Abstract— In this paper, the performance of three fair resource allocation strategies for packet data transmission in EDGE/GPRS networks are mathematically analyzed. The strategies aim at achieving, respectively, fair access, fair transmission rate and, simultaneous fair access and transmission rate. The differences in terms of QoS and GoS caused by link adaptation (LA) and multi-slot allocation experienced by data users in EGPRS networks is smoothed by these strategies through the use of multiple resource reservation and/or dynamic resource allocation strategies, which, at the same time, compensate users with poor radio channel conditions. Also, the performance of the proposed strategies is evaluated by means of teletraffic analysis tools in terms of packet blocking probabilities and throughput.

Keywords: EDGE, GPRS, Fairness, Access, Throughput, Transmission Rate.

I. INTRODUCTION.

EDGE/GPRS (Enhanced Data Rates for GSM Evolution for GPRS) is an improvement to the radio interface for GSM based cellular systems [1]. Techniques such as link adaptation and multi-resource allocation give support to the broadband services, but also produce marked differences among users. With link adaptation, EDGE/GPRS mobile stations can choose one of nine coding/modulation schemes, depending on the radio channel condition (i.e. propagation environment, fading, etc.), to obtain the largest possible throughput [1], [2], [13]. Each coding/modulation scheme has a different transmission rate, bringing advantages to users with good link quality over users with bad signal quality such as small resource holding time and larger throughput. Multi-resource allocation, on the other hand, increases the blocking probabilities of users with large bandwidth requirements.

In this paper, access and transmission rate fairness are used to smooth the differences in terms of packet blocking probabilities, transmission rate and throughput among EGPRS packets. Fairness has been studied before at different levels, however, most of the works reported in the literature have traditionally carried the study out at packet level. For example, in [14], [16], [17], [29], [30] and [32] packet scheduling in CDMA-based systems to achieve transmission rate fairness at data flow level has been studied. [18] uses the same technique to achieve fairness in GPRS networks. Further, studies searching for fairness at packet level have dealt with fair queuing and TCP transmission window are presented, respectively, in [19] and [22]. A performance evaluation of an EDGE network using a TCP transmission window technique by means of computer simulation is presented in [20]. Medium access fairness has been studied in [15], [24] and [27] in IEEE 802.11 and in EGPRS [21] networks. Several dynamic resource allocation strategies are studied in [12], [23], [25], [26] to achieve access fairness. In [12] link adaptation and resource re-allocations are used to improve throughput fairness. Finally, in [31] an analytical framework to evaluate capacity and fairness at the physical and network level in generic wireless data networks is presented. Contrary to most of the cited works, in this paper, a system level approach is considered to analyze EGPRS system performance. To the author’s knowledge, no similar evaluation of the performance of EGPRS networks at the system level has been reported in the literature. The analysis presented can be used to select the operation configuration of the EGPRS cellular network that fulfills the users’ and/or network operator’s requirements.

II. EGPRS SYSTEM MODEL

In EGPRS cellular systems radio channel quality depends, among others factors, on propagation conditions, mobile location, interference levels, etc. Hence, the optimum coding/modulation transmission scheme varies with the radio channel quality. In EGPRS, nine coding/modulation schemes have been defined at the radio link interface to determine the scheme that yields the greater throughput [1], [2], [13]. In addition, link adaptation and incremental redundancy are used. Table I shows the nine coding/modulation schemes defined in EDGE as well as the radio interface rate associated with each scheme. In this work it is assumed that the data packet size is exponentially distributed with mean 1500 Bytes so the mean holding time, when a single time slot is used, is the ratio of the mean data packet size in kbits ($D_{pk}$) and the transmission rate for the coding/modulation scheme in kbps ($R_{CMS,i}$). That is:

$$\frac{1}{\mu_{CMS,i}} = \frac{D_{pk}}{R_{CMS,i}}$$

(1)

<table>
<thead>
<tr>
<th>Coding/Modulation Scheme</th>
<th>Modulation</th>
<th>Transmission rate (kbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCS-9</td>
<td>8-PSK</td>
<td>59.2</td>
</tr>
<tr>
<td>MCS-8</td>
<td>8-PSK</td>
<td>54.4</td>
</tr>
<tr>
<td>MCS-7</td>
<td>8-PSK</td>
<td>44.8</td>
</tr>
<tr>
<td>MCS-6</td>
<td>8-PSK</td>
<td>29.6</td>
</tr>
<tr>
<td>MCS-5</td>
<td>8-PSK</td>
<td>22.4</td>
</tr>
<tr>
<td>MCS-4</td>
<td>GMSK</td>
<td>17.6</td>
</tr>
<tr>
<td>MCS-3</td>
<td>GMSK</td>
<td>14.8</td>
</tr>
<tr>
<td>MCS-2</td>
<td>GMSK</td>
<td>11.2</td>
</tr>
<tr>
<td>MCS-1</td>
<td>GMSK</td>
<td>8.8</td>
</tr>
</tbody>
</table>

1 To support this, in [34] it is stated that few studies exist in the literature that addresses the performance of a wireless network with LA by means of a teletraffic analysis (i.e., system level analysis).
A. Teletraffic Model

A homogenous case is assumed where all cells are statistically identical. Hence the overall system performance can be analyzed by focusing on only one given cell and considering the statistical behavior of this focused cell under the conditions that its neighboring cells exhibit their typical random behavior independently. The general guidelines of the model presented in [33]-[35] are adopted to analyze the strategies here evaluated. They have been widely used and accepted in the literature, and allow the strategies presented to be cast in the framework of multidimensional birth and death processes [5]-[6]. It is also considered that no user multiplexing is accepted in the literature, and allow the strategies presented to be evaluated. They have been widely used and

considered to be Poisson with mean arrival rate \( \lambda \). Due to the uniform distribution of users, the data packets arrival process can be treated as subdivisions of the total data packet arrival process \([28]\). Finally, it is assumed that no user multiplexing is used.

III. FAIR ACCESS STRATEGY

In order to achieve access fairness, every data packet must perceive the same access blocking probability regardless of its coding/modulation scheme (CMS-i). In a complete resource sharing strategy access fairness is accomplished by allocating the same fixed number of resources to all data users (i.e., Fixed Resource Allocation or FIRA) irrespective to the MCS requested. In this way, coding/modulation schemes have the same number of resources allocated; therefore the same blocking condition is applied to all data users.

A. Teletraffic analysis.

Let \( k_i \) represent the number of packets being transmitted in the cell under study using the coding/modulation scheme \( i \). For notation simplicity a system state vector is defined in (2).

\[
K = \{ k_1, k_2, k_3, k_4, k_5, k_6 \}
\]  

(2)

The number of resources occupied in the cell analyzed in the state \( K (L(K)) \), is given by

\[
L(K) = M_F \sum_{i=1}^{6} k_i \quad \text{where} \quad 0 \leq L(K) \leq N_C.
\]  

(3)

The valid state space \( \Omega_0 \) is defined as follows

\[
\Omega_0 = \{ K | 0 \leq L(K) \leq N_C; 0 \leq k_i \leq \lceil N_C / M_F \rceil \}.
\]  

(4)

The mean arrival rate of EGPRS packet using coding/modulation scheme \( i (\lambda_{CMS-i}) \), is given by

\[
\lambda_{CMS-i} = p_{CMS-i} \cdot \lambda_d .
\]  

(5)

The offered traffic per cell of EGPRS packets using coding/modulation scheme \( i (\lambda_{CMS-i}) \), is given by

\[
a_{CMS-i} = \frac{\lambda_{CMS-i}}{M_F \cdot \mu_{CMS-i}}.
\]  

(6)

The global blocking probability is therefore given by

\[
P(K) = \left( \prod_{i=1}^{6} \left( \frac{a_{CMS-i}}{k_i!} \right) \right) \quad \text{where} \quad \left( 0 \leq k_i \leq \lfloor N_C / M_F \rfloor \right)
\]  

(7)

The steady-state probability distribution is therefore given by

\[
P_0 = \left[ \sum_{\{k\} \in \Omega_0} \left( \sum_{i=1}^{6} \frac{a_{CMS-i}}{k_i!} \right) \right]
\]  

(8)

The mean arrival rate of EGPRS packet using coding/modulation scheme \( i (\lambda_{CMS-i}) \), is given by

\[
\lambda_{CMS-i} = p_{CMS-i} \cdot \lambda_d .
\]  

(5)

The offered traffic per cell of EGPRS packets using coding/modulation scheme \( i (\lambda_{CMS-i}) \), is given by

\[
a_{CMS-i} = \frac{\lambda_{CMS-i}}{M_F \cdot \mu_{CMS-i}}.
\]  

(6)

The blocking probability \( P_{CMS-i} \) for EGPRS packets using CMS-i scheme (for \( i = 1, 2, ..., 6 \)), is given by the sum of the steady state probabilities in which the number idle resources is less than the number of resources \( M_F \) needed to serve a GPRS packet arrival, hence

\[
P_{CMS-i} = \sum_{\{k\} \in \Omega_0} P(K).
\]  

(9)

The throughput for EGPRS packets using CMS-i scheme \( (Th_{CMS-i}) \) is given by

\[
Th_{CMS-i} = a_{CMS-i} \cdot M_F \cdot R_{CMS-i}.
\]  

(10)

The total throughput for EGPRS packets in the sistem is given by

\[
Th = \sum_{i=1}^{6} Th_{CMS-i}.
\]  

(11)

IV. FAIR TRANSMISSION RATE STRATEGY

The transmission rate fairness refers to the homogeneity in the transmission rate among EGPRS packets independently

TABLE II. CODING/MODULATION SCHEMES USED IN TELETRAFFIC ANALYSIS

<table>
<thead>
<tr>
<th>Index (CMS-i)</th>
<th>Proportion of packets requesting the CMS-i scheme ( (p_{CMS-i}) )</th>
<th>EDGE Coding/Modulation Scheme (MCS)</th>
<th>Mean holding time (ms) ( 1/\lambda_{CMS-i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/6</td>
<td>MCS-0</td>
<td>197.951</td>
</tr>
<tr>
<td>2</td>
<td>1/6</td>
<td>MCS-6</td>
<td>395.903</td>
</tr>
<tr>
<td>3</td>
<td>1/6</td>
<td>MCS-4</td>
<td>665.838</td>
</tr>
<tr>
<td>4</td>
<td>1/6</td>
<td>MCS-3</td>
<td>791.807</td>
</tr>
<tr>
<td>5</td>
<td>1/6</td>
<td>MCS-2</td>
<td>1046.316</td>
</tr>
<tr>
<td>6</td>
<td>1/6</td>
<td>MCS-1</td>
<td>1331.676</td>
</tr>
</tbody>
</table>

2 The methodology to calculate these proportions is explained in [33].
of the coding/modulation scheme used. To accomplish this objective, a radio link quality-based dynamic resource allocation strategy is proposed. The proposed fair transmission rate strategy allocates a higher number of time slots to EGPRS packets using coding/modulation scheme with small transmission rate. Table III summarizes the resource allocation for the different coding/modulation schemes analyzed. It also shows the total transmission rate and the mean holding time used for the different classes of packets. Note that in this work, for the sake of simplicity, the allocation of multiple time slots (up to six in the smallest transmission rate scheme) aims at obtaining a transmission rate close to 59 Kbps for every packet, regardless of the of the coding/modulation scheme (MCS-i) utilized.

A. Teletraffic analysis

The teletraffic analysis for transmission rate fairness is similar to that presented for access fairness. The same notation used in the previous case is used here but some changes are made. A new variable, \(c_i\), is introduced that represents the number of time slots allocated to a EGPRS packet using the CMS-i scheme, so that a vector can be defined with the number of time slots allocated to the different classes of EGPRS packets as follows:

\[
c = \{1, 2, 3, 4, 5, 6\} \tag{14}
\]

The number of resources occupied in the state \(K (L(K))\), is given by

\[
L(K) = \sum_{i=1}^{6} c_i \cdot k_i \tag{15}
\]

The valid state space \(\Omega_0\) is defined as follows

\[
\Omega_0 = \{K | 0 \leq L(K) \leq N_c, 0 \leq k_i \leq \left\lceil \frac{N_c}{c_i} \right\rceil \} \tag{16}
\]

The offered traffic per cell of EGPRS packets using coding/modulation scheme \(i (a_{MCS-i})\), is given by

\[
a_{MCS-i} = \frac{\lambda_{MCS-i}}{c_i \cdot \mu_{MCS-i}} \cdot \frac{k}{c_i} \tag{17}
\]

The steady-state probability distribution has a product form solution and it is given by

\[
P(K) = \left( \prod_{i=1}^{6} \left( \frac{a_{MCS-i}^{k_i}}{k_i} \right) P_0 (0 \leq k_i \leq N_c) \cap (L(K) \leq N_c) \right) \cdot \left( \prod_{i=1}^{6} \frac{a_{MCS-i}^{k_i}}{k_i} \right)
\]

\[
P(K) = \left[ \sum_{k=0}^{N_c} \sum_{k_1=0}^{N_c} ... \sum_{k_6=0}^{N_c} \prod_{i=1}^{6} \frac{a_{MCS-i}^{k_i}}{k_i} \right]^{-1} \tag{18}
\]

The blocking probability \(P_{MCS-i}\) for EGPRS packets using the CMS-i scheme (for \(i = 1, 2, ..., 6\)), is the sum of the state probabilities in which the number of idle resources is less than the number of resources \(c_i\) needed to serve a GPRS packet arrival using the CMS-i scheme. Therefore

\[
P_{MCS-i} = \sum_{\{k | L(K) \leq N_c\}} P(K) \tag{19}
\]

The throughput for EGPRS packets using coding/modulation scheme \(i (\theta_{MCS-i})\), is given by

\[
\theta_{MCS-i} = a_{MCS-i} \cdot c_i \cdot R_{MCS-i} \tag{20}
\]

V. SIMULTANEOUS FAIR ACCESS AND TRANSMISSION RATE (THROUGHPUT) STRATEGY

Access and transmission rate fairness comprises the fact that EGPRS data packets using any coding/modulation scheme must be treated by the system in equal manner, in terms of both access and transmission rate. To achieve this, a resource allocation policy based on both the multiple channel reservation ([3], [5], [6], [11]) and link quality-based dynamic resource allocation strategies are proposed. In order to accomplish simultaneously access and transmission rate fairness, the proposed strategy allocates a higher number of resources and at the same time restricts a higher number of resources to EGPRS packets using low transmission rate coding/modulation schemes. Table IV shows the number of restricted resources for each coding/modulation scheme to achieve access fairness. The table also shows the number of resources allocated, transmission rates and the mean holding time for each coding/modulation scheme. Note that the mean resource holding time for the different classes of packets is similar (around 200 ms).

A. Teletraffic analysis

The teletraffic analysis for simultaneously fair access and transmission rate strategy is presented in this sub-section. Bearing in mind that packets using different coding/modulation schemes have similar mean resource holding time, the approximation proposed in [6], [9]-[10] can be used. This analytical approach is based on a one-dimensional recursive formula [7]-[8]. However, in this work both the six-dimensional exact analysis, as well as the one-dimensional approach are used and a comparison among them is presented. The same assumptions used above are considered here, nonetheless, a new variable, \(r\), is introduced to define the number of resources restricted to EGPRS packets using coding/modulation scheme CMS-i (\(r\)).

\[
r = \{5, 4, 3, 2, 1, 0\} \tag{21}
\]

1) Exact analysis

The exact teletraffic analysis consist in a multi-dimensional birth-death process with order six, which valid state space \(\Omega_0\) is defined as follows

<table>
<thead>
<tr>
<th>Index (CMS-i)</th>
<th>Number of resources allocated (time slots)</th>
<th>Transmission rate (Kbps)</th>
<th>Total transmission rate (Kbps)</th>
<th>Mean holding time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>59.2</td>
<td>59.2</td>
<td>197.951</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>29.6</td>
<td>59.2</td>
<td>197.951</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>17.6</td>
<td>52.8</td>
<td>221.946</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>14.8</td>
<td>59.2</td>
<td>197.951</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>11.2</td>
<td>56</td>
<td>209.263</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>8.80</td>
<td>52.8</td>
<td>221.946</td>
</tr>
</tbody>
</table>
of the left hand side of (29) as the GPRS packet using CMS-i scheme birth rate and the factors of \( q(l) \) on the right hand side of (29) as the GPRS packet using CMS-i scheme death rate. Then, multiplying (29) by \( c_i \) and summing over \( i \) yields
\[
\sum_{i=1}^{N_c} \beta(i-c_i,l) \cdot \mu_{CMS-i} \cdot c_i \cdot q(l-c_i) = q(l) \cdot \sum_{i=1}^{N_c} c_i \cdot E\{k,l\} = q(l) \cdot E\left(\sum_{i=1}^{N_c} c_i \cdot k,l\right)
\]
(30)
where \( q(x) = 0 \) for \( x < 0 \) and \( \sum_{i=1}^{N_c} q(l) = 1 \) Equation (30) defines a one-dimensional recursion (for an arbitrary \( \lambda \)). The blocking probability \( (P_{CMS-i}) \) for EGPRS packets using CMS-i scheme (for \( i = 1, 2, \ldots, 6 \)), is the sum of the state probabilities in which the number of idle resources is less than the number of resources \( c_i \) needed to serve a GPRS packet arrival using CMS-i scheme. Therefore
\[
P_{CMS-i} = \sum_{m=0}^{N_c} q(N_c-m)
\]
(31)
The throughput can be calculated in the same way as in the previous analysis using (10) and (20).

VI. NUMERICAL RESULTS

In this section, the performance of the three fair resource allocation strategies is numerically evaluated in terms of the blocking probability and throughput. In the numerical evaluations, the parameters shown in Tables I to III are used and it is considered that \( N_c = 30 \). Fig. 1 shows that, in the fair access strategy, the EGPRS packet blocking probability increases as the fixed number \( M_F \) of resources allocated to the EGPRS packets increases. Hence, the lowest value of \( M_F \) gives the largest access capacity. In very much the same manner, an increase of \( M_F \) decreases the throughput in the fair access strategy (as shown in Fig. 2). This is due to the higher packet blocking probability. As the number of allocated resources to an EGPRS packet increases, fewer packets monopolize the resource available in the cell, causing starvation of resources and higher blocking probabilities. Fig. 3 shows that the use of link quality-based dynamic resource allocation causes differences in the packet blocking probabilities for the EGPRS packets using different coding/modulation schemes in the fair transmission rate strategy.

### TABLE IV. NUMBER OF RESOURCES ALLOCATED, RESOURCES RESTRICTED, TRANSMISSION RATES AND MEAN HOLDING TIME FOR EACH CODING/MODULATION SCHEME ANALYZED.

<table>
<thead>
<tr>
<th>Index (CMS-i)</th>
<th>Number of resources allocated (time slots)</th>
<th>Number of resources restricted (time slots)</th>
<th>Transmission rate (Kbps)</th>
<th>Mean holding time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>59.2</td>
<td>197.951</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>59.2</td>
<td>197.951</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
<td>52.8</td>
<td>221.946</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>2</td>
<td>59.2</td>
<td>197.951</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>1</td>
<td>56</td>
<td>209.263</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0</td>
<td>52.8</td>
<td>221.946</td>
</tr>
</tbody>
</table>
In particular, it is observed that packets using a coding/modulation scheme with higher transmission rates (and smaller number of resources) have a smaller blocking probability than packets using coding/modulation schemes with lower transmission rates (and higher number of resources). The throughput achieved by packets using coding/modulation scheme CMS-1 for the fair transmission rate strategy are unequal. As it is shown in Fig. 4, in spite of having similar transmission rates, such differences are due to the differences between the blocking probabilities for each CMS scheme. It is also observed in Fig. 4 that the results of the approximation approach perfectly agree the results of the exact analysis (see Fig. 1). Note that packet blocking probabilities and transmission rates for packets using the different CMS schemes are similar, consequently, throughput fairness among the different packets is achieved (see Fig. 4).

VII. COMPARISON OF FAIR RESOURCE ALLOCATION STRATEGIES

Figure 1 shows the blocking probabilities plotted versus the offered traffic for the different strategies. It can be observed that the fair access strategy using one resource for each packet achieves the lowest blocking probability (i.e. the largest access capacity). It is also observed that the highest blocking probability is reached by the fair access strategy assigning six resources to each EGPRS packet (achieving the access lowest capacity). It is also observed that the transmission rate fairness can be achieved at the expense of decreasing access capacity. Figure 2 shows the total throughput in the cell versus the offered traffic. From the figure it can be observed that the largest throughput is achieved by the fair transmission rate strategy, and the smallest is obtained by the fair access strategy assigning six time slots to serve each EGPRS packet. Table V shows the throughput achieved by each strategy evaluated at 20 Erlangs/cell.

VIII. CONCLUSIONS

In this paper, the performance of three fair resource allocation strategies for packet data transmission in EDGE/GPRS networks is mathematically analyzed. Giving their nature, a tradeoff between capacity and fairness exists in these strategies. Numerical analysis shows that the fair transmission rate strategy increases throughput approximately 64.06% (162.5%) \(262.06\%\) \(275.00\%\) \(503.44\%\) \(600\%\) in throughput, but increases 5200% (60.60%) \(-3.63\%\) \(-21.48\%\) \(-26.89\%\) \(-28.50\%\) access blocking probability relative to the fair access strategy with \(M_F = 1\) (2) \(3\) \(4\) \(5\) \(6\). As noticed, the fair transmission rate strategy achieves similar access blocking probability to the fair access strategy with \(M_F = 3\). However, the fair access strategy with \(M_F = 1\) (fair transmission rate strategy) \{fair access and transmission rate strategy\} increases 326.66% (600%) \{420\%\} in throughput and decreases 98.70% (31.81%) \{16.12\%\} access blocking probability relative to the fair access strategy with \(M_F = 6\), which is the worst case scenario. Depending on the needs and load of the network one strategy can be preferred over other. Particularly, systems with light (heavy) traffic operation may select a strategy that improves throughput (capacity). The selection is left up to the network operator. Results showed that throughput fairness is a consequence of simultaneous access and transmission rate fairness. It is showed that users with bad radio channel condition can be compensated using multiple resource reservation and/or dynamic resource allocation, achieving a homogenous data service.

REFERENCES

Figure 3. Packet blocking probabilities for the fair transmission rate strategy for the different classes of packets.


Figure 4. Throughput for the fair transmission rate and simultaneous fair access and transmission rate strategies for the different classes of packets.


TABLE V. PERFORMANCE COMPARISON OF FAIRNESS STRATEGIES (OFFERED TRAFFIC = 20 ERLANGS/CELL).

<table>
<thead>
<tr>
<th>Fairness strategy</th>
<th>Throughput (kbps)</th>
<th>Gain (compared with Access M=1)</th>
<th>Packet blocking probability</th>
<th>Relative Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Access M=1</td>
<td>320</td>
<td>-65%</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Access M=6</td>
<td>75</td>
<td>-76.56%</td>
<td>0.775</td>
<td>7650%</td>
</tr>
<tr>
<td>Transmission rate</td>
<td>525</td>
<td>64.06%</td>
<td>0.53</td>
<td>5200%</td>
</tr>
<tr>
<td>Access &amp; transmission rate</td>
<td>390</td>
<td>21.87%</td>
<td>0.65</td>
<td>6400%</td>
</tr>
</tbody>
</table>