Scheduling Combination and Headway Optimization of Bus Rapid Transit

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Abstract: The flexibility of bus rapid transit (BRT) in scheduling is one of the greatest differences with traditional buses. In order to improve BRT operation quality, the paper studied the headway optimization and scheduling combination of BRT vehicles. A model has been established to minimize passengers’ travel costs and vehicles’ operation cost, and constraints included passenger volume, time, and frequency. The scheduling combination was composed by normal, zone, and express scheduling. The model was solved by genetic algorithm of variable-length coding. The result of the numerical case shows that: the optimization results can save 69.92% cost. The sensitivity analysis shows that, under higher traffic volume or lower speed, the travel cost can be reduced through reasonable scheduling combination. The method has been proved scientifically and is feasible.

Key Words: transit operation; bus rapid transit; scheduling; genetic algorithm

1 Introduction

Traditional bus scheduling is a kind of fixed scheduling, which is checked on the terminals. Because of the congestion in urban city, public transport vehicles often arrive at station unevenly, which leads to instable quality and low attraction. BRT is constructed by exclusive lanes and intelligent transportation systems. Moreover, BRT can provide scheduling combination to meet passengers’ travel demand well and reduce vehicles operation costs. Most researches focused on traditional bus scheduling, such as determining the frequency by genetic algorithms or mixed algorithm [1–6]. Teng and Yang studied bus frequency under the APTS [7], which did not consider the characteristics of BRT scheduling. Zou studied regional scheduling with mixed scheduling but did not offer an algorithm [8]. Bai et al. optimized bus frequency by taboo algorithms [9], with no studies on combinations. Therefore, it is necessary to study the scheduling combination and headway optimization of BRT.

2 Scheduling forms of BRT

Scheduling combination is the special characteristics of BRT. According to vehicle operation form and stops number, the scheduling is divided into normal scheduling, zone scheduling, and express scheduling, and so on.

Normal scheduling: vehicles run along the routes and stop at every station from the initial stop to the end. The vehicle must run at fixed stations and complete the whole routes, as shown in Fig. 1(a). Zone scheduling is defined as vehicles only run on high-traffic volume section or zone (Fig. 1(b)). Express scheduling, that is, vehicles only stop at certain station with large passenger volume (Fig. 1(c)).

In traditional bus scheduling, the normal scheduling is the most popular form. Whereas in BRT operation, there are scheduling combinations, which are more in accordance with the trips needed on the corridor than traditional bus.

Fig. 1 (a) Express schedule; (b) normal schedule; (c) zone schedule
3 Optimization model of BRT scheduling

3.1 Problem description and assumption
This study focuses on BRT headway optimization and scheduling combination (normal, zone, and express) in a certain period, which examines the appropriate departure frequency and scheduling combination to minimize the objective function. First, the following assumptions are given:

(a) BRT vehicles run at constant speed, namely, the running time between stations is certain. (b) In the study period, the departure frequency is uniform. (c) The passenger arrival rate is uniform and unchanged in the given period. (d) The time to open and close vehicle doors are fixed. (e) There are enough vehicles in every fleet.

3.2 Symbol definition
In the study, the key variables are defined as follows:

\( i \) — BRT vehicle, \( i = 1, 2, \cdots, I \); 
\( j \) — stops on BRT routes, \( j = 1, 2, \cdots, N \); 
\( l \) — scheduling form, \( l = 1 \) means normal scheduling, \( l = 2 \) is zone scheduling, \( l = 3 \) represents express scheduling; 
\( t_i \) — the number of normal scheduling; 
\( t_l \) — the number of zone scheduling; 
\( t_s \) — the number of express scheduling; 
\( t_{l+s} \) — the total number of scheduling; 
\( d_{ij} \) — departure time of vehicle \( i \) at stop \( j \); 
\( a_{ij} \) — the arrival time of vehicle \( i \) at stop \( j \); 
\( h_{ij} \) — headway between vehicle \( i-1 \) and \( i \) at stop \( j \); 
\( t_j \) — vehicle running time between stop \( j-1 \) and \( j \); 
\( h \) — fixed headway; 
\( T_w \) — dwelling time at stop; 
\( c \) — acceleration and deceleration time; 
\( r_{jk} \) — passenger arrival rate from stop \( k \) to \( j \), \( 1 \leq j < k \leq N \); 
\( r_j \) — arrival rate at stop \( j \); 

\[ r_j = \sum_{k=j+1}^{N} r_{jk} \]

\( A_{ij} \) — the number of alighting passengers at stop \( j \) from vehicle \( i \); 
\( B_{ij} \) — the number of boarding passengers at stop \( j \) from vehicle \( i \); 
\( L_{ij} \) — passenger number on bus when vehicle \( i \) leaves stop \( j \); 
\( W_{ij} \) — the number of passenger from stop \( j \) to \( k \) on vehicle \( i \); 
\( s_{ij} \) — passenger number for stop \( k \) when vehicle \( i \) leaves; 
\( S_{ij} \) — the number of passengers on vehicle \( i \) leaves stop \( j \); 
\( \delta_{ij} \) — "0-1" variable, for the scheduling form \( l \), when vehicle \( i \) stops at \( j \), the value is 1, otherwise is 0; 
\( \delta_{ij} \) — "0-1" variable, for scheduling form \( l \), vehicle \( i \) stops at both \( j \) and \( k \), the value is 1, otherwise is 0; 
\( \delta_i \) — scheduling form of vehicle \( i \); 
\( C_v \) — value of passenger waiting cost (yuan/min); 
\( C_s \) — value of passenger on board cost (yuan/min); 
\( C_{op} \) — operation cost of vehicles (yuan/min); 
\( T \) — studied period.

3.3 Model formulation

3.3.1 Objective function

\[ \min Z = f_1 + f_2 + f_3 \]

\[ f_1 = C_i \sum_{j=1}^{N} \sum_{k=1}^{N} \left( \frac{r_{jk} \cdot h_{jk}}{2} + S_{ij} \cdot h_{ij} \right) \]

\[ f_2 = C_i \sum_{j=1}^{N} \sum_{k=1}^{N} \left[ T_{ij} (t_j + (\delta_{ij} + \delta_{ij}) \cdot c) + (L_{ij} - A_{ij}) \cdot \delta_{ij} \cdot T_w \right] \]

\[ f_3 = C_i \sum_{j=1}^{N} \sum_{k=1}^{N} \left[ T_j + (\delta_{ij} + \delta_{ij}) \cdot c + \delta_{ij} \cdot T_w \right] \]

The objective function consists of the passenger waiting cost, the passenger on board cost, and the vehicle operation cost. The passenger waiting cost includes the average passenger waiting cost and the skipping-station waiting cost. The former is calculated by the arrival rate at stop \( j \), and \( r_j \) multiplies the headway \( h_{ij} \) and half of the headway. The latter is calculated by the skipped passenger of the former vehicle \( S_{i-1,j} \) multiplied by the waiting time \( h_{ij} \).

The passenger on board cost is divided into the passenger on board time and the dwelling cost. The former is calculated by vehicle number multiplied by the running time. The running time includes the interval running time and the two consecutive dwelling times \((\delta_{ij} + \delta_{ij}) \cdot c\). The dwelling time is obtained by the on board number minus the alighting number multiplied by the dwelling time \( \delta_{ij} \cdot T_w \).

The vehicle operation cost consists of the running time \( t_j \), the closing and opening time of two consecutive station \((\delta_{ij} + \delta_{ij}) \cdot c\), and the dwelling time \( \delta_{ij} \cdot T_w \) if present.

Decision variables are \( h \) and \( \delta_i \) (\( i = 1, 2, 3 \)), which determine headway and scheduling form. Although \( \delta_i \) is not shown in the objective function, it is directly related to \( \delta_{ij} \).

3.3.2 Constraints

(1) Passenger number constraints

The passenger number on board of vehicle \( i \) at stop \( j \) equals to vehicle \( i \) and leaves \( j-1 \) plus boarding passengers at stop \( j \) minus alighting passenger at stop \( j \); \( L_{ij} = L_{ij-1} + B_{ij} - A_{ij} \).

The alighting passenger number of vehicle \( i \) at stop \( j \) equals to passenger number of the vehicle \( i \) at all the former \( j-1 \) station to the stop \( k \) multiplied by the vehicle stop or not:

\[ A_{ij} = \delta_{ij} \sum_{k=j+1}^{N} W_{ik} \cdot \delta_{ik} \]

The boarding passenger number of vehicle \( i \) at stop \( j \) equals to passenger number of vehicle \( i \) boarding at stop \( j \) while alighting at all the downstream stations multiplied by vehicle stop or not:

\[ B_{ij} = \delta_{ij} \sum_{k=j+1}^{N} W_{ik} \cdot \delta_{ik} \]

The transferring passenger number from stop \( j \) to stop \( k \) equals to the passenger number of former vehicle \( i-1 \) and leaves stop \( j \) to \( k \) plus the arrival passenger number in the waiting time. The arrival passenger number is calculated by the arrival rate \( r_{jk} \) multiplied by the headway \( h_{ij} \):
The left passenger number of vehicle $i$ from stop $j$ to $k$ equals to the passenger number left by the former vehicle plus the arrival passenger in the waiting time. The second part determines when the vehicle stops both at stops $j$ and $k$, and the left number is added; otherwise, there is no left passenger number:

$$s_{i,j,k} = s_{i,j,k}(1-\delta_{i,j}^\gamma) + r_{j,k} \cdot h_{i,j} \cdot (1-\delta_{i,k}^\gamma)$$

The left total passenger number of vehicle $i$ at stop $j$ equals to the sum of all the passengers boarding at stop $j$ alighting at the rest stops:

$$S_{i,j} = \sum_{k=1}^j s_{i,j,k}$$

(2) Time constraint

The arrival time of vehicle $i$ at stop $j$ is equivalent to the departure time of vehicle $i$ at stop $j-1$ plus running operation time, even if the dwelling time is stop at $j$ and $j-1$ or not:

$$a_{i,j} = a_{i,j-1} + t_j + (\delta_{i,j-1}^\gamma + \delta_{i,j}^\gamma) \cdot c$$

The departure time of vehicle $i$ at stop $j$ is equivalent to the arrival time plus the dwelling time. If it stops at the station, the dwelling time is considered; otherwise, the dwelling time is 0:

$$d_{i,j} = d_{i,j-1} + \delta_{i,j}^\gamma \cdot T_0$$

The headway of vehicle $i$ at stop $j$ is equivalent to the headway difference between vehicle $i$ and $i-1$ at stop $j$:

$$h_{i,j} = d_{i,j} - d_{i-1,j}$$

Headways of all vehicles at the terminals are equivalent to the uniform headway: $h_{i,1} = h_{i,1+1} = h$.

The headway of vehicle $i$ at stop $j$ equals to the uniform headway plus the dwelling time on the former two stations:

$$h_{i,j} = h + \sum_{l=1}^j (\delta_{i,l}^\gamma (T_0 + c)) - \sum_{l=1}^j (\delta_{i,l}^\gamma (T_0 + c))$$

With uniform headway the studied period is divided into $l$, headways: $l \cdot h = T$.

(3) Frequency constraints

When the service level and bus lanes capacity is considered, the bus headway needs restriction: $h_{\text{min}} \leq h \leq h_{\text{max}}$.

3.3.3 Boundary condition

The boundary condition makes a significant impact on the result. There are several conditions needed to be clarified: the last vehicle at last stop is defined to the arrival time, that is $d_{l,2N-1} = d_{l,2N}$. The first vehicle at the first stop equals to the departure time. $a_{1,1} = \max(t_1)$. The headway of first vehicle is defined as $h_{1,1} = 0$. The first running time $t_1$ is defined as $t_1 = 0$.

4 Genetic algorithm based solutions

Genetic algorithm is a random search algorithm, which traces its roots to biological evolution. The algorithm is first proposed by professor Holland, the University of Michigan, in 1975. Genetic algorithm is a highly efficient, parallel, and global search method, which has been widely used in combinatorial optimization, machine learning, signal processing, adaptive control, artificial life, and so on. The major steps of genetic algorithm include: parameter choice and initialization, fitness value and genetic operator, and so on.

4.1 Parameters choice and Initialization

(1) Coding

Coding is the key point in the model solution because the model contains a variety of variables, including the headway and scheduling form combination. Moreover, the decision-making variables are interrelated and the length is variable. How to determine coding and the code length is the special problem in the solution. Models include the maximum and minimum headway, which determines the length of coding interval. This solution is encoded by the variable-length coding genetic algorithm. To encode the scheduling form, the length of the scheduling form is bus frequency. For three different scheduling forms, the corresponding 01 is represented as normal scheduling, 10 as zone scheduling, and 11 as express scheduling. According to the coding length interval, three different models are produced. For example, when the headway interval is [3, 15] and we take 5, the possible coding is [01 10 11 01 10], which gives one of the scheduling combination.

(2) Initial population

The initial population is randomly generated. The first term is to determine the population size ($N$ chromosomes). The second is to randomly select $N$ points as the initial solution in optimizing space. For example, [1, 3] is randomly generated a initial population, such as a 20×3 population, in which population size is 20 and the frequency number is 3.

(3) Parameters choice

The crossing rate $P_c$ and the mutation rate $P_m$ should be identified.

4.2 Fitness value calculation

In the proposed model, the objective function is to minimize the value. The simple fitness function is utilized here, namely, $F(x) = M - f(x)$. $f(x)$ is the fitness function, $f(x)$ is the objective function, and $M$ is a large enough constant. Due to large numbers of parameters are involved, it is difficult to calculate the fitness function. With coding, known arrival rate, and the number of rest passengers on initial vehicles, we get the related parameters, fitness, and objective functions value.

4.3 Genetic operators

(1) Selection

Selection is to determine which individuals enter the next generation, for which Roulette gambling law is chosen.

(2) Crossing

The crossing rate is $P_c$, that is, two individuals cross at a certain location, which is similar to the gene splitting and reorganization.
(3) Mutation
Mutation is that individual in farther population overturned at each location with certain probability $P_m$, namely, from “1” to “0”, or “0” to “1”. Mutation can provide possible solutions by random search in the space and find global optimal solution to a certain extent.

5 Numerical examples

5.1 Parameters choice
According to the average income of residents and the working hours, the passengers waiting time value is identified ($C_w$) to be 0.4 yuan/min, and the on board time value is 0.2 yuan/min. With the survey in the bus companies, the operation cost value is defined to be 0.4 yuan/min. With the BRT system in Beijing, Hangzhou, and other cities, the dwelling time is 1 min, the acceleration and deceleration time is 40 s, and the running speed is 26 km/h. The number of stations is 12 and the studied period is 1 hour. According to Refs. [1], [2], and [7] and other relevant parameters of genetic algorithm, the crossing rate ($P_c$) is 0.8, the mutation rate ($P_m$) is 0.005, the population size is 20, and times 100. Passengers’ volume, different forms of bus stops, and the route parameters are shown in Fig. 2 and Tables 1 and 2.

5.2 Result analysis
In Table 3, 1 represents normal scheduling, 2 represents zone scheduling, and 3 represents express scheduling. From the result, with the headway decreased, the objective functions un-optimized decreased gradually because of the declined waiting time. After optimized combination, the headway, the express, and zone scheduling all increased. The combination can also reduce the total system cost. From Table 3, it can be observed that, after optimization, cost is saved by 69.92% at most and 29.52% at least. The optimization is very significant. The model and the solution of BRT headway and scheduling combination are feasible to save system cost greatly.

<table>
<thead>
<tr>
<th>Stop</th>
<th>Normal schedule</th>
<th>Zone schedule</th>
<th>Express schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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</tr>
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</tr>
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</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: “1” means stop, “0” means not stop. Stop or not need consider the factors including passenger OD, passenger volume, and the land use comprehensively.

<table>
<thead>
<tr>
<th>Stop</th>
<th>Distance (m)</th>
<th>Running time (min)</th>
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<tbody>
<tr>
<td>1–2</td>
<td>800</td>
<td>1.85</td>
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<tr>
<td>2–3</td>
<td>1000</td>
<td>2.31</td>
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<tr>
<td>3–4</td>
<td>750</td>
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<td>6–7</td>
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<td>1.38</td>
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<tr>
<td>11–12</td>
<td>800</td>
<td>1.85</td>
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<table>
<thead>
<tr>
<th>Headway (min)</th>
<th>Frequency</th>
<th>Combination</th>
<th>Objectives value</th>
<th>Objectives value (not optimized)</th>
<th>Saving cost (%)</th>
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<td>7662</td>
<td>29.52</td>
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</tbody>
</table>
5.3 Sensitivity analysis

(1) Impact of traffic volume on objective function
To verify the impact of traffic volume on the objective function, the trip volume in Fig. 2 is taken as the standard. Thus, the changes of objective function are examined in the case that trip volume is respectively 0.5, 1.5, and 2 times than the appointed standard (Fig. 3).

Figure 3 illustrates that with the increase of the traffic volume, travel costs are the largest when the headway is 20 min. Under the given traffic volume, small headways reduce the objective function value, which is mainly caused by different scheduling combinations. There exist much varied solutions, as the frequency decreases and headway increases. Fig. 3 provides the optimum value of the objective function under different cases. Despite that, the headway increases, and the system cost does not increase as a whole.

(2) Impact of travel speed on objectives function
From Fig. 4, it can be observed that the impact of speed on objective functions is twisted. When the speed reaches 30 km/h, the objective function reduces obviously. When the speed is 25–28 km/h, the objective functions with different headways perform differently, and some values increase as speed increases, which indicates that the scheduling combination make an impact on the cost evidently. Under the same speed, the reasonable combination of the scheduling can optimize the system cost.

6 Conclusions

This study focuses on BRT headway and scheduling combination optimization. The model is developed by minimizing passenger travel time and the vehicle operation time of BRT. Variable-length coding genetic algorithm was used to solve the model. The obtained optimization results indicate that the proposed approach is feasible. Sensitivity analysis shows that when the traffic volume increases and the travel speed decreases, a reasonable allocation of vehicles by scheduling form combination would reduce travel costs.

Acknowledgements

This research was funded by the New Century Excellent Researcher Award Program from Ministry of Education of China (NCET040946) and the Doctoral funding from Ministry of Education of China (20050710006).

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