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ACTION AT A DISTANCE AND COSMOLOGY: A Historical Perspective

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■ **Abstract** The first law of theoretical physics, the Newtonian law of gravitation, relies on the concept of action at a distance. The success of this law led to the concept being applied to electricity and magnetism, which were next to be explored in depth. Here the action at a distance had a limited success and ultimately had to be abandoned in favor of the increasingly more popular field theory. Nevertheless, in the 1940s, an attempt was made to revive the concept of action at a distance in a relativistically invariant way by Wheeler & Feynman (1945, 1949). It inspired a series of investigations in both electrodynamics and gravity in which the field concept was not used but the interaction was described as taking place directly between particles. As it impinged very intimately on cosmology, Hoyle was keenly interested in it. This review discusses the work by Hoyle, the author, and others on the development of electrodynamics and gravitation as direct particle theories. In this review, the author discusses how the work was started and went through stages of increasing sophistication, e.g., extending the Wheeler-Feynman electrodynamics to curved spacetime, its consequences in different cosmologies, and the issues arising from its quantization. The resolution of ultraviolet divergences in quantum electrodynamics is also briefly discussed. The parallel development of a Machian theory of gravitation followed the lead from electrodynamics. In both theories one sees a strong link between the large-scale structure of the universe and local physics, as might be expected from an action-at-a-distance framework.

1. HISTORICAL BACKGROUND

I still vividly recall a wet afternoon in Varenna on Lake Como in northern Italy in the summer of 1961. I was a student participant in one of the annual summer schools held in this scenic resort. Our school was on various aspects of gravitational theories and observations and the lecturers included Bob Dicke, Alfred Schild, Joshua Goldberg, Bruno Bertotti, and Fred Hoyle. We also had seminars from a few other scientists who passed by for a short duration. That day it was Hermann Bondi who lectured on in his inimitable style, although hampered by frequent sneezes brought about by hay fever. Despite these interruptions he conveyed to us that his topic was extremely interesting and highly unusual. What Bondi reported on that day concerned a very basic aspect of electrodynamics, an aspect that linked it to cosmology—a follow-up by a Canadian physicist named Jack Hogarth on the work done by John Wheeler and Richard Feynman in the 1940s on action at a distance electrodynamics (Wheeler & Feynman 1945, 1949). Briefly, the history may be summarized as follows: The story began 100 years before the Wheeler-Feynman paper of 1945, with no less a person than Gauss. In a letter to Weber on March 19, 1845, Gauss wrote the following:

> I would doubtless have published my researchs long since were it not that at the time I gave them up I had failed to find what I regarded as the keystone, Nil actum reputans si quid superesset agendum, namely, the derivation of the additional forces—to be added to the interaction of electrical charges at rest, when they are both in motion—from an action which is propagated not instantaneously but in time as is the case with light.

Gauss was referring to experiments in electrodynamics that suggested the simple Coulomb inverse square law of action at a distance was not adequate to explain all that was being observed. Although it was early days still for the elctromagnetic waves to be established, there were indications of delayed action at a distance. The question arose, then, of how to incorporate these effects into the basic Coulomb law that assumed (as in Newton's inverse square law of gravitation) that the electric and magnetic effects propagate instataneously across spatial separations, i.e., that action at a distance operated with infinite speed.

Gauss's attempts came two decades before the Maxwellian field theory[1] and six decades before special relativity. The success of these two theories shifted the emphasis from action at a distance to fields.

In this formulation there are two basic entities: electric charges and the electromagnetic field. If we have two charges a and b separated by a distance r, then movement of a sets off a wave in the field in its neighborhood. The wave travels outward from charge a with the speed c and reaches the charge b after the time interval r/c. The charge b therefore experiences the action of a after a delay of this interval. The Maxwellian equations comprehensively described this phenomenon through mathematical equations that led to the conclusion that the speed c with which these waves propagate is none other than the speed of light. Experiments by Hertz successfully confirmed this conclusion.

Special relativity then came along to establish the invariance of c under Lorentz transformations and thereby identified the Maxwellian field theory as the foundation on which all electromagnetic phenomena can be based. The march of these ideas left the earlier concept of action at a distance far behind. The difficulty so succinctly stated by Gauss had remained the hurdle it could not cross, although his perceptiveness in identifying the speed of light as playing a key role in the transmission of electromagnetic effects cannot be denied even today.

[1]Those who wish to refresh their knowledge of Maxwell's electromagnetic field theory may find it useful to look at chapters 18–21 of *Feynman Lectures in Physics*, Volume II.

It was not until well into the twentieth century that the problem posed by Gauss was solved. A theory in which action at a distance propagates with the speed of light in a relativistically invariant way was eventually put together. A beginning was made by Schwarzschild (1903), Tetrode (1922), and Fokker (1929a,b; 1932), who independently formulated the concept of delayed action at a distance. The action principle as formulated by Fokker may be written in the following form:

$$J = -\sum_a \int m_a \, da - \sum_{a<b}\sum \iint e_a e_b \delta\left(s_{AB}^2\right) \eta_{ik} da^i db^k. \tag{1}$$

In the above expression the charged particles are labeled, a, b,... with e_a and m_a the charge and mass of particle a. The worldline of a is given by the coordinate functions $a^i(a)$ of the proper time a. The spacetime is Minkowskian, so that

$$da^2 = \eta_{ik} \, da^i da^k, \tag{2}$$

with $\eta_{ik} = \mathrm{diag}(-1, -1, -1, +1)$. The first term of J therefore describes the inertial term. The second term describes the electromagnetic interaction between the worldlines of a typical pair of particles a, b. The delta function shows that the interaction is effective only when s_{AB}^2, the invariant square of distance between typical world points A, B on the worldlines of a and b, vanishes. This implies delayed action: $s_{AB}^2 = 0$ means that world points A and B are connected by a light ray.

This is therefore a prima facie representation of Gauss's conjecture.

2. THE ABSORBER THEORY OF RADIATION

Although this formulation met the requirement of relativistic invariance, it gave rise to other difficulties. The major difficulty is as follows: For a typical point A on the worldline of a there are two points B_+ and B_- on the world line of b for which $s_{AB}^2 = 0$. The effect of A is felt at B_+ (at a later time) and at B_- (at an earlier time). Similarly, since the action principle guarantees the equality of action and reaction, the reaction from B_+ and B_- is felt back at A. Thus there are influences propagating with the speed of light, not only into the future but also into the past. This led to a conflict with the principle of causality, which seems to hold in everday life. The other difficulties were of a less serious nature although not ignorable. For example, there was no self-action ($a = b$ is avoided in the double sum) and so there did not appear to be any obvious way of accounting for radiation damping.

These difficulties were removed in a ground-breaking paper by Wheeler & Feynman (1945) because they brought into discussion the important role of the universe as an absorber. In our above example, the reactions from B_+ and B_- arrive at A instantaneously, whatever the spatial separation of a and b. So it becomes necessary to take into account the reaction from the entire universe to A. Although the remote particles are expected to contribute less, their total number might be large enough to make the calculation nontrivial. The essence of the argument given by Wheeler & Feynman is described below.

To begin, define the 4-potential at X due to particle b by

$$A_i^{(b)}(X) = e_b \int \delta\left(s_{XB}^2\right)\eta_{ik}\, db^k, \tag{3}$$

and the corresponding direct-particle field by

$$F_{ik}^{(b)} = A_{k;i}^{(b)} - A_{i;k}^{(b)}. \tag{4}$$

A direct particle field is not an ordinary field because it does not have any independent degrees of freedom. The 4-potential identically satisfies the relations

$$A_{;k}^{(b)k} \equiv 0, \quad \Box A^{(b)k} \equiv \eta^{mn} A_{;mn}^{(b)k} = 4\pi j^{(b)k}, \tag{5}$$

where $j^{(b)k}(X)$ is the current density vector of the particle b at a typical point X, defined in the usual way. Thus although Equation 5 resembles the Maxwell wave equation and the guage condition, it represents identities.

The equation of motion of a typical charge a is obtained by varying its worldline and requiring $\delta J = 0$. We get the analogue of the Lorentz force equation in which the charge a is acted on by all other charges in the universe. It appears, therefore, that an alternative formulation of electrodynamics based on action at a distance has been found. However, this appearance is illusory.

We now turn to the difficulty introduced by the time symmetry of this formulation. Instead of being the retarded solution of Equation 5, Equation 3 is the time-symmetric half-advanced and half-retarded solution. The same applies to the direct-particle fields. Suppressing the indices i, k, we may write Equation 4 as

$$F^{(b)} = \frac{1}{2}\left[F_{\text{ret}}^{(b)} + F_{\text{adv}}^{(b)}\right]. \tag{6}$$

This field is present in the past as well as the future light cone of B.

Although the individual influence of a charged particle such as b is time symmetric, the combined influence of all the particles in the universe, excited by the movement of b, may not be so. This was the central thesis of the 1945 work of Wheeler & Feynman.

Wheeler & Feynman argued in the following way: If we move the charge b, it generates a disturbance that affects all other charges in the universe. Their reaction arrives back instantaneously. How did they calculate such a reaction in a universe of static Minkowski type with a uniform distribution of electric charges? They found that the reaction to the motion of charge b can be calculated in a consistent fashion and comes out to be

$$R^{(b)} = \frac{1}{2}\left[F_{\text{ret}}^{(b)} - F_{\text{adv}}^{(b)}\right]. \tag{7}$$

Thus a test particle in the neighborhood of charge b experiences a net total field

$$F^{(b)}_{\text{tot}} = F^{(b)} + R^{(b)} = F^{(b)}_{\text{ret}}. \tag{8}$$

This is the pure retarded field observed in real life. The self-consistency of the argument follows from the fact that the reaction $R^{(b)}$ has been calculated by adding the $\frac{1}{2}F^{(a)}$ fields of all particles $a \neq b$ that have been excited by this total field $F^{(b)}_{\text{tot}}$, for these influences arrive back at B instantaneously. Thus only the future light cone of B comes into play. The reaction from the universe on the future light cone cancels the advanced component of $F^{(b)}$ and doubles its retarded component.

Also, according to the Lorentz force equation, $R^{(b)}$ is the force arising from all other particles in the universe experienced by the particle B. This is nothing but the radiative reaction to the motion of b as obtained earlier by Dirac (1938) on empirical grounds. Dirac had shown that the Lorentz force produced by half the difference between the retarded and advanced fields of a charge, evaluated at the charge, gives the radiative damping force. Dirac's rule was difficult to understand within the context of the field theory, although it was known to give the right answer. In the present action at a distance framework the Dirac formula is understood as the natural consequence of a response of the universe to the local motion of the charge. Thus the theory not only gets around the problem of causality but it also accounts for the radiation damping formula.

Physically, what happens is the following: To the motion of b the future half of the universe acts as an absorber. It absorbs all the energy radiated by b, and in this process sends the reaction $R^{(b)}$, which does the trick. For this reason Wheeler & Feynman called this theory the absorber theory of radiation. The presence of the absorber is essential for the calculation to work. For example, it will not work in an empty universe surrounding the electric charge. In the above self-consistent derivation there was still one defect: it was not unique. Another self-consistent picture was possible in which the net field near every particle was the pure advanced field and the radiative reaction was of opposite sign to that of Equation 7. The two solutions are thus compared. In one we have the retarded solution and in the other we have the advanced solution. In the former, absorption in the future light cone is responsible, whereas in the latter, it is the absorption in the past that plays the crucial role. The important role of the absorbers is that they convert the time-symmetric situation of an isolated charge to a time-asymmetric one. It is, however, not possible to distinguish between the two without reference to some other independent time asymmetry.

Wheeler & Feynman (1945, 1949) realized this and linked the choice to the time asymmetry in thermodynamics. Given the usual thermodynamic time asymmetry, they argued that the latter situation would be highly unlikely (under the probability arguments of statistical mechanics) and that the usual asymmetry of initial conditions will favor the former (retarded) solution.

It was, however, pointed out by Hogarth (1962) that it is not necessary to bring thermodynamics into the picture at all. If one takes account of the fact that the universe is expanding, then it is clear that its past and future are naturally different.

The reaction from the absorbing particles in the future light cone (designated by Hogarth collectively as the future absorber) does not automatically come out equal and opposite to the reaction from the past absorber. Thus the two pictures do not always follow in an expanding universe. Hogarth found that for the retarded solution to hold, the future absorber must be perfect and the past absorber imperfect, and vice versa for the advanced solution.

An absorber is perfect if it entirely absorbs the radiation emitted by a typical charge. In the static universe discussed by Wheeler & Feynman (1945) both the past and future absorbers are perfect, and this leads to the ambiguity mentioned earlier. However, Hogarth (1962) found that the ambiguity is resolved if the cosmological time asymmetry is taken into account. He found, for example, that in most big-bang models that expand forever from an initial singularity, the advanced (and not the retarded) solution is valid. In the steady-state model (Bondi & Gold 1948, Hoyle 1948), only the retarded solution holds. In the big bang models that expand and contract, both the absorbers are perfect and the outcome is ambiguous. This was basically the work that Bondi reported on in Varenna.

3. THE ABSORBER THEORY IN AN EXPANDING UNIVERSE

Fred Hoyle and I were very impressed by this conclusion. Only a few months before we had argued against the claim by Martin Ryle and his group in Cambridge that their observations of radio source counts ruled out the steady state theory. Those arguments involved several details of observational errors and extrapolations of theory. Here, on the contrary, was a clear-cut conclusion that did not depend on such messy details. Because of its cosmological implications, we felt that this approach of action at a distance needed to be pushed further.

Although interesting, Hogarth's work was, however, incomplete in two aspects. First, he had not shown how to generalize the Fokker action to the curved spacetimes needed to describe the expanding world models. Second, he had used collisional damping to decide on the nature of absorbers, past and present, and this brought in thermodynamics by the back door. In fact, Feynman had criticized Hogarth's work on this very ground.

Later, Hoyle and I (Hoyle & Narlikar 1963) removed these deficiencies by first rewriting the Fokker action (Equation 1) in curve spacetime as is necessary for any cosmological discussion. The delta functions in Equation 1 have to be replaced by propagators that are two-point functions in spacetime, in fact the symmetric Green's functions for the wave operator. This enables one to define action at a distance in a typical expanding world model. We too arrived at conclusions similar to Hogarth's but we did so by using the radiative damping for producing absorption. This kept the asymmetry entirely within electrodynamics and cosmology and away from collective phenomena and thermodynamics.

However, a greater challenge lay ahead. If the action at a distance picture was to go further, on parity with the rival field theoretic picture, then it must be quantizable. Quantum electrodynamics presents a far richer set of phenomena than its classical counterpart. Our next step was to demonstrate that it is indeed possible to extend the entire picture into the quantum domain, and it is possible to describe the entire range of phenomena of quantum electrodynamics, such as spontaneous transition of the atomic electron, Compton scattering, pair creation, and even the Lamb shift and anomalous magnetic moment of the electron without recourse to field theory (Hoyle & Narlikar 1969, 1971). This therefore removes any possible objection to the concept of action at a distance insofar as it is applicable to electrodynamics. In fact, the work two decades later, described briefly in Section 7, shows that even the infinities introduced by the so-called self interactions can be eliminated in this formulation.

The crucial role played in the whole calculation is that of the response of the universe. In the classical calculation the steady-state universe generates the correct response so that the local electric charges interact through retarded signals. The response from the big-bang models is of the wrong type. We are thus able to distinguish between the different cosmological models and decide on their validity or otherwise on the basis of the Wheeler-Feynman theory. We also see why charges interact through retarded signals: They do so because of the response of the universe. In the Maxwell field theory, by contrast, the choice of retarded solutions of Maxwell's equations is made by an arbitrary fiat.

In the quantum calculation it also can be shown that an asymmetric phenomenon with respect to time, such as the spontaneous downward transition of an atomic electron, is caused by the response of the universe. By contrast, the quantization of the Maxwell electromagnetic field ascribes such an asymmetry to the quantum vacuum and to the rather abstract rules of quantization.

The direct-particle approach therefore achieves for electrodynamics what Mach (1893) sought to achieve for inertia. By bringing in the response of the universe to a local experiment in electrodynamics we have essentially incorporated Mach's principle into electromagnetic theory. Given the correct response of the universe, we can almost decouple our local system from it, although strictly speaking the theory would not be possible without a universe with the right kind of absorbing properties.

Can the same prescription be applied to the original Mach's principle per se, to inertia and gravitation? We discuss this problem in the following section. For this issue also attracted Hoyle and me in the 1960s, in a parallel manner to our work with electrodynamics.

4. INERTIA AS A DIRECT-PARTICLE FIELD

What is Mach's principle? There is no clear-cut statement of the principle ascribed to Mach (1893). The philosophical content of Mach's ideas may be summarized thus: The inertial mass of an object links its acceleration to the impressed forces. What is the frame of reference relative to which the motion is to be measured? Newton had postulated an abstract absolute space relative to which velocities and accelerations could be measured. As his rotating bucket experiment showed, one needed to add inertial forces to the second law of motion whenever one used a frame of reference accelerated relative to the absolute space. Mach felt that this preferred frame of reference (i.e., the absolute space) was none other than one determined by the distant parts of the universe. In fact he argued that without such a background frame the absolute space cannot be identified nor can the laws of motion be stated. For example, one cannot describe the motion of a single particle in an otherwise empty universe with no background. In this way he linked the universal background of matter to the concept of local inertial frame and the notion of inertia itself. For a while Einstein was impressed by this argument and hoped to incorporate it into general relativity. Later he felt somewhat disillusioned and came to believe that the idea of instantaneous effect of distant matter on local frames was irreconcilable with relativity. We now return to the problem of achieving a reconciliation between general relativity and Mach's principle. To this end we shall look for a theory with the following properties:

(*a*) It has Mach's principle built into one of its postulates.

(*b*) It is conformally invariant.

(*c*) It does not have the conceptual difficulties associated with the case of a single particle in an otherwise empty universe.

(*d*) For a universe containing many particles the theory reduces to general relativity for most physical situations.

Some discussion is needed as to why the theory should be conformally invariant. The reasons are twofold. First, when we take note of the local Lorentz invariance of special relativity, the natural units to use are those with the fundamental velocity $c = 1$. The quantum theory, with which our theory should be consistent, throws up another fundamental constant, the Planck constant related to the uncertainty principle. Thus it is natural to use units in which $\hbar = 1$. This makes the classical action J, for example, dimensionless, and the natural unit of mass is the Planck mass, which we shall quantify by

$$m_P = \sqrt{\frac{3c\hbar}{4\pi G}}. \tag{9}$$

All masses are then expressible as numbers in the unit of this mass. With our choice of units, only one independent dimension out of the three—length, mass, and time—remains. Taking it as the dimension of mass, length goes as a reciprocal of mass.

However, in a Machian theory we expect the particle masses to be functions of space and time and as such not necessarily constant. Therefore, the standards of lengths and time, intervals also may vary from one point to another. Hence we need laws of physics that are invariant with respect to this variation. Conformal invariance guarantees this.

Our second reason is based on the nature of action at a distance. Given that as in electrodynamics, the interaction propagates principally along null rays, we need an invariance that preserves the global structure of light cones. This again is guaranteed by conformal invariance. Just as Lorentz invariance identifies the light cone as an invariant structure locally, so does conformal invariance identify it globally.

A theory following these guidelines was developed by Hoyle and me (1964, 1966) and its broad features are described next. We begin with a second look at the Fokker action for electrodynamics, this time rewritten in a curved Riemannian space-time:

$$J = -\sum_a \int m_a \; da - \sum_{a<b}\sum 4\pi e_a e_b \iint \bar{G}_{i_A k_B} \, da^{i_A} \, db^{k_B}. \qquad (10)$$

Here, in going from Equation 1 to Equation 10, the first term of J needs a trivial modification: *da* is now computed with a Riemannian metric. The modification of the second term of J requires considerable thought. The $\delta(s_{AB}^2)\eta_{ik}$ is now replaced by $\bar{G}_{i_A k_B}$, a bivector propagator between A and B. It is the symmetric Green's function for the wave euqation

$$\Box \bar{G}_{ik_B} + R_i^l \bar{G}_{lk_B} = [-\bar{g}(X, B)]^{-1/2} \bar{g}_{ik_B} \delta_4(X, B). \qquad (11)$$

Here $\bar{G}_{ik_B}$ behaves as a vector both at X and B, respectively, with the indices i and k_B (the subscript X on i is suppressed for the convenience of writing). $\bar{g}_{ik_B}$ is the parallel propagator between X and B [see Synge (1960) for details] and $\bar{g}(X, B)$ its determinant. In the limit $g_{ik} \to \eta_{ik}$, $\bar{G}_{i_A k_B} \to \delta(s_{AB}^2)\eta_{ik}/4\pi$. The detailed structure of this propagator has been studied by DeWitt & Brehme (1960).

The electromagnetic part of J is conformally invariant but the mechanical part (the first term) is not. We now compare Equation 10 with the action for field theory of Maxwell and general relativity. This action, denoted by $J^{(F)}$, is given by

$$J^{(F)} = \frac{1}{16\pi G}\int R(-g)^{1/2}\, d^4x - \sum_a \int m_a \, da$$
$$- \frac{1}{16\pi}\int F^{lm} F_{lm} (-g)^{1/2} d^4x - \sum_a \int A_i \, da^i. \qquad (12)$$

The third and fourth term of $J^{(F)}$ represent the free-field term and the field-particle interaction term, respectively. In the direct-particle theory the fourth term of J replaces these two terms of the field theory. The fields as such lose their independent status and are replaced by propagators connecting particle world lines. What can we do about the first two terms of Equation 12? The second term already exists in Equation 10, and it is tempting to simply insert the first term into Equation 10 as representing gravity.

This procedure, however, is contrary to the spirit of the direct-particle picture. The first term of Equation 12, although containing geometrical information, also has the character of a field. Hence it is out of place.

The clue to the correct procedure that needs to be adopted is provided by a comparision of the last term of Equation 12 with the second term of Equation 10. If in the former we replace the potential A_i by a sum over the direct-particle potentials defined by a relation analogous to Equation 3 for a curved space, we shall recover something that looks like the latter. In the same way we now replace the masses m_a by direct-particle fields defined in the following manner:

$$m^{(b)}(X) = \int \lambda_b G(X, B)\, db, \quad \lambda_b = \text{a coupling constant}. \tag{13}$$

$$m_a(A) = \lambda_a \sum_{b \neq a} m^{(b)}(A), \quad \lambda_a = \text{a coupling constant}. \tag{14}$$

The propagator $G(X, B)$ has to be biscalar because masses are scalars, and we wish to preserve a symmetry between X and B. The action (Equation 10) is now changed to

$$J = -\sum_{a<b}\sum \iint \lambda_a \lambda_b G(A, B)\, da\, db - \sum_{a<b}\sum 4\pi e_a e_b \iint \bar{G}_{i_A k_B}\, da^{i_A}\, db^{k_B}. \tag{15}$$

In Equation 15 both electrodynamics and inertia are now appearing manifestly in direct particle interaction format.

What should be the exact form of $G(A, B)$? Taking a clue from electromagnetism, we expect it to be a symmetric Green's function of a scalar wave equation. However, we also want the equation to be conformally invariant. These two requirements fix the form of the scalar propagator uniquely to within a multiplicative factor. We shall take $G(A, B)$ to satisfy the scalar wave equation

$$\Box G(X, B) + \frac{1}{6} R(X) G(X, B) = [-\bar{g}(X, B)]^{-1/2} \delta_4(X, B). \tag{16}$$

The wave operator is uniquely fixed by the requirement of conformal invariance.

Turning from these purely formal aspects to those of interpretation, we note that Equation 13 and Equation 14 are essentially Machian ideas on inertia expressed mathematically. The mass of a particle a at its world point A is the sum of the contributions of all other particles in the universe. Thus requirement (a) has been met. Requirement (c) is also met because for a single particle in an otherwise empty universe there is no action. The minimum number of particles required to define J is two. Thus for each of the two particles the other provides the background in the Machian sense. The requirement of conformal invariance is also met by our choice of the propagator. It therefore remains to examine requirement (d). In addition, we have got around Einstein's difficulty with instantaneous action at a distance by choosing propagators limited to the light cones.

So far we have concentrated on inertia and ignored gravity. The action (Equation 15) does not contain the gravitational term

$$\frac{1}{16\pi G} \int R(-g)^{1/2}\, d^4x$$

explicitly. Yet as we shall see in the following section, the theory is fully capable of describing gravitational phenomena.

5. CONFORMAL GRAVITY

Returning to the action of Equation 12 we note that when we try to derive the Einstein field equations by the Hilbert action principle, we get the Einstein tensor from the first term. This term does not exist any more in the direct particle action of Equation 15. Shall we then get any gravitational term at all from Equation 15 if we sought to perform the metric veriation $g_{ik} \to g_{ik} + \delta g_{ik}$? A look at the electromagnetic part of Equation 12 does not inspire confidence that the answer to this question should be in the affirmative. There it is the third rather than the fourth term that contributes the energy tensor of electrodynamics, and it is the fourth term that was used in going over to Equation 15. Nevertheless, a closer examination shows that the terms in Equation 15 do give nontrivial answers when the metric variation is performed.

The reason for this is understood as follows: Consider the electromagnetic propagator $\bar{G}_{i_A k_B}$ connecting A and B, respectively, on the world lines of a and b. Suppose we perform a variation in the space-time metric of a compact region Ω. Because the propagator is a global property of space-time structure, it will change through this change in the structure of Ω. The change in the propagator is therefore expressible, in a first-order calculation, as a functional of δg_{ik} over the volume Ω.

In the electromagnetic case the answer may be expressed in the following form:

$$-\delta \sum_{a<b}\sum 4\pi e_a e_b \iint \bar{G}_{i_A k_B}\, da^{i_A}\, db^{k_B} = -\frac{1}{2}\int T^{ik}\delta g_{ik}(-g)^{1/2}\, d^4x, \qquad (17)$$

where

$$T^{ik} = \frac{1}{8\pi}\sum_{a<b}\sum \left[\frac{1}{2} g^{ik} F^{(a)mn}_{\text{ret}} F^{(b)}_{mn\,\text{adv}} - F^{(a)i}_{l\,\text{ret}} F^{(b)kl}_{\text{adv}} - F^{(b)i}_{l\,\text{ret}} F^{(a)kl}_{\text{adv}}\right]. \qquad (18)$$

The details of this derivation are given by Narlikar (1974).

It is interesting to note that this derivation resolves an ambiguity about the energy tensors of direct-particle electrodynamics. Wheeler & Feynman (1949) had discussed two tensors for this theory. Of these one was the canonical tensor given above by Equation 18 and the other was the Frenkel tensor whose form differs from that given in Equation 18 in having all the direct particle fields $F^{(a)lm}$ as the symmetric half-advanced-plus-half-retarded fields. Wheeler & Feynman had concluded the following:

> From the standpoint of pure electrodynamics it is not possible to choose between the two tensors. The difference is of course significant for the general theory of relativity, where energy has associated with it a gravitational mass. So far we have not attempted to discriminate between the two possibilities by way of this higher standard.

As mentioned above, the usual prescription of metric variation uniquely yields the canonical tensor. The fact that we could get a nontrivial answer to the variational problem and that this resolves a long-standing ambiguity reinforces our belief that we are proceeding along the correct path toward a theory of gravitation.

We now consider the variation of the first term of Equation 15 as $g_{ik} \rightarrow g_{ik} + \delta g_{ik}$. We ignore the second term and concentrate on gravitation alone. Also, for simplicity, we begin by putting $\lambda_a = 1$ for all a. This does not alter the essential features of the theory. The method is similar to that adopted for electromagnetism. We compute the change in the propagator $G(A, B)$ as the geometry changes in any compact region Ω. The details of this somewhat lengthy calculation are given elsewhere (see Hoyle & Narlikar 1974). We simply quote the result. The field equations turn out to be

$$\frac{1}{2}\phi\left(R_{ik} - \frac{1}{2}g_{ik}R\right) = -T_{ik} + \frac{1}{6}[g_{ik}\Box\phi - \phi_{;ik}] + \frac{1}{2}\left[m_i^{\text{ret}}m_k^{\text{adv}} + m_k^{\text{ret}}m_i^{\text{adv}} - g_{ik}m^{l\,\text{ret}}m_l^{\text{adv}}\right], \quad (19)$$

where,

$$m(X) = \textstyle\sum_a \int G(X, A)\, da, \quad m_i = \partial m/\partial x^i, \quad (20)$$

$$\phi(X) = m^{\text{adv}}(X)m^{\text{ret}}(X), \quad (21)$$

and m^{ret} and m^{adv} denote twice the retarded and advanced parts of $m(X)$, respectively. The energy tensor T_{ik} is the familar energy tensor for a system of particles $a, b, \ldots$ with masses as defined by the Machian prescription Equation 13 and Equation 14. Note that the masses are time symmetric. The function $m(X)$ satisfies the conformally invariant wave equation

$$\Box m + \frac{1}{6}Rm = N, \quad (22)$$

where

$$N(X) = \sum_a \int \delta_4(X, A)[-\bar{g}(X, A)]^{-1/2}\, da \quad (23)$$

is the invariant particle number density.

There are 10 equations in Equation 19 and one equation in Equation 22 for the 11 unknowns g_{ik} and m. However, the divergence and trace of Equation 19 identically vanish, showing that there are in fact five fewer independent equations. This is hardly surprising because four of these five are due to the general coordinate invariance (as in general relativity) whereas the fifth identity (the vanishing of trace) is due to conformal invariance. It is easy to verify that if $[g_{ik}, m]$ is a solution of these equations then so is $[\zeta^2 g_{ik}, \zeta^{-1}m]$ for an arbitrary well-behaved (i.e., of type C^2) nonvanishing finite function ζ of space and time. This arbitrary function is nothing but the expression of the arbitrariness of mass-dependent units discussed in Section 4.

Suppose now that it is possible to choose ζ such that

$$m^{\text{ret}}\zeta^{-1} = m_0 = \text{constant}. \quad (24)$$

Suppose also that the response of the universe is such as to cancel all advanced components and double the retarded ones so that the effective mass function is m^{ret}. Then the field equations are simplified to

$$R_{ik} - \frac{1}{2} g_{ik} R = -\kappa T_{ik}, \quad \kappa \equiv \frac{8\pi G}{c^4} = 6/m_0^2. \tag{25}$$

We shall later identify m_0 with the mass of the Planck particle. However, as seen above, we have arrived at the familiar equations of general relativity. The conformal frame for which Equation 24 and Equation 25 hold will be called the Einstein frame. We have thus completed the remaining part of the program outlined at the beginning of Section 3.

The following points are worth emphasizing in the above derivation of Einstein's equations, which is radically different from the standard ones (used for example, by Einstein in 1915 and by Hilbert later the same year).

1. The approach to Einstein's equations is via the wider framework of a conformally invariant gravitation theory. Only in the limit of many particles in a suitably responding universe do we arrive at Einstein's equations. In the other limit of zero or no particles there is no theory. Thus it brings out the reason why the Machian paradox of one particle in an empty universe is not valid in the context of Einstein's equations. This reason does not emerge in the standard derivations of Einstein's equations.
2. It is significant that the coupling constant κ is positive in this approach. This conclusion is unaffected by the change of sign of the coupling constants λ_a, λ_b, etc. (taken here as unity), nor is it affected by the choice of signature (i.e, $- - - +$ instead of $+ + + -$) of the spacetime metric. The choice of the conformally invariant scalar propagator leads to the coupling constant being positive, i.e., to gravity being attractive. In the standard derivation the coupling constant is fixed (in sign as well as magnitude) by a comparison with Newtonian gravity.
3. A considerable discussion has gone on in literature regarding the admissibility of the so-called λ-term in Einstein's equations. This is because this term could be accmmodated in Einstein's heuristic derivation or in Hilbert's action principle. It is worth emphasizing that the direct-particle approach to gravity given so far does not permit the λ-term. As we shall see below, this term does arise in a Machian way in the direct particle theory, provided we allow the wave equation (Equation 22) for inertia to be nonlinear. The present cosmological observations generally do seem to require the cosmological constant (Bagla et al. 1996, Narlikar & Padmanabhan 2001).
4. The condition (Equation 24) that leads to Einstein's equations needs to be reexamined carefully under two special circumstances. Near a typical particle a, we expect the mass function $m^{(a)}(X)$ to blow up so that $m(x) \to \infty$, as $X \to A$, on the worldline of a. In order to make $\zeta^{-1} m(X)$ finite at A, we therefore require $\zeta \to \infty$, as $X \to A$. However, we have already ruled out

such conformal functions by restricting ζ to finite values. Thus the transition to Einstein's equations is not valid as we tend to any typical source particle. The nature of the equations and their solutions near a particle in this theory have been discussed by Hoyle & Narlikar (1966) and by Islam (1968). The other aspect arises if there exist $m = 0$ hypersurfaces. Clearly we cannot choose a finite $\zeta(\neq 0)$ to make $m =$ constant >0 on these hypersurfaces. If we insist on driving such solutions into the Einstein frame, we end up having spacetime singularity there. This has been pointed out by Kembhavi (1978).

6. COSMOLOGICAL CONSTANT AND THE CREATION OF MATTER

In recent years, this theory has been further generalized and applied to cosmology to include the cosmological constant as well as an explicit description of the creation of matter (see Hoyle et al. 1995). Taking the cosmological constant into account first, we may ask whether Equation 22 is the most general conformally invariant wave equation satisfied by a scalar function $m(X)$. The answer is No! The most general such equation is

$$\Box m + \frac{1}{6} Rm + \Lambda m^3 = N, \tag{26}$$

where Λ is a constant. This of course makes the scalar Machian interaction nonlinear and more difficult to handle. However, it very naturally leads to the cosmological constant of the right magnitude at the present epoch. For if we assume that for a single particle, the value of this constant is unity, then in the sum of Equation 20 leading to $m(X)$, because of the presence of a large number $\mathcal{N}$ of particles within the cosmological horizon, the cube term is less effective by the factor $\mathcal{N}^{-2}$. Thus the factor Λ in Equation 26 is of this order. With the identification of m_0, with the only possible fundamental quantity with the dimension of mass, viz, the Planck mass

$$m_0 = \sqrt{\frac{3\hbar c}{4\pi G}}, \tag{27}$$

one can write the familiar cosmological constant in the Einstein equations as

$$\lambda = -3\Lambda m_0^2. \tag{28}$$

With around 2.10^{60} Planck particles in the horizon, we get the value of $\lambda \sim -2.10^{-56}$ cm^{-2}. It is of the right order, but it is negative. However, it leads to interesting and physically relevant cosmological models.

Let us consider the creation of matter next. The standard relativity theory starts with the assumption that matter can neither be created nor destroyed, that is, the worldlines of particles are endless. However, in the big bang singularity, all worldlines are incomplete, thus leading to a contradiction with the basic assumption.

Indeed the presence of the singularity signifies the breakdown of the basic rules such as the action principle from which the equations of general relativity are obtained.

To account for the creation of matter in a nonsingular fashion, we introduced the additional input into the theory that the particle worldlines can be with beginnings and ends. The same action as before then describes this theory but the endpoints generate extra terms in the field equations.

For details of this work see the paper by Hoyle et al. (1995). The field equations in the "constant mass" conformal frame then take the form

$$R_{ik} - \frac{1}{2}g_{ik}R + \lambda g_{ik} = -\kappa \left[T_{ik} - \frac{2}{3}\left(c_i c_k - \frac{1}{4}g_{ik}c^l c_l \right) \right]. \tag{29}$$

The scalar c-field arises from the contribution to inertia from ends of particle worldlines. These are the contributions the Planck particles created that last a very short timescale, $\sim 10^{-43}$ s.

Sachs et al. (1996) have solved these field equations and obtained a series of cosmological models that are a combination of two kinds: (*a*) models with creation of matter and (*b*) models without creation of matter. The generic solution is known as the Quasi Steady State Cosmology (QSSC). It is a nonsingular model that has a long-term de Sitter–type expansion with short-term oscillations superposed on it. The former represents the creative and the latter the noncreative mode. The QSSC is being proposed as an alternative to the standard hot big bang cosmology (Hoyle et al. 2000).

Two aspects of the QSSC may be mentioned here. First, because there is no equivalent of an "early hot era," the problems of origin of light nuclei and microwave background are handled differently. The details are described in Hoyle et al. (2000). The MBR is seen here as the relic starlight of all previous cycles (each having its own sequence of star formation to star death), it being fully thermalized by cosmic dust in the form of metallic whiskers. As shown recently (Narlikar et al. 2003), the theory explains the observed power spectrum of small scale fluctuations of MBR.

The second interesting aspect of the theory is that it has a negative cosmological constant. How then does the QSSC explain the m-z relation for high redshift supernovae? As discussed by Narlikar et al. (2002), the extra faintness of the distant supernovae is explained by the whisker dust in exactly the same amount as required for the thermalization of the MBR. This is an indication that a positive λ is not necessary for understanding the supernova data.

7. FINITE RESOLUTION OF THE SELF-ENERGY PROBLEM

After this excursion into gravitation, let me return to electrodynamics. In a recent paper Hoyle & Narlikar (1993) had shown that with suitable cosmological boundary conditions, such as the de Sitter horizon, there is a cutoff on high frequencies that otherwise lead to divergent integrals in the standard electromagnetic field theory. Thus the electron self-energy problem and the various radiative corrections of quantum electrodynamics can be handled without subtraction of one infinity from another. In this sense the direct particle theory fares better than the field theory. This effect comes about in the following way: How are the high-frequency contributions to various integrals of quantum electrodynamics that diverge in normal field theory calculations "damped" in the absorber theory? The de Sitter line element is used in the steady state theory (or its asymptotic form is used in the QSSC),

$$ds^2 = c^2dt^2 - \exp(2Ht)[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)] \tag{30}$$

can be written in a manifestly conformally flat form

$$ds^2 = (1 - H\tau)^{-2}[c^2d\tau^2 - dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)] \tag{31}$$

by a time transformation

$$H\tau = 1 - \exp(-Ht), \tag{32}$$

where H is the Hubble constant. The fact that $\tau < H^{-1}$ suggests that there is an event horizon in the future absorber. This property in turn tells us that the response of the future absorber is cut off at a frequency

$$k_{\max} = \omega_{\text{eff}}/HT, \tag{33}$$

where ω_{eff} is the effective frequency of future absorber and T is the time duration of the local process. For a medium with N as the number density of charged particles in the future absorber, ν the typical plasma frequency, and m the electron mass, one has

$$\omega_{\text{eff}} = [2\nu Ne^2/mH]^{1/2}, \tag{34}$$

where ν is defined in terms of the physical condition of the plasma. For a free electron, one may take $T = \hbar/mc^2$.

A cutoff at $k_{\max}$ results in a finite value for the observed mass of an electron in terms of its bare mass:

$$m_{\text{obs}} = m + \Delta m, \quad \Delta m = m \times [1 + (3e^2/2\pi)\ \ln(\hbar k_{\max}/mc^2)]. \tag{35}$$

For a free electron, the choice of $T \sim 10^{-21}$ s, $H \sim 3.10^{-18}\ \text{s}^{-1}$, and $\omega_{\text{eff}} \sim 80\ \text{s}^{-1}$ yields $\Delta m \sim 0.15m$. The important result is that radiative corrections are finite and no subtraction of one infinity from another is required. This cutoff necessarily results from the local-distant interaction that is characteristic of action at a distance.

8. EXPERIMENTAL SEARCH FOR ADVANCED POTENTIALS

If one assumes the above approach to electrodynamics to be valid, it follows that the nature of the accepted cosmological model should be consistent with the local experiments of electrodynamics. In particular, if the cosmological response is not such as to give pure retarded solutions, then it may be possible to detect advanced effects. There have been attempts to look for small advanced effects in local radiation experiments, although their interpretation itself is shrouded in controversy.

As we saw in earlier sections, no standard big bang cosmology satisfies the absorber condition to give unambiguous pure retarded solutions. It follows, therefore, that if one of these cosmologies is right then the pure retarded solution is untenable. Can it be that the incompleteness of future absorption would show itself through the presence of small fractions of advanced effects in local experiments?

Patridge (1973) attempted to detect such an effect in the radiation of a microwave source as it alternately radiated into free space and a local absorber. Patridge argued that advanced potentials lead to power gain rather than power loss in the source. Hence if a tiny fraction of radiation is via advanced potentials, the power drain from the source would be less than in the pure retarded case.

Patridge set up an arrangement in which radiation was blocked by a local absorber in one direction and was allowed to move freely in another. The argument was that the local absorber would ensure pure retarded effects, whereas the radiation into free space would travel long distances and through an incompletely absorbing universe. A switching arrangement allowed these possibilities alternately. Within the accuracy of the experiment (estimated at 1 part in 10^8) there was no difference in the two cases. Thus Patridge claimed to have found no evidence of advanced effects.

Subsequently, Heron & Pegg (1974) argued that Patridge's use of a static absorber would inevitably lead to a null result. Instead what they proposed was an experiment with a time asymmetric chopper absorber to alter the boundary conditions. This would allow them to alter the ratio of advanced to retarded components, leading to a possible detection of the former.

However, Davies (1975) has criticized the above approaches on the grounds that with proper inclusion of thermodynamics, attempts like these are bound to give null results. The objection raised by Davies goes in fact deeper than the specific issues relating to the proposed experiments. Davies has argued that one cannot bypass thermodynamics as proposed by Hogarth, Hoyle & Narlikar and that ultimately the thermodynamics asymmetry like that in the Boltzmann H theorem will have to be included in any realistic discussion of electrodynamic time asymmetry. In other words, Davies was reverting to the explanation of time asymmetry given by Wheeler & Feynman (1945) referred to in Section 2.

Although this could be a possible line of argument, it misses the entire spirit of the action-at-a-distance theory. First, it postulates ad hoc asymmetrical initial conditions that are basic to the H theorem. Second, once one decides to work within the action-at-a-distance framework the nonlocality of the problem forces one to take cognizance of the large scale structure of the universe, and the cosmological considerations of Sections 3 and 7 become relevant and unavoidable. The self-consistent mixture of advanced and retarded potentials is determined by including the response of the universe. Finally, rather than treat the statistical laws of thermodynamics as fundamental laws, attempts should be made to understand them as a consequence of other more fundamental arrows of time such as electrodynamics and cosmology.

Another aspect of the Patridge-type experiment relates to the deeper question of a relationship of thermodynamic and electrodynamic arrows to the expanding and contracting phases of a time-symmetric universe, such as the Friedmann model with $k = +1$. Do these time arrows reverse when the universe contracts? This question, so long as one sticks to the Wheeler-Feynman electrodynamics, is not uniquely answered, as we mentioned in Section 2. Recently, Gell-Mann & Hartle (1991) have discussed this problem in a different way. They investigate a way in which the rules of quantum mechanics might be adapted to impose a time symmetry on the boundary conditions. Thus when the universe enters the contracting phase, these microscopic degrees of freedom of the universe conspire to reverse the time-asymmetric processes. In a reanalysis of the Patridge experiment Davies & Twamley (1993) argue that its null result goes against the Gell-Mann-Hartle model but suggest that a more stringent test would be to repeat the Patridge experiment with a laser rather than a microwave antenna. This is because the universe is apparently transparent out to great distances at the GHz frequencies, and to include its absorptive effects along the future light cone in a more significant way much higher frequencies should be used.

There is one further hint of the possible role of the response of the universe in local phenomena, a role that takes us beyond electrodynamics. The discussions of quantum electrodynamics tell us that it is not proper to talk of a probability amplitude for a local microscopic system. The correct description of the physical behavior of the system follows from the probability calculation that includes the response of the universe. Thus one is dealing with a square of the amplitude type of expression rather than the amplitude itself.

This may explain the mystery that surrounds such epistemological issues such as the collapse of the wave function. What is missing from the usual discussion of the problem is the response of the universe. The wavefunction collapse represents the final course of action taken by the system consistent with the response of the universe. We suggest this idea as a way of understanding many other conceptual issues of quantum mechanics. It may well be that the real nonlocal hidden variables are contained in the response of the universe. For a detailed discussion of this idea see Hoyle (1982) and Narlikar (1993).

What has been the progress toward extending the action-at-a-distance formulation to other interactions? In the late 1960s Narlikar (1968) showed how to construct an action-at-a-distance counterpart for a field theory of arbitrary spin having a quadratic Lagrangian and linear field equations. Such a formulation will naturally have an absorber theory similar to the Wheeler-Feynman theory. Earlier

Narlikar (1962) had discussed an absorber theory involving neutrinos on lines similar to an absorber theory involving virtual photons assuming that the neutrinos travel with the speed of light and mediate in weak interactions.

9. CONCLUDING REMARKS

We thus have, in the case of electrodynamics and gravity, a close connection between the local and the distant parts of the universe. The connection is established through the Green's functions G, that is, the propagators of the interactions. These Green's functions establish the connection along the light cone and as such the action at a distance is not instantaneous. However, when one examines the details of the absorber theory of radiation in the electromagnetic case, it tells us that a signal from a local source at $r = 0$, sent out at $t = 0$, reaches a distant absorber particle at distance $r = R$ at $t = R/c$. The response of the absorber travels backward in time and so reaches the original source at $t = R/c - R/c = 0$, i.e., instantaneously. This mixture of advanced and retarded signals serves to create an instantaneous effect without violating relativistic invariance.

Does this, however, violate causality? The answer is yes, but at a very minute level. As was discussed by Dirac (1938), the radiation reaction formula has an advanced component, and this may be interpreted as generating pre-acceleration. Because, in this framework, one is looking at a self-consistent solution, this acausal effect is not disturbing. At the quantum level it may be more significant, as was pointed out by Hoyle & Narlikar (1995).

The quantum version of the Wheeler-Feynman theory involves an influence functional through which the local system interacts with the large-scale cosmological boundary conditions. This local + cosmological interaction appears as transition probability for a local system, wherein all cosmological variables are integrated out. Phenomena such as spontaneous transition or a collapse of the wavefunction are seen to arise from this interaction. This suggests that the attempts to explain some of these phenomena through local hidden variables have failed; the real clue to the mystery may lie in the response of the universe in the above fashion.

Experiments by Aspect et al. (1982a,b) inspired by Bell's inequality (1966) have generated considerable discussion on nonlocality of hidden variables, and apparent acausal effects across space-like separations. The response of the universe provides an additional factor that has been so far ignored in such discussions.

Coming to gravity and inertia, the ideas in Mach's Principle are capable of wider applications than thought earlier by Mach. One can use the Machian concept in electrodynamics where the response of the universe can play a key role in both classical and quantum electrodynamics. The action-at-a-distance framework used here is consistent with special relativity as well as with causality. The formalism can be used to give an expression to inertia as a direct long-range effect from the distant parts of the universe. From inertia one can arrive at a theory of gravity that is wider in its applications than general relativity. The theory can be extended to incorporate the cosmological constant and the concept of creation of matter without spacetime singularity. It leads to a viable cosmological model, known as the QSSC model.

Finally, the as yet unchartered territory for this framework lies in the direction of epistemological aspects of quantum mechanics, such as understanding the rationale behind the collapse of a wavefunction and the correlations found across spacelike separations by Aspect-type experiments.

These local $\leftrightarrow$ cosmological interactions occupied Fred Hoyle's interest right to the end. Although the bulk of our work on action at a distance was done in the 1960s, Fred liked to return to the topic from time to time, vide our work on the electron self energy problem in 1992–1993 and the genesis of the QSSC in 1993–1995. His autobiographical article in these *Annual Reviews* (Hoyle 1982) devoted a considerable part to the epistemological issues of quantum mechanics wherein the response of the universe to a local microscopic experiment played a crucial role in deciding its outcome.

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