are with angles in $[\theta_0 - 20^\circ, \theta_0 - 10^\circ]$ and $[\theta_0 + 10^\circ, \theta_0 + 20^\circ]$.
- $C = 24$ bits/symbol and $P_C = \tilde{L} M_{t,C}$ (the power is normalized by the power of the radar waveform).
- $\mathbf{G}_1$ and $\mathbf{G}_2$ are with entries i.i.d. $\mathcal{CN}(0, 0.1)$. $\mathbf{H}$ has entries i.i.d. $\mathcal{CN}(0, 1)$.
- Methods for comparison
  - the proposed method based on SOCP - "precoding with clutter mitigation (SOCP)"
  - the design of $(\mathbf{R}_x, \mathbf{\Phi})$ based on SDP - "precoding with clutter mitigation (SDP)"
  - precoding without consideration of clutter
  - uniform precoding, *i.e.*, $\mathbf{P} = \sqrt{L P_R / M_{t,R}} \mathbf{I}$

Numerical Results: radar SINR vs radar TX pwoer

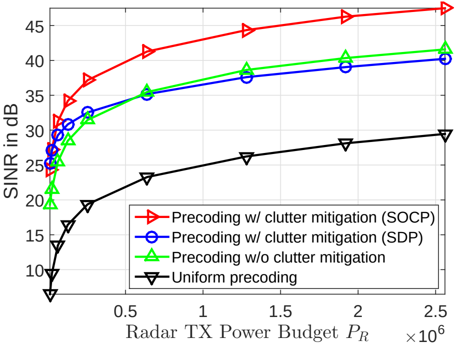

Figure 1: SINR performance under different values of radar TX power. CNR= 20 dB.

Precoding w/ CM > Precoding w/o CM > Uniform Precoding

- "Precoding w/o CM" focuses more power on the target than "Uniform precoding" does.
- "Precoding w/ CM" effectively reduces the power transmitted on the clutter while "Precoding w/o CM" does not.

The SOCP based precoding design outperforms the SDP based design.

Numerical Results: radar SINR vs clutter to noise ratios

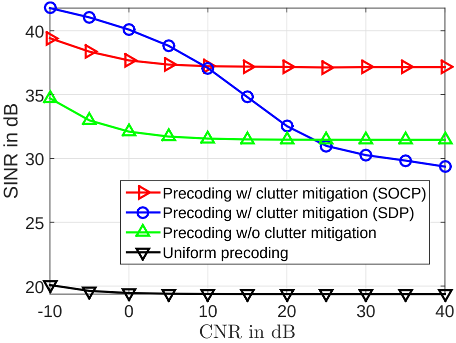

Figure 2: SINR performance under different clutter to noise ratios (CNR). $P_R = 2.56 \times 10^5$.

Precoding w/ CM > Precoding w/o CM > Uniform Precoding

The SOCP based precoding design is more tractable and computationally efficient than the SDP based design.

- The SOCP based precoding design outperforms the SDP based design when CNR is larger than 10dB.
- The CPU time required by the SDP method increase dramatically with $M_{t,R}$.

Outline

Conclusion

- We have proposed an efficient spectrum sharing method for a MIMO radar and a communication system operating in a scenario with clutter. The radar and communication system signals were optimally designed by minimizing a lower bound for the SINR at the radar receive antennas.
- We have shown that the radar precoder always has a rank one solution. Based on this key observation, the alternating iteration of the radar precoder has been solved by a sequence of SOCP problems, which are more efficient and tractable than applying SDP directly.
- Simulation results have shown that the proposed spectrum sharing method can effectively increase the radar SINR for various scenarios with clutter.

Thank You

Thank You!
Questions please