Circuit theory and microphotonic circuit design:
from resonant filters to light-powered nanomachines

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Abstract: Physically intuitive coupled-mode and equivalent electrical-circuit theories are described for synthesis of nanophotonic resonator/interferometer circuits, including a new phase law for general 4-ports. Synthesis of self-adaptive (highly-nonlinear) optomechanical systems based on light forces is introduced.

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1. Introduction
Microphotonic circuits based on strong-confinement (SC) waveguides and resonators are becoming an enabling technology providing filters for broadband chip-scale tunable optical add-drop multiplexers (T-OADMs) for telecom and on-chip interconnect applications [1-3], and high-efficiency modulators [4]. Furthermore, efficient resonance-enhanced nonlinear devices such as wavelength converters have been demonstrated [5], and recently a new class of optomechanical devices has been proposed that enable fundamentally new signal processing functionalities by exploiting resonantly-enhanced light forces in structures producing strong effective optical nonlinear responses [6].

Rigorous synthesis techniques are needed to allow the optimal design of such resonator-based nanophotonic devices for given response objectives. In this paper, we discuss coupled-mode theory and equivalent circuit models for the design of coupled-resonator filters, and in particular recently proposed loop-coupled resonator filters [7] that allow simultaneously square amplitude and linear phase responses. A new loop-coupling phase (LCP) physical parameter is introduced and shown to play a key role in the operation of these structures. Then, we discuss canonical representations for reflectionless 4-ports, and the derivation and design of a new class of interferometers, universally balanced interferometers (UBIs) [8]. The use of UBIs for dispersionless Vernier FSR multiplication is illustrated. Next, a new general law [9] for the phase response of arbitrary passive 4-ports is presented. It accounts for characteristic response distortions in microring filters with lossy couplers, has consequences for UBIs, and may have consequences in other fields that deal with near-Hermitian operators, such as quantum computation. Finally, we address synthesis methods for highly nonlinear optomechanical nanophotonic devices based on light forces [6].

2. Coupled-mode and equivalent-circuit synthesis of loop-coupled resonators; concept of loop-coupling phase
Loop-coupled-resonator structures, proposed in Fig. 1(a-b) [7,9], allow optimal add-drop responses for a given number of resonators used (in the sense of sharpest rolloff, maximally linearized phase or the chosen tradeoff between the two). They do this by allowing full pole-zero manipulation in a robust device architecture [7,9]. The resulting structures can provide simultaneously square-amplitude (flat-top) and linear-phase (dispersionless) bandpass responses (Fig. 1c-g) that are not achievable in series-coupled cavities.

Fig. 1. Loop-coupled resonator filters [7,9]: (a) microring-based and (b) standing-wave-cavity implementations, showing tilt or cavity-orientation selection of 0 or 180° loop coupling phase (LCP); (c) equivalent circuit allowing direct mapping to known response functions, as well as to coupling of modes in time (CMT), and (d) pairwise model showing that each capacitor corresponds to an optical resonator, and the pairwise resonance frequency corresponds to frequency splitting of coupled optical cavities; (e) flat-top amplitude response and simultaneously linear phase (flat group delay, zero dispersion) showing novel functionality of such an architecture relative to CROWS.
A physically intuitive model for both the synthesis and analysis of these devices is the coupling of modes in time (CMT) model. On the other hand, equivalent circuits allow us to map photonic geometries to previous equivalent structures in electrical engineering literature and take advantage of known response types that are synthesizable for a given device topology. Although the most straightforward mapping is that of coupled high-frequency resonators, we will discuss an intuitively simpler low-pass model, valid for degenerate coupled resonators [10,9] (Fig. 1c). This is allowed because the frequency response of a ring resonator repeats every free spectral range (FSR), so we may consider the response at (or near) zero frequency as a conceptual tool, because it is analogous to that at the frequency of interest. Then, each capacitor in Fig. 1c is a “resonator” at zero frequency [7,9]. It can be thought to have a parallel inductor of infinite inductance, i.e. an open circuit, so that the resonance frequency is $\omega = 1/\sqrt{LC} = 0$.

Then a pair of capacitors connected by an impedance inverter is a pair of coupled resonators. Since adjacent capacitors with zero energy (empty resonators) are short circuits ($V = 0$), seen from behind an impedance inverter they are open circuits, so we can determine the energy exchange between two resonators by considering each pair in isolation, and therefore trivially map impedance inverter values to coupled-mode theory (CMT) coupling coefficients (which are energy exchange rates in time). Models like these also bear out a new fundamental parameter of geometries that have loop-coupled cavities – the loop-coupling phase (LCP) [9]. This parameter has not been addressed in optical or analogous microwave devices, and shows that practical optical devices have a richer set of responses than microwave cavity structures. The LCP is shown to directly control the position of transmission zeros in the complex plane, and to be indispensable in the design of optimal responses.

3. Canonical representations of photonic circuits and universally balanced interferometers (UBIs) [8,9]

In addition to novel resonator configurations, planar nanophotonic circuits support fabrication of complex interferometers in a stable, repeatable way. An interesting general class of interferometers has been discovered that produces a near-inverse (“amplitude-only” inverse) of an arbitrary 2x2 operator without having to know what that operator is [9]. These are called universally balanced interferometers (UBIs) and comprise the arbitrary 2x2 (reflectionless) device, and form the “amplitude inverse” using a copy of it rotated by 180° preceded and followed by a 180° differential phase shift. The 2x2 device can be arbitrarily complex, and may even be non-reciprocal
It has a number of applications in the design of devices needed to provide new functionalities that are necessary to enable nanophotonic signal processing in telecom applications, including dispersionless Vernier FSR multiplication (illustrated in Fig. 2b-c [8]). Folded UBIs promise to reduce complexity but require integrated circulators [9]. The consideration of canonical representations of optical multiports can lead to more rigorous definition of the bounds of performance on devices, and to the delineation of general device classes such as UBIs.

4. The Characteristic Phase Inequality [9]

Another result stemming from such general considerations is a phase condition for a large class of generic 2-ports and 4-ports [9]. It is well known, with four relevant response functions in the scattering or transmission matrix, and that in the idealized lossless case the phase-response functions are rigorously pinned by energy conservation to an equality: \( \phi_1 + \phi_2 - \phi_1 - \phi_2 = \pm \pi \). We name here \( \Omega = \phi_1 + \phi_2 - \phi_1 - \phi_2 \) as the characteristic phase [9]. In practice, optical multiports have some loss, but a rigorous bound can still be derived for the characteristic phase that depends only on the maximum splitting ratio of the device, \( r_{\text{max}} \) (from either input port), and on its maximum transmission loss, \( \delta_{\text{max}} \) (for any combination and phasing of inputs) [9]:

\[
-1 \leq \cos(\Omega) \leq -1 + \frac{1}{2} \left( 1 + r_{\text{max}} \right) \left( 1 + \frac{1}{r_{\text{max}}} \right) \left( \frac{2}{\delta_{\text{max}}} - 1 \right)^2
\]

This bound is a fundamental result that applies to all passive 2-ports and reflectionless 4-ports in various wave systems, including optics, acoustics, quantum mechanics, and more generally a fundamental result for 2x2 matrices.

The CPI has implications for practical device design. For example, the CPI of lossy directional couplers describes the degree to which a microring-resonator filter using lossy couplers can have a distorted, skewed response, as previously observed [11]. A single-cavity response is known to be non-Lorentzian when the input coupling (not necessarily the cavity) is lossy. It also has implications for phase-error tolerance of UBIs.


The results in this paper have addressed design and synthesis concepts for linear photonic devices. The recent proposal of a new class of optomechanical devices that have moving parts that move in response to resonantly-enhanced optical forces has shown opportunities for numerous novel device concepts without simple analogue in other fields [6]. The motion in these systems corresponds to a macroscopic equivalent of nonlinear polarizability, while freely suspended parts allow them to have multiple possible “polarizabilities” (determined by state) in the linear regime (unlike traditional linear/nonlinear devices). These concepts promise new device types like self-adaptive, self-tuning resonant cavities (Fig. 3) [6], and as such have generated a need for systematic design and synthesis techniques, and “equivalent circuits” for such designs. Starting steps in this direction and a brief discussion of these structures and their design will be described.

References