Optimal Path Planning and Power Allocation for a Long Endurance Solar-Powered UAV

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Abstract—In this paper the problem of optimal path planning and power allocation for an Unmanned Aerial Vehicle (UAV) is explored. The UAV is equipped with photovoltaic cells on top of its wings and its energy sources are solar power and rechargeable batteries. The Sun incidence angle on the photovoltaic cells, which subsequently affects energy harvesting, is determined by the attitude of the UAV and the Sun position. The desired optimal path between two given boundary points, is aimed at increasing the amount of energy storage at the final point. Meanwhile, the charging state of the battery, resulting from the power allocation, needs to be determined along with the path planning procedure. Two approaches, nonlinear programming and model reduction, are proposed and their corresponding simulation results are presented and compared.

I. INTRODUCTION

The application of renewable energy sources on aerial vehicles is an emerging technology which provides substantial benefits for Unmanned Aerial Vehicles (UAVs) fuel consumption by collecting the solar radiation energy. As sun is the major energy source for solar powered UAVs, it is necessary to employ optimal strategies to maximize energy harvesting during the flight. The energy optimized path planning problem has been studied by Klesh and Kabamba [1], [2] for level flight where the only source of energy is solar. In these works, the UAV flies between two given locations where the sun angles are constant during the mission, under the assumption that the flight simulation time is short. In [3], periodic optimization has been used to obtain a perpetual flight, where the total energy is maximized such that the aircraft is constrained to move on the surface of a vertical cylinder. Sachs et al. [4] have also used periodic optimization to design a trajectory that allows the UAV to fly with minimum or even no storage equipment. In [4], the mission is decomposed into three phases. In the first phase, the UAV climbs to the altitude of 8.5 km when the solar energy is available and in the second phase it descends during night. In the last phase, it stays at low altitude performing level flight.

Solving this type of optimal control problems is complicated due to the nonlinear kinematics of aircraft and its feasibility constraints on maneuvers. The direct and indirect optimization methods are the two general approaches proposed in solving optimal path planning problems [5]. The indirect method, which is based on Hamiltonian and Lagrangian equations, has low rate of convergence and is sensitive to the initial guess. Since the kinematics of the system is complex, it is very difficult to obtain a proper initial guess. Another approach is direct optimization, such as collocation and nonlinear programming, which discretizes the trajectory into multiple segments and then solves the parameterized optimization problem. In addition to these approaches, the aircraft kinematics can be simplified based on the characteristics of different flight maneuvers, which is referred to as the model reduction approach in this paper. The subsequent generated trajectories are composed of several modes, for example, Kamgarpour et al. in [6] have assumed the operating state of the aircraft in each mode to solve a hybrid optimization problem.

In this paper, we examine both the optimal path planning and power allocation problems for a duration of one day and night of a solar powered UAV. The objective is to maximize the remaining energy stored in batteries at the end of the duration while satisfying the boundary conditions and feasibility constraints on the states of the aircraft and batteries. Since the UAV is equipped with electronic components, i.e., photovoltaic cells, batteries and electrical engine, the power allocation problem is to efficiently allocate the amount of power to or from each component during the flight. In our work the power allocation problem is handled as part of the optimal path planning problem, where the states and controls of the flight kinematics are coupled with the amounts of allocated power. The direct optimization method is described first to find an optimal trajectory as well as time history of power allocation for a 24-hour flying period UAV. In the second approach, the desired trajectory is divided into three phases with reduced models representing each phase and a combined analytical and numerical method is used to solve the corresponding optimization problem. The results are then compared with those obtained from the direct optimization method.

The paper is organized as follows. §II presents the aircraft kinematics and relevant background. In addition, the corresponding models for UAV subsystems and sun angles are introduced in §II. The optimal control problem formulation is presented in §III and a solution strategy is discussed. In §IV numerical results from nonlinear programming and their interpretation for a sample UAV path planning are presented. The model reduction method, including the corresponding simulation results are presented in §V followed by concluding remarks in §VI.
II. BACKGROUND AND MODEL

The models for the major components, as well as the aircraft kinematics required to characterize the UAV trajectory, are presented in this section. The major electric components considered in the UAV power system are solar cells, electric motor, and rechargeable batteries.

A. Aircraft Kinematics Model in Three Dimensions

The point mass kinematics in three-dimensions for an aircraft flying in still air is expressed as [7]:

\[
\begin{align*}
\dot{x} &= V \cos \psi \cos \gamma, \\
\dot{y} &= V \sin \psi \cos \gamma, \\
\dot{z} &= V \sin \gamma
\end{align*}
\]

\[
\dot{\psi} = \frac{g}{V} \left( \frac{n_h}{\cos \gamma} \right), \\
\dot{\gamma} = \frac{g}{V} (n_v - \cos \gamma)
\]

(2.1)

where \(x, y,\) and \(z\) are the aircraft position coordinates in a flat Earth-fixed reference frame, \(V\) is the aircraft velocity, \(\psi\) is the heading angle, and \(\gamma\) is the flight path angle. Parameters \(D\) and \(T\) represent drag and thrust forces, respectively. The vertical and horizontal load factors \(n_v\) and \(n_h\) are defined as \(n_h = \frac{L \sin \phi}{W}\) and \(n_v = \frac{L \cos \phi}{W}\), where \(W\) represents the aircraft weight, \(L\) represents lift force, and the bank angle \(\phi\) can be expressed as \(\phi = \tan^{-1} \left( \frac{n_h}{n_v} \right)\).

1) Aircraft Aerodynamics: The drag force in (2.1) is derived from standard lift and drag equations:

\[
C_L = \frac{nW}{2 \rho V^2 S}, \quad C = C_{D0} + KC_L, \quad D = \frac{1}{2} \rho V^2 SC_D
\]

(2.2)

where \(C_L\) is the lift coefficient, \(C_D\) is the drag coefficient, \(C_{D0}\) is the coefficient of parasitic drag, and \(K\) is the aerodynamic coefficient. The parameter \(n\) is the magnitude of the load factor, which can be determined by \(n_h\) and \(n_v\) such that \(n = \sqrt{n_h^2 + n_v^2}\). The air density \(\rho\) is a function of altitude, \(z\), and ambient temperature which is estimated from the U.S. Standard Atmosphere [8].

B. Electric Motor Output Power

The output power consumed by the UAV propeller engine can be expressed as \(P_{Eng} = \frac{TV}{\eta_{prop}}\), where \(\eta_{prop}\) is the efficiency of the propeller. The available thrust, \(T\), is limited to a maximum value, \(T_{\text{max}}\), which is predetermined by the characteristics of the aircraft engine.

C. Solar Radiation Energy

The energy of the solar gained from solar cells during time interval \([t_0, t_f]\) is \(E_{\text{Sun}} = \int_{t_0}^{t_f} P_{\text{Sun}} dt\), where \(P_{\text{Sun}}\) is the input power from solar cells. The power gained from solar cells is modeled based on sun incidence angle \(\theta\) and solar cell parameters [2] \(P_{\text{Sun}} = \eta_{\text{sol}} P_{\text{sd}} S \cos \theta\), where \(S\) is the wing area, \(\eta_{\text{sol}}\) is the efficiency of the solar cell, and \(P_{\text{sd}}\) is the solar spectral density. The incidence angle \(\theta\) depends upon the azimuth and elevation angles of the sun which are time varying during a day [9]. The Euler angles and the unit vector to the sun in the aircraft-fixed frame are depicted in Figure 1.

The incidence angle \(\theta\) is determined as \(\theta = \cos^{-1}(\hat{s}_A^T z_A)\), where \(z_A\) is the aircraft-fixed vertical axis defined as \(z_A = [0 \ 0 \ 1]^T\) and the unit vector to the sun in aircraft-fixed frame is expressed as \(s_A(t) = R_1(\phi)R_2(\psi)R_3(\gamma)s_E(t)\). The matrices \(R_1, R_2,\) and \(R_3\) represent rotation about the first, second, and third axis, respectively and \(s_E(t) = [\cos e(t) \cos a(t) \cos e(t) \sin a(t) \sin e(t)]^T\). Therefore, the sun incidence angle, and the subsequent the input power from the solar cells are functions of aircraft’s Euler angles and the solar elevation/azimuth angles.

D. Battery Model

The power output model of battery pack are based on the circuit representation of Lithium sulfur cells (Li-S) [10]. In this model, the battery pack output power is defined as:

\[
P_{\text{Batt}} = -V_{\text{OC}} Q S \dot{\text{SOC}} - R (Q S \dot{\text{SOC}})^2
\]

(2.3)

where \(V_{\text{OC}}\) is the battery pack open circuit voltage, \(R\) is the battery pack internal resistance, \(Q\) is the battery pack capacity, and \(\text{SOC}\) is the battery pack state of charge defined as the ratio of current charge capacity to maximum charge capacity and

\[
0.25 \leq \text{SOC} \leq 0.9
\]

(2.4)

The output power from batteries, \(P_{\text{Batt}}\), is positive during discharge state and negative during charge state such that

\[
P_{\text{Batt, Charge}} \leq P_{\text{Batt}} \leq P_{\text{Batt, Discharge}}
\]

(2.5)

From (2.3), the rate of charge, \(S \dot{\text{SOC}}\), can be represented as

\[
\frac{d}{dt} \text{SOC} = -\frac{V_{\text{OC}} - \sqrt{V_{\text{OC}}^2 - 4 P_{\text{Batt}} R}}{2QR}
\]

(2.6)

The discharge performance is a function of the cell temperature and the value of \(\text{SOC}\), based on the battery specification sheet [11]. However, the impact of \(\text{SOC}\) on the open circuit voltage, \(V_{\text{OC}}\), of the battery pack is negligible under normal.

III. PROBLEM STATEMENT

The problem of optimal energy path planning and power allocation between two given locations can now be formulated as an optimization problem, whereby the final battery
state of charge, $SOC(t_f)$, is used as the cost function. In addition, the state variables $[x, y, z, \psi, \gamma, V, SOC]^T$ and control variables $[n_v, n_h, T, P_{Batt}]^T$ are constrained to satisfy the aircraft kinematics (2.1), battery discharge rate (2.6), and boundary conditions. The mission is to fly between specified initial and final positions during the time interval $[t_0, t_f]$. Due to limited UAV maneuverability, solar sources, and battery capacity, constraints on state and control variables are introduced. Equations (2.5) and (2.4) represent the constraints associated with the battery performance.

Based on the above description, the optimal control problem can be formulated as:

$$\max_{(n_v, n_h, T, P_{Batt})} SOC(t_f)$$

subject to

$$\dot{x} = V \cos \psi \cos \gamma, \quad \dot{y} = V \sin \psi \cos \gamma, \quad \dot{z} = V \sin \gamma$$

$$\dot{\psi} = \frac{g}{V} \left( \frac{n_h}{\cos \gamma} \right), \quad \dot{\gamma} = \frac{g}{V} (n_v - \cos \gamma)$$

$$\dot{V} = \left( \frac{T - D}{mg} - \sin \gamma \right) g,$$

$$S\dot{OC} = \frac{V_{OC} - \sqrt{V_{OC}^2 - 4P_{Batt}R}}{2QR}$$

and

$$|\psi| \leq 2\pi, \quad |\dot{\psi}| \leq \dot{\Phi}_t, \quad |\gamma| \leq \frac{\pi}{2}, \quad V_{\text{stall}} \leq V \leq V_{\text{max}}$$

$$0.25 \leq SOC \leq 0.9, \quad 0 \leq T \leq T_{\text{max}}$$

$$|n_v| \leq n_{v_{\text{max}}}, \quad |n_h| \leq n_{h_{\text{max}}}$$

$$P_{\text{Batt}, \text{Charge}} \leq P_{\text{Batt}} \leq P_{\text{Batt}, \text{Discharge}}$$

$$P_{\text{Tot}} = P_{\text{Sun}} + P_{\text{Batt}} - P_{\text{Eng}} \geq 0,$$

with boundary conditions

$$[x(t_0), y(t_0), z(t_0), \psi(t_0), \gamma(t_0), V(t_0), SOC(t_0)]^T = [x_0, y_0, z_0, \psi_0, \gamma_0, V_0, SOC_0]^T$$

$$[x(t_f), y(t_f), z(t_f)]^T = [x_f, y_f, z_f]^T,$$  (3.10)

where $V_{\text{stall}}$ and $V_{\text{max}}$ are the stall and maximum feasible velocity of the UAV, respectively, and the heading angle has to satisfy $|\psi| \leq 2\pi$ based on its physical definition. Furthermore, $|\dot{\psi}| \leq \dot{\Phi}_t$ is required to generate a smooth trajectory where $\dot{\Phi}_t$ is the maximum changing rate of the heading angle. Powered-flight requires the total power, $P_{\text{Tot}}$, to be positive, which means the power consumption is less than or equal to power generated by solar cells and/or the battery pack. This constraint is expressed in the last equation of (3.9).

### IV. NONLINEAR PROGRAMMING APPROACH

One simulation example, assumed to be performed on April 1st, 2012, in San Diego, CA is illustrated in Figures 2 to 5(b) by solid lines and its corresponding boundary conditions are

$$[x_0, y_0, z_0] = [0, 0, 0] \quad (\text{km}), \quad [x_f, y_f, z_f] = [0, 0, 0] \quad (\text{km})$$

$$[\psi_0, \gamma_0] = [0, 0] \quad (\text{rad}), \quad V_0 = V_{\text{min}}, \quad SOC_0 = 0.9.$$  (4.11)

SNOPT [12] can be used to solve this problem. Since the simulation time of 24-hour is quite long (86400 seconds), some of the state and control variables need to be scaled to make the solution converge faster. This also avoids the singularities that may occur in estimating the underlying Hessian matrices. The path to be optimized is discretized into 24 nodes representing each hour and the time span is from 1am to midnight.

The time history of altitude in Figure 3(a) shows that the UAV flies at low level flight ($z \geq 100m$) during the early hours of the day where the minimum power required to maintain the powered flight is optimum. The aircraft stays at low altitude until 9 a.m., then gradually climbs to the summit when the sun is rising, and then descends to its initial altitude. A feature of the optimal output is that part of the solar energy is saved as potential energy during the day which is transformed to kinematic energy as the UAV glides down during night.

The projection of the 3D trajectory on the $\{x-y\}$ plane in Figure 2 is relatively smooth based on the fact that the time step in optimization algorithm is 1 hour. Figure 3(b) shows the UAV velocity during the flight implying that the aircraft flies at two specific velocity magnitudes at most of the time with small scale fluctuation. Figure 4(a) shows the smooth transitions of attitude angles between discrete points. The flight path angle $\gamma$, shown in Figure 4(b), is approximately zero during the early hours of the mission, implying the low altitude level flight during those hours. The flight path angle increases while the aircraft is ascending during the day and eventually reaches a constant negative value during the night, indicating a gradual descend.

In addition to the optimal trajectory and UAV attitude presented above, the optimal power allocation strategy for the periodic mission is shown in Figure 5(a). The available solar, battery pack, and electric motor power are presented by red, blue, and magnet lines respectively. The amount of power at the early hours of the day is approximately equivalent to the minimum power required at the low altitude flight. Around noon, the input power from the sun is large enough to compensate for both recharging the battery and providing sufficient power to ascend to higher altitude. Around 7pm, the engine is shut down ($T = 0N$) and the battery is fully charged. It is important to note that the required engine power $P_{\text{Eng}}$ is zero at night and the total power $P_{\text{Tot}}$ in (3.9) is non-negative at during the flying period.

Figure 5(b) indicates the time history of SOC status from the optimal power allocation output. Consistent with our presupposition, the UAV uses the battery power as a sole source of energy during night and recharge it in the day. The output indicates that the rechargeable battery pack reaches its maximum allowed capacity at the end of the mission, implying that the battery returns to its initial state of charge.
Therefore, this is a global optimum solution for the problem and the result of SOC allows the UAV to resume its operation in the next day.

V. MODEL REDUCTION

Based on the numerical results, the one-day mission can be divided into three phases starting from 1 a.m.: low altitude level flight, climb, followed by a descend flight. As discussed in section IV, the UAV flies at low altitude level to minimize the engine power consumption and start climbing to gain potential energy when sufficient amount of solar power is available to both charge the battery and maintain the engine operation. The UAV uses the stored potential energy to glide to the final location. In this section, the flight kinematics during each phase is represented by a first order approximation model and the duration of each phase is specified by the availability of the solar source.

A. Phase 1 (Low Altitude Level Flight)

The first phase starts at the initial time (1 a.m.) and lasts until sun rise. A common property of all phases is the bank angle profile. Since the time span is large and the aircraft altitude change is small, we can assume the steady flight condition for all three phases.

The minimum power consumption for steady flight with a given time span is when $\phi = 0, \gamma = 0$, and $V_{\text{min}} = \left( \frac{4K W^2}{3p^2S^2 C D_0} \right)^{1/4}$.

At steady flight the altitude change is constant or even zero in level flight, therefore the vertical acceleration of the aircraft is zero. According to Newton’s law $(L \cos(\phi) \cos(\gamma) - W = 0)$ and from (2.2) the power consumption can be expressed as

$$ P_{\text{Eng}} = \frac{1}{2} \eta_{\text{prop}} \left( C_{D0} + K \frac{W^2}{(\frac{1}{2} \rho V^2 S)^2} \cos^2(\phi) \cos^2(\gamma) \right). $$

Therefore, $P_{\text{Eng}}$ is always positive and if we minimize it over $\phi$ and $\gamma$ the optimal solution is obtained when $\cos^2(\phi) = 1$ and $\cos^2(\gamma) = 1$. Since $|\phi| \leq \frac{\pi}{2}$ and $|\gamma| \leq \frac{\pi}{2}$, the optimal solution is at $\phi^* = 0$, $\gamma^* = 0$ and the power consumption can be rewritten as

$$ P_{\text{Eng}} = \frac{1}{2} \rho SV^3 \left( C_{D0} + K \frac{W^2}{(\frac{1}{2} \rho V^2 S)^2} \right). $$

If the UAV flies at this constant velocity, the minimum required power provided from the battery to maintain the mission is constant during this phase and equal to the minimum engine power consumption. Therefore, with the information on the low altitude air density we can determine the velocity at which (5.12) is minimized, referred as $V_{\text{min}}$ which is expressed as

$$ V_{\text{min}} = \left( \frac{4KW^2}{3p^2S^2 C D_0} \right)^{1/4}. $$

This proposition applies to all three phases. Thus the desired bank angle and flight path angle during the entire mission is zero. This property simplifies the flight kinematics and solar power harvesting equation. Based on (2.6) and specified time span for this phase, $SOC(t_1)$ can be computed. The time instant $t_1$ refers to sunrise and the final time of low altitude level flight phase. Using the aforementioned assumptions, the aircraft kinematics in this phase is simplified as $\dot{x} = V_{\text{min}} \cos \psi$, $\dot{y} = V_{\text{min}} \sin \psi$ and the thrust from the engine is determined by $T = \frac{P_{\text{Batt}}(t_1) \eta_{\text{prop}}}{V_{\text{min}}}$. where $P_{\text{Batt}}$ is equal the minimum required power from the engine flying at the given altitude.

B. Phase 2 (Climb)

The second phase starts right after the first phase and ends at sunset. In order to maximize the aircraft mechanical energy, it is desirable for the UAV to reach its ceiling which is assumed to be 5km in this paper. Rapid changes in altitude are not desirable in order to maintain a smooth trajectory. Therefore, if we discretize the time horizon into intervals with one hour step size, the vertical acceleration of the aircraft and flight path angle are very small. In the simplified model, we assume that the vertical displacements at all intervals are the same and equal to $\Delta z$ which specifies the rate of climbing ($z_r = \frac{\Delta z}{\Delta t}$). From (2.1), the aircraft kinematics is reduced to

$$ \dot{x} = V \cos \psi, \quad \dot{y} = V \sin \psi, \quad \dot{z} = V \gamma $$

$$ \dot{V} = \frac{T - D}{m} - \gamma g. \quad (5.14) $$

At the beginning of this phase, the battery is almost discharged. Our objective here is to minimize the value of $P_{\text{Batt}}$ during the second phase to achieve the maximum recharging rate from the solar source. The $P_{\text{Batt}}$ is assumed to be negative when being recharged.

The maximum available power to recharge the battery while maintaining powered flight is the solution of the following optimal control problem with the constraint $\cos(a - \psi) = -1$:

$$ \min_{V,T} \left( T \int_{t_1}^{t_2} \left( \frac{TV}{\eta_{\text{prop}}} - \eta_{\text{sol}} P_{sa} S \cos(\frac{z_r}{V}) \right) dt \right) $$

$$ \text{s.t.} \quad V = \frac{T - D}{m} - \frac{z_r}{V g}, \quad V_{\text{stall}} \leq V \leq V_{\text{max}}, $$

where

$$ T - D = \frac{V W}{z_r} - \frac{\rho SV^3}{4K \pi r^2} \left( V - \sqrt{V^2 - 4KC_{D0} z_r^2 - \frac{8KS z_r(WV - z_r T))}{\rho SV^2}} \right). \quad (5.16) $$

From the last equation of (3.9), the minimum power provided from the battery to maintain powered flight occurs when $P_{\text{Tot}} = 0$, $P_{\text{Eng}}$ is minimum, and $P_{\text{Sun}}$ is maximum. Based on the fact that the bank angle is zero and flight path angle is very small, $P_{\text{Sun}}$ is reduced to

$$ P_{\text{Sun}} = \eta_{\text{sol}} P_{sa} S \left( \sin \epsilon - \gamma \cos \epsilon \cos(a - \psi) \right). \quad (5.17) $$
In the climb flight, the path angle $\gamma$ is always positive, hence the heading angle $\psi$ is specified to satisfy $\cos(\alpha - \psi) = -1$ during the climbing phase to maximize the solar power harvesting. Hence (5.17) is expressed as $P_{\text{Sun}} = \eta_{\text{eff}} P_{\text{ad}} S (\sin e + \gamma \cos e)$. The sun elevation angle $e$ is specified and thus we can ignore the term $\sin e$ in the cost function. Meanwhile, the flight path angle is determined by $\gamma = \frac{\dot{r}}{\dot{z}}$, assuming that $z_r$ is predetermined. Therefore, the minimization problem of $P_{\text{Batt}}$ will reduce to (5.15). The vertical acceleration of the aircraft is zero, thus based on Newton’s law, the lift force is expressed as $L = mg - (T - D) \dot{z}$. Therefore, the $T - D$ term can be derived from (2.2).

By substituting $T - D$ in (5.15), we can solve the minimization problem and find the optimal $V$ and $T$ during the climb. Then one can proceed to calculate the flight path angle and subsequently the altitude change.

C. Phase 3 (Glide)

In the third phase, the solar power is not available, however, the UAV can use its potential energy to glide through the night. The ideal scenario is that the electric engine is turned off during the entire phase and the value of $SO\dot{C}$ is constant. To avoid rapid changes in altitude, the flight path angle is assumed to be zero and the bank angle is zero for this phase as well. The aircraft kinematics at this phase is expressed as in (5.14), where $T = 0$ and $P_{\text{Batt}} = 0$. To further simplify the aircraft model without loss of generality, the flight path angle is assumed to be a constant negative number, which is constrained by the highest altitude of the UAV.

D. Solution Strategy

In order to compare the results from the reduced model with the direct collocation method, the time span is discretized into 24 nodes with one hour interval for two adjacent nodes. A feasible solution with maximum $SO\dot{C}(t_f)$ is guaranteed using the simplified model and its corresponding trajectory properties are discussed in the above sections. Therefore, the solution strategy will concentrate on satisfying the path constraints.

1) Path Planning Problem: A path satisfying periodical boundary conditions can guarantee the perpetual flight. We find the UAV trajectory by integrating backward the states of kinematics starting from the final point which is also the initial point for the periodical flight. Since the UAV fly at low altitude level flight after the third phase, we change the $z$ component of periodic boundary conditions to a low altitude of 100m. Flying at this altitude reduce the risk of collision with trees, buildings, and earth terrains. Without lack of generality, an arbitrary linear function is considered for heading angle which satisfies $\psi(t_2) = \alpha(t_2) - \pi$ and $\psi(t_f) = \psi_0$ in the third phase. The results from phase 3 will specify the state of UAV at time $t_2$. Given the state of UAV at time $t_2$ and trajectory characteristics introduced in subsection V-B, the aircraft kinematics (5.14) is integrated backward using Euler’s method. Based on the result from second phase, the path planning problem in phase 1 is expressed as a two point boundary value problem where the boundary conditions are specified as

$$\begin{align*}
[x(t_0), y(t_0), \psi(t_0)]^T &= [x_0, y_0, \psi_0]^T \\
[x(t_1), y(t_1), \psi(t_1)]^T &= [x_1, y_1, \alpha(t_1) - \pi]^T.
\end{align*}$$

This problem can be solved using a numerical approach such as shooting method. In this part, the algorithm runs feasibility evaluation to make sure that the UAV can reach the boundary condition with $V_{\text{min}}$ during the specified time interval.

2) Power Allocation Problem: In the power allocation problem, (2.6) is integrated forward. In the first phase $P_{\text{Batt}}$ and engine thrust are specified and there is no solar power available. Therefore, the $SO\dot{C}$ at time $t_1$ can be specified. The $T$ and $V'$ in second phase are calculated by solving the optimal control problem (5.15). Given the constraint in phase 2, the solar power is expressed as $P_{\text{Sun}} = \eta_{\text{sol}} P_{\text{ad}} S (\sin e + \gamma \cos e)$, and thus $P_{\text{Batt}}$ can be written as $P_{\text{Batt}} = \frac{\gamma V}{\eta_{\text{prop}}} - \eta_{\text{sol}} P_{\text{ad}} S (\sin e + \gamma \cos e)$. From (2.6), the $SO\dot{C}(t_2)$ is specified. Since the electric engine is turned off, the $SO\dot{C}$ does not change during phase 3 and it will maintain its level for the rest of the mission which is equivalent to $SO\dot{C}(t_f)$.

E. Numerical Results using the Simplified Model

In this section, the numerical results obtained from model reduction method are presented by marked lines and compared with the results obtained from NLP method in Figures 2 to 5(b). The UAV trajectory in the $\{x, y\}$ plane, as illustrated in Figure 2, is completely different from original results due to the arbitrary selection of heading angle in the last phase as reflected in Figure 4(a). Unlike the velocity profile obtained using original model, the velocity profile of UAV in the simplified model varies smoothly. However, the aircraft altitude and flight path angle in the first and last phases are almost identical as shown in Figures 3(a) and 4(b).

This analysis indicates that a more complicated maneuver other than the steady climb is required for both models to match on their altitude change and flight path angle profile in the second phase. However, according to the time history of $SO\dot{C}$ and power output in Figures 5(b) and 5(a), the $SO\dot{C}(t_f)$ is identical in both methods. Therefore, we can make the conclusion that there are many optimal paths leading to the maximum power storage for a battery pack with a favorable storage capacity.

VI. CONCLUSION

The main contribution of this paper is the investigation of optimal path planning and energy allocation problem for solar-powered UAVs in three dimensional space. The solutions using the direct optimization approach provides complete information of the trajectory, attitude, and battery status of the UAV whose kinematics and electrical models are integrated when solving the optimization problem. In addition, it was revealed from the simulation results that the UAV mission can be divided into three phases, level flight, climb, and glide. The corresponding maneuvers are
then represented by simplified aircraft kinematics to improve the efficiency of the numerical and analytical solutions. The reduced model is shown to yield similar power allocation strategy and optimal cost as the one obtained from the original model. We used a desktop computer with quad core processor (3.4 GHz) and the computation time for the direct optimization method and simplified model are 393.417 sec and 1.758 sec, respectively. This indicates that both methods are executable on a real time operating system, and that the computation speed using reduced model is much faster.

The future research will focus on different cost functions such as the weight of the battery pack and the flight range. In addition, other types of missions will be considered, imposing additional constraints on UAV path planning. For example, surveillance and reconnaissance missions require the UAV to meet specific way points along its trajectory.

REFERENCES


APPENDIX

The aircraft and battery pack parameters for a sample UAV which is used in numerical simulation are: $\eta_{sol} = 0.22$, $P_{sol} = 380 \text{ W}$, $b = 0.711 \text{ m}$, $S = 0.156 \text{ m}^2$, $m = 4.201 \text{ kg}$, $\eta_e = 0.992$, $C_{DD} = 0.011$, $\eta_{prop} = 0.7$, $Q = 26.4 \text{ Ah}$, $V_{OC} = 12.6 \text{ V}$, $R = 0.0125 \Omega$, $m_{Batt} = 1.28 \text{ kg}$, $P_{Batt, \text{Discharge/Charge}} = \pm 30 \text{ W}$. 

The optimal trajectory projection on the $\{x − y\}$ plane.

(a) Altitude (km)
(b) Velocity (m/s)

Fig. 2. The optimal trajectory projection on the $\{x − y\}$ plane.

(a) Heading Angle (rad)
(b) Flight Path Angle (rad)

Fig. 3. The time history of UAV altitude and velocity.

(a) Power Allocation (W)
(b) SOC

Fig. 4. The time history of power allocation and SOC.

Fig. 5. The time history of power allocation and SOC.

The time history of UA V altitude and velocity.

The time history of UA V heading and flight path angles.