A categorical approach towards metamodeling cyber-physical systems

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ABSTRACT

Cyber-physical systems, orchestrating computing, networking, and physical processes, are coupled, hybrid systems characterized by heterogeneous, interconnected subsystems. In civil engineering, the most common applications of cyber-physical systems are associated with structural health monitoring (SHM) and structural control. Given the complexity and heterogeneity of cyber-physical systems designed for structural health monitoring and control applications, it is hardly possible to reliably assess the quality of a cyber-physical system (CPS), which is needed for increased availability and for identifying potential sources of faults in a CPS. This conceptual paper proposes a metamodeling approach that can be used as a basis to model cyber-physical systems in order to assess the CPS quality based on category theory. Category theory is a mathematical theory that allows formalizing concepts of high-level abstractions on a meta level. Using the CPS metamodeling approach presented in this paper, each subsystem of a CPS can be modeled and assessed individually, entailing a holistic quality assessment of a CPS and its subsystems. However, since this paper is conceptual, it is intended to devote additional future research efforts to metamodeling and assessing cyber-physical systems, achieving a complete conceptual modeling framework for cyber-physical systems applied in structural health monitoring and control.

INTRODUCTION

The dynamic nature of physical processes in engineering applications has rendered computational models and data collected from physical processes essential for optimizing the performance of the applications [1,2]. Merging the digital and the physical world, cyber-physical systems integrate physical processes with computational and networking processes [3]. Cyber-physical systems have been widely applied in many engineering fields, such as civil engineering [4], construction [5], aerospace engineering [6], and avi-

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ation [7]. In general, a cyber-physical system (CPS) is composed of two subsystems: (i) a physical subsystem, which includes the physical processes and (ii) a computational subsystem, which contains the computational and networking processes. For a CPS employed in engineering applications, the physical processes represent the behavior of the physical elements of the application. The computational processes comprise the computational models created to describe the physical processes, while the networking processes cover the feedback loops between the computational models and physical or virtual elements used to control the physical processes (e.g. actuators, software modules).

In civil engineering, cyber-physical systems are basically employed for structural health monitoring (SHM) and control [8]. Moreover, since the control of physical processes is typically automated, wireless SHM systems are usually adopted for cyber-physical systems, leveraging high intelligence levels of wireless sensor nodes and the decentralized nature of wireless sensor networks [9]. The physical subsystem of a CPS in civil engineering is the structure itself [10]. The computational subsystem is the monitoring and control system, where the control elements are either actuators that modify the structural behavior or software modules that enable decision making. Examples of cyber-physical systems in civil engineering include structures equipped with active or semi-active tuned mass dampers [11] or wind turbines with wireless SHM and control systems installed, which can be accessed online via cyber infrastructures [12–14].

The behavior of both subsystems in cyber-physical systems is characterized by strong interactions, thus indicating high degrees of inter-subsystem and intra-subsystem coupling. In addition to this coupled nature, cyber-physical systems in civil engineering are predominantly used in structures of high significance often featuring unconventional and usually irregular structural systems, i.e. systems exhibiting complex dynamic behavior, which is difficult to accommodate (or even to anticipate). Increasing cost and complexity of engineering systems put additional demands on the task of CPS quality assessment. Quality assessment of engineering systems requires a consistent methodology for determining the level of performance of a given structural system on a mathematical basis. However, to introduce a consistent mathematical basis for quality assessment, a conceptual (or abstract, general) modeling methodology is necessary. Conceptual modeling methodologies enable considering models representing systems in general, i.e. regardless of specific boundary and initial conditions. Through this general consideration, the results obtained in one application field can be easily adopted to other fields, which is particularly important for CPS modeling, since subsystems of different nature are coupled. Therefore, for an overall assessment of cyber-physical systems, an approach to assess the quality of the respective models is proposed, using a metamodeling concept.

A metamodel is often defined as a model of a model. This definition is widely used, but it is wrong. A metamodel specifies the structure, the semantics, and the constraints for a family of models in a certain domain [15]. While a model is, simply speaking, an abstraction of phenomena in the real world, a metamodel is a further abstraction that specifies the properties of the model itself [16]. For metamodeling, lots of standards and recommendations exist, such as the standard ISO/IEC 24744 that introduces a software engineering metamodel for development methodologies [17]. In model-driven engineering, model-based architectures are commonly used for metamodeling, as proposed by the Object Management Group (OMG) [18]. OMGs Model-driven Architecture (MDA) provides a set of standards that includes the representation and exchange of models in a variety of modeling languages [19], OMGs Meta-Object Facility (MOF) is a standard for model-driven engineering that is used to express metamodels [20], and OMGs Common Warehouse Metamodel (CWM) defines a specification for metamodeling in the context of data warehousing [21]. In summary, metamodels have a lot in common with ontologies that are commonly used in engineering to define types, properties, and interrelationships of the objects to be modeled. However, given the metamodel definitions stated above, while every valid metamodel is an ontology, not every ontology is a metamodel.

This paper presents first ideas towards metamodeling of cyber-physical systems based on category theory. By utilizing the advantages of a strong mathematical formalism provided by category theory, the assessment of cyber-physical systems can be done objectively on a mathematical basis. The next section provides a motivation for the use of category theory, followed by a general concept of approaching cyber-physical system modeling using the categorical formulation. The paper concludes with a summary and an outlook on future research.

CATEGORICAL APPROACH TO CYBER-PHYSICAL SYSTEM MODELING

The behavior of CPS components in civil engineering is characterized by strong interactions and, therefore, by a high degree of coupling. As mentioned previously, the adoption of cyber-physical systems in civil engineering is often associated with unconventional structural systems (usually employed in structures of high significance, such as large financial centers); thus, ensuring the highest possible level of performance of cyber-physical systems is of utmost importance to the structures' stakeholders. In this direction, a methodology for assessing the quality of cyber-physical systems is necessary, accounting for the quality of the individual components of cyber-physical systems and of the complex coupling conditions between the components.

Fig. 1 indicates the two types of coupling appearing in cyber-physical system modeling:

- Coupling between cyber-physical subsystems (inter-subsystem coupling)
- Coupling among the components, or partial subsystems, of each cyber-physical subsystem (intra-subsystem coupling)

For an overall quality assessment of cyber-physical systems, besides the cyber-physical subsystems themselves, the coupling between the subsystems must be assessed. However, existing modeling methodologies for cyber-physical system modeling do not address the issue of inter and intra-subsystem coupling clearly, and particularly, do not assess the coupling quality [22].

The difference in the types of coupling appearing in cyber-physical systems puts additional constraints on a modeling methodology; the methodology must be general enough to cover all necessary couplings and models. The high level of abstraction required for the conceptual CPS modeling methodology can be achieved by the tools of category theory, which is an abstract mathematical theory providing a language that helps describing properties of different objects, e.g. subsystems or models, through a

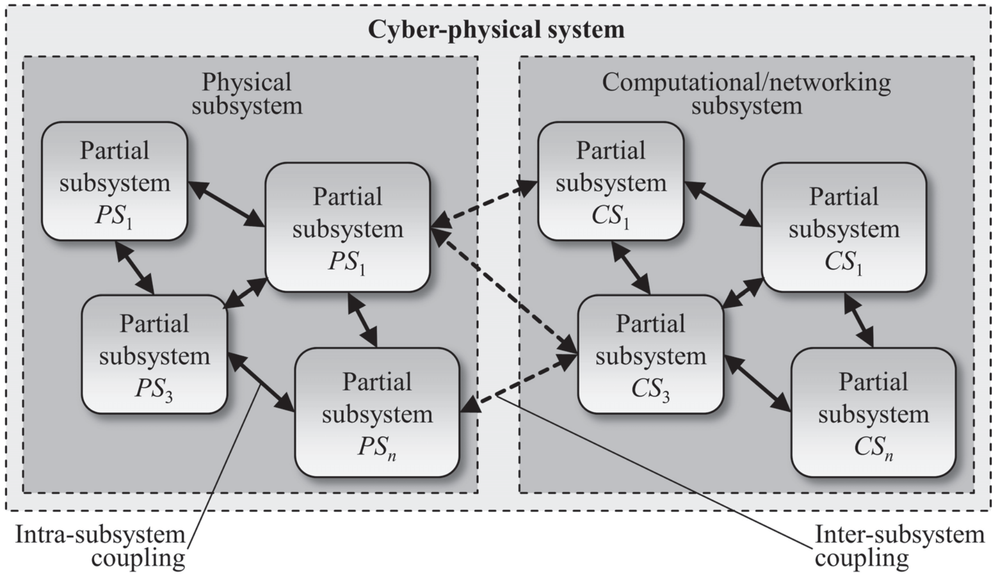

Figure 1. Coupling in cyber-physical systems

representation with diagrams of arrows. Each arrow represents a function (mapping) between two objects. A formal definition of a category is introduced as follows [23]:

Definition 1 *A category* $\mathbf{A}$ *consists of the following data:*

- *Objects:* $A, B, C, \ldots$
- *Arrows:* $f, g, h, \ldots$
- *For each arrow* f*, there are given objects* $\mathrm{dom}(f)$, $\mathrm{cod}(f)$ *called the domain and codomain of* f*. We write* $f\colon A \to B$ *or* $A \xrightarrow{f} B$ *to indicate that* $A = \mathrm{dom}(f)$ *and* $B = \mathrm{cod}(f)$*.*
- *Given arrows* $f\colon A \to B$ *and* $g\colon B \to C$*, that is, with* $\mathrm{cod}(f) = \mathrm{dom}(g)$ *there is given an arrow* $g \circ f\colon A \to C$ *called the composite of* f *and* g*.*
- *For each object* A*, there is given an arrow* $1_A\colon A \to A$ *called the identity arrow of* A*.*

The data is required to satisfy the following laws:

- *Associativity:* $h \circ (g \circ f) = (h \circ g) \circ f$ *for all* $f\colon A \to B$, $g\colon B \to C$, $h\colon C \to D$*.*
- *Unit:* $f \circ 1_A = f = 1_B \circ f$ *for all* $f\colon A \to B$*.*

A category is anything that satisfies this definition. It must be emphasized that in general the objects do not have to be sets and the arrows do not have to be functions. In this sense, a category is an abstract algebra of functions, or arrows (sometimes also called morphisms), with the composition operation $\circ$ as primitive.

Bringing Definition 1 into the context of cyber-physical system modeling entails defining categories and their corresponding structures. Since the motivation of this study is to describe the CPS modeling process in general, it seems to be natural to take models as objects in categories. Taking into account that a CPS combines subsystems of different nature, as shown in Fig. 1, it is clear that the corresponding categorical constructions will also be different, because of the different types of models used to represent physical and computational CPS subsystems.

First ideas on developing an abstract modeling methodology based on category theory have been introduced in [24], where mathematical models (or physics-based models) and their coupling have been discussed in detail. For advancing the categorical approach towards cyber-physical system modeling, some results achieved in [24] are extended herein.

Categories of different mathematical models are considered, denoted by $\mathbf{Model}_1$, $\mathbf{Model}_2, \ldots$. These categories contain mathematical models for one physical phenomenon relevant to cyber-physical systems, i.e. one category contains mathematical models describing solely one physical phenomenon (e.g. models of linear elasticity, models for damage propagation). A clear separation of categories is important for defining the coupling of models. The structure of a category of mathematical models is given by the following definition.

Definition 2 (Objects of a category of mathematical models) *Let* $\mathbf{Model}_1$ *be a category of mathematical models describing a given physical phenomenon. Then, for all objects of* $\mathbf{Model}_1$ *the following assumptions hold*

(i) objects are finite sets – set of assumptions of a mathematical model, denoted by $\mathbf{Set}_{\mathrm{A}}$*, where* A *is the corresponding mathematical model;*

(ii) morphisms are relations between these sets;

(iii) for each set of assumptions and its corresponding model exists an invertible mapping $\mathbf{Set}_{\mathrm{A}} \overset{S}{\mapsto} \mathrm{A}$*;*

(iv) all objects are related to mathematical models acting in the same physical domain.

The next step is to introduce a relation between different objects in one category. The relation serves as a tool to relate different models regardless of the specific problem considered. To satisfy this requirement, the complexity of models is used, which is defined as follows.

Definition 3 (Complexity of mathematical models) *Let* A *and* B *be mathematical models in a category* $\mathbf{Model}_1$*. We say that model* A *has higher complexity than model* B *if and only if* $\mathbf{Set}_{\mathrm{A}} \subset \mathbf{Set}_{\mathrm{B}}$*, but* $\mathbf{Set}_{\mathrm{B}} \not\subset \mathbf{Set}_{\mathrm{A}}$*. In addition, we say that models* A *and* B *are equal if and only if* $\mathbf{Set}_{\mathrm{A}} = \mathbf{Set}_{\mathrm{B}}$*.*

According to this definition, the complexity of models is a binary relation in a category of mathematical models, which can be used to order objects within the category. Moreover, the complexity of models is used to order not only partial models (i.e. single models) in cyber-physical systems, but also coupled mathematical models. However, in some cases categories with totally and partially ordered models must be distinguished [24].

The ideas of using category theory for the modeling purposes shown in Definitions 2 and 3 can be summarized in the following steps:

(i) describe objects of a category;

(ii) describe relations (morphisms) between the objects;

(iii) interpret the categorical construction from a practical point of view.

The first two points are standard steps in applying category theory as a language to describe the structure and the relations between different objects, but the third point is necessary to highlight the correspondence between the introduced categorical structure and practical modeling.

In this work, the three modeling steps are applied to a CPS, first describing the CPS partial subsystems and then the couplings between the subsystems. The results presented in [24] can directly be used in this study, since mathematical models are widely used for modeling physical subsystems within cyber-physical systems. However, mathematical models are not the only type of models used in CPS modeling; surrogate models (or data-based models) are also very popular in modern engineering, particularly when using classical physics-based models is computationally expensive. Therefore, it is proposed to describe surrogate models in the language of categories, entailing, because of the different nature of surrogate models, a different categorical construction. It is further proposed to formalize the coupling between mathematical and surrogate models as well.

Another point for an extension of the categorical approach is to represent computational and networking subsystems in the language of category theory. It must be noted that category theory has been used since many years in computer science, due to its direct relation to typed versions of lambda calculus [25]. Therefore, the theory of computation based on typed versions of lambda calculus can be included into the categorical modeling framework if necessary. However, a major objective of this study is to consider, for example, sensors and actuators being parts of a CPS as well as the communication between the sensors/actuators as subsystems modeled by category theory. According to the three modeling steps, a possible direction can be to use a natural link between category theory and graph theory, which is frequently used to model sensor networks [26]. This approach has been used in [27], where the formalization of distributed and dynamic resources allocation problem for modeling wireless sensor networks has been presented via category theory. Thus, the next step is to extend existing results in modeling cyber-physical systems with graph theory towards a more formal description by category theory.

SUMMARY AND CONCLUSIONS

Cyber-physical systems are widely used in engineering applications in an attempt to merge physical processes with computational and networking processes. A cyber-physical system (CPS) consists of two subsystems, the physical subsystem including the physical processes and the computational subsystem comprising the computational and networking processes. In civil engineering, cyber-physical systems are structures equipped with structural health monitoring (SHM) and control systems. Because SHM and control systems are usually installed on structures of high significance, frequently featuring unconventional structural systems that exhibit unfavorable dynamic behavior, assessing the quality of cyber-physical systems is of high importance. For CPS quality assessment, a consistent methodology based on a well-defined mathematical basis is required. Moreover, for defining the mathematical basis, a conceptual modeling methodology is necessary. In this paper, a conceptual methodology for modeling cyber-physical systems has been presented. The conceptual modeling methodology draws from category theory, which is an abstract mathematical theory for describing objects as well as the coupling conditions between objects. Using category theory, the model categories for each subsystem of a CPS have been defined, while the coupling between objects of each category (intra-subsystem coupling) and between the categories themselves (inter-subsystem coupling) has been formalized. Last but not least, it must be emphasized that the steps towards metamodeling cyber-physical systems presented in this paper, although very promising, are purely conceptual at the current state of research, and further substantial research efforts are required to further elaborate and validate the conceptual approach.

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