The arrow of time: from universe time-asymmetry to local irreversible processes

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Abstract

In several previous papers we have argued for a global and non-entropic approach to the problem of the arrow of time, according to which the "arrow" is only a metaphorical way of expressing the geometrical time-asymmetry of the universe. We have also shown that, under definite conditions, this global time-asymmetry can be transferred to local contexts as an energy flow that points to the same temporal direction all over the spacetime. The aim of this paper is to complete the global and non-entropic program by showing that our approach is able to account for irreversible local phenomena, which have been traditionally considered as the physical origin of the arrow of time.

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I. INTRODUCTION

In several previous papers[1,2,3,4,5,6] we have argued for a global and non-entropic approach to the problem of the arrow of time, according to which the "arrow" is only a metaphorical way for expressing the geometrical time-asymmetry of the universe. We have also shown that, under definite conditions, this global time-asymmetry can be transferred to local contexts as an energy flow that points to the same temporal direction all over the spacetime. However, many relevant irreversible local phenomena were still unexplained by our approach. The account of them is necessary to reach a full answer to the problem of the arrow of time, since they have been traditionally considered as the physical origin of such an arrow. The aim of this paper is to complete the global and non-entropic program by showing that our approach is able to account for those local irreversible phenomena.

For this purpose, the paper is organized as follows. In Section II we introduce the precise definition of the basic concepts involved in the discussion: time-reversal invariance, irreversibility and arrow of time. In Section III we summarize our global and non-entropic approach to the problem of the arrow of time, according to which the arrow is given by the time-asymmetry of spacetime. In this section we also explain how the global arrow is transferred to local contexts as an energy flow defined all over the spacetime and which, as a consequence, represents a relevant physical magnitude in local theories. Section IV is devoted to show how the energy flow breaks the time-symmetry of the pair of solutions, one the temporal mirror image to the other, resulting from different time-reversal invariant fundamental laws. In particular, we consider quantum mechanics, quantum field theory, and the case of Feynman graphs and quantum measurements. In section V, irreversibility at the phenomenological level is discussed: we show that, when phenomenological theories are analyzed in fundamental terms, a second irreversible solution evolving towards the past can always be identified; the energy flow is what breaks the just discovered time-symmetry of the pair. Finally, in Section VI we draw our conclusions.

II. BASIC CONCEPTS

It is surprising that, after so many years of debates about irreversibility and time's arrow, the meanings of the terms involved in the discussion are not yet completely clear: the main obstacle to agreement is conceptual confusion. For this reason, we begin with disentangling the basic concepts of the problem.

A. Time-reversal invariance

Even a formal concept as time-reversal invariance is still object of controversies (see Albert's recent book[(7)], and Earman's criticisms[(8)]). We define it as follows:

> **Definition 1:** A dynamical equation is *time-reversal invariant* if it is invariant under the application of the time-reversal operator $\mathcal{T}$, which performs the transformation $t \rightarrow -t$ and reverses all the dynamical variables whose definitions in function of t are non-invariant under the transformation $t \rightarrow -t$.

On the basis of this definition, we can verify by direct calculation that the dynamical equations of fundamental physics are time-reversal invariant. Let us see some examples:

- **Classical mechanics:** In ordinary classical mechanics, the basic magnitudes (position $\mathbf{x}$, velocity $\mathbf{v}$ and acceleration $\mathbf{a}$) change as

$$\mathcal{T}\mathbf{x} = \mathbf{x}, \quad \mathcal{T}\mathbf{v} = -\mathbf{v}, \quad \mathcal{T}\mathbf{a} = \mathbf{a} \tag{1}$$

  In general, mass m and force $\mathbf{F}$ are conserved magnitudes, that is, they are not functions of t; then,

$$\mathcal{T}m = m, \quad \mathcal{T}\mathbf{F} = \mathbf{F} \tag{2}$$

  Since $\mathbf{F} = -\nabla V(\mathbf{x})$, where V is a potential, the energy $H = \frac{1}{2}m\mathbf{v}^2 + V(\mathbf{x})$ is invariant under the action of $\mathcal{T}$:

$$\mathcal{T}H = H \tag{3}$$

  Analogously, in Hamiltonian classical mechanics, the position $\mathbf{q} = (x_1, ..., x_n)$ and the momentum $\mathbf{p} = (p_1, ..., p_n)$ change as

$$\mathcal{T}\mathbf{q} = \mathbf{q}, \quad \mathcal{T}\mathbf{p} = -\mathbf{p} \tag{4}$$

- **Electromagnetism:** Here the charge q is not function of t since it is also a conserved magnitude; so,

$$\mathcal{T}q = q \tag{5}$$

Then, the charge density ρ and the current density $\mathbf{j} = \rho\mathbf{v}$ change as

$$\mathcal{T}\rho = \rho, \quad \mathcal{T}\,\mathbf{j} = -\mathbf{j} \tag{6}$$

Since the Lorentz force is defined as $\mathbf{F} = q\mathbf{E} + \mathbf{j} \times \mathbf{B}$, from eqs. (2), (5) and (6) the electric field $\mathbf{E}$ and the magnetic induction $\mathbf{B}$ change as

$$\mathcal{T}\,\mathbf{E} = \mathbf{E}, \quad \mathcal{T}\,\mathbf{B} = -\mathbf{B} \tag{7}$$

- **Quantum mechanics:** In order to apply the time-reversal operator to quantum mechanics, the configuration representation has to be used: $x \sim x, \quad p \sim -i\hbar\frac{\partial}{\partial x}$. Since we want to obtain $\mathcal{T}x = x$ and $\mathcal{T}p = -p$ as in the classical case, we impose that the wave function change as $\mathcal{T}\phi(x) = \phi(x)^*$. But this requirement makes the quantum time-reversal operator *antilinear* and *antiunitary* (by contrast with the linearity of $\mathcal{T}$ in classical mechanics). In order to express this difference, the time-reversal operator in quantum mechanics is denoted by $\mathbf{T}$:

$$\mathbf{T}\phi(x) = \phi(x)^* \tag{8}$$

In fact, if ω are the eigenvalues of the Hamiltonian H, with the linear $\mathcal{T}$ we would obtain $\mathcal{T}e^{-i\omega t} = e^{i\omega t}$ and, therefore, $\mathcal{T}\omega = -\omega$, which would lead to unacceptable negative energies. On the contrary, with the antilinear $\mathbf{T}$,

$$\mathbf{T}e^{-i\omega t} = e^{-i\omega t}, \quad \mathbf{T}H = H, \quad \mathbf{T}\omega = \omega \tag{9}$$

- **Quantum field theory:** In this chapter of physics, the linear and unitary operator $\mathbf{P}$, corresponding to space-inversion, and the antilinear and antiunitary operator $\mathbf{T}$, corresponding to time-reversal, apply to the quadri-momentum P^μ as (we will return on this point in Section IV.B)

$$\mathbf{P}iP^\nu\mathbf{P}^{-1} = i\mathcal{P}^\nu_\mu P^\mu, \quad \mathbf{T}iP^\nu\mathbf{T}^{-1} = i\mathcal{T}^\nu_\mu P^\mu \tag{10}$$

where

$$\mathcal{P}^\mu_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \mathcal{T}^\mu_\nu = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{11}$$

As a consequence of the definition of time-reversal invariance, given a time-reversal invariant equation L, if $f(t)$ is a solution of L, then $\mathcal{T}f(t)$ is also a solution. In previous papers[2,4], we have called these two mathematical solutions "*time-symmetric twins*": they are twins because, without presupposing a privileged direction of time, they are only conventionally different; they are time-symmetric because one is the temporal mirror image of the other. The traditional example of time-symmetric twins is given by electromagnetism, where dynamical equations always have advanced and retarded solutions, respectively related with incoming and outgoing states in scattering as described by Lax-Phillips theory[9]. The two twins are identical and cannot be distinguished at this stage since, up to now, there is no further criterion than the time-reversal invariant dynamical equation from which they arise. Conventionally we can give a name to each solution: "advanced" and "retarded", "incoming" and "outgoing", etc. But these names are just conventional labels and certainly do not establish a non-conventional difference between both time-symmetric solutions.

In general, the dynamical equations of *fundamental* physics are time-reversal invariant, e.g. the dynamical equation of classical mechanics, the Maxwell equations of electromagnetism, the Schrödinger equation of quantum mechanics, the field equations of quantum field theory, the Einstein field equations of general relativity. However, not all axioms of fundamental theories are time-reversal invariant; this is the case of the Postulate III of quantum field theory (see Section IV.B) and the measurement postulate of quantum mechanics (see Section IV.C). On the other hand, many non fundamental laws are non time-reversal invariant, as the phenomenological second law of thermodynamics (see Section V). One of the purposes of this paper is to explain these apparent "*anomalies*".

B. Irreversibility

Although the concepts of reversibility and irreversibility have received many definitions in the literature on the subject, from a very general viewpoint a reversible evolution is usually conceived as a process that can occur in the opposite temporal order according to the dynamical law that rules it: the occurrence of the opposite process is not excluded by the law. The typical irreversible processes studied by physics are decaying processes, that is, time evolutions that tend to a final equilibrium state from which the system cannot escape: the irreversibility of the process is due precisely to the fact that the evolution leaving the equilibrium state is not possible. For these cases, reversibility can be defined as:

> **Definition 2:** A solution $f(t)$ of a dynamical equation is *reversible* if it does not reach an equilibrium state (namely, if $\nexists \lim_{t\to\infty} f(t)$) where the system remains forever.

For instance, according to this definition, in classical mechanics a solution of a dynamical equation is reversible if it corresponds to a closed curve in phase space (even if these curves are closed through a point at infinite); if not, it is irreversible.

It is quite clear that time-reversal invariance and reversibility are different concepts to the extent that they apply to different mathematical entities: time-reversal invariance is a property of dynamical equations and, *a fortiori*, of the set of its solutions; reversibility is a property of a single solution of a dynamical equation. Furthermore, they are not even correlated, since both properties can combine in the four possible cases (see Castagnino, Lara and Lombardi[(2)]). In fact, besides the usual cases time-reversal invariance-reversibility and non time-reversal invariance-irreversibility, the remaining two combinations are also possible:

- **Time-reversal invariance and irreversibility.** Let us consider the pendulum with Hamiltonian
$$H = \frac{1}{2m}\, p_\theta^2 - \frac{k^2}{2}\cos\theta \tag{12}$$
The dynamical equations are time-reversal invariant since $\mathcal{T}\,\theta = \theta$:
$$\dot{\theta} = \frac{\partial H}{\partial p_\theta} = \frac{p_\theta}{m} \qquad\qquad \dot{p_\theta} = -\frac{\partial H}{\partial \theta} = -\frac{k^2}{2}\sin\theta \tag{13}$$
Therefore, the set of trajectories in phase space is symmetric with respect to the θ-axis. However, not all the solutions are reversible. In fact, when $H = \frac{k^2}{2}$, the solution is irreversible since it tends to $\theta = \pi$, $p_\theta = 0$ ($\theta = -\pi$, $p_\theta = 0$) when $t \to \infty$ ($t \longrightarrow -\infty$) (see Tabor[(10)]): it corresponds to the pendulum reaching its unstable equilibrium state for $t \to \infty$ ($t \longrightarrow -\infty$). For $H < \frac{k^2}{2}$ (oscillating pendulum) and $H > \frac{k^2}{2}$ (rotating pendulum), the evolutions are reversible.

- **Non time-reversal invariance and reversibility.** Let us now consider the modified oscillator with Hamiltonian
$$H = \frac{1}{2m}\, p^2 + \frac{1}{2}\, K(p)^2\, q^2 \tag{14}$$

where $K(p) = K_+$ when $p \geq 0$, $K(p) = K_-$ when $p < 0$, and K_+ and K_- are constants. This means that $\mathcal{T}\,K_+ = K_-$. As a consequence, if $K_+ \neq K_-$, the dynamical equations are non time-reversal invariant since, for $p \geq 0$,

$$\dot{p} = -K_+^2 q \quad \mathcal{T}\dot{p} = -\mathcal{T}\,K_+^2\,\mathcal{T}\,q = -K_-^2\,q \neq -K_+^2 q \tag{15}$$

and for $p < 0$,

$$\dot{p} = -K_-^2 q \qquad \mathcal{T}\dot{p} = -\mathcal{T}\,K_-^2\,\mathcal{T}\,q = -K_+^2\,q \neq -K_-^2 q \tag{16}$$

Nevertheless, the solutions $q(t)$ and $p(t)$ are, for $p \geq 0$,

$$q(t) = C_1 \cos(\omega_+ t + \alpha_{+n}) \quad p(t) = C_1 m\omega_+ \sin(\omega_+ t + \alpha_{+n}) \tag{17}$$

and for $p < 0$,

$$q(t) = C_2 \cos(\omega_- t + \alpha_{-n}) \quad p(t) = C_2 m\omega_- \sin(\omega_- t + \alpha_{-n}) \tag{18}$$

where $\omega_\pm = \frac{K_\pm^2}{m}$ and the constants $\alpha_{\pm n}$ change from one cycle n to the next cycle $n+1$ in such a way that the solutions turn out to be continuous. In Fig. 1 we display the time-asymmetric solutions $q(t)$ for this example. It is clear that these solutions have no limit for $t \to \pm\infty$: each trajectory is reversible since it is a closed curve in phase space.

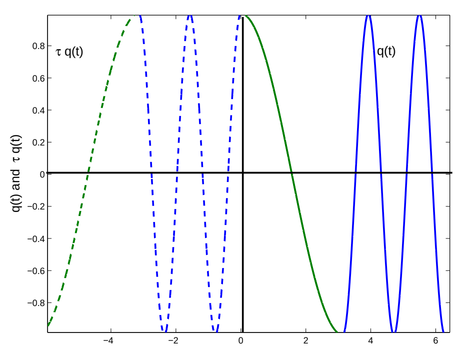

Fig. 1: Time-asymmetric solutions for $q(t)$

Once both concepts are elucidated in this way, *the problem of irreversibility* can be clearly stated: *how to explain irreversible evolutions in terms of time-reversal invariant laws.* When explained in these terms, it turns out to be clear that there is no conceptual puzzle in the problem of irreversibility: nothing prevents a time-reversal invariant equation from having irreversible solutions. Nevertheless, the solution of the problem of irreversibility does not provide yet an adequate distinction between the two directions of time. In fact, if an irreversible evolution is a solution of a time-reversal invariant law, there will always exist its time-symmetric twin, that is, another irreversible solution that is its temporal mirror image. For instance, if there is an irreversible solution leading to equilibrium towards the future, necessarily there exists another irreversible solution leading to equilibrium towards the past, and there is no non-conventional criterion for selecting one of the temporally opposite evolutions as the physically relevant.

In general, a privileged direction of time is presupposed when irreversible processes are studied. In fact, when we talk about entropy increasing processes, we suppose an entropy increase *towards the future*; or when we consider a process going from non-equilibrium to equilibrium, we implicitly locate equilibrium *in the future*. In general, any evolution that tends to an attractor is conceived as approaching it towards the future. This means that the distinction between past and future is usually *taken for granted*, and this fact usually hides the existence of the second irreversible twin of the pair. However, when the time-reversal invariant theory is developed without projecting our time-asymmetric intuitions, the pair of time-symmetric twins becomes manifest.

C. Arrow of time

The problem of the arrow of time owes its origin to the intuitive asymmetry between past and future. We experience the time order of the world as "directed": if two events are not simultaneous, one of them is earlier than the other. Moreover, we view our access to past and future quite differently: we remember past events and predict future events. On the other hand, we live in a world full of processes that never occur in the opposite direction: coffee and milk always mix together, we always get older, and regrettably we never see the reversed processes. Therefore, if we conceive the problem of the arrow of time as the question "Does the arrow of time exist?", we can legitimately solve it on the basis of our best grounded experiences: there is a non merely conventional difference between the two directions of time, and the privileged direction, that we call "future", is the direction of those well known processes.

However, this is not the problem of the arrow of time as conceived in the foundations of physics since the birth of thermodynamics. In this context, the difficulty consists in finding a *physical correlate* of the experienced difference between the two temporal directions. If such a temporal asymmetry did not exist, there would be no need to ask physics for its explanation. It is precisely due to the "directedness" of our experience of time that we want to find this feature accounted for by physical theories. But, then, we cannot project our time-asymmetric experiences and observations into the solution of the problem without begging the question. In this paper, we will address the problem of the arrow of time within the limits of physics: we will not discuss our experiences about time and processes. Our question will be: *Do physical theories pick out a preferred direction of time*?

The main difficulty to be encountered in answering this question relies on our anthropocentric perspective: the difference between past and future is so deeply rooted in our language and our thoughts that it is very difficult to shake off these temporally asymmetric assumptions. In fact, traditional discussions around the problem of the arrow of time in physics are usually subsumed under the label "the problem of the direction of time", as if we could find an exclusively physical criterion for singling out the privileged direction of time, identified with what we call "future". But there is nothing in the dynamical laws of physics that distinguishes, in a non-arbitrary way, between past and future as we conceive them in our ordinary language and our everyday life. It might be objected that physics implicitly assumes this distinction with the use of temporally asymmetric expressions, like "future light cone", "initial conditions", "increasing time", and so on. However this is not the case, and the reason relies on the distinction between "conventional" and "substantial".

> **Definition 3:** Two objects are *formally identical* when there is a permutation that interchanges the objects but does not change the properties of the system to which they belong.

In physics it is usual to work with formally identical objects: the two semicones of a light cone, the two spin directions, etc.

> **Definition 4:** We will say that we establish a *conventional* difference between two objects when we call two formally identical objects with two different names.

This is the case when we assign different signs to the two spin directions, or different names to the two light semicones of a light cone, etc.

> **Definition 5:** We will say that the difference between two objects is *substantial* when we assign different names to two objects that are not formally identical. In this case, although the particular names we choose are conventional, the difference is substantial (see Penrose[11], Sachs[12]).

For instance, the difference between the two poles of the theoretical model of a magnet is conventional since both poles are formally identical; on the contrary, the difference between the two poles of the Earth is substantial because in the North Pole there is an ocean and in the South Pole there is a continent (and the difference between ocean and continent remains substantial even if we conventionally change the names of the poles). In mathematics, given a segment $[\overline{A, B}]$, if we call its end points "A" and "B" (or "B" and "A", since the names are always conventional), we are establishing a conventional difference between them because the points A and B are formally identical; on the contrary, if we call the end points of an arrow $[\overrightarrow{A, B}]$ "A" and "B", we are expressing a substantial difference between both points since the tail A is not formally identical to the head B.

Once this point is accepted, it turns out to be clear that, given the time-reversal invariance of the fundamental laws, physics uses the labels "past" and "future" in a conventional way. Therefore, the problem cannot be posed in terms of identifying the privileged direction of time named "future", as we conceive it in our ordinary language: the problem of the arrow of time in physics becomes the problem of finding a *substantial difference between the two temporal directions grounded only on physical theories*. But if this is our central question, we cannot project our experiences about past and future for solving it. When we want to address the problem of the arrow of time from a perspective purged of our temporal intuitions, we must avoid the conclusions derived from subtly presupposing temporally asymmetric notions. As Huw Price[13] claims, it is necessary to stand at a point outside of time, and thence to regard reality in atemporal terms: this is his "*view from nowhen*". This atemporal standpoint prevents us from using temporally asymmetric expressions in a non-conventional way: the assumption about the difference between past and future is not yet legitimate in the context of the problem of the arrow of time.

But then, what does "the arrow of time" mean when we accept this constraint? Of course, the traditional expression coined by Eddington has only a metaphorical sense: its meaning must be understood by analogy. We recognize the difference between the head and the tail of an arrow on the basis of its intrinsic properties; therefore, we can substantially distinguish between both directions, head-to-tail and tail-to-head, *independently of our particular perspective and our pretheoretical intuitions.* Analogously, we will conceive *the problem of the arrow of time* in terms of *the possibility of establishing a substantial distinction between the two directions of time on the basis of exclusively physical arguments.*

III. THE PROBLEM OF THE ARROW OF TIME: A GLOBAL AND NON-ENTROPIC APPROACH

On the basis of the distinction between conventional and substantial differences, and of the need of an atemporal standpoint, we have proposed and developed a global and non-entropic approach to the problem of the arrow of time in several previous papers. Here we will only summarize the main points of our argument.

A. Why global and non-entropic

Let us begin with explaining in what sense our approach moves away from the traditional local and entropic way of addressing the problem.

1. Why global?

The traditional local approach owes its origin to the attempts to reduce thermodynamics to statistical mechanics: in this context, the usual answer to the problem of the arrow of time consists in defining the future as the direction of time in which entropy increases. However, already in 1912 Paul and Tatiana Ehrenfest[(14)] noted that, when entropy is defined in statistical terms on the underlying classical dynamics, if the entropy of a closed system increases towards the future, such increase is matched by a similar one towards the past: if we trace the evolution of a non-equilibrium system back into the past, we obtain states closer to equilibrium. This old discussion can be generalized to the case of any kind of evolution arising from local time-reversal invariant laws. In fact, as we have seen in the previous section, any time-reversal invariant equation gives rise to a pair of time-symmetric twins $f(t)$ and $\mathcal{T}f(t)$, which are only conventionally different to each other.

Of course, the existence of time-symmetric twins is a result of a formal property of the equation. When it represents a local dynamical law, the solutions are conceived as representing two possible evolutions relative to that law, because local physics assumes a one-to-one mapping between possible evolutions and solutions of the dynamical law. But since both solutions are only conventionally different, they do not supply a substantial distinction between the two directions of time. In the face of this problem, one might be tempted to solve it by simply stating that both solutions describe the same process from temporally reversed viewpoints. However, the fact that a single process be described by $e(t)$ and $e(-t)$ means that "t" and "$-t$" are only two different names for the same temporal point. Therefore, the time represented by $[0, \infty)$ and the time represented by $(-\infty, 0]$ would be not conventionally different, but strictly identical. Then, time itself would not have the topology of $\mathbb{R}$, as in local physical theories, but the topology of $\mathbb{R}^+$. And since in $\mathbb{R}^+$ the directions $0 \rightarrow \infty$ and $\infty \rightarrow 0$ are substantially different, the substantial difference between the two directions of time would turn out to be imposed "by hand" in local theories, which should include the specification of an absolute origin of time. Moreover, this move would break the Galilean or the Lorentz invariance of those theories; in particular, the non-homogeneity of time would lead them to be non-invariant under time-translation and this, in turn, would amount to resign the local principle of energy conservation.[1]

Summing up, local theories do not offer a non-conventional criterion for distinguishing between the time-symmetric twins and, therefore, between the two directions of time. When this fact is accepted, general relativity comes into play and the approach to the problem of the arrow of time turns out to be global.

2. Why non-entropic?

When, in the late nineteenth century, Boltzmann developed the probabilistic version of his theory in response to the objections raised by Loschmidt and Zermelo (for historical details, see Brush(15)), he had to face a new challenge: how to explain the highly improbable current state of our world. In order to answer this question, Boltzmann offered the first global approach to the problem.[2] Since that seminal work, many authors have related the temporal direction past-to-future to the gradient of the entropy function of the universe: it has been usually assumed that the fundamental criterion for distinguishing between the two directions of time is the second law of thermodynamics (see, for instance, Reichenbach(17), Feynman[18], Davies[19,20]).

The global entropic approach rests on two assumptions: that it is possible to define entropy for a complete cross-section of the universe, and that there is an only time for the universe as a whole. However, both assumptions involve difficulties. In the first place, the definition of entropy in cosmology is still a very controversial issue: there is no consensus regarding how to define a global entropy for the universe. In fact, it is usual to work only with the entropy associated with matter and radiation because there is not yet a clear idea about how to define the entropy due to the gravitational field. In the second place, when general relativity comes into play, time cannot be conceived as a background parameter which, as in pre-relativistic physics, is used to mark the evolution of the system. Therefore, the problem of the arrow of time cannot legitimately be posed, from the beginning, in terms of the entropy gradient of the universe computed on a background parameter of evolution.

Nevertheless, there is an even stronger argument for giving up the traditional entropic approach. As it is well known, entropy is a phenomenological property whose value is compatible with many configurations of a system. The question is whether there is a more fundamental property of the universe which allows us to distinguish between both temporal directions. On the other hand, if the arrow of time reflects a substantial difference between both directions of time, it is reasonable to consider it as an intrinsic property of time, or better, of spacetime, and not as a secondary feature depending on a phenomenological property. For these reasons we will follow Earman's "*Time Direction Heresy*"[21], according to which the arrow of time is an intrinsic property of spacetime, which does not need to be reduced to non-temporal features.

B. Conditions for a global and non-entropic arrow of time

In general relativity, the universe is a four-dimensional object, physically described by the geometrical properties of spacetime, embodied in the metric tensor $g_{\mu\nu}$, and the distribution of matter-energy throughout the spacetime, embodied in the energy-momentum tensor $T_{\mu\nu}$. Both properties are physical, and they are related by the Einstein field equations in such a way that the universe can be physically described in geometrical terms or in matter-energy terms. We will use a geometrical language for presenting the conditions for the arrow of time only because it makes the explanation more intuitive.

As it is well known, many different spacetimes, of extraordinarily varied topologies, are consistent with the field equations. And some of them have features that do not admit a unique time for the universe as a whole, or even the definition of the two directions of time in a global way. Therefore, the possibility of defining a global arrow of time requires two conditions that the spacetime must satisfy: time-orientability and existence of a global time.

1. Time-orientability

A spacetime $(M, g_{\mu\nu})$ is *time-orientable* if there exists a continuous non-vanishing vector field $\gamma^{\mu}(x)$ on the manifold M which is everywhere non-spacelike (see Hawking and Ellis[22]). By means of this field, the set of all light semicones of the manifold can be split into two equivalence classes, C_+ (semicones containing the vectors of the field) and C_- (semicones non containing the vectors of the field). It is clear that the names "C_+" and "C_-" are completely conventional, and can be interchanged as we wish: the only relevant fact is that, for all the semicones, each one of them belongs to one and only one of the two equivalence classes. On the contrary, in a non time-orientable spacetime it is possible to transform a timelike vector into another timelike vector pointing to the opposite temporal direction by means of a continuous transport that always keeps non-vanishing timelike vectors timelike; therefore, the equivalence classes, C_+ and C_- cannot be defined in an univocal way.

2. Global time

Time-orientability does not guarantee yet that we can talk of *the* time of the universe: the spacetime may be non globally splittable into spacelike hypersurfaces such that each one them contains all the events simultaneous with each other. The *stable causality condition* amounts to the existence of a *global time function* on the spacetime (see Hawking and Ellis[22]), that is, a function $t : M \to \mathbb{R}$ whose gradient is timelike everywhere. This condition guarantees that the spacetime can be *foliated* into hypersurfaces of simultaneity ($t = const$), which can be ordered according to the value of t (see Schutz[23]).

C. The definition of the global and non-entropic arrow

1. Time-asymmetry

As Grünbaum[24] correctly points out, the mere "oppositeness" of the two directions of a global time, and even of the two equivalence classes of semicones, does not provide a non-conventional criterion for distinguishing the two temporal directions. Such a criterion is given by the time-asymmetry of the spacetime. A time-orientable spacetime $(M, g_{\mu\nu})$ with global time t is *time-symmetric* with respect to some spacelike hypersurface $t = \alpha$ if there is a diffeomorphism d of M onto itself which (i) reverses the temporal orientations, (ii) preserves the metric $g_{\mu\nu}$, and (iii) leaves the hypersurface $t = \alpha$ fixed. Intuitively, this means that the spacelike hypersurface $t = \alpha$ splits the spacetime into two "halves", one the temporal mirror image of the other. On the contrary, in a time-asymmetric spacetime there is no spacelike hypersurface $t = \alpha$ from which the spacetime looks the same in both temporal directions: the properties in one direction are different than the properties in the other direction, and this fact is expressed by the metric $g_{\mu\nu}$. But, according to the Einstein field equations, this also means that the matter-energy of the universe is asymmetrically distributed along the global time, and this is expressed by the energy-momentum tensor $T_{\mu\nu}$. Therefore, no matter which spacelike hypersurface is used to split a time-asymmetric spacetime into to "halves", the physical (geometrical or matter-energy) properties of both halves are substantially different, and such a difference establishes a substantial distinction between the two directions of time.

Now we can assign different names to the substantially different temporal directions on the basis of that difference. For instance, we can call one of the directions of the global time t "positive" and the class of semicones containing vectors pointing to positive t "C_+", and the other direction of t "negative" and the corresponding class "C_-". Of course, the particular names chosen are absolutely conventional, we can use the opposite convention, or even other names ("A" and "B", "black" and "white", or "Alice" and "Bob"): the only relevant fact is that both directions of time are substantially different to each other, and the different names assigned to them express such a substantial difference.

2. The meaning of time-reversal invariance in general relativity

The metric $g_{\mu\nu}$ and the energy-momentum tensor $T_{\mu\nu}$ of a particular spacetime are, of course, a solution of the Einstein field equations which, being fundamental laws, are time-reversal invariant. Then, there exists another solution given by $\mathcal{T}\, g_{\mu\nu}$ and $\mathcal{T}\, T_{\mu\nu}$, the time-symmetric twin of the previous one. So, the ghost of symmetry threatens again: it seems that we are committed to supplying a non-conventional criterion for picking out one of both solutions, one the temporal mirror image of the other. However, in this case the threat is not as serious as it seems.

As it is well known, time-reversal is a symmetry transformation. Under the active interpretation, a symmetry transformation corresponds to a change from one system to another; under the passive interpretation, a symmetry transformation consists in a change of the point of view from which the system is described. The traditional position about symmetries assumes that, in the case of discrete transformations as time-reversal or spatial reflection, only the active interpretation makes sense: an idealized observer can rotate himself in space in correspondence with the given spatial rotation, but it is impossible to "rotate in time" (see Sklar[(25)]). Of course, this is true when the idealized observer is immersed in the same spacetime as the observed system. But when the system is the universe as a whole, we cannot change our spatial perspective with respect to the universe: it is as impossible to rotate in space as to rotate in time. However, this does not mean that the active interpretation is the correct one: the idea of two identical universes, one translated in space or in time regarding the other, has no meaning. This shows that both interpretations, when applied to the universe as a whole, collapse into conceptual nonsense (for a full argument, see Castagnino, Lombardi and Lara[(1)]).

In fact, in cosmology symmetry transformations are neither given an active nor a passive interpretation. Two models $(M, g_{\mu\nu})$ and $(M', g'_{\mu\nu})$ of the Einstein equations are taken to be equivalent if they are *isometric*, that is, if there is a diffeomorphism $\theta : M \to M'$ which carries the metric $g_{\mu\nu}$ into the metric $g'_{\mu\nu}$ (see Hawking and Ellis[(22)]). Since symmetry transformations are isometries, two models related by a symmetry transformation (in particular, time-reversal) are considered equivalent descriptions of one of the same spacetime. Therefore, by contrast with local theories, when the object described is the universe as a whole, it is not necessary to supply a non-conventional criterion for selecting one solution of the pair of time-symmetric twins. This fundamental difference between general relativity and the local theories of physics is what allows the global approach to the problem of the arrow of time to provide a solution that cannot be offered by local approaches.

3. The generic character of the global and non-entropic arrow

As we have seen, the global entropic approach explains the arrow of time in terms of the increasing entropy function of the universe: as a consequence, this position has to posit a low-entropy initial state from which entropy increases. Then, the problem of the arrow of time is pushed back to the question of why the initial state of the universe has low entropy. But a low-entropy initial state is extraordinarily improbable in the collection of all possible initial states. Therefore, the global entropic approach is committed to supply an answer to the problem of explaining such an improbable initial condition (see, for instance, the arguments of Davies(19,20) and of Penrose and Percival(26); see also the well known criticisms directed by Price(13) to the global entropic approach).

In our global and non-entropic approach there are not improbable conditions that require to be accounted for. On the contrary, in previous papers (Castagnino and Lombardi(4,5)) we have proved that the subset of time-symmetric spacetimes has measure zero in (or is a proper subspace of) the set of all possible spacetimes admissible in general relativity. This result can be intuitively understood on the basis of the evident fact that symmetry is a very specific property, whereas asymmetry is greatly generic. Therefore, in the collection of all the physically possible spacetimes, those endowed with a global and non-entropic arrow of time are overwhelmingly probable: the non existence of the arrow of time is what requires an extraordinarily fine-tuning of all the variables of the universe.

These arguments, based on theoretical results, are relevant to the problem of finding a substantial difference between the two directions of time grounded on physical theories. Of course, theories are undetermined by empirical evidence, and this underdetermination is even stronger in cosmology, where the observability horizons of the universe introduce theoretical limits to our access to empirical data. In fact, on the basis of the features of the unobservable regions of the universe, it may be the case that our spacetime be time-symmetric, or lacking a global time, or even non time-orientable. Of course, this does not undermine the overwhelmingly low probability of time-symmetry. But since probability zero does not amount to impossibility, we cannot exclude the case that we live in a time-symmetric, or even in a non time-orientable universe. In that case, the global and non-entropic arrow of time would not exist and, therefore, the explanation of the local time-asymmetries to be presented in the next sections would not apply. It is quite clear that this case cannot be excluded on logical nor on theoretical grounds. However, the coherence of our overall explanation of the arrow of time and of the local irreversible phenomena, whose theoretical account we were looking for, counts for its plausibility. On the other hand, it is difficult to see what theoretical or empirical reasons could be used to argue for the fact that we live in a universe lacking a global arrow of time; on the contrary, the cosmological models accepted in present-day cosmology as the best representations of our actual universe (Big Bang-Big Rip FRW models) are clearly time-asymmetric (see Caldwell *et al.*[(27)]). As always in physics and, in general, in science, there are not irrefutable explanations. In particular, any statistical argument admits probability zero exceptions. Nevertheless, we can make reasonable decisions about accepting or rejecting a particular explanation on the basis of its fruitfulness for explaining empirical evidence and its coherence with the knowledge at our disposal.[3]

D. Transferring the global arrow to local contexts

1. From time-asymmetry to energy flow

As we have seen, the time-asymmetry of spacetime establishes a substantial difference between the two directions of time. This time-asymmetry is a physical property of the spacetime that can be equivalently expressed in geometrical terms ($g_{\mu\nu}$) or in terms of the matter-energy distribution ($T_{\mu\nu}$). However, in none of both descriptions it can be introduced in local theories which, in principle, do not contain the concepts of metric or of energy-momentum tensor. For this reason, if we want to transfer the global arrow of time to local contexts, we have to translate the time-asymmetry embodied in $g_{\mu\nu}$ and $T_{\mu\nu}$ into a feature that can be expressed by the concepts of local theories. This goal can be achieved by expressing the energy-momentum tensor in terms of the four-dimensional energy flow.

As it is well known, in the energy-momentum tensor $T_{\mu\nu}$, the component T_{00} represents the matter-energy density and the component T_{0i}, with $i = 1$ to 3, represents the spatial energy flow. Thus, $T_{0\alpha}$ can be viewed as a spatio-temporal matter-energy flow that embodies, not only the flow of matter-energy in space but also its flow in time; let us call it "energy flow" for simplicity. In turn, $T_{\mu\nu}$ satisfies the dominant energy condition if, in any orthonormal basis, $T_{00} \geq |T_{\alpha\beta}|$, for each $\alpha, \beta = 0$ to 3. This is a very weak condition, since it holds for almost all known forms of matter-energy (see Hawking and Ellis[(22)]) and, then, it can be considered that $T_{\mu\nu}$ satisfies it in almost all the points of the spacetime. The dominant energy condition means that, for any local observer, the matter-energy density T_{00} is *non-negative* and the energy flow $T_{0\alpha}$ is *non-spacelike*. Therefore, in all the points of the spacetime where the condition holds, T_{00} points to the same temporal direction and, since it gives the temporal direction of $T_{0\alpha}$, the energy flow is endowed with the same feature.

The first point to stress here is that the dominant energy condition is immune to time-reversal. In fact, if we apply the time-reversal operator $\mathcal{T}$ to $T_{\mu\nu}$, we obtain

$$\begin{aligned} \mathcal{T}\,T_{00} &= T_{00} \qquad\qquad \mathcal{T}\,T_{i0} = -T_{i0} \\ \mathcal{T}\,T_{0i} &= -T_{0i} \qquad\qquad \mathcal{T}\,T_{ij} = T_{ij} \end{aligned} \tag{19}$$

Therefore, time-reversal does not change the sign of T_{00} and, as a consequence, if the condition is satisfied by $T_{\mu\nu}$, it is also satisfied by $\mathcal{T}\,T_{\mu\nu}$.

The second point that has to be emphasized is that, without a substantial difference between the two temporal directions, the term "positive" applied to T_{00} by the dominant energy condition is merely conventional. The relevant content of the condition is that the energy flow $T_{0\alpha}$ is *non-spacelike* (that is, the matter-energy does not flow faster than light), and that it points to the same temporal direction in all the points where the condition is satisfied. Therefore, we can choose any temporal direction as "positive", and the condition will preserve its conceptual meaning.

A third point that deserves to be stressed is the application of the condition to spacetimes where T_{00} points to the "opposite" temporal direction than in the original spacetime. Let us consider the two spacetimes with $T'_{\mu\nu} = -T_{\mu\nu}$ and $T''_{\mu\nu} = -\mathcal{T}\,T_{\mu\nu}$:

$$\begin{aligned} T'_{00} &= -T_{00} \qquad\qquad T''_{00} = -T_{00} \\ T'_{0i} &= -T_{0i} \qquad\qquad T''_{0i} = T_{0i} \end{aligned} \tag{20}$$

It is clear that, if the dominant energy condition is satisfied by $T_{\mu\nu}$, it is also satisfied by $T'_{\mu\nu}$ and $T''_{\mu\nu}$, with the only change in the conventional decision about the "positive" time direction. This is not surprising since the models with $T_{\mu\nu}$, $T'_{\mu\nu}$ and $T''_{\mu\nu}$ are isometric and, therefore, they are equivalent descriptions of one and the same universe: to say that the temporal direction in one of them is "opposite" to the time direction in the other is senseless (see Subsection III.C.2).

Once these delicate points have been understood,[4] it turns out to be clear that the non-conventional content of the dominant energy condition is not the substantial identification of a temporal direction as the direction of the energy flow, but the coordination of the temporal direction of that flow in all the points of the spacetime where the condition holds. In other words, if $T_{\mu\nu}$ satisfies the condition in all (or almost all) the points of the spacetime, we can guarantee that the energy flow is always contained in the semicones belonging to only one of the two classes, C_+ or C_-, arising from the partition introduced by time-orientability. At this point, somebody might suppose that the satisfaction of the condition, by itself, solves the problem of the arrow of time: "if the energy flow points to the same temporal direction all over the spacetime, let us define that direction as the future and that's all, we don't need time-asymmetry". But this conclusion forgets the conventionality of the direction selected as "positive" for the energy flow. In fact, even if the energy flow is always contained in the semicones belonging to, say, C_+, in a time-symmetric spacetime the difference between C_+ and C_- is merely conventional. Only when we can substantially distinguish between the two temporal directions in terms of the time-asymmetry of the spacetime, we can use the energy flow pointing to the same direction all over the spacetime for expressing that substantial difference. In short, the arrow of time is *defined* by the time-asymmetry of the spacetime, and *expressed* by the energy flow.

Up to now we have not used the words "past" and "future". It is clear that, since they are only labels, their application is conventional. Now we can introduce the usual convention in physics, which consists in calling the temporal direction of T_{00} and, therefore, also the temporal direction of the energy flow $T_{0\alpha}$ "positive direction" or "future"; then, it is said that, under the dominant energy condition, $T_{0\alpha}$ is always contained in the semicones belonging to the positive class C_+. With this convention we can say that the energy flows towards the future for any observer and in any point of the spacetime.[5] Of course, we could have used the opposite convention and have said that the energy always flows towards the past. But, in any case, no matter which terminological decision we make, past is substantially different than future, and the arrow of time consists precisely in such a substantial difference grounded on the time-asymmetry of spacetime.

2. Breaking local time-symmetries

As we have said, any time-reversal invariant equation leads to a pair of time-symmetric twins, that is, two solutions symmetrically related by the time-reversal transformation: each twin, which in some cases represents an irreversible evolution, is the temporal mirror image of the other twin. From the viewpoint of the local theory that produces the time-symmetric twins, the difference between them is only conventional: both twins are nomologically possible with respect to the theory. And, as the Eherenfests pointed out, this is also true for entropy when computed in terms of a time-reversal invariant fundamental law. The traditional arguments for discarding one of the twins and retaining the other invoke temporally asymmetric notions which are not justified in the context of the local theory. For instance, the retarded nature of radiation is usually explained by means of *de facto* arguments referred to initial conditions: advanced solutions of wave equations correspond to converging waves that require a highly improbable "conspiracy", that is, a "miraculous" cooperative emitting behavior of distant regions of space at the temporal origin of the process (see Sachs[12]).

It seems quite clear that this kind of arguments, even if admissible in the discussions about irreversibility, are not legitimate in the context of the problem of the arrow of time, to the extent that they put the arrow "by hand" by presupposing the difference between the two directions of time from the very beginning. In other words, they violate the "nowhen" requirement of adopting an atemporal perspective purged of our temporal intuitions and our time-asymmetric observations, like those related with the asymmetry between past and future or between initial and final conditions. Therefore, from an atemporal standpoint, the challenge consists in supplying a non-conventional criterion, *based only on theoretical arguments*, for distinguishing between the two members of the pair of twins. The desired criterion, that can be legitimately supplied neither by the local theory nor by our time-asymmetric experiences, can be grounded on global considerations.

If we adopt the usual terminological convention according to which the future is the temporal direction of the energy flow and $C_+(x)$ denotes a future semicone, then the energy flow is contained in the future semicone $C_+(x)$ for any point x of the spacetime. On the other hand, in any pair of time-symmetric twins, the members of the pair involve energy flows pointing to opposite temporal directions, with no non-conventional criterion for distinguishing between them. But once *we have established the substantial difference between past and future on global grounds* and have decided that energy flows towards the future, we have a substantial criterion for discarding one of the twins and retaining the other as representing the relevant solution of the time-reversal invariant law. For instance, given the usual conventions, in the case of electromagnetism only retarded solutions are retained, since they describe states that carry energy towards the future.

Another relevant example is the creation and decaying of unstable states. From an equilibrium state ρ_*, an unstable non-equilibrium state $\rho(t=0)$ is created by an antidissipative process with evolution factor $e^{\gamma t}$ and $\sigma < 0$. This unstable non-equilibrium state $\rho(t=0)$ decays towards an equilibrium state ρ_* through a dissipative process with evolution factor

$e^{-\gamma t}$ and $\sigma > 0$. When considered locally, the pairs of twins (antidissipativea and dissipative, $\sigma < 0$ and $\sigma > 0$) are only conventionally different. However, since past is substantially different than future and, according to the usual convention, the energy flow always goes from past to future, the unstable states are always created by energy pumped from the energy flow coming from the past, while unstable states decay returning this energy to the energy flow pointing towards the future. Therefore, the energy flow introduces a substantial difference between the two members of each pair.

In the next sections, we will analyze the breaking of the symmetry introduced by the energy flow in different local laws, coming from fundamental theories and from phenomenological theories; in this last case, the first step will be to bring into the light the second twin usually hidden in the formalism.

IV. FUNDAMENTAL THEORIES

A. Quantum mechanics

The so-called irreversible quantum mechanics is based on the use of rigged Hilbert spaces, due to the ability of this formalism to model irreversible physical phenomena such as exponential decay or scattering processes (see Bohm and Gadella[28]). The general strategy consists in introducing two subspaces, Φ_- and Φ_+, of the Hilbert space $\mathcal{H}$. The vectors $|\phi\rangle$ of the subspaces Φ_- and Φ_+ are characterized by the fact that their projections $\langle\omega|\phi\rangle$ on the eigenstates ω of the energy are functions of the Hardy class from above and from below respectively. These subspaces yield two rigged Hilbert spaces:

$$\Phi_- \subset \mathcal{H} \subset \Phi_-^\times \qquad \Phi_+ \subset \mathcal{H} \subset \Phi_+^\times \tag{21}$$

where $\Phi_-^\times$ and $\Phi_+^\times$ are the anti-dual of the spaces Φ_- and Φ_+ respectively.

It is quite clear that, up to this point, this general strategy amounts to obtain two time-symmetric structures from a time-reversal invariant theory. In fact, quantum mechanics formulated on a Hilbert space $\mathcal{H}$ is time-reversal invariant since

$$\mathbf{T}\mathcal{H} = \mathcal{H} \tag{22}$$

where $\mathbf{T}$ is the antilinear and antiunitary time-reversal operator (see Section II.A). But if quantum mechanics is formulated on spaces $\Phi_\pm$, it turns out to be a non time-reversal invariant theory since

$$\mathbf{T}\,\Phi_{\pm} = \Phi_{\mp} \tag{23}$$

Moreover, in the analytical continuation of the energy spectrum of the system's Hamiltonian, there exists at least a pair of complex conjugate poles, one in the lower half-plane and the other in the upper half-plane of the complex plane. Such poles correspond to a pair of Gamov vectors:

$$|\Psi_G^-\rangle \in \Phi_-^\times \qquad\qquad |\Psi_G^+\rangle \in \Phi_+^\times \tag{24}$$

These vectors are proposed to describe irreversible processes and are taken to be the representation of the exponentially growing and decaying part of resonant unstable states, respectively. But the symmetric position of the poles with respect to the real axis in the complex plane is a clear indication of the fact that Gamov vectors are a case of time-symmetric twins.

In his detailed description of scattering processes, Arno Bohm breaks the symmetry between the twins by appealing to the so-called "preparation-registration arrow of time", expressed by the slogan "*No registration before preparation*" (see Bohm, Antoniou and Kielanowski[29,30]). The key idea behind this proposal is that observable properties of a state cannot be measured until the state acting as a bearer of these properties has been prepared. For instance, in a scattering process, it makes no sense to measure the scattering angle until a state is prepared by an accelerator. On this basis, Bohm proposes the following *interpretational postulate*: the vectors $|\varphi\rangle \in \Phi_-$ represent the states of the system and the vectors $|\psi\rangle \in \Phi_+$ represent the observables of the system in the sense that observables are obtained as $O = |\psi\rangle\langle\psi|$. The time $t = 0$ is considered as the time at which preparation ends and detection begins. The preparation-registration arrow imposes the requirement that the energy distribution produced by the accelerator, represented by $\langle\omega|\varphi\rangle$, be zero for $t > 0$, and the energy distribution of the detected state, represented by $\langle\omega|\psi\rangle$, be zero for $t < 0$:

$$\begin{aligned} |\varphi\rangle \ &\in\ \Phi_- \qquad\qquad \langle\omega|\varphi\rangle = 0 \quad \text{for } t > 0 \\ |\psi\rangle \ &\in\ \Phi_+ \qquad\qquad \langle\omega|\psi\rangle = 0 \quad \text{for } t < 0 \end{aligned} \tag{25}$$

The time evolution, traditionally represented by the group U_t on the Hilbert space $\mathcal{H}$, is here represented by two semigroups U_t^- and U_t^+: (i) U_t^- is U_t restricted to Φ_- and, then, it is valid only for $t > 0$, and (ii) U_t^+ is U_t restricted to Φ_+ and, then, it is valid only for $t < 0$

(see Bohm *et al.*[31], Bohm and Wickramasekara[32]):

$$U_t^- \ : \ \Phi_- \to \Phi_- \qquad |\varphi(t)\rangle = U_t^- |\varphi_0\rangle \quad \text{for } t > 0$$
$$U_t^+ \ : \ \Phi_+ \to \Phi_+ \qquad |\psi(t)\rangle = U_t^+ |\psi_0\rangle \quad \text{for } t < 0 \tag{26}$$

As a consequence, the two Gamov vectors $|\Psi_G^-\rangle \in \Phi_-^\times$ and $|\Psi_G^+\rangle \in \Phi_+^\times$ turn out to be representations of growing and decaying states respectively: the evolution of the *growing Gamov vector* $|\Psi_G^-\rangle$ can be defined only for $t < 0$, and the evolution of the *decaying Gamov vector* $|\Psi_G^+\rangle$ can be defined only for $t > 0$.

As we can see, Bohm's approach breaks the symmetry between the time-symmetric twins by means of an interpretational postulate based on the preparation-registration arrow. Of course, this strategy supplies a solution to the problem of irreversibility to the extent that it permits the representation of irreversible growing and decaying processes. However, it does not offer a theoretical way out of the problem of the arrow of time, since the preparation-registration arrow is introduced as a postulate of the theory. In fact, such an arrow presupposes the distinction between past and future from the very beginning: in the past ($t < 0$) the system is prepared, in the future ($t > 0$) the system is measured, and $|\Psi_G^-\rangle$ and $|\Psi_G^+\rangle$ represent the corresponding growing and decaying processes respectively, both evolving toward the future. It is clear that such a postulate is based on our pretheoretical assumption that preparation is temporally previous than registration. But, from an atemporal viewpoint, we could reverse that interpretational postulate: we could consider that Φ_+ is the space of states and Φ_- is the space of vectors by means of which the observables are obtained; in this case we would obtain the temporal mirror image of the original theory, where $|\Psi_G^+\rangle$ and $|\Psi_G^-\rangle$ represent growing and decaying states respectively, both evolving toward the past. In other words, the two possible interpretational postulates restore the time-symmetry since they lead to two non time-reversal invariant theoretical structures, one the temporal mirror image of the other. Bohm's strategy of choosing the future directed version of the postulate introduces the arrow of time only on the basis of pretheoretical considerations (for a detailed discussion, see Castagnino, Gadella and Lombardi[33,34]) .

Nevertheless, it is possible to find a theoretical justification for the preparation-registration arrow and, as a consequence, for choosing the future directed version of the interpretational postulate: the breaking of the symmetry between the time-symmetric twins is supplied by the energy flow represented by $T_{0\alpha}$. The preparation of the states $|\varphi\rangle \in \Phi_-$

acting as bearers of properties requires energy coming from other processes. Since the energy flow comes from the past and goes to the future, the states are prepared by means of energy coming from the past, that is, from previous processes; the growing Gamov vector $|\Psi_G^-\rangle$ represents precisely the growing process occurring at $t < 0$, which absorbs the energy coming from the past. On the other hand, the registration of the observable properties represented by $|\psi\rangle \in \Phi_+$ provides energy to other processes. Again, since the energy flow comes from the past and goes to the future, the measurement of observables emits energy toward the future, that is, provides energy to latter processes; the decaying Gamov vector $|\Psi_G^+\rangle$ represents precisely the decaying process occurring at $t > 0$, which emits energy toward the future. Summing up, the energy flow coming from the past and directed towards the future supplies the criterion for selecting the future directed version of Bohm's interpretational postulate, and turns quantum mechanics into a non time-reversal invariant theory without the addition of pretheoretical assumptions.

B. Quantum field theory

It is quite clear that the two classes of light semicones C_+ and C_- are a pair of time-symmetric twins. Quantum field theory breaks the symmetry of this pair from the very beginning, by introducing non time-reversal invariance as a primitive assumption. In this section we will explain how this symmetry-breaking can be derived from the global time-asymmetry of the universe.

1. The non time-reversal invariance of axiomatic QFT

In any of its versions, axiomatic QFT includes a non time-reversal invariant postulate (see Bogoliubov *et al.*(35), Roman(36), and also Haag(37), where it is called Postulate III), which states that the spectrum of the energy-momentum operator P^μ is confined to a future light semicone, that is, its eigenvalues p^μ satisfy

$$p^2 \geq 0 \qquad\qquad p^0 \geq 0 \tag{27}$$

This postulate says that, when we measure the observable P^μ, we obtain a *non-spacelike classical* p^μ *contained in a future semicone*, that is, a semicone belonging to C_+.

It is clear that condition $p^0 \geq 0$ selects one of the elements of the pair of time-symmetric twins C_+ and C_- or, in other words, of the pair $p^0 \geq 0$ and $p^0 \leq 0$ that would arise from the theory in the absence of the time-reversal invariance breaking postulate. By means of this postulate, QFT becomes a non time-reversal invariant theory. In turn, since QFT, being both quantum and relativistic, can be considered one of the most basic theories of physics, the choice introduced by condition $p^0 \geq 0$ is transferred to the rest of physical theories. But such a choice is established from the very beginning, as an unjustified assumption. The challenge is, then, to *justify* the non time-reversal invariant postulate by means of independent theoretical arguments.

Let us recall that, in the energy-momentum tensor, $T_{0\alpha}$ represents the spatio-temporal matter-energy flow and $T_{\alpha 0}$ represents the linear momentum density. Since $T_{\mu\nu}$ is a symmetric tensor, $T_{\mu\nu} = T_{\nu\mu}$ and, therefore, $T_{0\alpha} = T_{\alpha 0}$; in other words, the matter-energy flow is equal to the linear momentum density. This means that, if $T_{0\alpha}$ can be used to express the global arrow of time under the dominant energy condition, this is also the case for the linear momentum density $T_{\alpha 0}$. But it is precisely the linear momentum density $T_{\alpha 0}$ the magnitude corresponding to the classical p^μ of QFT; thus, at each point x of the spacetime, $T_{\alpha 0}(x) \in C_+(x) \Longrightarrow p^\mu \sim T_{\alpha 0}(x) \in C_+(x)$.

In conclusion, the fact that p^μ at each point x of the local context and, therefore, for every classical particle, must be contained in the future light semicone $C_+(x)$ turns out to be a consequence of the global time-asymmetry of the spacetime when the dominant energy condition holds everywhere. In other words, the non time-reversal invariant postulate can be justified on global grounds instead of being imposed as a starting point of the axiomatic version of QFT.

2. The non time-reversal invariance of ordinary QFT

In the ordinary version of QFT, the classification of one-particle states according to their transformation under the Lorentz group leads to six classes of four-momenta. Among these classes, it is considered that only three have physical meaning: these are precisely the cases that agree with the non time-reversal invariant postulate of the axiomatic version of QFT. In other words, the symmetry group of QFT is the orthochronous group (see Weinberg[(38)]), where space-reversal $\mathcal{P}$ but not time-reversal $\mathcal{T}$ is included. This is another way of expressing the non time-reversal invariance of QFT. In this case, the non time-reversal invariance is introduced not by means of a postulate, but on the basis of empirical arguments that make physically meaningless certain classes of four-momenta. However, to the extent that special relativity and standard quantum mechanics are time-reversal invariant theories, those arguments give no theoretically grounded justification for such a breaking of time-reversal invariance. Nevertheless, as we have seen in the previous subsection, this justification can be given on global grounds.

Let us make the point in different terms. The quantum field correlates of $\mathcal{P}$ and $\mathcal{T}$, **P** and **T**, are defined as

$$\mathbf{P} i P^{\nu} \mathbf{P}^{-1} = i\mathcal{P}_{\mu}^{\nu} P^{\mu} \qquad\qquad \mathbf{T} i P^{\nu} \mathbf{T}^{-1} = i\mathcal{T}_{\mu}^{\nu} P^{\mu} \tag{28}$$

where **P** is a linear and unitary operator and **T** is an antilinear and antiunitary operator (see Section II.A). In fact, if **T** were linear and unitary, we could simply cancel the i's and, then, from eq. (28), $\mathbf{T}P^0\mathbf{T}^{-1} = -P^0$: the action of the operator **T** on the operator P^0 would invert the sign of P^0, with the consequence that the spectrum of the inverted energy-momentum operator would be contained in a past light semicone. Precisely, for $\nu = 0$, $P^{\nu} = H$, where H is the energy operator; then, if **T** were linear and unitary, $\mathbf{T}H\mathbf{T}^{-1} = -H$ (in contradiction with eq. (3)) with the consequence that, for any state of energy E there would be another state of energy $-E$. The antilinearity and the antiunitarity of **T** avoid these "anomalous" situations, in agreement with the conditions imposed by the non time-reversal invariant postulate and, at the same time, make QFT non time-reversal invariant. Once again, there are good empirical reasons for making **T** antilinear and antiunitary, but not theoretical justification for such a move.

Summing up, in ordinary QFT it is always necessary to make a decision about the time direction of the spectrum of the energy-momentum operator P^{μ}. The point that we want to stress here is that, either in the case of the non time-reversal invariant postulate of the axiomatic version of QFT or in the case of the usual version of QFT, the decision can be justified on global grounds, as a consequence of the time-asymmetry of the spacetime.

3. Weak interactions

Finally, it is worth reflecting on the role of weak interactions in the problem of the arrow of time. The CPT theorem states that **CPT** is the only combination of charge-conjugation **C**, parity-reflection **P** and time-reversal **T** which is a symmetry of QFT. In fact, it is well known that weak interactions break the **T** of the CPT theorem. According to a common opinion, it is precisely this empirical fact the clue for the solution of the problem of the arrow of time: since the **T** symmetry is violated by weak interactions, they introduce a non-conventional distinction between the two directions of time (see Visser[39]). The question is: Is the breaking of **T** what distinguishes both directions of time in QFT? As we have seen, the operator **T** was designed precisely to avoid that certain tetra-magnitudes, such as the linear momentum p^μ, have the "anomalous" feature of being contained in a past light semicone: the action of the operator **T** onto the energy-momentum operator P^μ preserves the time direction of P^μ and, therefore, of its eigenvalues. It is this fundamental fact what makes QFT non time-reversal invariant, and not the incidental violation of **T** by weak interactions. This non time-reversal invariance of QFT, based on the peculiar features of the operator **T**, distinguishes by itself between the two directions of time, with no need of weak interactions (see discussion in Castagnino and Lombardi[4]). In other words, even if weak interactions did not exist, QFT would be a non time-reversal invariant theory which would distinguish between the two directions of time.[6] The real problem is, then, to justify the non time-reversal invariance of a theory which is presented as a synthesis of two time-reversal invariant theories such as special relativity and quantum mechanics. But this problem is completely independent of the existence of weak interactions and the breaking of **T** introduced by them. Summing up, weak interactions do not play a role as relevant in the problem of the arrow of time as it is usually supposed.

C. Feynman graphs and quantum measurements

1. Feynman graphs

Let us consider the Feynman graphs of Fig. 2, where the horizontal direction ideally corresponds to the time axis, the vertical direction represents a spatial axis, $|\Phi_2\rangle$ is a two-particle state and $|\Phi_n\rangle$ is an n-particle state.

As we have argued in the previous sections, at this point there is no substantial criterion to select the past-to-future direction on the time axis. Thus, we cannot even consider motions (e.g. convergence to the vertex or divergence from the vertex) along the lines of the graph because both graphs are formally identical: one is the temporal mirror image of the other. Moreover the probabilities of both processes are equal:

$$p_1 = |\langle\Phi_2|\Phi_n\rangle|^2 = |\langle\Phi_n|\Phi_2\rangle|^2 = p_2 \tag{29}$$

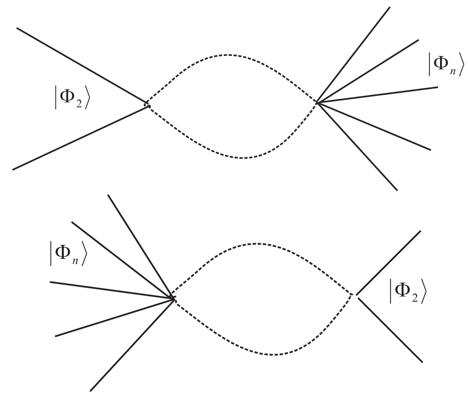

Fig. 2: Feynman time-symmetric twins

This fact shows that the probabilities are not affected by the direction of time. The time-symmetry of both graphs results from the time-reversal invariance of the physical laws on which the graphs are based. If we wanted to distinguish them, we should say that in one graph the state $|\Phi_2\rangle$ is at the left (i.e. in the past) of the state $|\Phi_n\rangle$, and in the second graph $|\Phi_2\rangle$ is at the right (i.e. in the future) of $|\Phi_n\rangle$. But this argument requires a theoretical reason to say which one of the states is at the left of the other.

If we want to turn the merely conventional difference between the two graphs of Fig. 3 into a substantial difference, we have to consider the energy flow trough the process. In fact, if we represent such a flow by means of arrows (they are not fermion arrows!!!), we obtain the following figure: Now both process are substantially different: in the first one

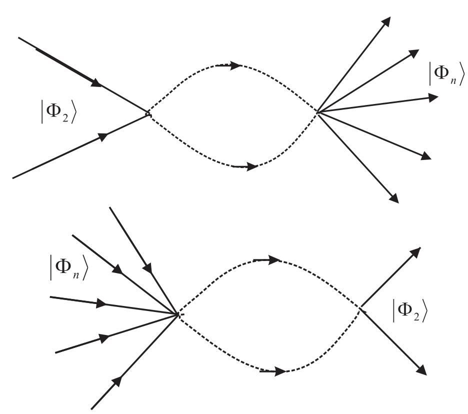

Fig. 3: Feynman twins with broken symmetry

two arrows converge to the target point and n arrows diverge from it, while in the second one n arrows converge and only two diverge. As we can see, the temporal mirror image of one of the graphs is not the other: both graphs are not formally identical because the flow of energy introduces a substantial difference in the pair of time-symmetric twins. On the basis of this substantial difference between the two graphs, now we can define the first one as the typical quantum scattering process and call $|\Phi_2\rangle$ the "prepared state" and $|\Phi_n\rangle$ the "detected state". Only on these theoretical grounds we can say that the arrow of time goes *from preparation to registration in a quantum scattering process.*

2. von Neumann quantum measurements

The argument above can be easily applied to the case of quantum measurement. Let us consider the two graphs of Fig. 4, representing a typical von Neumann measurement, where

$$|\Phi_0\rangle = |\varphi_i\rangle|A_0\rangle \qquad |\Phi_n\rangle = |\varphi_i\rangle|A_i\rangle \tag{30}$$

being $|\varphi_i\rangle$ the state that we want to measure, and the $|A_i\rangle$ the eigenstates of the pointer observable. In both cases, the measurement can be performed on the basis of the correlation $|\varphi_i\rangle \longleftrightarrow |A_i\rangle$. As in the case of the Feynman graphs, in the measurement situation the arrow of time is usually introduced by saying that it goes from the preparation state $|\Phi_0\rangle$ to the set of measured states represented by $|\Phi_n\rangle$. But, as in the previous subsection, this amounts to putting the arrow by hand, without theoretical grounds.

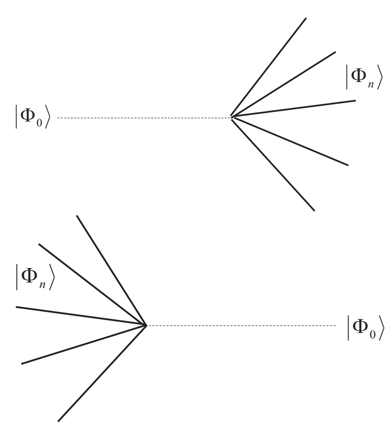

Fig. 4: Measurement time-symmetric twins

Once again, if the energy flow trough the process is considered, the two members of the pair of graphs of Fig. 4 turn out to be substantially different and can be represented as

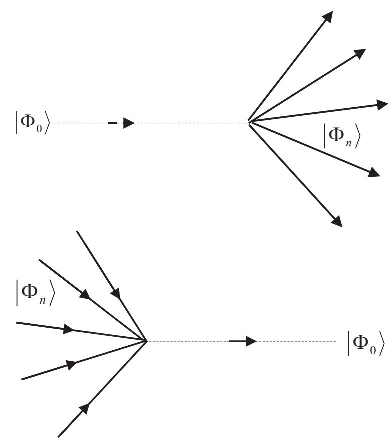

Fig. 5: Measurement twins with broken symmetry

in Fig. 5. On the basis of this substantial difference, now we can define the first graph as representing the typical quantum measurement, and call $|\Phi_0\rangle$ the "prepared state" and $|\Phi_n\rangle$ the "measured state". Analogously to the previous case, the arrow of time goes *from preparation to registration in a quantum measurement process.*

V. PHENOMENOLOGICAL THEORIES

A. Phenomenological twins

Phenomenological theories are usually non time-reversal invariant; so, the solution directed towards the future is taken as the physically relevant. However, in those theories the complexity of the fundamental models is hidden in some phenomenological coefficients that are assumed as positive; but if these coefficients are deduced from underlying fundamental theories, always a negative counterpart can be discovered. This means that, when a phenomenological theory is explained in fundamental terms, the hidden time-reversal invariance becomes manifest, and the corresponding pair of time-symmetric twins can be identified. Let us give some examples:

- **The dumped oscillator**: This is the paradigmatic example. Let us consider the equation of the dumped harmonic oscillator

$$\ddot{x} + \omega^2 x + \frac{\zeta}{m}\dot{x} = 0 \tag{31}$$

where $-\zeta\dot{x}$ is the viscosity term, which is opposed to the motion if the bulk viscosity is $\zeta \geq 0$. If we make the ansatz $x = e^{\alpha t}$, we obtain the solution

$$x(t) = e^{-\frac{\gamma}{2}t} e^{\pm i\widetilde{\omega}t} x(0) \tag{32}$$

where $\gamma = \frac{\zeta}{m} \geq 0$ and $\widetilde{\omega} = \sqrt{\omega^2 - \left(\frac{\gamma}{2}\right)^2}$ (we are just considering the dumped oscillation case where $\omega^2 - \left(\frac{\gamma}{2}\right)^2 \geq 0$). As a consequence, the resulting evolution is a dumped motion towards equilibrium: for $t \to \infty$, $x(t) \to 0$. The energy obtained by this process is dissipated towards the future, e.g. in the form of heat. This evolution is one of the phenomenological twins. The second twin is the time-reversal version of eq. (32), where ζ and γ are *negative.* This seems strange at first sight, but the existence of this *antidissipative* solution is a necessary consequence of the time-reversal invariance of the fundamental laws underlying the process. If in the first twin there was a flow of energy dissipated towards the future, in the second twin the energy that amplifies the oscillations comes *from the past.*

- **The Fourier law:** The Fourier law in thermodynamics tells us that the heat transport goes from a higher temperature region to a lower temperature region:

$$\mathbf{J} = -K\nabla T \tag{33}$$

If this equation were deduced from a fundamental theory, it would result time-reversal invariant. In turn, we know that the temperature T and the gradient ∇ do not change sign with time reversal, and that for the flow, $\mathcal{T}\mathbf{J} = -\mathbf{J}$. Therefore, if eq. (33) has to result time-reversal invariant, then $\mathcal{T}K = -K$: the negative coefficient K will lead to the second time-symmetric twin.

- **Perturbative master equation in quantum Brownian motion:** In Paz and Zurek[40], this equation is computed and the coefficient $\gamma(t)$ is given in eq. (3.13). Since this equation is deduced from the time-reversal invariant equations of quantum mechanics, it has to be also time-reversal invariant. It is easy to show that, if we perform the time-reversal $t \to -t$, we found $\gamma(-t) = -\gamma(t)$. This means that in this case $\gamma(t)$ can be either positive or *negative* and, therefore, antidissipative processes are as possible as dissipative ones.

- **Perturbative master equation for a two-level system coupled to a bosonic heat bath:** In Paz and Zurek(40), this equation is computed and the coefficient $f(t)$ (which plays the role of $\gamma(t)$) is given in eq. (3.21). Again, it is easy to verify that, under the time-reversal $t \to -t$, we obtain $f(-t) = -f(t)$, and also in this case $f(t)$ can be either positive or *negative:* antidissipative processes are as possible as dissipative ones.

- **Perturbative master equation for a particle coupled with a quantum field:** In Paz and Zurek(40), this equation is computed (eq. (3.25)) and the coefficient $\Gamma(x, x', t)$ (which plays the role $\gamma(t)$) is given in the dipole approximation of eq. (5.2). Once more, it is easy to verify that, under the time-reversal $t \to -t$, we obtain $\Gamma(x, x', -t) = -\Gamma(x, x', t)$, and also in this case $\Gamma(x, x', t)$ can be either positive (dissipative processes) or *negative* (antidissipative processes).

- **Quantum field theory in curved spacetime**: An example from a completely different chapter of physics comes from the theory of fields in curved spacetime (Birrel and Davies(41)). Let us consider a flat FRW universe and a scalar field $\psi(t, \mathbf{x}) = e^{-i\frac{\mathbf{k}}{a}\cdot\mathbf{x}}\varphi(t)$, where a is the scale factor of the universe, $\mathbf{k}$ is the linear momentum and $\varphi(t)$ is the time evolution factor that satisfies

$$\ddot{\varphi}(t) + 3H\dot{\varphi}(t) + \left[m^2 + \left(\frac{\mathbf{k}}{a}\right)^2\right]\varphi(t) = 0 \tag{34}$$

where $H = \frac{\dot{a}}{a}$ is the Hubble coefficient and m is the mass of the scalar field. We see that the last equation is similar to eq. (31) if we make the analogy $\zeta = 3mH$. When $H > 0$, the universe describes a dissipative evolution in such a way that $\varphi(t)$ vanishes for $t \to \infty$ (Castagnino *et al.* (42); see also Castagnino *et al.*(43) for the similar case of fluctuations in a FRW background). But even if $H > 0$ in an expanding universe, in a *contracting* universe $H < 0$. This shows that the time-reversal invariant equations of general relativity do not exclude a negative viscosity that leads to the second time-symmetric twin.

B. Viscosity and thermal conductivity

In this subsection we will consider how the pair of time-symmetric twins arises in the case of the equations of a viscous fluid, when they are deduced from the time-reversal invariant equations of classical mechanics. When Newton's Law is applied to a small fluid volume, the Navier-Stokes equations are obtained:

$$\rho\left(\frac{\partial}{\partial t}+u_i\frac{\partial}{\partial x_i}\right)u_j=F_j-\frac{\partial P_{ij}}{\partial x_i} \tag{35}$$

where ρ is the fluid density, u_i is the velocity, P_{ij} is a potential, the F_j are the external forces, and the $G_j=-\frac{\partial P_{ij}}{\partial x_i}$ are the internal forces, that is, the forces due to the neighboring fluid elements. If we consider the case of a non-rotational fluid, we obtain the following expression for P_{ij} (see Huang[44]):

$$P_{ij}=\delta_{ij}p-\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right) \tag{36}$$

where p is the pressure and μ is the viscosity.

Since eq. (36) was obtained exclusively by means of classical mechanics, it is necessarily time-reversal invariant. On this basis, we can infer the behavior of the viscosity μ under the application of the time-reversal operator $\mathcal{T}$. For the l.h.s. of eq. (36),

$$\mathcal{T}P_{ij}=P_{ij} \tag{37}$$

because P_{ij} is a potential that verifies $G_j=-\frac{\partial P_{ij}}{\partial x_i}$, where the G_j are the internal forces of the fluid. On the other hand, for the first term of the r.h.s. of eq. (36),

$$\mathcal{T}\delta_{ij}p=\delta_{ij}p \tag{38}$$

since p is the pressure, i.e. force per unit area. And for the second term,

$$\mathcal{T}\left(-\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}-\frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right)\right)=-\mathcal{T}(\mu)\left(-\frac{\partial u_i}{\partial x_j}-\frac{\partial u_j}{\partial x_i}+\frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right) \tag{39}$$

Given eqs. (37) and (38), this second term must be also invariant under the application of $\mathcal{T}$. Therefore, the viscosity must change as $\mathcal{T}\mu=-\mu$. This shows that the time-reversal invariant fundamental laws underlying the phenomenological equations allow for either positive and negative values of μ. In the usual formulation of the phenomenological equation, only the positive values are considered; but when such an equation is derived from fundamental laws, the second twin that leads to an antidissipative process becomes manifest.

We can go further to track the origin of these negative values up to the microscopic level. Let us consider the Maxwell-Boltzmann distribution:

$$f = \frac{n}{(2\pi m\theta)^{\frac{3}{2}}} e^{-\frac{m}{2\theta}(v-V)^2} \tag{40}$$

where $\theta = kT$ and V, the most probable speed of a molecule of the gas, coincides with the maximum of f, $f_{\max} = v^2 f(v)$. This distribution applies when the gas is in equilibrium. The standard method for studying a gas near equilibrium consists in considering a family of these distributions in the neighborhood of any point of the gas and solving the equations to different orders of corrections to the Maxwell-Boltzmann function. Then, by using the transport equations for a gas with n molecules per unit volume to the first order approximation of eq. (40), valid for a gas close to the equilibrium state (see Huang[44]), we arrive to a statistical expression for the viscosity μ and for the thermal conductivity K:

$$\mu = \frac{n\theta\lambda}{V} \qquad\qquad K = \frac{5n\theta\lambda}{2V} \tag{41}$$

where λ is the mean free path. But, again, if we apply the time-reversal operator to $f_{\max}$, we obtain $\mathcal{T} f_{\max} = v^2 f(-v)$; now the most probable speed is $-V$ as we can see in Fig. 6:

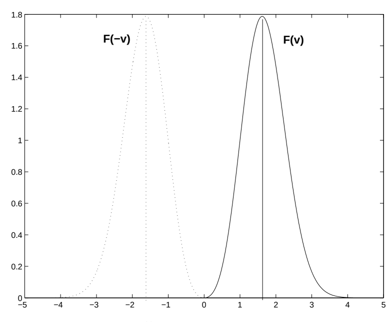

Fig. 6: Time-symmetric graphs of $f_{\max}$

Since $\theta = kT = \frac{2}{3}\epsilon$, where ϵ is the kinetic energy of the gas particles, and

$$\epsilon = \frac{\int_0^\infty d^3p : \frac{p^2}{2m} f(p)}{\int_0^\infty d^3p : f(p)} \tag{42}$$

clearly we verify eq. (3), i.e. $\mathcal{T}\epsilon = \epsilon$; therefore, $\mathcal{T}\theta = \theta$. Nevertheless, from eqs. (41) we can see that $\mathcal{T}K = -K$ and $\mathcal{T}\mu = -\mu$: both μ and K change their sign under the application of the time-reversal operator, result that again unmasks the second twin of the time-symmetric pair.

C. Phenomenological thermodynamics and phenomenological entropy

If the entropy of phenomenological thermodynamics is to be derived from fundamental laws, the equation that makes it to grow only towards the future (the second law) has to be one element of a pair of time-symmetric twins: as the Eherenfests pointed out many years ago, there must exist the time-reversed twin that makes entropy to grow towards the past. The problem consists in discovering the second twin by appealing to the fundamental definitions underlying the phenomenological approach.

The entropy balance equation reads

$$\frac{\partial \rho_S}{\partial t} + \nabla \cdot \mathbf{J}_S = \sigma \tag{43}$$

where ρ_S is the entropy per unit volume, $\mathbf{J}_S$ is the entropy flow and σ is the entropy production per unit volume. If the X^A are the thermodynamic forces or affinities such that $X^A = \nabla \gamma^A$, where γ^A are the thermodynamic variables (i.e. the coordinates of the thermodynamic space, $A, B, ... = 1, ..., n$, being n the number of thermodynamics variables [7]), and the J_A are the thermodynamic flows, σ reads

$$\sigma = \sum_A J_A X^A \tag{44}$$

Then, the Onsanger-Casimir relations near equilibrium read (Castagnino *et al.* [(42)])

$$J_A = \sum_B M_{AB} X^B \tag{45}$$

where M_{AB} is a matrix containing the constant phenomenological coefficients (as the coefficient γ of eq. (32), or the bulk viscosity ζ, the shear viscosity η, the heat conduction χ and all the remaining coefficients of the previous subsection), such that

$$M_{AB} = L_{AB} + f_{AB}, \qquad L_{AB} = L_{BA}, \qquad f_{AB} = -f_{BA} \tag{46}$$

Therefore, the entropy production results

$$\sigma = \sum_{AB} M_{AB} X^A X^B = \sum_{AB} L_{AB} X^A X^B \tag{47}$$

The phenomenological second law of thermodynamic states that

$$\sigma \geq 0 \tag{48}$$

As a consequence, L_{AB} is a positive definite matrix, that is, all the constant phenomenological coefficients $(\gamma, \zeta, \eta, \chi, \mu, K)$ are positive. This means that the second law describes dissipative processes corresponding to the future-directed twin of a time-symmetric pair.

Of course, the corresponding antidissipative twin is obtained simply changing the signs. However, the existence of such a second twin can also be proved by considering the original definition of entropy:

$$dS = \frac{dH}{T} \tag{49}$$

In this definition, $\mathcal{T}H = H$ (eq. (3)) and, since $T \geq 0$, $\mathcal{T}T = T$; therefore, $\mathcal{T}S = S$. Moreover, $S = \rho_S V$ and, therefore, $\mathcal{T}\rho_S = \rho_S$. And since $\mathbf{J}_S = \rho_S \mathbf{v}$, then $\mathcal{T}\mathbf{J}_S = -\mathbf{J}_S$. By applying these results to eq. (43), we can conclude that

$$\mathcal{T}\sigma = -\sigma \tag{50}$$

This means that, when thermodynamics is expressed in terms of fundamental definitions, the time-symmetric twin of the second law comes to the light: the evolutions with $\sigma < 0$ are nomologically possible and, in some cases, L_{AB} is a negative definite matrix. This mirror image behavior corresponds to the change of signs of the phenomenological coefficients γ, ζ, η, χ, μ, K studied in the previous subsection.[8]

If we use just the first twin, we are in the realm of phenomenological thermodynamics; if we use both twins, we are in the realm of fundamental thermodynamics. However, usually we only see dissipative process; thus, something must break the time-symmetry of the twins.

1. Breaking the time-symmetry with the second law

Let an oscillator be initially in motion in a gas atmosphere at rest. The oscillator gradually looses its energy and finally stops, while the initially motionless molecules get in motion. This is a paradigmatic dissipative process with factor $e^{-\gamma t}$ and $\sigma > 0$. But as the fundamental physical equations are time-reversal invariant, the opposite process is also possible according to the laws. If initially the oscillator were at rest but the molecules were in *exactly the opposite motion than in the final state of the previous case* (i.e. their velocities were inverted by a Maxwell demon), the evolution would be antidissipative with factor $e^{\gamma t}$

and $\sigma < 0$. It seems to be obvious that dissipative evolutions are much more frequent that antidissipative ones, because the latter are produced by infrequent "conspiracies". However, this is not a consequence of the dynamical laws but of the initial conditions. We might say that non-conspirative initial conditions are easy to be produced but conspirative initial conditions (demon conditions) are very difficult to be obtained. Nevertheless, this is just a practical problem: we are macroscopic beings and, for this reason, moving a single oscillator is a simple task for us, whereas endowing a great number of molecules with the precise "conspirative" motion is extremely difficult (we need the help of the microscopic Maxwell demon). This means that the limitation is practical and not resulting from some physical law. In fact, sometimes practical limitations can be overcome and the inversion of velocities can be obtained, as in the case of the spin-echo experiments (see Balliant[(45)], Levstein *et al.*[(46)]).

Some authors base their definition of the arrow of time and their foundation of the second law in these practical reasons, that is, the absence of "conspiracies" (see, for instance, Sachs[(12)]). However, such a position forces them to face a long list of well known criticisms. In fact, since antidissipative processes are not ruled out by the fundamental laws of physics, "*A violation of irreversibility is not forbidden as a matter of principle but because it is highly improbable"* (Balliant[(45)], p.408); therefore, "*Irreversibility is not an absolute concept, but is partially subjective by nature depending in the complexity of the system and on the details and ingenuity of our observations"* (Balliant[(45)], p.412). This means that the appeal to practical limitations is a non-theoretical, even a non-objective way to break the symmetry of the phenomenological pair of time-symmetric twins.

2. Breaking the time-symmetry with the energy flow

The future-directed energy flow, expressing the global time-asymmetry of the spacetime, supplies a theoretically grounded mean for breaking the symmetry of the twins.

Let us begin with the case of the Fourier law where, as we have shown, the negative coefficient K leads to the second time-symmetric twin. If the energy flow is future-directed all over the spacetime, at each point x the quadrivector $\mathbf{J}$, representing the heat flow, lies in the future light semicone $C_+(x)$. In the case of thermodynamics, the classical limit $v/c \to 0$ has to be applied; as a consequence, the semicone $C_+(x)$ becomes a future semiplane $P_+(x)$, but this limit does not affect the orientation of $\mathbf{J}$. Then, although the time-reversal invariant fundamental laws lead to a pair of time-symmetric twins corresponding to the two possible orientations of **J**, the energy flow breaks the symmetry by selecting the quadrivector **J** corresponding to $P_+(x)$; therefore, it explains the positive value of the coefficient K and, *a fortiori*, the second law of thermodynamics.

From a more general viewpoint, we have seen that the time-symmetric twin of the second law $\sigma \geq 0$ is $\sigma \leq 0$: when $\sigma > 0$, the process produces an energy flow directed towards the future; when $\sigma < 0$, the process requires an energy flow pumped from the past. Let us note that, if we did not have a criterion for defining the past-to-future direction of the energy flow, the previous assertions would be senseless: from an "atemporal" viewpoint that does not commits a *petitio principii* by presupposing a privileged direction of time, "past" and "future" are only conventional labels (see Fig. 6). On the contrary, with the energy flow pointing to the same direction all over the spacetime, we can legitimately say that $\sigma > 0$ corresponds to a dissipative decaying process evolving from non-equilibrium to equilibrium as $e^{-\gamma t}$, and $\sigma < 0$ corresponds to an antidissipative growing process evolving from equilibrium to non-equilibrium as $e^{\gamma t}$. The two processes, which in principle are only conventionally different, turn out to be substantially different due to the future-directed energy flow that locally expresses the global time-asymmetry of the universe (see Fig. 7).

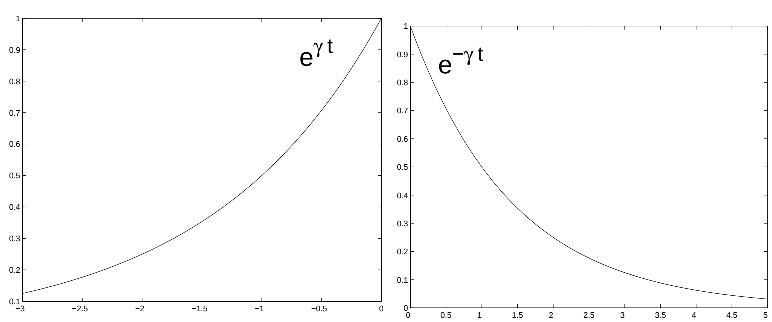

Fig. 7: Time-symmetric twins

D. Entropy and energy-momentum tensor: relativistic imperfect fluids

Finally, let us consider how the energy-momentum tensor and the entropy are related in the case of relativistic imperfect fluids, as a further example of time-symmetric twins

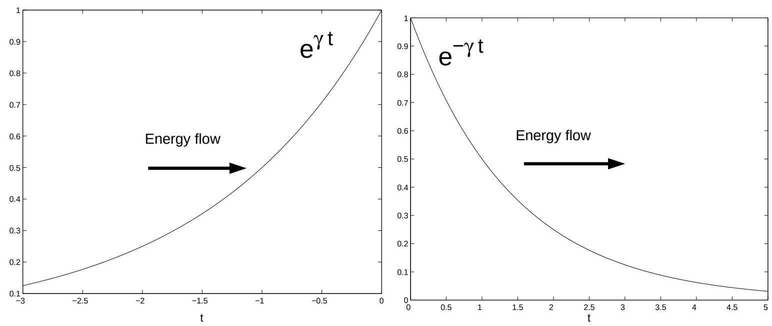

Fig. 8: Time-symmetric twins with broken symmetry

whose symmetry can be broken by a future-directed energy flow. For a universe containing a relativistic imperfect fluid, the energy-momentum tensor reads

$$T^{\mu\nu} = pg^{\mu\nu} + (p+\rho)U^{\mu}U^{\nu} + \Delta T^{\mu\nu} \tag{51}$$

where p is the pressure, $g^{\mu\nu}$ is the metric tensor, ρ is the energy-matter density, U^{μ} is the absolute velocity of the fluid, and $\Delta T^{\mu\nu}$ is a term due to the imperfection of the fluid. In a comoving (free falling) frame, $U^0 = 1$, $U^i = 0$, $\Delta T^{00} = 0$, and the corresponding eq. (44) reads

$$\sigma = -\left(\frac{1}{T}\dot{U}_i + \frac{1}{T^2}\frac{\partial T}{\partial x^i}\right)\Delta T^{i0} - \frac{1}{T}\frac{\partial U_i}{\partial x^j}\Delta T^{ij} \tag{52}$$

where T is the absolute temperature, $i, j... = 1,2,3$ are the spatial indices, and the Einstein summation convention is used as before. From this equation we want to obtain ΔT^{i0} and ΔT^{ij}. But here we will not appeal to the traditional argument, which relies on the second law $\sigma \geq 0$ (see Weinberg[(47)]). On the contrary, we will use a fundamental law as eq. (44), which will bring to the light the second element of the pair of time-symmetric twins; the second law will be a consequence of the symmetry breaking in the pair.

Let us define

$$\Delta T_0^{ij} = \delta^{ij}\Delta T_k^k, \qquad \Delta T_1^{ij} = \Delta T^{ij} - \Delta T_0^{ij} \tag{53}$$

ΔT_0^{ij} and ΔT_1^{ij} are the two irreducible components of the symmetric tensor ΔT^{ij} under the spatial rotation group $SO(3)$, while ΔT^{i0} is an irreducible vector under the same group. The thermodynamic space is flat or, at least, locally flat (near equilibrium). Therefore, since the theory must be invariant under the rotation of $SO(3)$, the matrix $M_{AB} = M_{(\mu\nu)(\kappa\lambda)}$ must be spherically symmetric for each irreducible component, and eq. (47) reads

$$\sigma = M_{(\mu\nu)(\kappa\lambda)}\Delta T^{\mu\nu}\Delta T^{\kappa\lambda} = \chi\Delta T^{i0}\Delta T_{i0} + \mu\Delta T_0^{ij}\Delta T_{ij0} + \eta\Delta T_1^{ij}\Delta T_{ij1} \tag{54}$$

where we have attributed an arbitrary scalar χ, μ, and η to each component. This equation has to be satisfied by any arbitrary values of χ, μ, and η; then, by means of eq. (52) we obtain

$$\Delta T^{i0} = -\chi \left(T\dot{U}_i + \frac{\partial T}{\partial x^i} \right) \qquad \Delta T^{ij} = -\eta \left(\frac{\partial U_i}{\partial x^j} + \frac{\partial U_j}{\partial x^i} - \frac{2}{3} \overline{\nabla} . \overline{U} \delta_{ij} \right) - \mu \overline{\nabla} . \overline{U} \delta_{ij} \quad (55)$$

Having obtained ΔT^{i0} and ΔT^{ij} without appealing to the second law $\sigma \geq 0$, now we can deduce the expression for σ from eqs. (52) and (55):

$$\sigma = \frac{\chi}{T^2} \left(\overline{\nabla} T + T\dot{\overline{U}} \right)^2 + \frac{\eta}{2T} \left(\frac{\partial U_i}{\partial x^j} + \frac{\partial U_j}{\partial x^i} - \frac{2}{3} \overline{\nabla} . \overline{U} \delta_{ij} \right) \left(\frac{\partial U_i}{\partial x^j} + \frac{\partial U_j}{\partial x^i} - \frac{2}{3} \overline{\nabla} . \overline{U} \delta_{ij} \right) + \frac{\mu}{T} \left(\overline{\nabla} . \overline{U} \right)^2 \quad (56)$$

Up to this point, we have made no assumptions about the values of the coefficients χ, μ and η; then, they can be either positive, leading to $\sigma > 0$, or negative, leading to $\sigma < 0$: both situations, one the temporal mirror image of the other, are nomologically possible according to the fundamental laws.

Also in this case, the symmetry can be broken by means of the energy flow. Once we have established a substantial difference between the two directions of time and we have used, following the traditional convention, the label ”future” for the direction of the energy flow, we can say that:

- For *dissipative* processes, $\chi > 0$, $\mu > 0$ and $\eta > 0$; as a consequence, since the ”geometrical” factors between parenthesis in eq. (56) are all non negative, then $\sigma > 0$ (the second law).

- For *antidissipative* processes, $\chi < 0$, $\mu < 0$ and $\eta < 0$; as a consequence, $\sigma < 0$. In the regions of the universe where this condition holds, the second law will be not locally valid.

E. The status of the second law

We can summarize the results of this section by saying that, according to the fundamental laws of physics, either dissipative processes with $\sigma > 0$ and antidissipative processes with $\sigma < 0$ are nomologically possible. The symmetry between both in principle formally identical situations is broken only by an energy flow which points to the same direction all over the universe and expresses the global time-asymmetry of the spacetime.

It seems quite clear that this way of breaking the symmetry in the pair of time-symmetric twins is theoretically grounded and not relying on merely contingent practical limitations. It is also completely general, since it can be applied to any pair of time-symmetric twins of physics. Furthermore, since the energy flow points to the same direction all over the spacetime, this symmetry breaking accounts for the otherwise unexplained fact that the different arrows of time, defined in the different chapters of physics (electromagnetic arrow, thermodynamic arrow, cosmological arrow, etc.), all point to the same time direction.

This conclusion allows us to assess the status of the second law of thermodynamics. As we have seen, when arguments are based exclusively on fundamental laws, pairs of time-symmetric twins appear in all the chapters of physics. In the particular case of phenomenological thermodynamics, the twin of the second law can also be discovered. So, the traditional second law $\sigma \geq 0$ only arises when the time-symmetry is broken by the future-directed energy flow. But this way of breaking the symmetry is common to all the pairs of time-symmetric twins, from electromagnetism to quantum field theory. Therefore, the second law is not endowed with a privileged character with respect to the arrow of time, as usually supposed: the thermodynamic arrow, as all the other arrows, is a consequence of the global time-asymmetry of the universe. In this sense, *the second law can be inferred on the basis of global considerations*, in the same way as the irreversible evolutions of quantum mechanics or the non time-reversal invariant postulate of quantum field theory.

VI. CONCLUSIONS

In this paper we have completed the following tasks:

- We have disentangled the concepts of time-reversal invariance, irreversibility and arrow of time, dissipating the usual confusions between the problem of irreversibility and the problem of the arrow of time.

- We have defined the arrow of time as the global time-asymmetry of spacetime, which is locally expressed as a future-directed energy flow all over the universe.

- We have shown how, in different chapters of physics, the time-reversal invariant fundamental laws lead to pairs of time-symmetric twins whose elements are only conventionally different in the light of such laws.

- We have shown how the future-directed energy flow is what breaks the symmetry of all the pairs of time-symmetric twins of physics, giving rise to the different arrows of time traditionally treated in the literature on the subject.

With this work we have tried to contribute to the resolution of the problem of the direction of time, one of the most longstanding debates on the conceptual foundations of theoretical physics.

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[1] We are grateful to one of the referees for drawing our attention on the relevance of discussing this point.

[2] Boltzmann[16] wrote: "*The universe, or at least a big part of it around us, considered as a mechanical system, began in a very improbable state and it is now also in a very improbable state. Then if we take a smaller system of bodies, and we isolate it instantaneously from the rest of the world, in principle this system will be in an improbable state and, during the period of isolation, it will evolve towards more probable states*".

[3] We are grateful to one of the referees for stressing the need to discuss this point.

[4] We are grateful to one of the referees for drawing our attention on the need to explain these subtle points.

[5] If we could compute the global entropy of the universe, with this convention we could say that the entropy increases towards the future. But this would require that the technical difficulties for defining such an entropy were overcome (see Subsection III.A.2).

[6] Of course, this leaves open a different problem: to explain why, among all the elementary interactions, only weak interactions break time-reversal invariance. It has be suggested that weak interactions define the arrow of time, but it is not clear at all how this microscopic phenomenon could break the time-asymmetry of the twins arising at the macroscopic level.

[7] We are considering a space of just one dimension for simplicity.

[8] Also in eq. (44), $\mathcal{T}J_A = -J_A$ since it is a flow, and $\mathcal{T}X_A = X_A$ since it is a force or affinity; then, $\mathcal{T}M_{AB} = -M_{AB}$. And since $\gamma, \zeta, \eta, \chi, \mu$ and K are contained in M_{AB}, they also have to change their signs.