ON DISTANCE EDGE COLOURINGS OF A CYCLIC MULTIGRAPH

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We shall use the distance chromatic index defined by the present author in early nineties, cf. [5] or [4] of 1993. The edge distance of two edges in a multigraph $M$ is defined to be their distance in the line graph $L(M)$ of $M$. Given a positive integer $d$, define the $d^{+}$-chromatic index of the multigraph $M$, denoted by $q^{(d)}(M)$, to be equal to the chromatic number $\chi$ of the $d$th power of the line graph $L(M)$,

$$q^{(d)}(M) = \chi(L(M)^d).$$

Then the colour classes are matchings in $M$ with edges at edge distance larger than $d$ apart.

Call $C$ to be a cyclic multigraph if $C$ consists of a cycle on $n$ vertices with possibly more than one edge between two consecutive vertices.

The following problem was presented in [6].

**Problem.** Given an integer $d \geq 2$ and a cyclic multigraph $C$, find (or estimate) $q^{(d)}(C)$, the $d^{+}$-chromatic index of $C$.

In other words, generalize the following formula due to Berge [1] for the ordinary chromatic index ($q = q^{1}$)

$$q(C) = \max \left\{ \frac{\Delta(C) + \left\lceil \frac{e(C)}{4\Delta(C)} \right\rceil}{2}, \Delta(C) \right\}$$

for odd $n$,

$$\frac{\Delta(C)}{2}$$

for even $n$,

where $\Delta(C)$ and $e(C)$ are the maximum degree among vertices and the size of $C$, respectively.
Remarks 1. $2^+$-chromatic index $q^{(2)}$ is known under the name *strong chromatic index*, estimations of $q^{(2)}(C)$ being studied in [2, 3].

2. In [5] it is proved that
\[
q^{(d)}(pC_n) = \begin{cases} 
  pn & \text{if } n \leq 2d + 1, \\
  \left\lfloor \frac{pn}{d+1} \right\rfloor & \text{if } n \geq d + 1
\end{cases}
\]

where $pC_n$ is the cyclic multigraph $C$ with all edge multiplicities equal to $p$.

3. Let $M$ be a loopless multigraph whose underlying graph is a forest. Then $q^{(d)}(M)$, the $d^+$-chromatic index of $M$, can be seen to be equal to the diameter-$d$ cluster (or diameter-$d$ edge-clique) number of $M$ (i.e., the density of the $d$th power, $L(M)^d$, of the line graph of $M$). This extends the known corresponding results on a tree [5] and on $q^{(2)}(M)$ in [2].

References


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