Characterizing the performance of baseball bats

Alan M. Nathan[a)]
Department of Physics, University of Illinois, Urbana, Illinois 61801

(Received 12 July 2002; accepted 26 September 2002)

The characterization of the performance of baseball bats is presented from a physics point of view. The goal is to define a set of laboratory measurements that can be used to predict performance in the field. The concept of a model-independent collision efficiency, which relates the post-collision ball speed to the initial ball and bat speeds, is introduced and its properties are investigated. It is shown to provide a convenient link between laboratory and field measurements. Other performance metrics are presented, related to the collision efficiency, and evaluated according to their predictive power. Using a computational model, it is shown that bat performance depends on the interplay of the elasticity of the ball–bat collision, the inertial properties of the ball and bat, and the bat swing speed. It is argued that any method of determining performance needs to take all of these factors into account. A new method is proposed and compared with commonly used existing methods.

[DOI: 10.1119/1.1522699]

I. INTRODUCTION

The game of baseball has evolved in the last 20 years to the point where the traditional solid wooden bat is used primarily by professional players, where the rules do not permit other options, whereas the bat of choice at the amateur level is the hollow aluminum bat. There is a common perception among players and fans that the aluminum bat is a much more effective hitting instrument than the wooden bat. Much of the evidence is anecdotal and is therefore difficult to evaluate. In addition, there is a large body of statistical data[1] showing that the number of runs per game and home runs per game is decidedly larger when aluminum bats are used. There are also laboratory and field measurements that demonstrate the improved effectiveness of aluminum bats. The laboratory studies are largely unpublished. The most recent field study of which we are aware is the batting cage study of Greenwald *et al.*[2] Using high speed video techniques, the bat and ball were tracked during the interval prior to and after the ball–bat collision, so that the pre-collision and post-collision ball and bat speeds could be accurately measured. It was demonstrated conclusively that the average hit ball speed for a selection of aluminum bats was larger than that for a particular wooden bat. Some of the aluminum bats performed similarly to the wooden bat, but some outperformed the wooden bat by a statistically significant amount.

In view of the perceived advantage of aluminum over wood, there is a desire among baseball and softball organizations to regulate the performance of nonwooden bats in order to bring the game back into balance between offense and defense and to reduce injuries due to high batted ball speeds. In general, the organizations would like to define a set of laboratory measurements that can be used to determine metrics of performance in the field. The ultimate goal is to specify upper limits on those metrics as a way of regulating field performance. The two most commonly used metrics are the collision efficiency e_A and the coefficient of restitution e, both of which will be carefully defined in Sec. II.

The traditional approach to characterizing the performance of a striking instrument, such as a baseball bat or tennis racket, has been to discuss it in the context of the coefficient of restitution, as done for example by Kirkpatrick.[3] But Kirkpatrick recognized very well that bat performance also depended on the inertial properties of the ball and bat. The concept of using a model-independent collision efficiency to characterize the performance of bats seems to have been first recognized by Hester and Koenig in a 1993 publication that is not generally accessible.[4] They recognized that the ball exit speed is related to the initial speeds of the ball and bat by a single parameter, the collision efficiency, although those words were not actually used. They regarded e_A as essentially a "black box" that can be measured in the laboratory and then directly used to relate batted ball speed to the speed of the pitched ball and bat. This concept was rediscovered by Carroll,[5] packaged in a slightly different form called the Ball Exit Speed Ratio (BESR), and subsequently used by the NCAA as its primary bat performance metric.[6] Using a variety of ball–bat collision models, he derived the expression that relates the ball exit speed to the initial speeds of the ball and bat in terms of the BESR and showed how the BESR is related to the coefficient of restitution and ball-to-bat mass ratio. It is useful to point out that the concept of a model-independent collision efficiency is also used in the context of the collision between a tennis ball and racket.[7,8]

The most extensive study of bat performance standards was done recently by Smith,[9] who developed a computational model to investigate a typical wood and a typical aluminum bat with the goal of evaluating various commonly used procedures. Perhaps his most important finding was that the two performance metrics, the coefficient of restitution and the collision efficiency, produced seemingly contradictory results in that the performance of one bat relative to another depended on which metric was used and the manner in which the bats were tested. He concluded that these metrics do not accurately reflect a bat's performance in the field. Other important conclusions were that laboratory measurements should be done at relative ball–bat speeds more typical of game conditions than is commonly used and that the measurements should be done at the "sweet spot" rather than at the center of percussion. Recommendations were presented for an improved bat assessment method based on bat and ball speeds before and after the collision.

In this paper, we discuss the evaluation of bat performance from a physics point of view. In Sec. II we start by defining

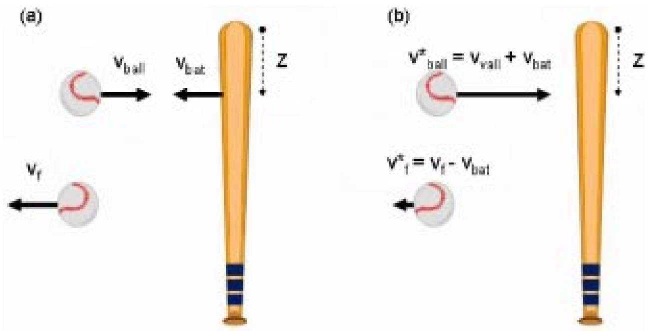

Fig. 1. Schematic of the baseball–bat collision in (a) the usual field frame and in (b) the bat rest frame, referred to in the text as the * frame. Relative to the field frame, the * frame moves to the left with velocity v_{bat}. The relative signs of the velocity vectors are positive if they point in the direction shown. The impact location z is the distance along the axis of the bat as measured from the end of the barrel.

various performance metrics, putting them on a sound physics footing, and elucidating their properties. We describe in Sec. III the computational models that will be used to study the issues related to performance, including both a dynamic model for the ball–bat collision as well as a phenomenological model for the swing of the bat. These models will be used in Sec. IV to investigate several examples of bats in order to lay the groundwork for an evaluation of the various metrics and procedures currently used to characterize the performance of bats. In Sec. V a new procedure for characterizing the performance of bats is proposed and used as a standard against which other commonly used procedures are compared and critiqued. We conclude with a summary of our important points in Sec. VI.

II. BAT PERFORMANCE METRICS

The two commonly used metrics for characterizing the performance of bats will be defined and discussed: the collision efficiency and the ball–bat coefficient of restitution. Later we will address the issue of how well these metrics serve as predictors of performance. First, however, it is necessary to have a quantitative working definition of ‘‘performance.’’ We tentatively define performance in terms of the velocity of the struck ball just after the ball–bat collision, herein denoted as the ball exit velocity v_f. More precisely, for specified values of the initial ball and bat velocities, v_{ball} and v_{bat}, respectively, where the latter refers to the bat speed at the point of impact, one bat is said to perform better than another bat if it produces a higher v_f. This criterion agrees with the common sense definition of performance. For the moment, we ignore the fact the v_{bat} itself is likely to depend on properties of the bat such as the mass and mass distribution. We will address this issue in Sec. III and will modify our tentative definition of performance in Sec. III D.

A. The collision efficiency e_A

We consider the collision of a baseball of initial velocity v_{ball} with a bat of initial velocity v_{bat}, resulting in a post-collision ball velocity v_f. The collision is shown schematically in Fig. 1 in both the usual field frame and in the frame of reference in which the bat is initially at rest at the impact location (the * frame). Our sign convention is that the velocities are all positive if they point in the usual directions: v_{bat} and v_f both opposite in direction to v_{ball}. Moreover, it is assumed that all velocity vectors are collinear.

We seek to derive a model-independent formula that relates v_f to v_{ball} and v_{bat}. We start by defining the collision efficiency e_A as the ratio of final to initial ball speed in the * frame:

$$e_A \equiv \frac{v_f^*}{v_{ball}^*} \quad \text{(collision efficiency)}. \tag{1}$$

Because $v_{ball}^* = v_{ball} + v_{bat}$ and $v_f^* = v_f - v_{bat}$ (see Fig. 1), e_A is given by

$$e_A = \frac{v_f - v_{bat}}{v_{ball} + v_{bat}}. \tag{2}$$

By inverting Eq. (2), we arrive at our desired result:

$$v_f = e_A v_{ball} + (1 + e_A) v_{bat}. \tag{3}$$

The collision efficiency is related to the so-called Ball Exit Speed Ratio (BESR), the performance metric described by Carroll[5] and used by the NCAA,[6] by

$$\text{BESR} = e_A + \tfrac{1}{2}, \tag{4}$$

leading to a more symmetrical form of Eq. (3):

$$v_f = (\text{BESR} - \tfrac{1}{2}) v_{ball} + (\text{BESR} + \tfrac{1}{2}) v_{bat}.$$

Both e_A and the BESR have an equivalent meaning physically, because they differ only by a constant additive factor.

The utility of the collision efficiency comes from the following observations. Equations (2) and (3) are exact expressions that are independent of any model of the ball, bat, or the collision between them. They are derived using nothing other than the definition of e_A followed by a change of inertial reference frame. Although the equations are identical to those derived previously by Hester[4] and Carroll,[5] the present derivation, which is identical to that of Brody,[7] is simpler and makes clear their model independence. For any combination of ball and bat speeds, Eq. (3) allows a prediction of v_f if e_A is known. Moreover the inverted form, Eq. (2), can be used to infer e_A from measurements of v_f. In either case, no additional information about the ball or bat is necessary. Also our derivation of Eq. (3) makes it clear that the velocity dependence of e_A is only on the relative ball–bat velocity, $v_{rel} = v_{ball} + v_{bat}$, so that any combination of v_{ball} and v_{bat} with the same v_{rel} will give the same result for e_A. This dependence has the practical significance that when measuring e_A, it makes no difference whether the bat (or ball) is initially at rest or moving, as long as the desired value of v_{rel} is achieved. Moreover, it is an empirical fact that although e_A depends on v_{rel}, it does so only weakly,[10] especially for velocities relevant to the game of baseball. This fact derives from the weak dependence of the coefficient of restitution, defined below, on v_{rel}.[11] This weak dependence means that the dependence of v_f on v_{ball} and v_{bat} comes primarily from the explicit factors in Eq. (3) rather from e_A itself. Nevertheless, when using laboratory measurements to predict field performance, it is highly desirable for the v_{rel} used in the laboratory to be close to that expected in the field, as we will show with a numerical example in Sec. II C.

The collision efficiency is expected to be a strong function of the impact location along the axis of the bat.[10] It is largest in the region of the bat commonly (but somewhat imprecisely) referred to as the "sweet spot," a region close to the nodes of the lowest frequency bending vibrations (~4–6 in. from the barrel end of the bat). Henceforth, this region will be referred to as the "sweet spot zone."

As shown in a recent study of the dynamics of the ball–bat collision[10] as well as in earlier studies, for a typical high speed ball–bat impact at the barrel end of the bat, neither e_A nor v_f depend on how the bat is supported at the knob end. In particular, it does not matter whether the bat is free, hand-held, pivoted, or clamped, nor does it matter how firmly the hand-held bat is gripped. The essential physics is that on the short time scale of the collision, the ball does not "know" what is happening at the far end of the bat. The practical significance is that measurements of e_A on a pivoted or clamped bat in the laboratory should allow predictions of performance in the field where the bat is hand-held.

B. The coefficient of restitution *e*

The ball–bat coefficient of restitution (COR), e, is the ratio of relative ball–bat speed after the collision to that before the collision and is another metric of bat performance. Equivalently e is a measure of the elasticity of the collision, because $1-e^2$ is the fraction of the initial kinetic energy in the center of mass (c.m.) frame that is dissipated. This result follows from the proportionality of the kinetic energy in the c.m. frame to the square of the relative velocity, as shown in many introductory textbooks.[12] Here "elasticity" is used in the particle physics sense in which an elastic collision between two bodies is one in which no energy is transferred to the internal degrees of freedom of the bodies. The energy dissipated in a nonelastic collision is that part of the initial kinetic energy that appears neither in the rebound kinetic energy of the ball nor in the rigid-body recoil of the bat. For a perfectly elastic collision, $e=1$. For ball–bat collisions in the sweet spot zone at speeds typical of the game of baseball, $e\approx 0.45-0.50$.[11] Due to the excitation of bending vibrations in all bats and to the so-called "trampoline effect" in hollow metal bats, e is generally different from the ball coefficient of restitution e_0, which is the ratio of rebound to initial speed when the ball collides with a massive rigid surface. Sometimes e_0 is referred to as the "ball–wall coefficient of restitution."[9] Although e_0 is a property of the ball alone, e is a joint property of the ball and bat. The ratio e/e_0 is called the Bat Performance Factor (BPF),

$$\mathrm{BPF}\equiv\frac{e}{e_0}\quad\text{(Bat Performance Factor)}. \tag{5}$$

In the sweet spot zone, the BPF is typically close to 1 for wooden bats, but sometimes significantly larger (1.10–1.20) for aluminum bats. The BPF is yet another performance metric and is the one used by the American Society for Testing and Materials (ASTM)[13] to characterize the performance of bats.

The COR and collision efficiency are not independent metrics, but are related by[10]

$$e_A=\frac{e-r}{1+r}, \tag{6}$$

which is also a model-independent relation that is easily derived using conservation laws. Here, r is the "bat recoil factor," which depends only on the inertial properties of the ball and bat. It is defined as ratio of ball mass m to the effective bat mass M_{eff},

$$r=\frac{m}{M_{\mathrm{eff}}}\quad\text{(bat recoil factor)}. \tag{7}$$

The use of an effective bat mass is a convenient device that allows the conservation equations to be solved by replacing the actual extended bat with an equivalent point bat whose mass depends on impact location.[14] For a free bat, the conservation of both linear momentum and angular momentum about the c.m. implies

$$\frac{1}{M_{\mathrm{eff}}}=\frac{1}{M}+\frac{(z-z_{\mathrm{c.m.}})^2}{I_{\mathrm{c.m.}}}\quad\text{(free bat)}, \tag{8}$$

whereas for a pivoted bat, conservation of angular momentum about the pivot point implies

$$\frac{1}{M_{\mathrm{eff}}}=\frac{(z-z_p)^2}{I_p}\quad\text{(pivoted bat)}. \tag{9}$$

M is the actual bat mass, $I_{\mathrm{c.m.}}$ and I_p are the moments of inertia of the bat about the c.m. and pivot point, respectively, z is the impact location, $z_{\mathrm{c.m.}}$ is the location of the c.m., and z_P is the position of the pivot point. From the parallel axis theorem, $I_p=I_{\mathrm{c.m.}}+M(z_{\mathrm{c.m.}}-z_p)^2$. All distances are along the axis of the bat from the barrel end, as shown in Fig. 1.

It is instructive to examine energy balance in the collision. Using Eqs. (3) and (6)–(8), it is straightforward to show that in the frame of reference in which the bat is initially at rest, the post-collision energy is partitioned as follows:

$$f_{\mathrm{ball}}=\left(\frac{e-r}{1+r}\right)^2, \tag{10a}$$

$$f_{\mathrm{bat}}=r\left(\frac{1+e}{1+r}\right)^2, \tag{10b}$$

$$f_{\mathrm{dis}}=\frac{1-e^2}{1+r}. \tag{10c}$$

Here, f_{ball}, f_{bat}, and f_{dis} are the fraction of initial kinetic energy going to the outgoing kinetic energy of the ball, the rigid-recoil kinetic energy of the bat, and dissipation, respectively. It is easily verified that $f_{\mathrm{ball}}+f_{\mathrm{bat}}+f_{\mathrm{dis}}=1$. Note that for an infinitely massive bat, $r=0$, so that $f_{\mathrm{ball}}=e^2$, $f_{\mathrm{bat}}=0$, and $f_{\mathrm{dis}}=1-e^2$. In the subsequent discussion, it will be helpful to think of r as controlling the recoil energy of the bat, just as $1-e^2$ controls the dissipation.

From Eqs. (8) and (9), we see that M_{eff} and therefore r depend on the impact location z. For a free bat, the effective mass *decreases* with increasing distance of the impact from the c.m., because the linear impulse to the bat gives rise to a larger angular impulse about the c.m. Equation (6) shows that for a given e, e_A is maximized when r is small, which minimizes the bat recoil energy. For a ball incident on an infinitely massive bat, $r=0$ and $e_A=e$. For this reason e_A is sometimes referred to as the "apparent COR,"[15] a terminology that sometimes leads to confusion as to the physical significance of e_A. More generally, $e_A\leqslant e$ because some of the initial energy goes into the recoil of the bat.

In parallel with our discussion of e_A, we next discuss several observations about e relevant to its utility as a metric of bat performance. First, whereas e_A is sufficient as a predictor of performance, e is not, because it is also necessary to know r [see Eqs. (3) and (6)]. As we will show later with examples, bats with the same e but different mass distributions can perform quite differently. This difference should not be surprising, because e only accounts for the dissipated energy, whereas r is needed to account for the bat recoil energy. As with e_A, the velocity dependence of e is only on v_{rel}. Empirical data on ball–wall collisions[12] show that e_0 drops roughly linearly from about 0.55 at $v_{\mathrm{rel}}=60$ mph to about 0.45 at $v_{\mathrm{rel}}=160$ mph. Moreover e is a strong function of the impact location along the axis of the bat.[10]

Unlike e_A, e depends on how the bat is supported at the knob end. This dependence follows immediately from Eq. (6) and the fact that the recoil factor r does depend on the means of support (for example, free, pivoted). To our knowledge, this dependence has not been pointed out in the literature. The physics behind this dependence is interesting and subtle. Looking at the collision in the frame in which the bat is initially at rest, the collision transfers energy to the bat in the form of both rigid-body kinetic energy (which is related to r) and vibrational energy (which is related to e). Because e_A is independent of the support, so too must be the sum of these energies, although neither of these are individually independent of the support. As an extreme example, consider the collision of ball with a bat that is clamped at the handle, in which case r is identically zero, because the bat can neither translate nor rotate. Therefore, all of the bat energy appears as vibrations, resulting in a smaller e than for a free bat but the same e_A. The essential physics is that the partitioning of the bat energy into rigid body and vibrational modes is artificial on the short time scale of the collision, because the different modes only get sorted out long after the ball has left the bat. Said differently, on time scales short compared to a vibrational period, a vibrational mode is indistinguishable from a rigid-body mode. The practical consequence of this is that one must proceed cautiously whenever using laboratory measurements of e with a pivoted bat to predict performance with a free bat or hand-held bat.

Related to the preceding point is the center of percussion (COP), which is sometimes discussed in the context of baseball bat performance. Two points are said to be COP conjugates of each other if an impact at one of the points results in no change in rigid-body motion at the other point. In other words, for a bat initially at rest, a collision at the first point would cause the bat to rotate about the second point. These two points, z_1 and z_2, are related by[16]

$$(z_1-z_{\mathrm{c.m.}})(z_{\mathrm{c.m.}}-z_2)=\frac{I_{\mathrm{c.m.}}}{M}. \tag{11}$$

As pointed out by Brody[16] and more recently by Smith,[9] the COP has no relevance for bat performance, although it may have relevance to the post-collision "feel" of the bat in the hands of the batter.[17] It may also have relevance for the interpretation of e because, as is easily shown, the bat recoil factor r for a free and pivoted bat, Eqs. (8) and (9), respectively, is equal when the impact point z is the COP conjugate to the pivot point z_p. Therefore, for collisions at (or near) the COP, e is the same (or almost the same) for a free and pivoted bat, thereby facilitating its interpretation as a performance metric.

C. Numerical examples

For a typical ball–bat collision in the sweet spot zone, $e \approx 0.5$ and $r \approx 0.25$, so that $e_A \approx 0.2$ (or BESR ≈ 0.7), implying that the collision is very inefficient. Moreover, Eq. (3) takes the form $v_f=0.2v_{\mathrm{ball}}+1.2v_{\mathrm{bat}}$, showing that bat swing speed matters considerably more than pitched ball speed in determining v_f, a fact known intuitively to most players. The reasons for the large asymmetry between v_{ball} and v_{bat} are due in part to the low collision efficiency and in part to the extra factor of 1 in the pre-factor multiplying v_{bat}. This factor arises from the transformation from the bat rest frame to the lab frame; in essence, it arises because the bat is initially moving in the direction of the outgoing ball.

Given the relative smallness of e_A and the presence of the factor of 1, the exit ball speed is less sensitive to e_A than what one might otherwise have thought. In particular, it is less sensitive to the ball–bat COR than often thought. Indeed, there is much current interest in the effect of "juiced" balls and bats on the ball exit speed, where juiced refers to a larger than normal COR. This issue has been discussed by Cross[15] in the context of tennis, but it is worthwhile reiterating the essential point here. By combining Eqs. (3) and (6), it is straightforward to show that if the COR is changed by an amount δe, the ball exit speed is changed by an amount δv_f given by

$$\delta v_f=\frac{v_{\mathrm{rel}}}{1+r}\,\delta e. \tag{12}$$

Cross's observation is that the COR matters more for groundstrokes than for serves, because v_{rel} is typically twice as large in the former case than in the latter. The same holds true for baseball (similar to groundstrokes) and slow-pitch softball (similar to serves). For example, changing e by 10%, from 0.50 to 0.55, increases v_f by about 6 mph for baseball and 3 mph for slow-pitch softball, where v_{rel} is typically 160 and 80 mph, respectively.

Finally, we address the question of the dependence of e_A on v_{rel}. If e varies approximately linearly between 0.55 at 60 mph and 0.45 at 160 mph, then e_A varies between about 0.24 and 0.16, respectively. Therefore if a laboratory measurement of e_A at 60 mph is used to predict v_f at 160 mph, v_f will be overestimated by around 12.8 mph. This discrepancy is reduced to about 2.6 mph if the measurement is done at 140 mph. Therefore, depending on the overall accuracy desired in characterizing performance in the field, the laboratory measurements should be done close to the field value of v_{rel}. This observation is in accord with that of Smith.[9] A related point is that the rules of the game allow for a variation in e_0. For example, the NCAA allows balls in the range $e_0=0.525$–0.555 to be used in regulation games, as measured at 60 mph. When comparing the performance of one bat to another in a laboratory test, it is therefore important either to use balls with nearly identical values of e_0 or to have a method of correcting for a different e_0. Lacking the latter, we emphasize the former.

D. Summary

To summarize the key points of this section, performance is tentatively defined as the ball exit velocity v_f achieved for a given v_{ball} and v_{bat}. Two metrics of bat performance have been defined: the collision efficiency e_A (or BESR), and the ball–bat COR e (or BPF). Equations (3) and (6) show that for a given v_{bat} and v_{ball}, v_f depends on two general properties of the ball–bat collision: the elasticity of the collision (through e) and the inertial properties of the ball and bat (through r, which controls the recoil energy of the bat). Contrary to the conclusion reached by Smith,[9] the quantity e_A properly takes into account *both* of these properties. It is measured directly via Eq. (2) and used as a predictor for v_f via Eq. (3); for a fixed v_{rel}, no additional information is needed. On the other hand, e takes into account *only* the elasticity. It is measured indirectly by first measuring e_A [Eq. (2)], then using knowledge of r and Eq. (6) to infer e. Similarly, e by itself is not sufficient for predicting v_f because it is also necessary to know r. Finally, we remark that we have thus far considered performance only for a given v_{ball} and v_{bat} and have postponed a consideration of the relationship between bat speed and the inertial properties of the bat to Sec. III.

III. COMPUTATIONAL MODELS

To investigate further the merits of the metrics described in Sec. II as predictors of field performance, it is necessary to go beyond generalities and to study specific examples. Ideally, we would like to investigate these issues experimentally. However, as with the study of Smith,[9] the approach here is to investigate this question using computational models. First, a dynamical model is developed to simulate the baseball–bat collision.[10] Next, a mostly phenomenological model is developed to simulate the bat swing. Then a simple example is presented followed by a definition of ‘‘standard game conditions’’ and a redefinition of performance. The formalism of this section will be used in Sec. IV to study the essential issues in characterizing bat performance.

A. Model for the ball–bat collision

The model used to simulate the ball–bat collision is only slightly modified from that described in an earlier publication.[10] Briefly, the bat is modeled as a nonuniform Timoshenko beam, from which the eigenvalue problem can be solved to find the normal mode frequencies and shapes for transverse bending vibrations. The ball is modeled as a nonlinear spring with losses simulated by a hysteresis curve. The parameters describing the force-versus-compression curve are adjusted to reproduce approximately the collision time and ball coefficient of restitution e_0. The collision is treated by dynamically coupling the ball to the bat so that the force that they mutually exert on each other compresses the ball and bends the bat. By expanding the motion of the bat into a sum over normal modes (including rigid body modes), we obtain a set of $N+1$ coupled second-order (in time) differential equations of motion, where N is the number of normal modes of the bat that are included. In practice, because the collision times are $\geqslant 0.5$ ms, only modes with frequencies less than a few kilohertz need be included (that is, the lowest four to six modes for a typical wooden bat). For given initial values of v_{ball} and v_{bat}, standard numerical techniques are used to integrate the coupled equations until the ball and bat separate and v_f is determined. The kinematic equations, Eqs. (2) and (6), along with the recoil factor r, can then be used to determine e_A and e.

This general technique needs to be augmented for a hollow aluminum bat to take into account the ‘‘shell modes,’’ which correspond to a radial deformation of the bat with a $\cos n\theta$ azimuthal dependence. The most important of these modes is the lowest $n=2$ mode, which typically is at 2–3 kHz. It is the mode responsible for the characteristic ‘‘ping’’ of the bat. It also plays a dominant role in the trampoline effect, whereby some of the collision energy that would otherwise have been stored and mostly dissipated in the compression of the ball is instead stored in this mode. Because the frequency of this mode is larger than the inverse of the collision time, the stored energy is mostly returned to the ball at the end of the collision. Therefore, the overall dissipation is less than it would otherwise have been; that is, $e>e_0$ or BPF>1. The physics behind the trampoline effect will be explored in depth in a future publication.

B. Model for the swing

Thus far, we have only considered bat performance for a specified bat speed. In order to predict performance in the field, however, it is necessary to recognize the important role played by the swing of the bat. Because we generally believe that a lighter bat is likely to be swung faster than a heavier bat, there must be some relationship between the inertial properties of a bat and bat speed. Moreover, because the motion of the bat involves some mixture of translational and rotational motion,[18] v_{bat} will depend on the impact location z. Finally, for a given bat, both the overall speed of the swing and the relationship between bat speed and impact location are likely to be different for different batters. Indeed, one of the primary reasons why some players hit more home runs than others is that they are able to generate higher bat speed. The importance of bat speed can be appreciated by the following numerical example. For a typical collision efficiency of 0.2, an increase of 1 mph in v_{bat} results in an increase of 1.2 mph in v_f. Predicting absolute bat speed for a specified bat and batter is not easily amenable to a physics analysis. However, predicting how *relative* bat speed depends on *relative* inertial properties of the bat is something that is subject to such an analysis. We proceed by formulating a model that we will attempt to constrain with experimental data.

We start with the assumption that a batter swings the bat by rotating it about a fixed point on the axis of the bat. As shown by Adair,[18] this is not a good approximation for the full swing; however, it may be a reasonable approximation for the period just prior to the ball–bat collision. Indeed, recent experiments,[2,19] in which swung bats were tracked using high speed video under batting cage conditions, show that just prior to the collision, the bat is instantaneously rotating about a point about 1 in. from the knob toward the barrel. With this assumption, a single parameter, the angular velocity ω about the rotation point, is sufficient to determine v_{bat} as a function of z. Following Cross and his analysis of the swing of a tennis racket,[20] we assume that ω has a power-law dependence on I_{knob}, the moment of inertia of the bat about a point on the axis of the bat 1 in. from the knob,

$$\omega \sim I_{\mathrm{knob}}^{-n}. \tag{13}$$

We consider the two limiting cases of $n=0$, which implies a constant bat speed, that is, independent of I_{knob}, and $n=0.5$, which implies a constant bat kinetic energy. These two cases lead to an optimum bat weight (that is, a bat weight that maximizes v_f) which is unrealistically large or small, respectively.[3,18] It therefore seems reasonable that the variation of ω with I_{knob} lies somewhere between these two extremes, as suggested by Adair.[21] Recent data[2,19] support this hypothesis by showing that $n \approx 0.3$ for I_{knob} in the range $(1.5-2.0)\times 10^4$ oz in.^{2} This value is conveniently about halfway between the two extreme cases and is consistent with $n=1/3$, which would be expected if the batter puts a constant power into the bat.

It should be pointed out that both of the recent experiments had very limited data sets, and additional research in this area is highly desirable. Moreover, our model for the swing, Eq. (13), is by no means the only possible way to parametrize the dependence of swing speed on the inertial properties of the bat. Adair[19] and independently Koenig[22] have proposed a model in which the batter puts a constant kinetic energy into the bat-plus-batter system. With that hypothesis, the scaling becomes $\omega \sim (I+I_0)^{-0.5}$, where I is the moment of inertia of the bat and I_0 is the moment of inertia of the batter, both taken about a vertical axis through the center of the batter's body. This model is more physical than that of Eq. (13), because it has a reasonable limit as $I_{\text{knob}} \to 0$. Over the narrow range of I_{knob} investigated, the recent data can be equally well described by this hypothesis. Moreover, none of the conclusions reached below depend substantially on which model is used, provided we restrict its use to the range over which it has been tested. It is not the goal of the present work to advocate for one swing model or another but only that some model is needed to account for the experimentally determined fact that swing speed depends on the inertial properties of the bat. For illustrative purposes, we take Eq. (13) as our working assumption, with the exponent bounded by the limits $n=0$ and $n=0.5$.

C. A simple example

To demonstrate the interplay among the various parameters, calculations were done on a generic wooden bat (Louisville Slugger R161, 34 in., 31 oz), the results of which are shown in Fig. 2. It was assumed that $v_{\text{ball}}=90$ mph, the ball–wall COR (e_0)=0.50, and the bat is swung by rotating it with an angular velocity of 45 rad/s about a point on the axis of the bat 1 in. from the knob toward the barrel, implying that $v_{\text{bat}}=70$ mph at a point 6 in. from the tip of the bat. The dependence of e on impact location is determined primarily by the location of the nodes of the bending vibrations, with the maximum occurring roughly between the first and second nodes. The maximum value is close to $e_0=0.50$, as expected if very little energy goes into bending vibrations. However, e falls off sharply at impact locations removed from that point, as more and more energy gets dissipated in bending vibrations. Because the bat recoil factor r for the free bat is minimized at the c.m., which is about 12 in. from the tip, the location of the peak of e_A is a little further from the tip than the peak of e. Otherwise, e_A mainly follows e. The profile of v_f is determined both by e_A and by v_{bat}. In the absence of a dependence of v_{bat} on impact location, the profile of v_f would exactly follow that of e_A. However, because v_{bat} is larger closer to the tip, the peak of the v_f profile is closer to the tip than that of the e_A profile. This example shows clearly that the impact location giving the maximum value of v_f depends on how the bat is swung.

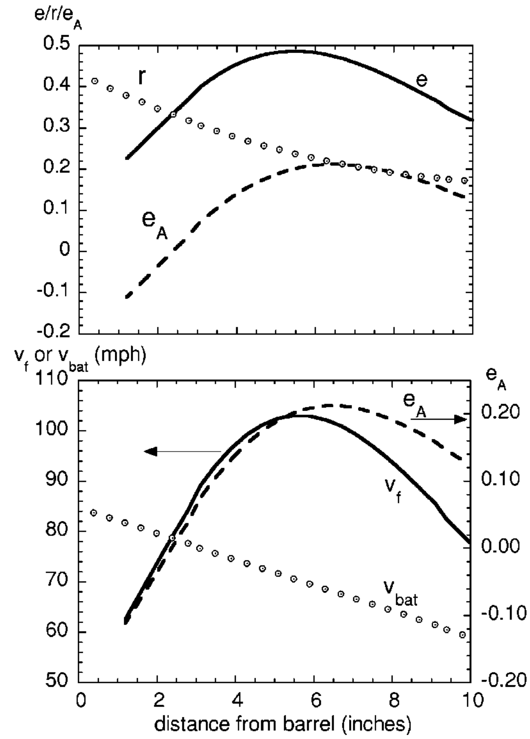

Fig. 2. Results for a generic wood bat. It is assumed that the incident ball speed is 90 mph, the initial angular velocity of the bat about the knob is 45 rad/s, and that the ball COR $e_0=0.50$. The upper plot shows e, r, and e_A; the lower plot shows e_A, v_{bat}, and v_f.

D. Specification of standard game conditions

As seen from the previous example, and as will be seen in the more detailed examples in Sec. IV, the interplay between e_A and v_{bat} is important in characterizing the performance of bats. In order to compare the field performance of one bat to another, it is necessary to define carefully the conditions under which they are compared by specifying e_0, v_{ball}, and v_{bat}. In the discussion below, it will be assumed that $e_0=0.50$ and $v_{\text{ball}}=90$ mph, and that the bat is swung by rotating it about a point 1 in. from the knob with an angular velocity given by

$$\omega=\omega_0\left[\frac{I_0}{I_{\text{knob}}}\right]^n, \tag{14}$$

with $\omega_0=45$ rad/s, $I_0=1.7\times 10^4$ oz in.^{2}, and n in the range 0–0.5. Together these parameters define what we will call "standard game conditions." We now redefine "performance" to be the maximum exit velocity achieved under standard game conditions. That is, one bat will be said to perform better than another bat if its maximum v_f under standard game conditions is larger. We emphasize that it is *not necessary* to measure e_A or BPF under these standard conditions, as long as the desired v_{rel} and range of impact locations is obtained.

IV. SOME EXAMPLES AND DISCUSSION

We next use our computational models of the ball–bat collision and bat swing to evaluate how well the metrics described earlier predict performance. We investigate four different bats, whose properties are given in Table I. All bats

Table I. Properties of our standard wood bat (R161) and several different aluminum bats. The quantities $z_{\mathrm{c.m.}}$, $I_{\mathrm{c.m.}}$, and I_{knob} are the location of the center of mass, the moment of inertia about the center of mass, and the moment of inertia about the knob, respectively. The seventh and eighth columns show the frequencies and barrel nodes, respectively, for the lowest two bending vibrations. All distances are measured with respect to the barrel of the bat.

Bat	Length (in.)	Mass (oz)	$z_{\mathrm{c.m.}}$ (in.)	$I_{\mathrm{c.m.}}$ (oz in.^{2})	I_{knob} (oz in.^{2})	f_1/f_2 (Hz)	$\mathrm{Node}_1/\mathrm{Node}_2$ (in.)
R161	34	31	11.3	2539	17 137	164/551	6.8/5.2
EA70	34	31	13.1	2757	15 033	221/721	7.6/5.5
Barrel-loaded	34	31	12.2	2857	16 269	216/703	6.8/4.8
Knob-loaded	34	31	14.6	3418	13 913	191/691	7.8/5.6

are 34 in. long and weigh 31 oz. One bat is the generic wooden bat described above. Another is a generic aluminum bat (Easton EA70) with shell modes adjusted to give it a peak BPF 8.5% larger than that of the wooden bat. The remaining two bats are versions of an EA70 bat that have been modified by making the shell about 10% thinner (and consequently a somewhat larger BPF), then adding an additional 2.3 oz weight either to the knob end (a so-called ''knob-loaded'' bat) or to the tip end (a ''barrel-loaded'' bat). These modified bats have the same weight but very different weight distributions. All calculations were done under the standard game conditions defined above and assuming a bat satisfying free boundary conditions. To facilitate comparison of the loaded bats with each other, the reference moment of inertia in Eq. (14) was chosen to be that of the barrel-loaded bat. The results of the calculations are shown in Figs. 3 and 4.

We first compare the R161 wooden and EA70 aluminum bats (see Fig. 3). Despite the fact that the aluminum bat has a 9% larger peak BPF and a 12% larger peak collision efficiency, the peak ball exit velocity is identical for the two bats when they are swung identically (that is, for $n=0$). That is, despite the large difference in peak values of the BPF and e_A, the two bats perform nearly identically. The reason can be traced to the very different mass distributions of the two bats. The hollow aluminum bat has a more uniform mass distribution, resulting in a shift of the c.m. and of the lowest vibrational nodes away from the tip (see Table I). Therefore, both the BPF and e_A reach their maximum further from the tip, where the bat speed is lower. In effect, the advantage of larger BPF is nearly offset by the fact that there is less mass at impact locations where the bat speed is high. However, for any scenario in which the aluminum bat is swung faster than the wooden bat ($n>0$), the aluminum bat outperforms the wooden bat by up to about 6 mph for $n=0.5$, although the actual difference is likely to be closer to 3 mph.

We next compare the knob-loaded and barrel-loaded aluminum bats (Fig. 4). These two bats have nearly identical peak values of BPF, yet the barrel-loaded bat clearly outperforms the knob-loaded bat when they are swung identically. The reason is the same as for the preceding example: The barrel-loaded bat has more mass in the vicinity of the impact location than the knob-loaded bat. Indeed for any realistic scenario for the bat swing ($n<0.5$), the barrel-loaded bat performs better.

Several important conclusions arise from these examples. First, for any ball–bat collision, v_f depends on an interplay among the three contributing factors of ball–bat elasticity (e or BPF), ball and bat inertial properties (through the bat recoil factor r), and v_{bat} (which also depends on the bat inertial properties). Any method of determining performance that does not take into account all three of these factors may lead to false conclusions. Both of the examples show the importance of the mass distribution of the bat in determining performance. In particular, they show that two bats with the same BPF will not necessarily perform identically. They also show that neither the BPF nor e_A is a consistently reliable metric of relative bat performance, in agreement with the findings of Smith.[9] Finally, the examples show the impor-

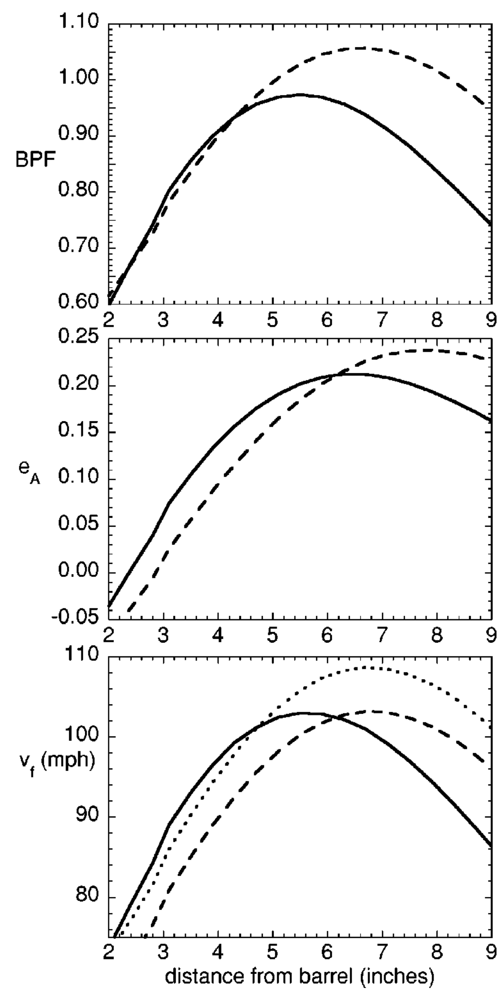

Fig. 3. Comparison of R161 wood bat (solid line) and EA70 aluminum (dashed and dotted lines) bats. The top, middle, and bottom plots show the BPF, the collision efficiency, and the ball exit speed, respectively. For the latter plot, the dashed and dotted curves are for bat speeds calculated with $n=0$ and 0.5, respectively.

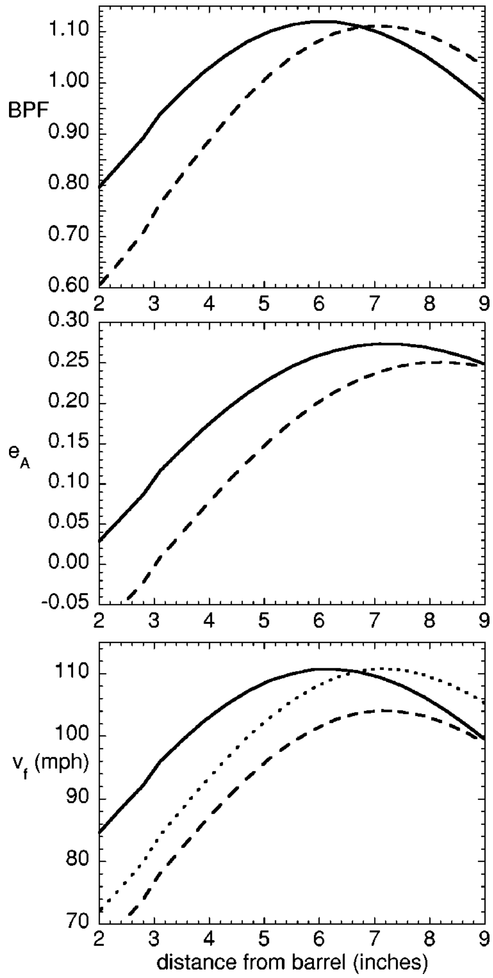

Fig. 4. Comparison between barrel-loaded (solid line) and knob-loaded (dashed and dotted lines) aluminum bats. The top, middle, and bottom plots show the BPF, the collision efficiency, and the ball exit speed, respectively. For the latter plot, the dashed and dotted curves are for bat speeds calculated with $n=0$ and 0.5, respectively.

tance of having a reasonable model for the bat swing when characterizing the performance of a bat. In particular, any testing procedure that characterizes performance at a fixed bat speed ($n=0$) would incorrectly conclude that the R161 wood bat outperforms the EA70 aluminum bat, which is almost surely not the case, and would overestimate the advantage of the barrel-loaded bat over the knob-loaded bat.

V. CHARACTERIZING THE PERFORMANCE OF BATS

In light of the examples and conclusions of Sec. IV, we next investigate possible procedures for characterizing the performance of bats. The goal of any such procedure is to define a set of laboratory measurements that will allow the performance of the bat to be predicted under standard game conditions. Said differently, we want to predict v_f for a hand-held bat under game conditions based on laboratory measurements done under similar but not necessarily identical conditions. We first present a proposal for a procedure that satisfies this criterion. We then describe and critique four procedures that are commonly used by groups that seek to regulate the performance of bats.

A. Proposed procedure

We first need to decide on the standard game conditions appropriate to the particular game. This means deciding on values of e_0, v_{ball}, I_0, and ω_0, all of which may differ depending on the nature of the participants. For example, it is reasonable to expect that v_{ball}, I_0, and ω_0 will be smaller for youth baseball than for professional baseball. Fixing these parameters then determines the range of values of v_{rel} for which testing is to be done. The procedure involves two distinct steps.

(1) Measure e_A over a range of impact locations in the barrel of bat. Equation (2) is used to extract e_A from v_f, which is measured using whatever technique is most convenient (for example, ball on stationary bat or bat on stationary ball).

(2) Using the measured e_A, calculate v_f under standard game conditions using Eq. (3) for whatever value or range of values of the bat-swing scaling parameter *n* that are deemed appropriate. The range of locations over which e_A is measured needs to be large enough to encompass the peak value of v_f.

Using this procedure, the maximum value of v_f from step 2 is taken as the measure of performance of the bat. If desired, it can be used to compare one bat to another and/or to limit the performance by specifying an upper limit.

B. Commonly used procedures

We next describe and critique four commonly used procedures for characterizing the performance of bats. We note from the outset that all but one of these techniques only consider the case $n=0$; that is, all bats are assumed to be swung with identical speed, independent of any differences in their inertial properties. It is an important feature of our proposed procedure that alternate values of *n* in the range 0–0.5 be considered.

(1) *ASTM method:* The ASTM method is based on a BPF standard.[13] The BPF is determined for a collision in which a 60 mph ball impacts a stationary bat (that is, $v_{\text{rel}}=60$ mph) which is free to pivot about a point on the axis of the bat 6 in. from the knob; the impact location is at the COP conjugate to the pivot point. The measured quantity is the rotational speed of the recoiling bat, which is related to v_f by conservation of angular momentum about the pivot point. Then Eqs. (2) and (6) (or their equivalent) are used along with *r* and e_0, both independently measured, to determine the BPF. Typically, baseball or softball regulatory agencies will require bats to have a BPF less than some standard value (for example, 1.20) in order to be used in officially sanctioned games. The ASTM also provides formulas, equivalent to our Eqs. (3), (6), (7), and (8) that allow a prediction of v_f for any combination of v_{ball} and v_{bat}, but for impacts at the COP only.

Besides the $n=0$ restriction, the ASTM method has three additional deficiencies. First, it only considers the BPF as a metric of performance. In effect, one bat is considered to perform better than another if it has a higher BPF. As we have already seen, this is not necessarily the case, given our carefully defined definition of performance. Second, it specifies that measurements be performed at the relatively low v_{rel} of 60 mph, which is well below that typical of field conditions. Third, it only specifies measurements at a single impact location, which does not necessarily coincide with the peak of v_f under standard game conditions. Moreover, the method is not easily adaptable to impacts at locations other than the COP. Recall that impacts at the COP result in the bat rotating smoothly about the pivot point. For impacts at other locations, the reaction force at the pivot will affect the subsequent recoil motion of the bat, possibly compromising the measurement of the recoil rotational speed. This problem would be alleviated by measuring the ball exit speed rather than the bat recoil speed.

(2) *ASA method:* The method used by the Amateur Softball Association for slow-pitch softball is a two-part process similar in spirit to our proposed procedure. First the ASTM measurement technique is used to determine the collision efficiency in a 60 mph collision at the COP. Second the collision efficiency is used along with standard conditions to calculate v_f, which is required to fall below some maximum value.[23] An important feature of this method is that the bat speed for standard conditions scales as $1/M^{0.25}$. The procedure could be improved by using a measurement speed closer to game conditions (80–90 mph), by scaling bat speed with $I_{\rm knob}$ rather than M, and by relaxing the restriction to measurements at the COP.

(3) *NCAA method:* The NCAA method[6] is based on a BESR standard. The BESR is determined for a collision in which a 70-mph ball impacts a bat that is swung so that the bat speed is 66 mph at a reference point located 6 in. from the barrel end of the bat. The measured quantity is v_f, which is used together with Eqs. (2) and (4) to determine the BESR. The collision is measured at several impact locations in the vicinity of the reference point until the maximum v_f is found. The BESR at the collision point corresponding to the maximum v_f is required to be less than 0.728 ($e_A \leqslant 0.228$). For the case in which the maximum v_f occurs exactly at the reference point, this corresponds to $v_f \leqslant 97$ mph. However, if the location of the maximum v_f is closer to (further from) the barrel end of the bat than the reference point, the corresponding maximum v_f is greater (less) than 97 mph, because $v_{\rm bat}$ is higher (lower) than 66 mph.

The NCAA method is improved relative to the ASTM method because it relies on e_A as the primary metric of performance, it allows measurements over a range of impact locations, and because it uses a more realistic (but for baseball, still somewhat low) value of $v_{\rm rel}$. Still, it would be better to translate the e_A measurement into a prediction of v_f, as in our proposed procedure, as well as to utilize bat speed prescriptions other than $n=0$.

(4) *Modified NCAA method:* The modified NCAA method is similar to the NCAA method, except that it is a v_f-based standard. For the same incident ball and bat swing speed as in the unmodified standard, the standard requires that the maximum $v_f \leqslant 97$ mph. If the maximum v_f occurs at the 6 in. reference point, the corresponding BESR$=0.728$ and the two methods are equivalent. If it occurs closer to (further from) the barrel end of the reference point, the corresponding BESR is less than (greater than) 0.728. The actual standard used to certify bats for the NCAA is the *modified* NCAA standard. Of the currently used procedures, this one is closest to the proposed procedure. It could be improved by allowing for a higher $v_{\rm rel}$ and by relaxing the $n=0$ restriction. With regard to the latter point, despite the fact that wood and aluminum bats used by the NCAA are legislated to perform nearly identically in the laboratory, where they are swung with the same speed, aluminum is still perceived to outperform wood in the field. Evidently, the lower moment of inertia aluminum bats can be swung faster than wood bats. Finally, it is useful to point out that this method actually combines the two steps of the proposed procedure by testing under conditions closely resembling game conditions.

VI. CONCLUSIONS

We have defined, discussed, and evaluated various metrics for characterizing the performance of bats. Having done so, we now enumerate the important conclusions resulting from this study.

(1) The relationship of the hit ball speed v_f to the pitched ball speed $v_{\rm ball}$ and the bat swing speed $v_{\rm bat}$ for a given $v_{\rm rel}$ and impact location depends on only a single parameter, the model-independent collision efficiency e_A defined in Eq. (3). Measurement of e_A in the laboratory [Eq. (2)] can be used to predict v_f in the field at the same $v_{\rm rel}$. Another commonly used metric, the ball–bat coefficient of restitution e has less predictive power, because it alone cannot be used to predict v_f; it requires knowledge of the bat recoil factor r [Eq. (7)].

(2) The only reasonable metric of performance is the maximum v_f under specified field conditions, including specification of the coefficient of restitution of the ball e_0, $v_{\rm ball}$, and $v_{\rm bat}$. All other metrics are indirect and likely to lead to contradictory results. The specification for $v_{\rm bat}$ needs to take into account the fact that lighter bats can be swung faster than heavier bats.

(3) Because the elasticity of the ball–bat collision depends on $v_{\rm rel}$ and on e_0, it is important that laboratory measurements be performed at $v_{\rm rel}$ and e_0 close to that expected for field conditions.

Using these conclusions, we have proposed a new procedure for a set of laboratory measurements that will allow performance of the bat to be predicted under specified game conditions, and we have critically analyzed various other procedures that are currently in use.

ACKNOWLEDGMENTS

I thank Professor Rod Cross for encouragement and for a critical reading of a preliminary version of this manuscript. I also thank Professor Ken Johnson for his enthusiasm and insight, Dr. J. J. (Trey) Crisco for providing me with his swing speed data, and Professor Lloyd Smith for many lively discussions. This work was supported in part by the U. S. National Science Foundation under Grant No. PHY-00-72044. This paper is dedicated to the memory of Ted Williams, 1918–2002, quite possibly the greatest hitter who ever lived.

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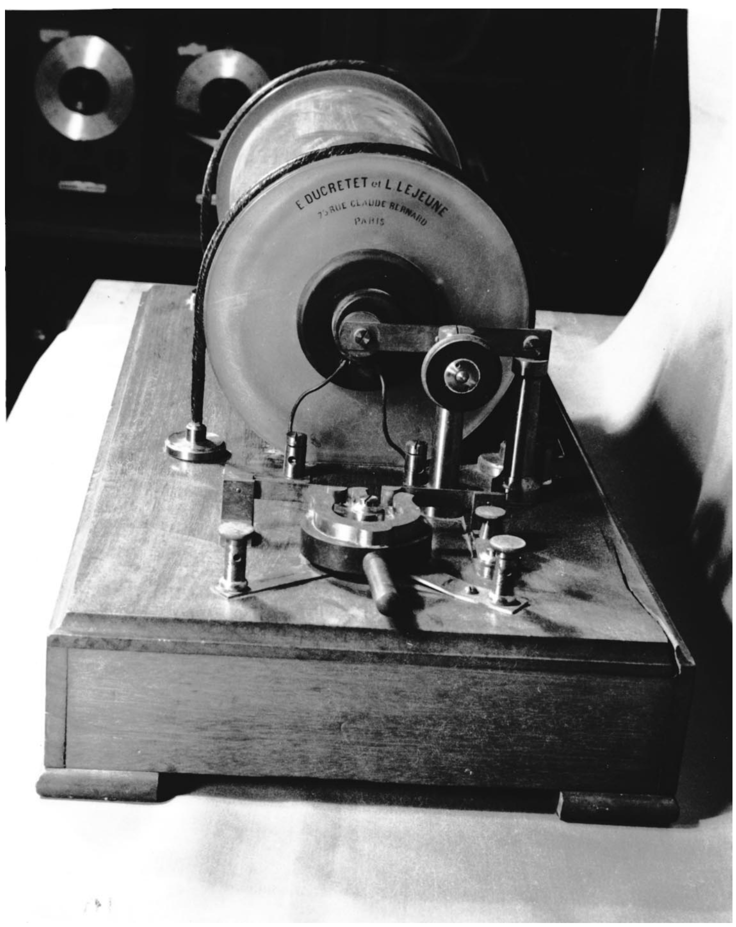

Induction Coil. The classic induction coil is used to provide a high voltage to excite discharges in rarified gases. The usual picture shows the multiple-turn secondary coil that surrounds the core made of a bundle of parallel iron wires (to avoid eddy currents) and the primary coil. This view shows the feedback mechanism used to interrupt the current in the primary coil, invented by Charles Grafton Page ca. 1840. Here, the magnetic field pulls in a flexible metal strip, breaking the primary current. This makes the magnetic field collapse, causing the strip to spring back and reestablish the current. This French coil, made with glass end plates on the coils, is at St. Mary's College in Notre Dame, Indiana. (Photograph and notes by Thomas B. Greenslade, Jr., Kenyon College)