A STUDY OF SCHWINGER-DYSON EQUATIONS FOR YUKAWA AND WESS-ZUMINO MODELS

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ABSTRACT

We study Schwinger-Dyson equation for fermions in Yukawa and Wess-Zumino models, in terms of dynamical mass generation and the wavefunction renormalization function. In the Yukawa model with γ_5-type interaction between scalars and fermions, we find a critical coupling in the quenched approximation above which fermions acquire dynamical mass. This is shown to be true beyond the bare 3-point vertex approximation. In the Wess-Zumino model [1], there is a neat cancellation of terms leading to no dynamical mass for fermions. We comment on the conditions under which these results are general beyond the rainbow approximation and also on the ones under which supersymmetry is preserved and the scalars as well do not acquire mass. The results are in accordance with the non-renormalization theorem at least to order α in perturbation theory. In both the models, we also evaluate the wavefunction renormalization function, analytically in the neighbourhood of the critical coupling and numerically, away from it.

1 Introduction

Despite the success of quantum field theory in the description of the behaviour of elementary particles in the perturbative regime of interactions, it still remains a challenge to understand the non-perturbative domain satisfactorily. One of the methods which has gained attention in this regard in recent years is the study of Schwinger-Dyson equations (SDEs) [2]. Despite the difficulties involved in finding a non-perturbative truncation of these equations, this approach has been very successful in addressing issues like dynamical mass generation for fundamental fermions when they are involved in sufficiently strong interactions [3]. Moreover, recent attempts, e.g., [4, 5, 6] to improve the reliability of the approximations used have increased the credibility of the results obtained through such studies.

Application of Schwinger-Dyson formalism to supersymmetric (SUSY) models has been less extensive. In supersymmetric Quantum Electrodynamics (SQED), based upon the arguments of non-renormalization theorem and gauge invariance, it is expected to be impossible to obtain dynamical mass generation for fermions [7] though some studies [8] argue that it is probably possible to break chiral symmetry dynamically in SQED. We postpone the study of SQED for a future work. In this paper, we take the simplest SUSY model, i.e the Wess-Zumino model and attempt to solve the corresponding Schwinger-Dyson equations for the fermion and scalar propagators. We believe this exercise will provide us with a deeper insight into how the role of supersymmetry in the context of dynamical mass generation translates into the language of Schwinger-Dyson equations.

Such a study should provide us with a better starting point for more complicated SUSY theories such as SQED and SQCD. In the latter theory, a need also exists to further explore connections between the Holomorphic approach and that of the Schwinger-Dyson equations [9].

We first study the Yukawa model with one real scalar and one Majorana fermion, which can be considered as a truncated Wess-Zumino model. We discuss this model in some detail for two reasons, first being that it is interesting in its own right because, after all, it is Yukawa interactions which are responsible for giving masses to fermions in the Standard Model (SM). Secondly, extending the Yukawa model by doubling the scalar degrees of freedom provides us with a clear understanding of how supersymmetry works.

We use the quenched approximation. Keeping in mind the perturbative expansion of the 3-point vertex beyond the lowest order and its transformation under charge conjugation symmetry, we propose an *ansatz* for the full effective vertex. One of the advantages of using this vertex is that the equations for the mass function $\mathcal{M}(p^2)$ and the wavefunction renormalization $F(p^2)$ decouple completely in the neighbourhood of the critical coupling, α_c above which mass is generated for the fermions, and partly above it. We solve both the equations to find analytical expressions for $F(p^2)$ and the anomalous mass dimensions in the neighbourhood of α_c. The results show that non-perturbative interaction of fermions with fundamental scalars can give masses to fermions in a dynamical way provided the interaction is strong enough. We use numerical calculation to draw Euclidean mass of the fermions as a function of the coupling, and confirm that it obeys Miransky scaling. We also evaluate $F(p^2)$ numerically. We then extend the particle spectrum by doubling the number of scalars and imposing relations for the couplings that define the Wess-Zumino model. Due to the presence of the additional symmetry, we are able to extract useful information beyond the rainbow approximation.

2 The Yukawa Model

Consider a massless Lagrangian with one Majorana fermion and one real scalar interacting with each other through a γ_5-type interaction:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2} i(\bar{\psi}\gamma^\mu \partial_\mu \psi) - \frac{1}{2} g^2 A^4 + ig\bar{\psi}\gamma_5 \psi A \; . \tag{1}$$

The corresponding Schwinger-Dyson equation for the fermion propagator, $S_F(p)$, is displayed in Fig. 1. Motivated by the success of the quenched approximation in QED and QCD, we neglect the fermion loops. Moreover, as a first step towards truncating the infinite set of Schwinger-Dyson equations, we drop all 4-point functions. The full scalar propagator can then be replaced by its bare counterpart. Using Feynman rules, the Schwinger-Dyson equation can be written as:

$$-\,iS_F^{-1}(p) \;=\; -iS_F^{0^{-1}}(p) \;-\; \int \frac{d^4k}{(2\pi)^4}\,(-2g\gamma_5)\;(iS_F(k))\;(-2g\gamma_5\Gamma_A(k,p))\;(\frac{i}{q^2}) \quad , \tag{2}$$

where $q = k - p$ and $S_F(p)$ can be expressed in terms of two Lorentz scalar functions, $F(p^2)$, the wavefunction renormalization and $\mathcal{M}(p^2)$, the mass function, so that

$$S_F(p) \;=\; \frac{F(p^2)}{\not p - \mathcal{M}(p^2)} \qquad . \tag{3}$$

The bare propagator $S_F^0(k) = 1/(\not p)$, where the bare mass has been taken to be zero. We can project out equations for $F(p^2)$ and $\mathcal{M}(p^2)$ by taking the trace of Eq. (2) having multiplied by $\not p$ and 1 in turn. On Wick rotating to Euclidean space,

$$F(p^2) \;=\; 1 - \frac{\alpha}{\pi^3}\frac{1}{p^2}\int d^4k \, \frac{F(k^2)F(p^2)}{k^2+\mathcal{M}^2(k^2)} \, \frac{k\cdot p}{q^2} \;\; \Gamma_A(k,p) \tag{4}$$

$$\mathcal{M}(p^2) \;=\; \frac{\alpha}{\pi^3}\int d^4k \, \frac{\mathcal{M}(k^2)}{k^2+\mathcal{M}^2(k^2)} \, \frac{F(k^2)F(p^2)}{q^2} \, \Gamma_A(k,p) \tag{5}$$

where $\alpha = g^2/4\pi$.

It is here that we cannot proceed any further without making an *ansatz* for $\Gamma_A(k,p)$. Any *ansatz* for the 3-point vertex must fulfill at least the following requirements:

- Perturbatively, we must have $\Gamma_A(k,p) = 1 + \mathcal{O}(g^2)$.

- It must be symmetric in k and p.

Moreover, as the SDEs relate the 2-point function with the 3-point function, the expression for the full vertex is expected to involve functions $F(p^2)$ or/and $\mathcal{M}(p^2)$.

The commonly used ansatz in the non-perturbative study of the SDEs is the bare vertex ansatz. For the Yukawa Model under discussion, it implies $\Gamma_A(k,p) = 1$. It agrees with the lowest order perturbation theory. The only truncation of the complete set of Schwinger-Dyson equations known so far that avoids any assumptions other than the smallness of the coupling at every level of this approximation is the perturbation theory. Therefore, it is natural to assume that physically meaningful solutions of the Schwinger-Dyson equations must agree with perturbative results in the weak coupling regime. It requires, e.g., that every non-perturbative *ansatz* chosen for the full vertex must reduce to its perturbative counterpart when the interactions are weak. Bare vertex fulfills this requirement to the lowest order in perturbation theory. Any other vertex which fulfills this condition and does not violate other requirements is at least as good as the bare vertex. One of the simplest non-perturbative vertices can be constructed by realizing that Eq.(4) yields the following expansion of $F(p^2)$ in perturbation theory:

$$F(p^2) = 1 + \mathcal{O}(\alpha) \tag{6}$$

Therefore, a simple candidate for the 3-point vertex can be written as:

$$\Gamma_A(k,p) = \frac{1}{F(k^2)F(p^2)} \quad . \tag{7}$$

Perturbatively, it gives

$$\Gamma_A(k,p) = \frac{1}{[1+\mathcal{O}(\alpha)][1+\mathcal{O}(\alpha)]} = 1 + \mathcal{O}(\alpha) \tag{8}$$

Therefore, to the lowest order in perturbation theory, our vertex *ansatz* reduces to the bare vertex.

It is exceedingly complicated to ensure that at higher orders, the non-perturbative vertex reduces to its perturbative counterpart in the weak coupling regime. We do not aim at it in this paper. However, we demonstrate that even up to next to lowest order, i.e, to $\mathcal{O}(\alpha)$, our *ansatz* is correct to the extent that both the *ansatz* and the real vertex have the logarithmically divergent behaviour in the ultraviolet regime.

Fig. 2 represents the perturbative expansion of the 3-point fermion-scalar vertex to $\mathcal{O}(\alpha)$. Using Feynman rules, we can write it as follows:

$$-2g\gamma_5\Gamma_A = -2g\gamma_5 + \int \frac{d^4w}{(2\pi)^4}\,(-2g\gamma_5)\,iS_F(p-w)\,(-2g\gamma_5)\,iS_F(k-w)\,(-2g\gamma_5)\,\frac{i}{w^2} \tag{9}$$

which can be simplified to:

$$\Gamma_A \;=\; 16\pi i\alpha^2\left[\not k \not p J^{(0)} - (\not k\gamma^\nu + \gamma^\nu \not p)J^{(1)}_\nu + K^{(0)}\right] \tag{10}$$

where

$$J^{(0)} \;=\; \int d^4w\,\frac{1}{w^2\,(p-w)^2\,(k-w)^2} \tag{11}$$

$$J^{(1)}_\mu \;=\; \int d^4w\,\frac{w_\mu}{w^2\,(p-w)^2\,(k-w)^2} \tag{12}$$

$$K^{(0)} \;=\; \int d^4w\,\frac{1}{(p-w)^2\,(k-w)^2} \tag{13}$$

The exact analytical expressions for these three integrals are known [10, 11]. They involve basic functions of momenta k and p and a spence function. We believe that it is highly non-trivial to construct a non-perturbative vertex which reduces to this complicated form in the weak coupling regime. However, asymptotic behaviour can be reproduced to some extent. Simple power counting reveals that the integrals $J^{(0)}$ and $J^{(1)}_\mu$ are perfectly well-behaved in the ultraviolet regime. However, $K^{(0)}$ is logarithmically divergent. We now show that our vertex *ansatz* also exhibits this behaviour. Using the fact that perturbatively $\mathcal{M}(p^2) = 0$, we can re-write Eq. (4) as follows:

$$F(p^2) \;=\; 1 \;-\frac{\alpha}{\pi^3}\frac{1}{p^2}\int d^4k\,\frac{F(k^2)F(p^2)}{k^2}\;\frac{k\cdot p}{q^2} \tag{14}$$

where we have employed Eq. (6), and the Feynman rule for the vertex. Carrying out angular and radial integrations respectively and retaining the leading log terms, we get

$$F(p^2) \;=\; 1+\frac{\alpha}{2\pi}\ln\frac{p^2}{\Lambda^2} \tag{15}$$

Therefore, the proposed vertex ansatz can be written perturbatively as follows

$$\Gamma_A(k,p) = \frac{1}{F(k^2)F(p^2)} = 1+\frac{\alpha}{\pi}\ln\frac{k^2p^2}{\Lambda^2}+\mathcal{O}(\alpha^2) \tag{16}$$

which is logarithmically divergent in the ultraviolet regime just as the real vertex to $\mathcal{O}(\alpha)$. Therefore, perturbatively our vertex *ansatz* is more realistic than the bare vertex.

An added advantage of using the proposed vertex *ansatz* is that Eq.(5) can be solved independently of Eq.(4). Therefore, it serves a purpose similar to that of Mandlestam's choice [12] of the 3-gluon vertex in studying the Schwinger-Dyson equation of the gluon propagator.

As the unknown functions F and $\mathcal{M}$ do not depend upon the angle between k and p, we can perform angular integration to arrive at

$$F(p^2) \;=\; 1\;-\frac{\alpha}{2\pi}\int dk^2\,\frac{1}{k^2+\mathcal{M}^2(k^2)}\;\left[\frac{k^4}{p^4}\theta(p^2-k^2)+\theta(k^2-p^2)\right] \tag{17}$$

$$\mathcal{M}(p^2) \;=\; \frac{\alpha}{\pi}\int dk^2\,\frac{\mathcal{M}(k^2)}{k^2+\mathcal{M}^2(k^2)}\;\left[\frac{k^2}{p^2}\theta(p^2-k^2)+\theta(k^2-p^2)\right]\quad . \tag{18}$$

Such equations are known to have a non-trivial solution for the mass function above a critical value of the coupling $\alpha = \alpha_c$. In the neighbourhood of the critical coupling, when the generated mass is still small, we can put $\mathcal{M}^2 = 0$. Then Eqs. (17,18) decouple from each other completely. The leading log solution for $F(p^2)$ is then:

$$F(p^2) \;=\; 1+\frac{\alpha}{2\pi}\ln\frac{p^2}{\Lambda^2}\qquad . \tag{19}$$

As for the mass function, multiplicative renormalizability demands a solution of the type $M(p^2) \simeq (p^2)^{-s}$. Substituting this in Eq. (18) and performing radial integration, we find

$$s = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - \frac{\alpha}{\alpha_c}} \tag{20}$$

where $\alpha_c = \pi/4$. For $\alpha > \alpha_c$ the solution of the mass function enters the complex plane indicating that a phase transition has taken place from perturbative to non-perturbative solution corresponding to the dynamical generation of mass. Numerically, above α_c, we solve Eq. (18) in a two-step process. We first use the iterative method to get close to the solution and then refine the answer by converting the integral equation into a set of simultaneous nonlinear equations to be solved by Newton-Raphson method. In Fig. 3., we have drawn the Euclidean mass M(which can be taken to be $\mathcal{M}(0)$) as a function of the coupling α. We see that it obeys Miransky scaling law and can be fitted to the form

$$\frac{M}{\Lambda} = \exp\left[-\frac{A}{\sqrt{\frac{\alpha}{\alpha_c} - 1}} + B \right] \tag{21}$$

very well by the choice $A = 0.97\pi$ and $B = 1.45$. These numbers are close to the ones found in [13] although the value of the critical coupling is of course different. The slight mismatch in the values of A and B is due to the fact that in the logarithmic grid of momenta, we choose 30 points per decade and do not extrapolate the result to an infinite number of points, an exercise carried out in [13]. We also compute $F(p^2)$ for various values of α and find that the closer we approach α_c, where the generated mass is still small, starting from a larger value of α, the numerical result gets closer and closer to the analytical result as expected, Fig. 4.

3 The Wess-Zumino Model

We now extend the particle spectrum by doubling the number of scalars to discuss the massless Wess-Zumino model, characterized by the following Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu A)^2 + \frac{1}{2}(\partial_\mu B)^2 + \frac{1}{2}(i\bar{\psi}\gamma^\mu\partial_\mu\psi) - \frac{1}{2}g^2(A^2 + B^2)^2 - g\bar{\psi}(B - iA\gamma_5)\psi \; . \tag{22}$$

The Schwinger-Dyson equation for the fermion propagator in this model is depicted in Fig. 5. Before we embark on solving this equation, we must re-address the validity of the ansatz for the 3-point vertex which we made for the Yukawa case. To the one loop level in perturbation theory, one now has contributions from both the scalars as depicted in

Fig. 6. Therefore, e.g., for scalar A, we can write

$$-2g\gamma_5\Gamma_A = -2g\gamma_5 + \int \frac{d^4w}{(2\pi)^4} (-2g\gamma_5)\, iS_F(p-w)\, (-2g\gamma_5)\, iS_F(k-w)\, (-2g\gamma_5)\, \frac{i}{w^2} + \int \frac{d^4w}{(2\pi)^4} (-2ig)\, iS_F(p-w)\, (-2g\gamma_5)\, iS_F(k-w)\, (-2ig)\, \frac{i}{w^2} = 0 \quad . \quad (23)$$

The same is the case for Γ_B, i.e., in the presence of both the scalars A and B, none of the 3-point vertices gets modified at $\mathcal{O}(\alpha)$ in perturbation theory. Therefore, in the Wess-Zumino case, it is more reasonable to use the bare vertex instead of the *ansatz* earlier made. Though one would now expect to solve coupled integral equations for $F(p^2)$ and $\mathcal{M}(p^2)$, a miraculous cancellation of terms takes place as evident from the following Schwinger-Dyson equation for the fermion propagator:

$$\frac{\not p - \mathcal{M}(p^2)}{iF(p^2)} = \frac{\not p}{i} - \frac{\alpha}{\pi^3} \int d^4k \left[\frac{F(k^2)}{\not k - \mathcal{M}(p^2)} \frac{1}{q^2} \right] + \frac{\alpha}{\pi^3} \int d^4k \left[\gamma_5 \frac{F(k^2)}{\not k - \mathcal{M}(p^2)} \gamma_5 \frac{1}{q^2} \right] . \quad (24)$$

Taking the trace of this equation, we get

$$\mathcal{M}(p^2) = 0 \quad .$$

As the cancellation of terms takes place at the very beginning, it is easy to see that dynamical mass generation will remain an impossibility for the full vertex and the full scalar propagator as long as they are identical for both the scalars. The vertex corrections for A and B have been proven to be equal up to $\mathcal{O}(\alpha^2)$ [14]. We shall shortly see that the same is true for the full scalar propagator at least up to $\mathcal{O}(\alpha)$. This is in accordance with the arguments based on non-renormalization theorem. SUSY plays a role in providing same number of bosonic and fermionic degrees of freedom. We have seen from the case of the Yukawa Model that without this equality, it will not be possible to prevent dynamical mass generation. Secondly, SUSY imposes relations on couplings of the two scalars with the fermions. This relationship is crucial in preventing dynamical mass generation.

As far as wavefunction renormalization $F(p^2)$ is concerned, its leading log behaviour gets modified slightly, by the inclusion of the other scalar, to:

$$F(p^2) = 1 + \frac{\alpha}{\pi} \ln \frac{p^2}{\Lambda^2} \quad . \quad (25)$$

Although it is an interesting conclusion in its own right that supersymmetry prevents dynamical mass generation for fermions in the Wess-Zumino model, another important issue to probe will be whether supersymmetry itself remains intact, i.e., whether the scalars can also be kept massless. This is what we discuss now. The Schwinger-Dyson equation for the scalar (for example A) has been depicted in Fig. 7. A scalar propagator, unlike a fermion, needs only one unknown function to describe it. But we shall prefer to split it into two parts and write the full scalar propagator as follows:

$$S_A(p) = \frac{F_A(p^2)}{p^2 - \mathcal{M}_A^2(p^2)} \quad . \tag{26}$$

The non-zero value of the mass function $\mathcal{M}_A(p^2)$ will be responsible for shifting the pole from $p^2 = 0$ to some finite value, generating the mass for the scalar dynamically. $F_A(p^2)$ on the other hand is the scalar wavefunction renormalization. The SD-equation for the scalar propagator in Euclidean space can now be written as:

$$\begin{aligned}\frac{p^2 + \mathcal{M}_A^2(p^2)}{F_A(p^2)} &= p^2 + \frac{3\alpha}{2\pi^3} \int d^4k \, \frac{F_A(k^2)}{k^2 + \mathcal{M}_A^2(k^2)} + \frac{\alpha}{2\pi^3} \int d^4k \, \frac{F_B(k^2)}{k^2 + \mathcal{M}_B^2(k^2)} \\ &\quad - \frac{2\alpha}{\pi^3} \int d^4k \, \frac{F(k^2)F(q^2)}{k^2 q^2} \, \Gamma_A(k,p) \; k \cdot q\end{aligned} \tag{27}$$

where we have used the fact that the fermions do not acquire mass. If we want to preserve supersymmetry and do not want the scalars to acquire mass, we must have:

$$\mathcal{M}_A(p^2) = \mathcal{M}_B(p^2) = 0 \quad . \tag{28}$$

We are then left with:

$$\frac{1}{F_A(p^2)} = 1 + \frac{\alpha}{2\pi^3 p^2} \int \frac{d^4k}{k^2} \left[3F_A(k^2) + F_B(k^2) - 4\frac{k \cdot q}{q^2} \, F(k^2)F(q^2)\Gamma_A(k,p) \right] \tag{29}$$

and there is a similar equation for the scalar B:

$$\frac{1}{F_B(p^2)} = 1 + \frac{\alpha}{2\pi^3 p^2} \int \frac{d^4k}{k^2} \left[3F_B(k^2) + F_A(k^2) - 4\frac{k \cdot q}{q^2} \, F(k^2)F(q^2)\Gamma_B(k,p) \right] . \tag{30}$$

These equations should yield a solution for $F_A(p^2)$ and $F_B(p^2)$ such that it does not change the position of the pole for the scalar propagator and that the quadratic divergences cancel. It is well-known that it does happen in perturbation theory to $\mathcal{O}(\alpha)$. In fact, one can evaluate $F_A(p^2)$ and $F_B(p^2)$. The leading log expression for these functions to $\mathcal{O}(\alpha)$ is

$$F_A(p^2) = F_B(p^2) = 1 + \frac{\alpha}{\pi} \ln \frac{p^2}{\Lambda^2} \tag{31}$$

which is exactly the same expression as that for $F(p^2)$ for the fermion propagator. This result indicates that supersymmetry need not be broken.

4 Conclusions

We have studied the Schwinger-Dyson equations for the Yukawa (a scalar interacting with a fermion with a γ_5 type interaction) and the Wess-Zumino models. In the simple Yukawa model, we propose a vertex *ansatz* which we argue should perform better than the bare vertex. In the quenched approximation, we find dynamical mass generation for fermions above a critical value of the coupling $\alpha_c = \pi/4$. The generated Euclidean mass obeys Miransky scaling. When we extend this Yukawa model to equate the scalar and fermionic degrees of freedom (Wess-Zumino model), we find that a neat cancellation of terms occurs and there is no mass generation for the fermions. This fact remains true beyond the rainbow approximation and is supported by perturbative calculations available for the 3-point vertex to $\mathcal{O}(\alpha^2)$ and of the scalar propagator. This result was expected on the basis of non-renormalization theorem. The two approaches will remain in agreement provided the full vertex and the full scalar propagator as long as they are identical for both the scalars to higher orders in perturbation theory as well.

If supersymmetry has to be preserved, the scalars should also acquire no mass dynamically. Studying the Schwinger-Dyson equations for the scalars, we observe that such a solution is allowed and in fact leads to the wavefunction renormalization function for the scalars which is exactly the same as that for the fermion.

It is more interesting to see the role of supersymmetry in more complicated theories such as SQED. The studies so far carried out in superfield and component formalism seem to arrive at different conclusions. We plan to present our work in this context in a future publication.

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Figure Captions

Fig. 1. Schwinger-Dyson equation for the fermion propagator in the Yukawa model. The solid lines represent fermions and the dashed the scalar. The solid dots indicate full, as opposed to bare, quantities.

Fig. 2. One loop perturbative expansion for the vertex in the Yukawa model.

Fig. 3. The dynamically generated mass (M/Λ) versus the 3-point coupling α in the Yukawa model. The critical coupling is $\alpha_c = \pi/4$, above which the mass can be seen to be bifurcating away from the chirally symmetric solution. $\diamond$s represent the numerical result and $+$s the numerical fit to the form $M = \Lambda \exp\left[-A/\sqrt{\alpha/\alpha_c - 1} + B\right]$ with $A = 0.97\pi$ and $B = 1.45$.

Fig. 4. The wavefunction renormalization function $F(p^2)$ in the Yukawa model for various values of the coupling α. The solid line corresponds to the analytical expression $1 - \alpha/4\pi + (\alpha/2\pi)\ln(p^2/\Lambda^2)$ in case of no mass generation for $\alpha = 0.78$.

Fig. 5. Schwinger-Dyson equation for the fermion propagator in the Wess-Zumino model.

Fig. 6. One loop perturbative expansion for the vertex in the Wess-Zumino model. We have shown the case for scalar A. A similar diagram exists for scalar B.

Fig. 7. Schwinger-Dyson equation for the scalar propagator in the Wess-Zumino model.

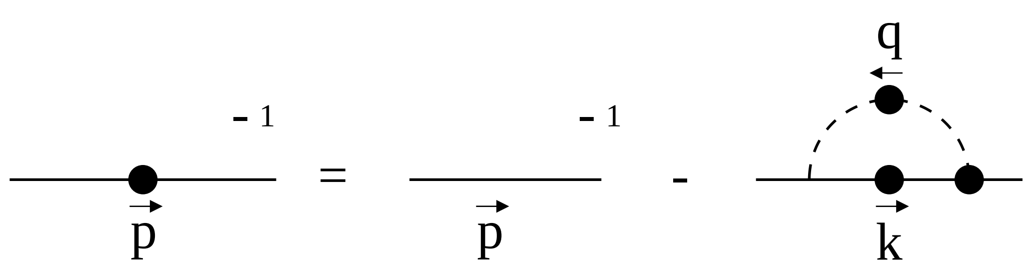


Fig. 1.

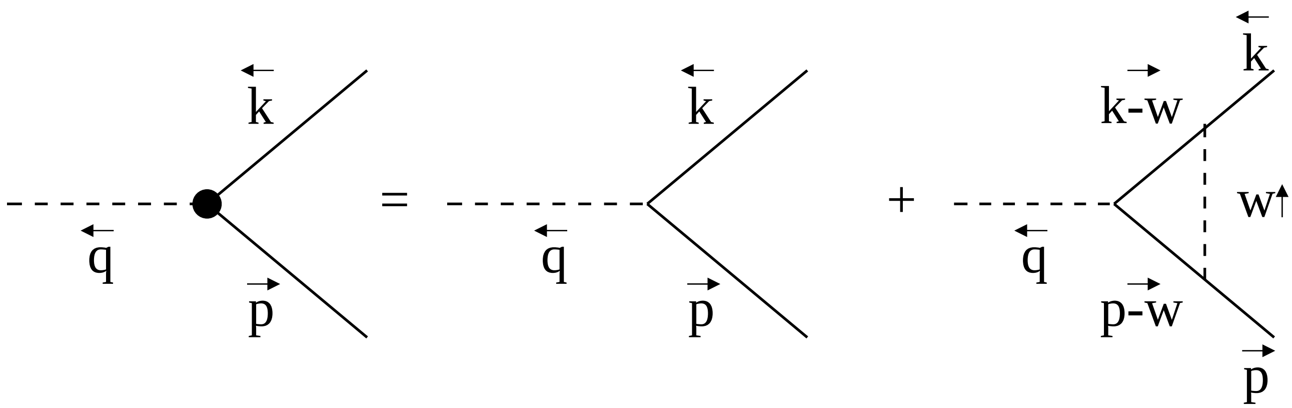
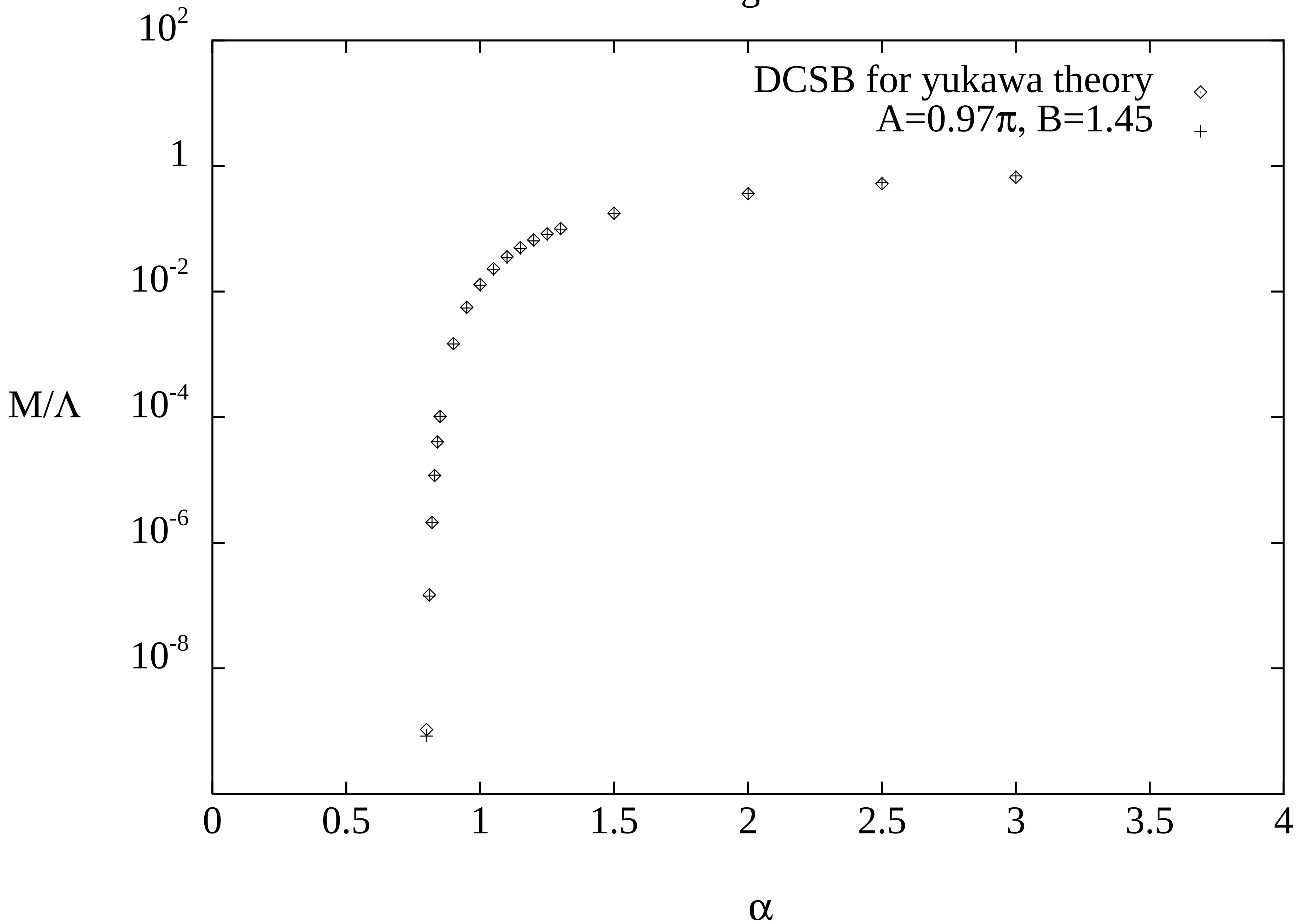


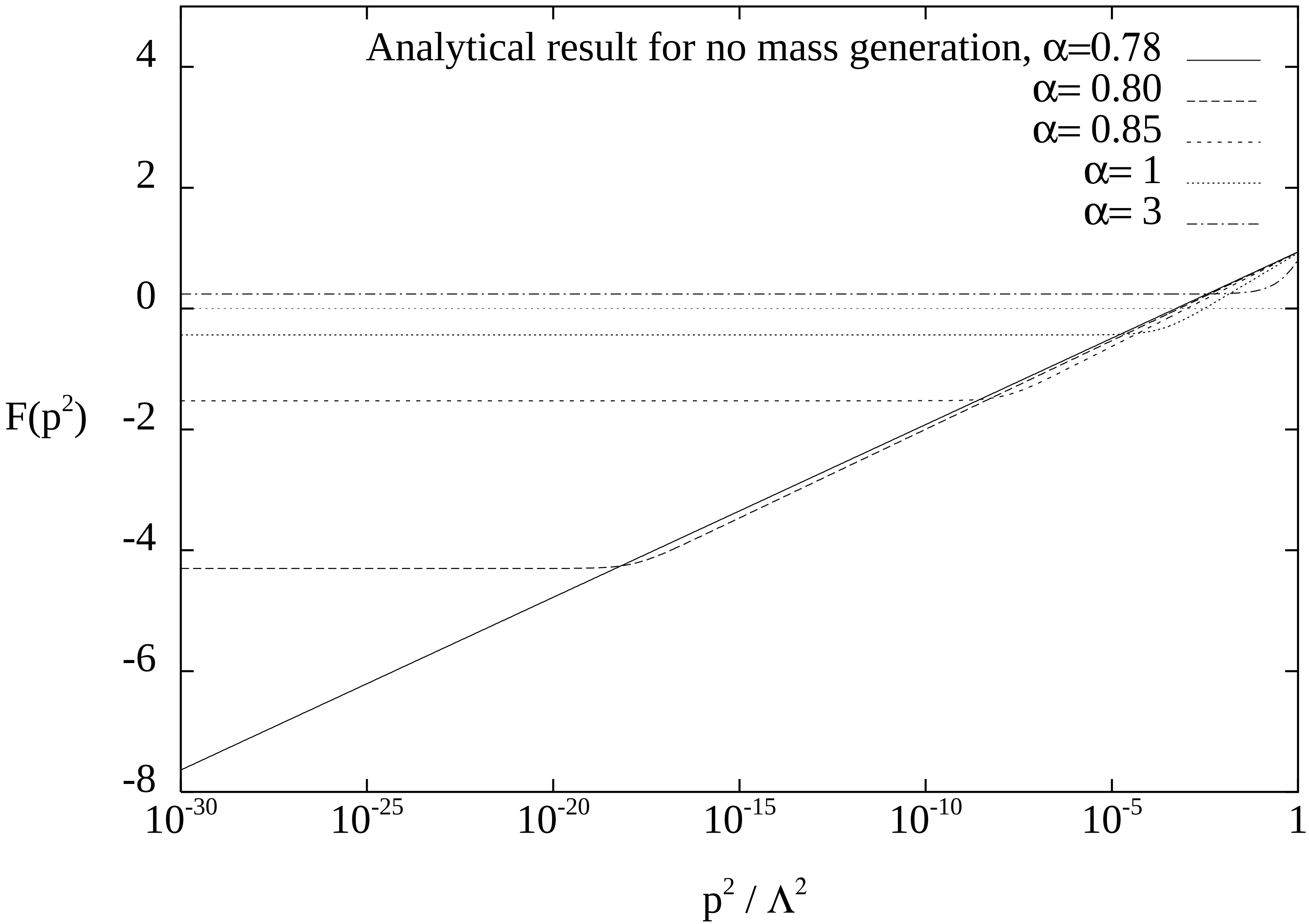
Fig. 2.

Fig. 3.



DCSB for yukawa theory
A=0.97π, B=1.45
M/Λ
10^2
1
10^-2
10^-4
10^-6
10^-8
0
0.5
1
1.5
2
2.5
3
3.5
4
α

Fig. 4.



Analytical result for no mass generation, α=0.78
α= 0.80
α= 0.85
α= 1
α= 3
4
2
0
-2
-4
-6
-8
F(p²)
10⁻³⁰
10⁻²⁵
10⁻²⁰
10⁻¹⁵
10⁻¹⁰
10⁻⁵
1
p² / Λ²

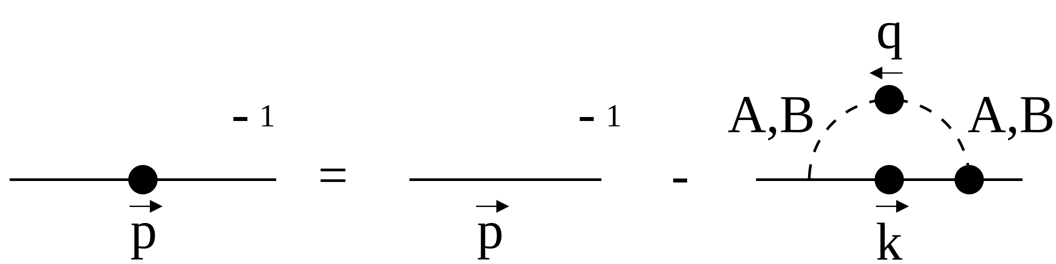


Fig. 5.

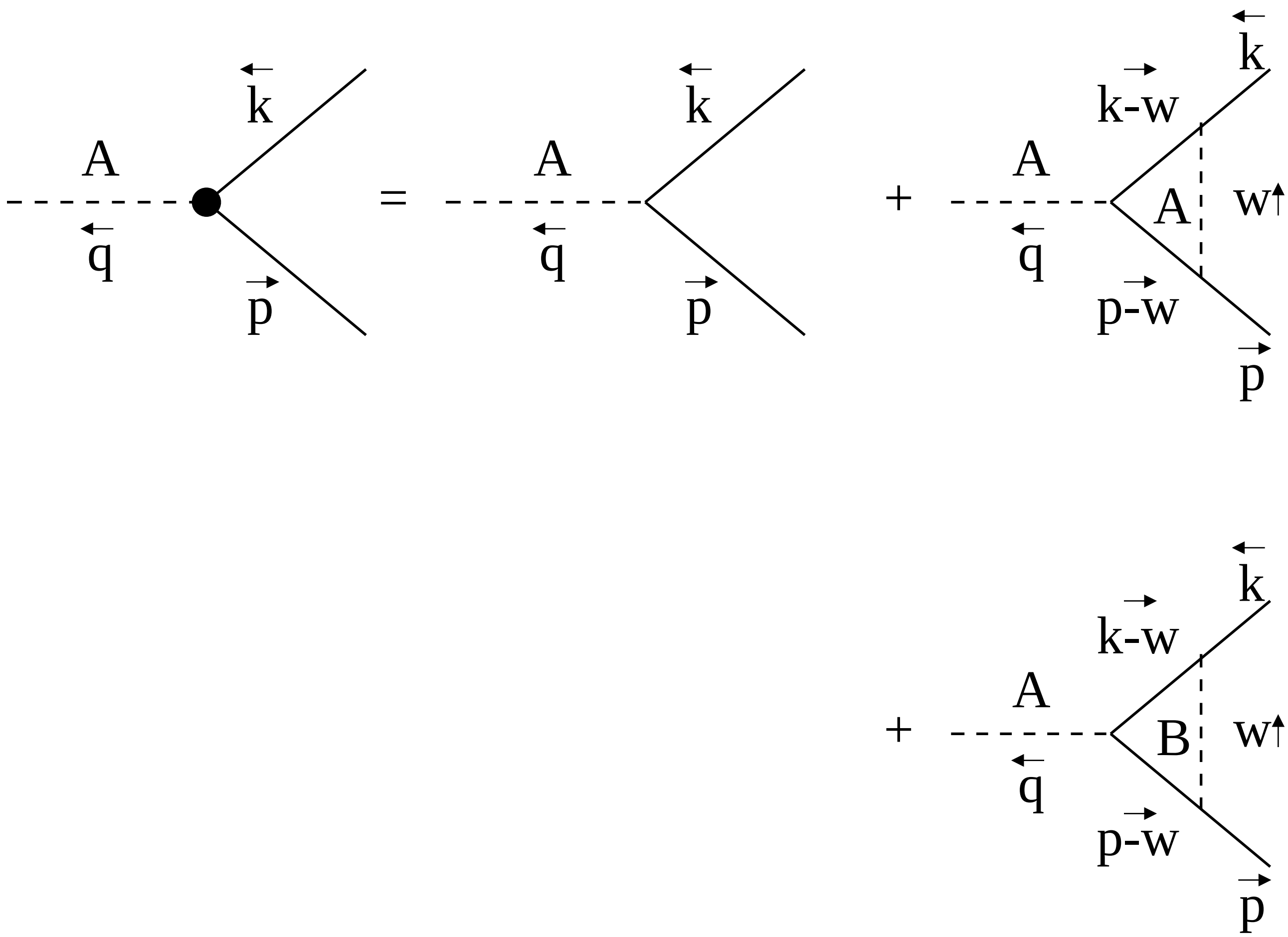


Fig. 6.

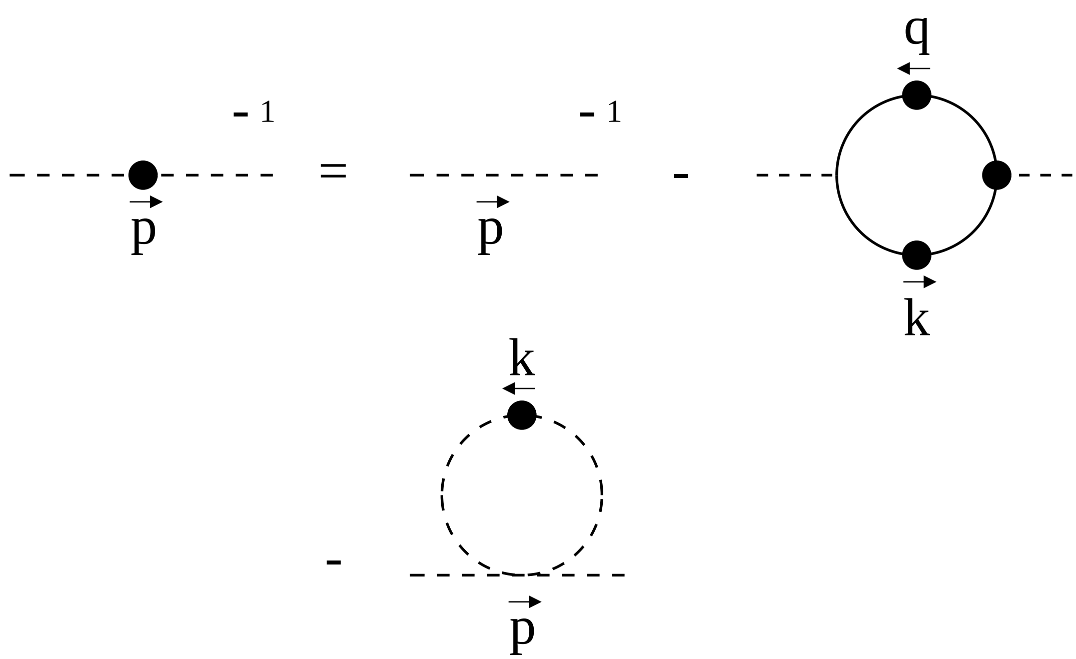


Fig. 7.