Pilot Contamination Reduction Scheme in Massive MIMO Multi-cell TDD Systems*

Cuifang ZHANG*, Guigen ZENG

College of Communication and Information Engineering, Nanjing University of Posts and Telecommunications, Nanjing 210003, China

Abstract

In this paper we consider massive MIMO Multi-cell TDD systems. In order to eliminate the effect of pilot contamination, a new scheme is proposed. In the same cell all transmitted signals from base station or user are multiplied same precoding sequences. The precoding sequences are orthogonal for different cell. The received signals at base station or terminals are multiplied the conjugate transpose of corresponding precoding sequences. Using the new scheme, the effect of pilot contamination is eliminated and SIR grows indefinitely for an unlimited number of base station antennas. When the interference of adjacent cell is very serious and the number of base station antennas is very large, the numerical results show that the new scheme provides noticeable improvement compared with MRC, LP, TSP scheme. Numerical results verify the effectiveness of the new scheme.

Keywords: Pilot Contamination; Massive MIMO; TDD Systems; Orthogonal Sequences

1 Introduction

Now multiuser MIMO using very large antenna arrays (so called massive MIMO) [1-3] has become a research hot spot. In massive MIMO Multi-cell time-division duplex (TDD) systems [4-6], users in the same cell use orthogonal pilots. The same set of orthogonal pilots is repeatedly used in all cells. It is shown in [7] that signal to interference ratio (SIR) does not grow indefinitely along with a very large number of base station antennas. The reason is that the channel estimates in the desired cell are contaminated by users from other cells who are assigned non-orthogonal pilot sequences.

The pilot contamination problem has been researched in massive MIMO Multi-cell TDD systems. [7] uses a linear pre-coder which is a version of conjugate transpose of uplink channel estimate. The inter-cell interference can’t be eliminated completely. The main idea of [8] is that each base station linearly combines messages aimed to users from different cells that re-use

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*Corresponding author.

Email address: zhangcf@njupt.edu.cn (Cuifang ZHANG).

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the same pilot sequence. The proposed method leads to the effective elimination of inter-cell interference. But it depends on the accuracy of estimated slow fading coefficients. [9] proposes a multi-cell MMSE-based precoding method. The author formulates an optimization problem for each precoding matrix. This method can reduce both intra-cell interference and inter-cell interference. But the algorithms are very complex. In [10], time-shifted pilot scheme (TSP) is proposed. In TSP scheme, cells are divided into groups. Once users in a same group finish their pilot sequence during each coherence interval, they start receiving downlink data while a different group starts sending pilots. All the interference coming from cells in different groups vanishes. But the interference in the same group still exists. [11] proposes a coordinated pilot assignment strategy. The effect of pilot contamination vanishes completely under certain conditions on the channel covariance. But the gains depend on system parameters such as the typical angle spread measured at the base station and the number of base station antennas. In [12] a pilot assignment method is further proposed by maximizing the achievable downlink sum rate. This method is not feasible when pilots are sent simultaneously by all users in the system.

This paper is organized as follows. In Section 2, we describe the multi-cell system model and TDD transmission protocol within a coherence interval. In Section 3, we analyze pilot contamination problem. In Section 4, we propose a new scheme. Using the new scheme, the effect of pilot contamination vanishes and SIR grows indefinitely for an unlimited number of base station antennas. Later in Section 5, we analyze the feasibility of the new scheme compared with MRC, LP, TSP scheme. In Section 6, we present numerical results. Finally, we provide our conclusion in Section 7. Notation: In this paper, $\| \cdot \|$ denotes the two-norm, $(\cdot)^T$ denotes the transpose and $(\cdot)^+$ denotes the conjugate transpose. $E[\cdot]$ stands for expectation.

2 System Model

![Fig. 1: System model between the $m$-th antenna of the $i$-th cell and the $k$-th user in the $l$-th cell](image1)

![Fig. 2: TDD transmission protocol within a coherence interval](image2)

This system model is shown in Fig. 1. We consider a cellular system consisting of $L$ non-cooperative hexagonal cells where share the same band of frequencies. Each cell consists of a $M$-antenna base station and $K$ single-antenna users. The number of antenna at base station grows without limit. The propagation coefficient between the $m$-th antenna of the $i$-th cell and the $k$-th user in the $l$-th cell is $g_{mikl} = h_{mikl} \sqrt{\beta_{ikl}}$. $\beta_{ikl}$ is slow fading coefficient which is composed of path loss and shadow fading, and is assumed invariant for different antennas of the same base station. Fast fading coefficients $h_{mikl}$ are independent and identically distributed (i.i.d.) zero-mean, circularly-symmetric complex Gaussian, i.e. $h_{mikl} \sim CN(0,1)$. The corresponding fast
fading coefficients form fast fading vector \( h_{ikl} = (h_{1ikl}, \ldots, h_{Mikl})^T \), and \( h_{ikl} \sim CN(0, I_M) \). The channel coefficients of the \( i \)-th base station form the channel vector \( g_{ikl} = (g_{1ikl}, \ldots, g_{Mikl})^T \).

We assume a TDD transmission protocol. It is assumed that the channel stays constant for \( T \) OFDM symbols within the coherence interval. Over \( \tau \) OFDM symbols the base station only learn the channel for \( K \) (\( K \leq \tau \)) users in a same cell. Every coherence interval is organized in four parts, as shown in Fig. 2: (1) the transmission of \( \tau_{up} \) uplink data; (2) the transmission of \( \tau \) uplink pilots; (3) the computation of the channel estimate, processing of uplink data and the precoding matrix (assumed to occupy one symbol); (4) the transmission of \( \tau_{down} \) downlink data. Therefore each coherence interval has length \( T = \tau_{up} + 1 + \tau + \tau_{down} \).

3 Analysis of Pilot Contamination

It is assumed that transmission and reception are synchronous in massive MIMO Multi-cell TDD systems. We consider that all users (in all cells) synchronously transmit uplink training sequences \( \sqrt{\tau} \psi_k = \sqrt{\tau}(\psi_{k1}, \ldots, \psi_{k\tau}) (\tau \geq K) \). We assume that \( K \) users in the same cell use orthogonal pilots \( (\psi_i \psi_j^+ = \delta_{ij}) \). The same set of orthogonal pilots is repeatedly used in all cells. We assume that the average power (during transmission) at the base station is \( p_f \) and the average power (during transmission) at each user is \( \rho_r \).

It is assumed that the \( k \)-th users in all cells synchronously send \( \psi_k \). The \( i \)-th base station receives the \( M \times \tau \) matrix

\[
Y_i = \sum_{l=1}^{L} \sum_{k=1}^{K} \sqrt{\rho_r} \tau g_{ikl} \psi_k + z_i
\]

where \( z_i \in \mathbb{C}^{M \times \tau} \) is the additive noise. The estimate of the vectors \( g_{ikl} \) is

\[
\hat{g}_{iki} = Y_i \cdot \frac{\psi_i^+}{\sqrt{\rho_r \tau}} = \sum_{l=1}^{L} \sum_{k=1}^{K} g_{ikl} \psi_k \psi_i^+ + z_i \cdot \frac{\psi_i^+}{\sqrt{\rho_r \tau}} = g_{iki} + \sum_{l=1, l \neq i}^{L} g_{ikl} + z_i'
\]

where \( z_i' \sim CN(0, \frac{1}{\rho_r \tau} I_M) \). The estimate of \( g_{iki} \) is contaminated. The reason is that the channel estimates at the base station contain not only the desired channel vector and additive white noise, but also the channel vectors of users from other cells who use same the pilot sequences.

4 The New Pilot Contamination Reduction Scheme

In order to eliminate the effect of pilot contamination, we present a new scheme. It is assumed that \( \{a_1, \ldots, a_L\} \) is a set of orthogonal sequences \( (a_j a_j^+ = \delta_{ij}) \). All transmitted signals from base station or user in the \( i \)-th cell are multiplied sequence \( a_i = (a_{i1}, \ldots, a_{i\tau'}) \). The value of \( \tau' \) is related to the number of cell \( (\tau' \geq L) \). The \( a_i \) sequence is known as precoding sequence. Orthogonal sequences \( \{a_1, \ldots, a_L\} \) can be obtained by Hadamard matrix or Walsh matrix. The precoding sequences of different cell are orthogonal. The received signals at base station or terminals are multiplied the conjugate transpose of corresponding precoding sequences. Now we discuss the effects of the proposed scheme for uplink and downlink in detail.
### 4.1 Downlink data transmission

During downlink data transmission, the base station uses normalized beamforming vector:

\[
\omega_{ki} = \frac{\hat{g}_{iki}}{\|\hat{g}_{iki}\|} = \frac{\hat{g}_{iki}}{\alpha_{ki}\sqrt{M}}
\]  

(3)

The scalar \(\alpha_{ki} = \frac{\|\hat{g}_{iki}\|}{\sqrt{M}}\) is a normalization factor. From (2), we can obtain

\[
\lim_{M \to \infty} \alpha_{ki}^2 = \lim_{M \to \infty} \frac{1}{M} \left( \sum_{l=1}^{L} \beta_{ikl}h_{ikl}^+h_{ikl} + \sum_{l=1}^{L} \sqrt{\beta_{ikl}}h_{ikl}^+z_i^l + z_i^l \sum_{l=1}^{L} \sqrt{\beta_{ikl}}h_{ikl} + z_i^l \cdot z_i^l \right)
\]  

(4)

To compute the value of (4) as \(M \to \infty\), we use the following well known lemma.

**Lemma 1** Let \(x, y \in \mathbb{C}^{M \times 1}\) be two independent vectors with distribution \(\mathcal{CN}(0, \mathbf{I})\). Then

\[
\lim_{M \to \infty} \frac{x^+y}{M} \to \mathbf{0} \quad \text{and} \quad \lim_{M \to \infty} \frac{x^+x}{M} \to \mathbf{c}
\]  

(5)

Using the fact that \(h_{ikl} \sim \mathcal{CN}(0, \mathbf{I}_M)\) and applying Lemma 1, we can obtain \(\lim_{M \to \infty} \frac{h_{ikl}^+h_{ikl}}{M} \to 1\) and \(\lim_{M \to \infty} \frac{\hat{g}_{iki}^+\hat{g}_{iki}}{M} \to \beta_{ikl}\). Using the fact that the channel vectors of different users are independent and applying the above Lemma 1, we can derive the value of \(\alpha_{ki}^2\), i.e. \(\lim_{M \to \infty} \alpha_{ki}^2 = \sum_{l=1}^{L} \beta_{ikl} + \frac{1}{\tau_{pr}}\).

We assume that channel has reciprocity in TDD systems. Let the information symbols to be transmitted to users in the \(l\)-th cell be \(q_l = [q_{k1}, \ldots, q_{KL}]^T\), satisfying \(E[q_l] = 0\) and \(E[q_lq_l^+] = \mathbf{I}\). So \(q_{kl}\) is the signal intended to the \(k\)-th user in the \(l\)-th cell. We consider that all transmitted signals from base station in the \(l - \tau\)th cell are multiplied sequence \(a_l = (a_{l1}, \ldots, a_{L\tau})\) (\(\tau > L\)). The \(k\)-th user of the \(i\)-th cell receives the signal

\[
x_{ki} = \sum_{l=1}^{L} \sum_{k'=1}^{K} \sqrt{\mathbf{P}f} g_{ikl}^+\omega_{kl}(q_{kl}a_l) + z_{ki}
\]  

(6)

The additive noises \(z_{ki}\) for all users are i.i.d. \(z_{ki} \sim \mathcal{CN}(0, 1)\). The estimate of the signal intended to the \(k\)-th user in the \(i\)-th cell is

\[
\hat{q}_{ki} = \left[\sum_{l=1}^{L} \sum_{k'=1}^{K} \sqrt{\mathbf{P}f} g_{ikl}^+\omega_{kl}(q_{kl}a_l) + z_{ki}\right]a_{k}^+ = \sum_{l=1}^{L} \sum_{k'=1}^{K} \sqrt{\mathbf{P}f} g_{ikl}^+\omega_{kl}(q_{kl}a_l) a_{k}^+ + z_{ki} \cdot a_{k}^+
\]  

(7)

Because the precoding sequences of different cell are orthogonal,

\[
\hat{q}_{ki} = \sqrt{\mathbf{P}f} g_{ikl}^+\omega_{kl}q_{kl} + \sum_{k'=1, k' \neq k}^{K} \sqrt{\mathbf{P}f} g_{ikl}^+\omega_{kl'}q_{kl'} + z_{ki} \cdot a_{k}^+
\]  

(8)

In the term (a), the case \(k = k'\) is considered first.

\[
Q_{ki} = E[|\sqrt{\mathbf{P}f} g_{ikl}^+\omega_{kl}q_{kl}|^2] = \frac{p_f}{\alpha_{ki}^2 M} \left|h_{ikl}^+\hat{g}_{iki}\right|^2 = \frac{p_f}{\alpha_{ki}^2 M} \sum_{l=1}^{L} h_{ikl}^+h_{ikl} + h_{ikl}^+z_{ki}^2
\]  

(9)
transmitted signals from user in the

4.2 Uplink data transmission

\[ h_{ik_l}(i \neq l_1) \text{ and } z_i' \text{ are independent. Applying Lemma 1 to the terms inside the absolute value, we get} \]
\[
\lim_{M \to \infty} \frac{1}{M} \left( \sum_{l_1=1}^{L} h_{ik_l} h_{ik_l l_1} + h_{ik_l} z_i' \right) = \lim_{M \to \infty} \frac{1}{M} \left( h_{ik_l} h_{ik_l} + \sum_{l_1=1, l_1 \neq i}^{L} h_{ik_l} h_{ik_l l_1} + h_{ik_l} z_i' \right) \to 1 \quad (10)
\]

So \( \lim_{M \to \infty} Q_{ki} \to p f^2 \beta^2_{ki} \). In the term (b), the case \( k \neq k' \) is considered.

\[ Q_{k'i} = E[|\sqrt{p_f} g_{ik_l}^+ \omega_{k'1}^i q_{k'i}|^2] = \frac{p_f \beta^2_{ki}}{\alpha_{k'i}^2 M} |h_{ik_l} \hat{g}_{ik'i}|^2 = \frac{p_f \beta^2_{ki}}{\alpha_{k'i}^2 M} \sum_{l_1=1}^{L} h_{ik_l}^+ h_{ik_l l_1} + h_{ik_l}^+ z_i' |^2 \quad (11)\]

\( h_{ik_l}, h_{ik_l l_1} \) and \( z_i' \) are independent for any \( l_1 \). Applying Lemma 1 to the terms inside the absolute value, we get

\[
\lim_{M \to \infty} \frac{1}{M} \left( \sum_{l_1=1}^{L} h_{ik_l}^+ h_{ik_l l_1} + h_{ik_l}^+ z_i' \right) = \lim_{M \to \infty} \frac{1}{M} \left( h_{ik_l}^+ h_{ik_l} + \sum_{l_1=1, l_1 \neq i}^{L} h_{ik_l}^+ h_{ik_l l_1} + h_{ik_l}^+ z_i' \right) \to 0 \quad (12)
\]

So \( \lim_{M \to \infty} Q_{k'i} \to 0 \). The effects of uncorrelated receiver noise and fast fading are eliminated completely. The interference from users in other cells who use the same pilot sequence vanishes.

\((c) = z_{k'i} a_{i}^{+}\). The term \(c\) isn’t related to the number of base station antennas. As shown in [7, 8], the variance of the additive noise is unitary regardless of the number of base station antennas, thus rendering the effect of the noise null in the asymptotic region. According to above analysis, the downlink SIR of the \( k\)-th user in \( i\)-th cell grows indefinitely as \( M \to \infty \), i.e. \( \lim_{M \to \infty} SIR_{ki}^{D_{new}} \to \infty \). The effect of pilot contamination is eliminated.

4.2 Uplink data transmission

We assume that the \( k\)-th user in the \( l\)-th cell transmits the signal \( s_{kl} \). \( K \) users of the \( l\)-th cell transmit signals \( S_l = [s_{l1}, \ldots, s_{lK}]^T \), satisfying \( E[S_l] = 0 \) and \( E[S_l S_l^+] = I \). We consider that all transmitted signals from user in the \( l\)-th cell are multiplied sequence \( a_l = (a_{l1}, \ldots, a_{lT'}) (T' \geq L) \). The \( i\)-th base station receives the signal

\[ y_i = \sum_{l=1}^{L} \sum_{k=1}^{K} \sqrt{p_r} g_{ik_l}(s_{kl} a_l) + v_i \quad (13)\]

Where \( v_i \in M_{x_T'} \) is the additive noise. \( v_i \sim CN(0, 1) \). Since multiplication on the receiver does not modify transmit power, normalization is not required in the uplink. To decode the signal \( s_{kl} \), the \( i\)-th base station processes its received signal by using MRC and multiplying the conjugate-transpose of \( a_l \). We can obtain the estimate of \( s_{kl} \)

\[
\hat{s}_{kl} = \frac{1}{M} \left( \hat{g}_{ik_l} y_i a_i^{+} \right) = \frac{1}{M} \left[ \sum_{l_1=1}^{L} \sum_{l=1}^{K} \sum_{k=1}^{K} \sqrt{p_r} g_{ik_l} (s_{kl} a_l) a_i^{+} \right] + \frac{1}{M} \left| z_i^{+} a_i^{+} \right| (d) + \frac{1}{M} \sum_{l_1=1}^{L} g_{ik_l} v_i a_i^{+} + \frac{1}{M} \sum_{l=1}^{L} \sum_{k=1}^{K} \sqrt{p_r} g_{ik_l} (s_{kl} a_l) a_i^{+} + \frac{1}{M} \sum_{l_1=1}^{L} g_{ik_l} v_i a_i^{+} + \frac{1}{M} \sum_{l=1}^{L} \sum_{k=1}^{K} \sqrt{p_r} g_{ik_l} (s_{kl} a_l) a_i^{+} (f) + \frac{1}{M} \sum_{l_1=1}^{L} g_{ik_l} v_i a_i^{+} + \frac{1}{M} \sum_{l=1}^{L} \sum_{k=1}^{K} \sqrt{p_r} g_{ik_l} (s_{kl} a_l) a_i^{+} + \frac{1}{M} \sum_{l_1=1}^{L} g_{ik_l} v_i a_i^{+} + \frac{1}{M} \sum_{l=1}^{L} \sum_{k=1}^{K} \sqrt{p_r} g_{ik_l} (s_{kl} a_l) a_i^{+} (g) \]

\[ (d) + \frac{1}{M} \sum_{l_1=1}^{L} g_{ik_l} v_i a_i^{+} + \frac{1}{M} \sum_{l=1}^{L} \sum_{k=1}^{K} \sqrt{p_r} g_{ik_l} (s_{kl} a_l) a_i^{+} (f) + \frac{1}{M} \sum_{l_1=1}^{L} g_{ik_l} v_i a_i^{+} + \frac{1}{M} \sum_{l=1}^{L} \sum_{k=1}^{K} \sqrt{p_r} g_{ik_l} (s_{kl} a_l) a_i^{+} (g) \]
Using the fact that the channel vectors of different users are independent and the precoding sequences of different cell are orthogonal, and applying Lemma 1, we obtain

$$\lim_{M \to \infty} (d) = \lim_{M \to \infty} \frac{1}{M} \left[ \sqrt{\rho} g_{ik_i}s_{ki}a_i^+ \right] = \lim_{M \to \infty} \frac{1}{M} \sqrt{\rho} \beta_{ik_i}h_{ik_i}s_{ki} \to \sqrt{\rho} \beta_{ik_i}s_{ki}$$  \hspace{1cm} (15)

In the term (e) (f), $v_i' = v_i a_i^+ \sim CN(0, I_M)$. Using Lemma 1,

$$\lim_{M \to \infty} (e) = \lim_{M \to \infty} \frac{1}{M} z_i' v_i' \to 0 \hspace{1cm} (16)$$

$$\lim_{M \to \infty} (f) = \lim_{M \to \infty} \frac{1}{M} \sum_{l_1=1}^{L} g_{ik_{l_1}} (v_i a_i^+) = \lim_{M \to \infty} \frac{1}{M} \sum_{l_1=1}^{L} g_{ik_{l_1}} v_i' \to 0 \hspace{1cm} (17)$$

$$\lim_{M \to \infty} (g) = \lim_{M \to \infty} \frac{1}{M} \sum_{l=1}^{L} g_{ik_l}(v_i a_i^+) = \lim_{M \to \infty} \frac{1}{M} \sum_{l=1}^{L} g_{ik_l} v_i' \to 0 \hspace{1cm} (18)$$

According to above analysis, we can obtain $\hat{s}_{ki} = \sqrt{\rho} \beta_{ik_i}s_{ki}$. The interference from users in other cells who use the same pilot sequence vanishes. The effects of uncorrelated receiver noise and fast fading are eliminated completely along with $M \to \infty$. The uplink SIR of the $k$-th user in $i$-th cell grows indefinitely along with a very large number of base station antennas, i.e. $\lim_{M \to \infty} SIR_{ki}^{\text{new}} \to \infty$. The effect of pilot contamination is eliminated.

### 5 Analysis and Comparison of the New Scheme

We analyze the feasibility of the new scheme by the sum throughput of one cell during downlink data transmission. We compare the sum throughput obtained by LP, Time-Shifted Pilots (TSP) and the new scheme. According to TDD transmission protocol in Fig. 2, the sum throughput of the $i$-th cell obtained by the new scheme is

$$C_i^{D_{\text{new}}} = \sum_{k=1}^{K} \frac{\tau_{\text{down}}}{T} \cdot \frac{1}{T'} \cdot \log_2(1 + SIR_{ki}^{D_{\text{new}}})(\text{bits/sec/Hz})$$ \hspace{1cm} (19)

Here the factor $\in$ accounts for the effect of guard intervals, cyclic prefix and particular modulation constellation, equal for both schemes. All transmitted signals from base station in the same cell are multiplied same precoding sequences. So the sum throughput of the $i$-th cell obtained by new scheme is multiplied the factor $\frac{1}{T'}$. Because $SIR_{ki}^{D_{\text{new}}}$ grows indefinitely along with a very large number of base station antennas, $C_i^{D_{\text{new}}}$ constantly grows along with $M \to \infty$. In [10], this scheme partitions the $L$ cells into groups of cells $A_1 \ldots A_\Gamma$. $\Gamma$ is the number of groups. So the sum throughput of the $i$-th cell obtained by TSP scheme is

$$C_i^{D_{\text{TSP}}} = \sum_{k=1}^{K} \frac{\tau_{\text{down}}}{T} \cdot \log_2(1 + \frac{\beta^2_{ik_i}}{\sum_{l \in A_i, l \neq i} \beta^2_{lk_i}})(\text{bits/sec/Hz})$$ \hspace{1cm} (20)

Under the TSP scheme, all the interference coming from cells in different groups vanishes. But base station in the same group still can cause interference. $C_i^{D_{\text{TSP}}}$ depends on the slow fading
coefficients in the same group. In [7], the sum throughput of the $i$-th cell obtained by LP scheme is

$$C_{DL}^{LP} = \sum_{k=1}^{K} \tau_{\text{down}} \cdot \log_2 (1 + \frac{\beta_{ki}^2}{\sum_{l=1, l \neq i}^{L} \beta_{kl}^2})(\text{bits/sec/Hz})$$

(21)

$C_{DL}^{LP}$ and $C_{DL}^{TSP}$ tend to a constant value and does not grow indefinitely along with $M \to \infty$.

We compare the sum throughput obtained by MRC [7], Time-Shifted Pilots (TSP) and the new scheme during uplink data transmission. $C_{U}^{\text{new}}$, $C_{U}^{\text{TSP}}$ and $C_{U}^{\text{MRC}}$ denote the sum throughput of the $i$-th cell obtained by the new scheme, TSP and MRC, respectively. The conclusion of uplink transmission is similar to downlink transmission. Under the new scheme, all transmitted signals from user in the same cell are multiplied same precoding sequences. So $C_{U}^{\text{new}}$ is multiplied the factor $\frac{1}{\tau_t}$ too. Because $SIR_{ki}^{\text{new}}$ grows indefinitely along with $M \to \infty$, $C_{U}^{\text{new}}$ constantly grows along with $M \to \infty$. In [7, 10], $C_{U}^{\text{TSP}}$ and $C_{U}^{\text{MRC}}$ don’t grow indefinitely along with $M \to \infty$, and they depend on the slow fading coefficients.

6 Numerical Results

We evaluate the performance of the proposed scheme by means of MATLAB simulation. For all simulations, we assume $p_f = 20dB$ and $\rho_r = 10dB$. From the standpoint of pilot contamination, we consider the worst possible case, i.e. the same set of orthogonal pilots is used in all cells and the interference of adjacent cell is very serious ($\beta_{kj}(j \neq l)$ is big). For all $k$, $\beta_{kj} = 1$ if $j = l$; $\beta_{kj} = a$ if $j \neq l$. $C$ denotes the sum throughput achieved by one cell.

At first, we analyze simulation results of downlink transmission. We consider a TDD transmission protocol with $L = \tau_t = 4$, $K = \tau = 4$, $\tau_{\text{up}} = 4$, $\Gamma = 2$, $\tau_{\text{down}} = 8$ and $T = 17$. We compare the sum throughput of one cell achieved by LP, TSP and new scheme. As shown in Fig. 3, (1) $C_{D}^{\text{new}}$ changes little with the different slow fading coefficients. The reason is that the interference from users in other cells who use the same pilot sequence vanishes completely with $M \to \infty$ in the new scheme; (2) $C_{D}^{\text{new}}$ is obviously better than $C_{D}^{\text{LP}}$. The reason is that users in other
cells who use the same pilot sequence constitute interference in LP scheme. $C^{DLP}$ quickly tends to a constant value along with increase of $M$; (3) When $(\beta_{l,k})_{j \neq l}$ is big and the number of base station antennas is very large, $C^{D_{new}}$ provides noticeable improvement compared with TSP scheme along with increase of $M$. For example $a = 0.8$ and $M > 500$, $C^{D_{new}}$ is obviously better than $C^{D_{TSP}}$. The reason is that $C^{D_{TSP}}$ does not grow indefinitely along with $M \rightarrow \infty$ while $C^{D_{new}}$ constantly grows along with $M \rightarrow \infty$. The simulation conclusion confirms the analysis of (19), (20) and (21).

In Fig. 4, we consider a TDD transmission protocol with $K = \tau = 2$, $\tau_{up} = 4$, $\tau_{down} = 8$, $a = 0.7$ and $T = 15$. $C^{D_{new}}$ increases along with the reduction of $L$. The reason is that the value of $\tau'$ is related to the number of cell ($\tau' \geq L$). The new scheme provides noticeable improvement with the reduction of $L$ and a very large number of base station antennas.

From Fig. 5, Fig. 6, the conclusion of uplink transmission is similar to downlink transmission. From above analysis, the new scheme is feasible and better than MRC, LP and TSP scheme when interference of adjacent cell is very serious and the number of base station antennas is very large.

7 Conclusion

In this paper we analyze pilot contamination problem in massive MIMO Multi-cell TDD systems. In order to eliminate the effect of pilot contamination, we propose a new scheme. Using the new scheme, the effects of uncorrelated receiver noise and fast fading are eliminated completely, and the interference from users in other cells who use the same pilot sequence vanishes along with a very large number of base station antennas. The sum throughput of one cell obtained by the new scheme constantly grows along with $M \rightarrow \infty$. When interference of adjacent cell is very serious and the number of base station antennas is very large, the numerical results show that the new scheme provides noticeable improvement compared with MRC, LP, TSP scheme. The new scheme provides improvement with the reduction of $L$, too. It is show that the new scheme is feasible and low-complex.
References


