Bitcoin and Secure Computation With Money

How to Use Bitcoin to Play Internet Poker

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Goals

MPC enhancements

- Impose fairness in MPC without an honest majority.
- Secure (reactive) MPC with money inputs and outputs
  - For example: poker.
- Efficiency improvements to the MPC itself:
  - Transform semi-honest secure MPC to MPC secure in the malicious setting, while penalizing caught deviations.

Formal model that incorporates coins

Functionality $\mathcal{F}_{\square}$ versus functionality $\mathcal{F}^{\star}_{\square}$ with coins

- If party P_i has (say) secret key sk_i and sends it to party P_j, then both P_i and P_j will have the string sk_i.
- If party P_i has coins(x) and sends $y < x$ coins to party P_j, then P_i will have coins$(x - y)$ and P_j will have extra coins(y).

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- Ideally, all the parties deem coins to be valuable assets.
- It is possible to define the *secure computation with coins* model directly, or with (UC) ideal functionalities.
- Sending coins(x) may require a broadcast that reveals at least the amount x (not in zk-SNARK cryptocurrency like ZeroCash).
- We give proofs using the simulation paradigm (but not in this talk).

Claim-or-Refund for two parties P_s,P_r (implicit in [Max11],[BBSU])

The $\mathcal{F}^{\star}_{\mathrm{CR}}$ Claim-or-Refund ideal functionality

1. The sender P_s deposits (locks) her coins(q) while specifying a timebound τ and a circuit $\phi(\cdot)$.
2. The receiver P_r can claim (gain possession) of the coins(q) by publicly revealing a witness w that satisfies $\phi(w) = 1$.
3. If P_r didn't claim within time τ, coins(q) are refunded to P_s.

How to realize $\mathcal{F}^{\star}_{\mathrm{CR}}$ via Bitcoin

- The feature that is needed is "timelock" transactions.
- Technically: Bitcoin nodes agree to include a transaction with timelock field τ only if current block index/timestamp is $> \tau$
- It is possible to have more expressive schemes that allow not-yet-reached timelock transactions to reside on the blockchain (or local mempool), but this is prone to DoS.

$\mathcal{F}^{\star}_{\mathrm{CR}}$ via Bitcoin

High-level description the $\mathcal{F}^{\star}_{\mathrm{CR}}$ implementation in Bitcoin

- P_s controls TX_{old} that resides on the blockchain.
- P_s creates a transaction TX_{new} that spends TX_{old} to a Bitcoin script that can be redeemed by P_s and P_r, or only by P_r by supplying a witness w that satisfies $\phi(w) = 1$.
- P_s asks P_r to sign a timelock transaction that refunds TX_{new} to P_s at time τ (conditioned upon both P_s and P_r signing).
- After P_r signs the refund, P_s can safely broadcast TX_{new}.

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1. P_s is safe because P_r only sees Hash(TX_{new}), and therefore cannot broadcast TX_{new} to cause P_s to lose the coins.
2. P_r can safely sign the random-looking data Hash(TX_{new}) because the protocol uses a freshly generated (sk_R, pk_R) pair.

The structure of Bitcoin transactions

How standard Bitcoin transactions are chained

- TX_{old} = earlier TX output of coins(q) is redeemable by pk_A
- id_{old} = Hash(TX_{old})
- $PREPARE_{\text{new}} = (id_{\text{old}}, q, pk_B, 0)$ 0 means no timelock
- $TX_{\text{new}} = (PREPARE_{\text{new}}, \texttt{Sign}_{sk_A}(PREPARE_{\text{new}}))$
- id_{new} = Hash(TX_{new})
- Initial minting transaction specifies some pk_M that belongs to a miner, and is created via *proof of work*.

Realization of $\mathcal{F}^{\star}_{\mathrm{CR}}$ via Bitcoin

The $\mathcal{F}^{\star}_{\mathrm{CR}}$ transaction

- $PREPARE_{\text{new}} = (id_{\text{old}}, q, (pk_S \wedge pk_R) \vee (\phi(\cdot) \wedge pk_R), 0)$
- $\phi(\cdot)$ can be $\texttt{SHA256}(\cdot) == Y$ where Y is hardcoded.
- $TX_{\text{new}} = (PREPARE_{\text{new}},\ \texttt{Sign}_{sk_S}(PREPARE_{\text{new}}))$
- $id_{\text{new}} = \text{Hash}(TX_{\text{new}})$
- P_s sends $PREPARE_{\text{refund}} = (id_{\text{new}}, q, pk_S, \tau)$ to P_r
- P_r sends $\sigma_R = \texttt{Sign}_{sk_R}(PREPARE_{\text{refund}})$ to P_s
- P_s broadcasts TX_{new} to the Bitcoin network
- If P_r doesn't reveal w until time τ then P_s creates $TX_{\text{refund}} = (PREPARE_{\text{refund}},\ (\texttt{Sign}_{sk_S}(PREPARE_{\text{refund}}), \sigma_R))$ and broadcasts it to reclaim her q coins

Fairness with penalties

Definition of fair secure multiparty computation with penalties

- An honest party never has to pay any penalty
- If a party aborts after learning the output and doesn't deliver output to honest parties $\Rightarrow$ every honest party is compensated

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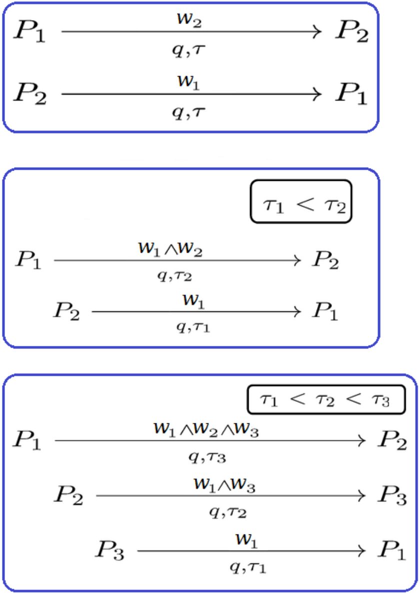
Outline of $\mathcal{F}_f^\star$ – fairness with penalties for any function f

- $P_1, \ldots, P_n$ run secure *unfair* MPC for $f(x_1, \ldots, x_n)$ that
  1. Computes shares $(y_1, \ldots, y_n)$ of the output $y = f(x_1, \ldots, x_n)$
  2. Computes $\mathrm{Tags} = (\mathsf{com}(y_1), \ldots, \mathsf{com}(y_n))$ $= (\mathtt{hash}(y_1), \ldots, \mathtt{hash}(y_n))$
  3. Delivers (y_i, Tags) to every P_i
- $P_1, \ldots, P_n$ deposit coins and run fair reconstruction (fair exchange) with penalties to swap the y_i's among themselves.

Fair exchange in the $\mathcal{F}^{\star}_{\mathrm{CR}}$-hybrid model - the ladder construction

"Abort" attack:

P_2 claims without deposting



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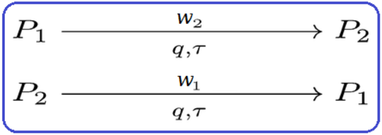
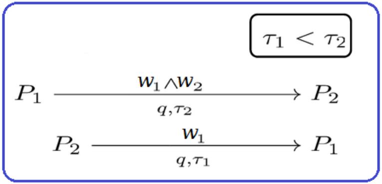
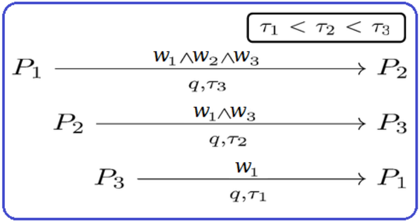
Fair exchange:

P_1 claims by revealing w_1

$\Rightarrow$ P_2 can claim by revealing w_2







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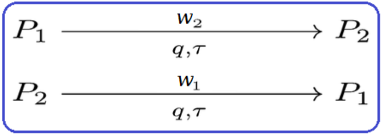

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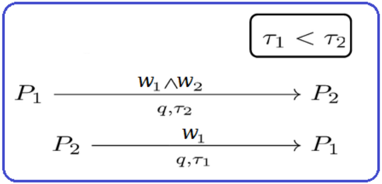
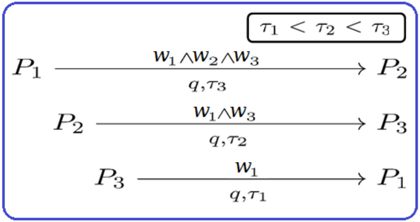


Malicious coalition:

Coalition P_1, P_2 obtain w_3 from P_3

P_2 doesn't claim the top transaction

P_3 isn't compensated



Fair exchange in the $\mathcal{F}^{\star}_{\text{CR}}$-hybrid model - the ladder construction (contd.)

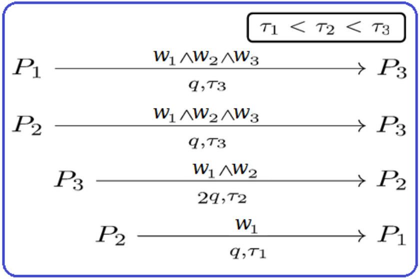
Fair exchange:

Bottom two levels:

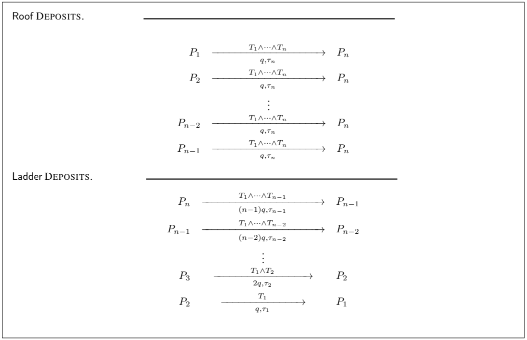
P_1, P_2 get compensated by P_3

Top two levels:

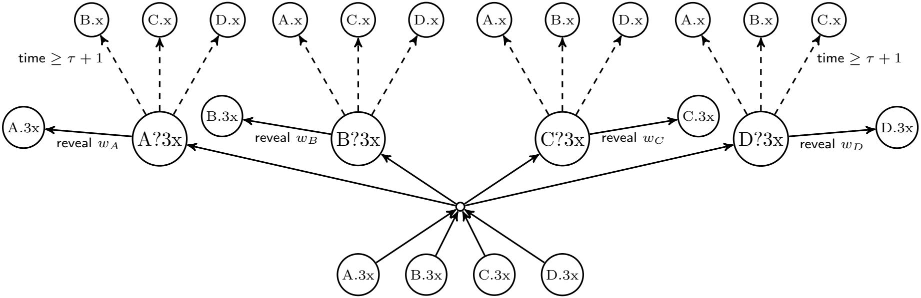
P_3 gets her refunds by revealing w_3



Full ladder:



Multilock



In principle, jointly locking coins for fair exchange can work well:

1. $M =$ "if P_1, P_2, P_3, P_4 sign this message with inputs of coins$(3x)$ each then their $3x$ coins are locked into 4 outputs of coins$(3x)$ each, where each P_i can redeem output T_i with a witness w_i that satisfies ϕ_i, and after time τ anyone can divide an unredeemed output T_i equally to $\{P_1, P_2, P_3, P_4\} \setminus \{P_i\}$"
2. P_1, P_2, P_3, P_4 sign M and broadcast it, and after M is confirmed, each P_i redeems coins(x) by revealing w_i

Practicality of multiparty fair exchange with penalties in Bitcoin

- Due to a design flaw, to implement $\mathcal{F}^{\star}_{\mathrm{ML}}$ in the current Bitcoin protocol an *unfair* secure MPC needs to be invoked, where the input of P_i is $inp_i = \mathsf{Sign}_{sk_i}(\mathsf{PREPARE}_{\mathsf{lock}})$, and the output to all parties is $\mathsf{SHA256d}(\mathsf{PREPARE}_{\mathsf{lock}}, inp_1, \ldots, inp_n)$.

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- $\mathcal{F}^{\star}_{\mathrm{ML}}$ requires $O(1)$ Bitcoin rounds and $O(n^2)$ transaction data (and $O(n^2)$ signature operations), while the ladder requires $O(n)$ Bitcoin rounds and $O(n)$ transactions data.

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Recap:

- Multiparty fair computation can be implemented in Bitcoin via the ladder construction.
- Multiparty fair computation can be implemented in Bitcoin via $\mathcal{F}^{\star}_{\text{ML}}$ with one superfluous unfair MPC.
- Multiparty fair computation can be implemented via $\mathcal{F}^{\star}_{\text{ML}}$ directly with an enhanced Bitcoin protocol.

Comparison with other ways to achieve fairness

Gradual release

- Even with only 2 parties, the number of rounds depends on a security parameter.

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- Disadvantage of Bitcoin: funny money?

Secure cash distribution and poker

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"Paradoxical" Abilities 1983-

- Exchanging Secret Messages without Ever Meeting
- Simultaneous Contract Signing Over the Phone
- Generating exponentially long pseudo random strings indistinguishable from random
- Proving a theorem without revealing the proof
- $\Longrightarrow$ Playing any digital game without referees
- Private Information Retrieval

Secure cash distribution with penalties

Ideal 2-party secure (non-reactive) cash distribution functionality:

1. Wait to receive $(x_1, \text{coins}(d_1))$ from P_1 and $(x_2, \text{coins}(d_2))$ from P_2.
2. Compute $(y, v) \leftarrow f(x_1, x_2, d_1, d_2)$.
3. Send $(y, \text{coins}(v))$ to P_1 and $(y, \text{coins}(d_1 + d_2 - v))$ to P_2.

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- In the general case, each party P_i has input $(x_i, \text{coins}(d_i))$ and receives output $(y, \text{coins}(v_i))$.
- Use-cases: generalized lottery, incentivized computation, ...

Blackbox secure cash distribution

- Blackbox realization of secure cash distribution in the $\mathcal{F}^{\star}_{\mathrm{CR}}$-hybrid model.
- Assume that input coins amount of P_i is m_i-bit number.

Step 1: commit to random secrets (preprocessing)

Invoke secure MPC where all $i \in [n], j \in [n] \setminus \{i\}, k \in [m_i]$:

- P_i picks random witness $w_{i,j,k} \leftarrow \{0,1\}^{\lambda}$ (also random $r_{i,j,k}$).
- P_i computes $c_{i,j,k} \leftarrow \text{commit}(1^{\lambda}, w_{i,j,k}, r_{i,j,k})$.
- P_i n-out-of-n secret shares each witness $w_{i,j,k}$.
- P_i outputs $c_{i,j,k}$ and the i-th share of each $w_{i,j,k}$ to each P_j.

Then, each P_i makes $\mathcal{F}^{\star}_{\mathrm{CR}}$ transaction $P_i \xrightarrow[2^k,\tau]{w_{i,j,k}} P_j$

Blackbox secure cash distribution (contd.)

Assume that the input coin amounts is $d = (d_1, \ldots, d_n)$ and the string inputs are $(x_1, x_2, \ldots, x_n)$.

Step 2: compute the cash distribution

Invoke secure MPC (unfair for now) for the cash distribution:

- Compute the output coin amounts $v = (v_1, v_2, \ldots, v_n)$.
- Derive numbers $b_{i,j}$ that specify how many coins P_i needs to send P_j according to the input coins d and output coins v.
- Let $(b_{i,j,1}, b_{i,j,2}, \ldots, b_{i,j,m_i})$ be the binary expansion of $b_{i,j}$.
- For all i, j, k, if $b_{i,j,k} = 1$ then reconstruct $w_{i,j,k}$ and concatenate it to the output.
- Compute $y = f(x_1, x_2, \ldots, x_n)$ and output y too.

Then, use fair exchange with penalties (with time limit $< \tau$) to deliver the output to all parties, so that $\mathcal{F}^{\star}_{\mathrm{CR}}$ claims will ensue.

Reactive secure cash distribution

Ingredients needed:

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- Blackbox secure cash distribution as described, with refunds at time τ that exceeds the see-saw time limits, and hence with circuits specified at start that are checked in the final rounds.

The see-saw construction: 2 parties

ROOF DEPOSIT.

$$P_1 \xrightarrow[q, \tau_{m,2}]{\mathrm{TT}_{m,2}} P_2 \qquad (\mathsf{Tx}_{m,2})$$

SEE-SAW DEPOSITS. For $r = m - 1$ to 1:

$$P_2 \xrightarrow[2q, \tau_{r+1,1}]{\mathrm{TT}_{r+1,1}} P_1 \qquad (\mathsf{Tx}_{r+1,1})$$

$$P_1 \xrightarrow[2q, \tau_{r,2}]{\mathrm{TT}_{r,2}} P_2 \qquad (\mathsf{Tx}_{r,2})$$

FLOOR DEPOSIT.

$$P_2 \xrightarrow[q, \tau_{1,1}]{\mathrm{TT}_{1,1}} P_1 \qquad (\mathsf{Tx}_{1,1})$$

The see-saw construction: multiparty

ROOF DEPOSITS. For each $j \in [n-1]$:

$$P_j \xrightarrow[q,\tau_{2n-2}]{\text{TT}_n} P_n$$

LADDER DEPOSITS. For $i = n-1$ down to 2:

- Rung unlock: For $j = n$ down to $i+1$:

$$P_j \xrightarrow[q,\tau_{2i-1}]{\text{TT}_i \wedge U_{i,j}} P_i$$

- Rung climb:

$$P_{i+1} \xrightarrow[i\cdot q,\tau_{2i-2}]{\text{TT}_i} P_i$$

- Rung lock: For each $j = n$ down to $i+1$:

$$P_i \xrightarrow[q,\tau_{2i-2}]{\text{TT}_{i-1} \wedge U_{i,j}} P_j$$

FOOT DEPOSIT.

$$P_2 \xrightarrow[q,\tau_1]{\text{TT}_1} P_1$$

The see-saw construction: multiparty (contd.)

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- The circuits verify a signed extension of the entire execution transcript, and that this extension conforms with the protocol.
- $\Rightarrow$ needs more expressive scripting language than vanilla Bitcoin, but not Turing complete scripts because the round bounds are known in advance.

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    - Computes commitments to shares of all the cards.
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- Make the cash distribution transactions whose circuits verify the signatures of a transcript, then scan it while performing arithmetic calculations.
- The $\mathcal{F}^{\star}_{\mathrm{CR}}$ circuit in each round of the see-saw will verify signatures of a transcript, then enforce betting rules or expect a party to reveal a share of a card.
- For example: if all partied called and the top card on the deck should be revealed, then the next see-saw circuits will require each party to reveal her share of the top card.

Some open questions

- Lower bound of linear number of rounds for fairness with penalties in the $\mathcal{F}^{\star}_{\mathrm{CR}}$-hybrid model?
- Bounds for the minimal deposit amounts? Rational analysis?
- Constructing secure cash distribution with penalties from *blackbox* secure MPC and $\mathcal{F}^{\star}_{\mathrm{CR}}$?

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