Supply Side Story: Risks, Guarantees, Competition and Information Asymmetry*

Mehmet Gümüş
Desautels Faculty of Management, McGill University, Montreal, Quebec H3A 1G5
mehmet.gumus@mcgill.ca

Saibal Ray
Desautels Faculty of Management, McGill University, Montreal, Quebec H3A 1G5
saibal.ray@mcgill.ca

Haresh Gurnani
Department of Management, University of Miami, Coral Gables, Florida 33124
haresh@miami.edu

The risk of supply disruption increases as firms seek to procure from cheaper, but unproven, suppliers. We model a supply chain consisting of a single buyer and two suppliers, both of whom compete for the buyer’s order and face risk of supply disruption. One supplier is comparatively more reliable but also more expensive, while the other unreliable one is cheaper and faces higher risk of disruption. Moreover, the risk level of the unreliable supplier may be private information, and this lack of visibility increases buyer’s purchasing risk. In such settings, the unreliable supplier often provides a price and quantity (P&Q) guarantee to the buyer. Our objective is to study the underlying motivation for the guarantee offer and its effects on the competitive intensity and the performance of the chain partners. Our model also includes a spot market that can be utilized by any party to buy or to sell. The spot market price is random, partially depends on the available capacity of the two suppliers and has a positive spread between buying and selling prices. We analytically characterize the equilibrium contracts for the two suppliers, and the buyer’s optimal procurement strategy. First of all, our analysis shows that P&Q guarantee allows the unreliable supplier to better compete against the more reliable one by providing supply assurance to the buyer. More importantly, when information asymmetry risk is high, use of guarantee may enable the unreliable supplier to credibly signal her true risk, thereby improving visibility into the chain. This signal can also be used by the buyer to infer the expected spot market price. In spite of the above benefits, a guarantee offer in an asymmetric setting may not be always desirable for the buyer. Rather, it can reduce competition between the suppliers resulting in higher costs for the buyer.

Key words: Supply risk management; Asymmetric information; Signaling; Guarantees; Stochastic spot market

1. Introduction

As supply chains expand to new geographies to seek lower cost solutions, the risk associated with unproven suppliers has emerged as an issue of concern for top management (Aberdeen 2007). Among the strategies used to deal with this concern, a popular one is performance-based contracts between chain partners (Bernstein and de Vericourt 2008). While such contracts come in many forms, we focus on supplier guarantees in terms of price and quantity attributes (P&Q guarantee).

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These guarantees are contractual assurances from suppliers to provide a certain minimum quantity/capacity at a fixed price for the buyer. We observe variants of such contracts in commodity sectors (Stevenson 2006; Creti and Fabra 2007), in PCB industry (Heyes 2008), in fresh foods (USDA 2001) as well as in spare part supply services (Alstom 2009). A typical guarantee offer for commodity-type electronics items, e.g., memory, may resemble what follows (Wang 2010):

Supplier agrees to provide (—) amount of capacity upon (—) days notice to ensure an uninterrupted flow of goods to the buyer. Supplier will be responsible for all costs incurred by the buyer due to supplier’s insufficient capacity.... During the term of this agreement, the prices for the goods will be (—) and Supplier agrees to absorb all changes in costs (exceptions...)

That there is a need for effective strategies to deal with supply risk is undeniable. The increase in outsourcing to low-cost countries with unproven suppliers has exposed supply chains to the risk of disruptions. This can mainly be attributed to exogenous factors such as political uncertainty, natural disasters, financial breakdown, terrorism and strikes (e.g., recent earthquake in Japan (New York Times 2011) and financial crisis of 2008 (USA Today 2008)). The fragmentation of supply chains also gives rise to lack of visibility across the chain (i.e., information asymmetry), which exacerbates the supply risk. Recent surveys suggest that more than 40% of buyers lack visibility into their tier-1 suppliers, and it increases to 75% for tier-2 suppliers (IndustryWeek 2009).

Interestingly, so far, the literature has focussed on analyzing how buyer-led contracts are able to manage supply disruption risk and the associated information asymmetry. In contrast, we study the role of supplier-initiated contracts, such as P&Q guarantees, especially when there are multiple suppliers competing for the buyer’s order. Moreover, although such guarantees are used in practice, their system-wide effects have not yet been theoretically studied in the literature. The motivation of our research is to address these gaps. Specifically, we study the following research questions.

- What motivates an unreliable supplier, competing with other more reliable suppliers, to offer a P&Q guarantee? Under what conditions will she do so?
- How effective are P&Q guarantees in dealing with information asymmetry and supply risk? Especially, when and how do they provide the buyer visibility into the supply system?
- How does provision of a P&Q guarantee affect the competitive intensity among suppliers and the cost or profit performance of the chain partners? In particular, is the buyer always better off with this type of guarantee from an unreliable supplier?

To answer the above questions, we study the procurement strategy for a buyer who uses two competing suppliers to satisfy a fixed end-customer demand in a risk-neutral setting. Both suppliers face exogenous disruption risks in their supply processes. One of them (supplier $S_M$) is relatively
“more” reliable, although she also incurs higher marginal costs\(^1\). Moreover, the extent of her supply risk is public knowledge. The other supplier (supplier \(S_L\)) is cheaper than \(S_M\), but is “less” reliable because of higher disruption risk. In contrast to \(S_M\), \(S_L\)’s level of risk is private information for her. Our framework also includes a spot market exhibiting randomness in price driven by supply risk and which all three parties can access for buying/selling purposes. The two suppliers first compete horizontally to decide on their contract terms for the buyer. The buyer then decides on the optimal order allocation strategy. Both suppliers include their per-unit prices in the contracts. However, \(S_L\) may decide to also include an additional P&Q guarantee in her contract terms.

We develop four models on the basis of whether \(S_L\) offers a P&Q guarantee or not, and whether there is information asymmetry about the extent of risk facing \(S_L\) or not. We analytically characterize and compare the equilibrium strategies for the above models in order to answer our research questions. First, we show that when there is no information asymmetry, a P&Q guarantee from \(S_L\) does not affect the equilibrium order allocation strategy of the buyer or the expected costs or profits of the three chain partners. But, such a guarantee has a significant impact on the expected performance when information about \(S_L\)’s level of risk is not available to other parties. In that case, we establish that \(S_L\) uses the guarantee to credibly signal her true level of risk. Indeed, such guarantees may afford perfect visibility into the risk of the supply system and expected spot market price for the buyer, especially when i) \(S_M\) is highly reliable, or when ii) \(S_M\)’s risk is of the medium range while there is considerable uncertainty about \(S_L\)’s potential risk. The visibility helps the buyer in the former case; but, unfortunately, it may be harmful for him in the latter scenario. Specifically, a P&Q guarantee can then weaken competition between the suppliers, resulting in higher contract prices, and, hence, higher expected costs for the buyer. This also implies that, in an asymmetric setup, \(S_M\) may prefer to compete with a reliable (guarantee-offering) \(S_L\). We also show that P&Q guarantees could even increase the expected cost incurred by the total system.

2. Related Literature

Our research falls within the general theme of managing supply risk (refer to reviews by Tang 2006 and Vakharia and Yenipazarlı 2009). However, to the best of our knowledge, ours is the first one in the literature that studies supplier-initiated signaling contracts for a decentralized supply chain facing supply disruption risk. There are three streams of research directly related to our paper.

The first stream investigates guarantee contracts. Contracting has been an active area of research in the operations area (Cachon 2003). As regards the issue of supply guarantee contracts, there are papers dealing with guarantees about attributes like delivery times (Bernstein and de Vericourt

\(^1\) Capacity and inventory are synonymous in our setting; so, we use them interchangeably throughout the paper.
2008). However, studies related to P&Q-type guarantees in the presence of supply risk is rather sparse. The only papers on this topic are in economics domain and study assurances regarding supply or price, but not both, in the utility sector (e.g., Creti and Fabra 2007). But, their context is different from ours, and they do not model horizontal competition or information asymmetry.

The second relevant stream is related to exogenous supply risk. This stream started with the seminal paper by Karlin (1958) who investigated yield risk in agricultural sector. Subsequently, a number of papers have studied different facets of exogenous supply randomness (e.g., Ciarallo et al. 1994, Farmer 1994 and references therein). Note that all the above papers use centralized decision-making frameworks. Recently, Babich et al. (2007) extend this stream by modeling exogenous supplier default risk in a decentralized context. In their paper, multiple suppliers actively compete via wholesale prices for a retailer’s order, and the extent of risk faced by the suppliers is common knowledge. Evidently, we add to this literature stream by incorporating information asymmetry about $S_L$’s disruption risk in the model framework.

The most relevant stream for us is the third one that addresses the issue of information asymmetry among decentralized channel partners. In the supply chain contracting literature, prior work has examined information asymmetry in terms of supplier cost (e.g., Corbett et al. 2004; Cachon and Zhang 2006) or retail demand (e.g., Cachon and Lariviere 2001; Özer and Wei 2006). Our research is more closely associated with papers that model information asymmetry about supplier risk or reliability (see Gurnani and Shi 2006; Chaturvedi and Martinez-de Albéniz 2008; Tomlin 2009; Yang et al. 2009). Gurnani and Shi (2006) consider a bargaining approach where a buyer and a supplier have different estimates about supply reliability. In their model, the players do not update their beliefs and the contract terms reflect their relative beliefs about supply reliability. In contrast, Tomlin (2009) considers the case of a buyer who has forecast of a supplier’s yield distribution and analyzes a Bayesian model of supply learning for the buyer to evaluate the effects of learning on sourcing and inventory strategies. In Yang et al. (2009), the buyer designs a menu contract, and subsequently private information about supplier reliability is revealed through contract choices made by the supplier. Chaturvedi and Martinez-de Albéniz (2008) extend Yang et al. (2009) by also including supplier’s cost as private information. Note that Yang et al. (2009) as well as Chaturvedi and Martinez-de Albéniz (2008) use a screening approach to model the buyer’s contracting problem. A novel feature of our paper is that, since the P&Q contract is offered by the supplier ($S_L$), the resulting problem is a signaling game as opposed to a screening one. Moreover, both suppliers actively compete for the buyer’s order through their contract terms in our framework; such competition is less relevant in a screening contract. Note that a significant body
of economics literature examining signaling contracts has emerged since the seminal paper on job-market signaling by Spence (1973). We refer the readers to Riley (2001) for a detailed review. However, this stream of literature is not concerned with the issue of supply risk.

3. Model Framework

Our model framework involves a single-product supply chain consisting of one buyer, two suppliers ($S_M$ and $S_L$) and a spot market. One supplier ($S_M$) is well-known to the buyer as a supply source, while the other one ($S_L$) is new and unproven as far as the buyer is concerned. Moreover, the two suppliers and the buyer are all risk-neutral entities. Since our aim is to shed light on the effects of supply uncertainty, we assume that the end-consumer demand for the buyer is known to be $Q$, the selling price to end-consumers is constant, and the buyer must satisfy the entire demand.

One of the distinguishing features of our paper is that we capture the inherent difference between the two suppliers through three factors - their levels of risks (or equivalently, reliabilities), information available to the other chain partners about their risk levels, and their marginal costs for supplying the products. First of all, in our model setting, both suppliers face risks of disruption to their own supply systems. Because of this, although each can potentially access a maximum capacity of $Q$ units for this particular buyer, at the time of presenting the contract terms, each is uncertain about how much capacity would be actually available to them for use (if, and when, needed). In order to focus on the strategic interaction between the channel partners, we assume the supply disruption risk facing $S_M$ and $S_L$ to be exogenous. Furthermore, we also assume that there is information asymmetry about the risk associated with $S_L$. Specifically, suppose that the level of risk facing $S_L$ can be either high ($h$-type) or low ($l$-type); the exact type is known only to $S_L$ (private information), whereas both the buyer and $S_M$ only have a-priori probabilistic beliefs about the type\(^2\). We use superscript $\theta$ to denote the type of supply risk, and define $\epsilon^\theta_L$ as the random variable representing risk of type $\theta \in \{h, l\}$. The supply risk of $S_M$, which is independent of $S_L$’s risk, is represented by the random variable $\epsilon_M$ and the distribution of $\epsilon_M$ is common knowledge. In order to develop analytical managerial insights, we follow the recent supply risk literature (e.g., Babich et al. 2007, Yang et al. 2009, and references therein) in assuming that both supply uncertainties are of “all-or-nothing” types. That is,

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\epsilon_M = \begin{cases} 
0 & \text{with prob. } \alpha_M \\
Q & \text{with prob. } 1 - \alpha_M 
\end{cases}
\quad \text{and} \quad
\epsilon^\theta_L = \begin{cases} 
0 & \text{with prob. } \alpha^\theta_L \\
Q & \text{with prob. } 1 - \alpha^\theta_L 
\end{cases}
$$

(1)

where $\theta \in \{h, l\}$. The a-priori belief for the buyer and $S_M$ about the uncertainty level being of type $\theta$ is given by $r^\theta$, where $r^h + r^l = 1$ ($r^\theta$ and distribution of $\epsilon^\theta_L$ are common knowledge).

\(^2\) However, we assume that the ex-post effect of the disruption is observable by all parties.
Second, as far as the comparison of the level of risk between the two suppliers is concerned, we assume that $\alpha_L^h > \alpha_L^l > \alpha_M$, i.e., the $l$-type $S_L$ has a lower disruption risk than the $h$-type, and $S_M$ is known to be more reliable than even the $l$-type $S_L$. Lastly, the higher reliability of $S_M$ comes at a premium - $S_M$ incurs a marginal cost of $c_M$ for only those units she actually supplies to the buyer, whereas $S_L$ incurs $c_L(\leq c_M)$ per unit only for her exact supply quantity\(^3\).

Another important feature of our model is a spot market that can be accessed by the buyer and the two suppliers only for buying or selling purposes\(^4\). The price in this spot market is stochastic with two sources of randomness. The first comprises exogenous factors such as macroeconomic conditions. The price is also affected by an endogenous element - the actual amount of capacity available for possible use with the two suppliers, which is dependent on the realization of random variables $\epsilon_M$ and $\epsilon_L^\theta, \theta \in \{h,l\}$. Specifically, the greater the availability, the lower is the spot market price (refer to Kazaz and Webster 2010, and references therein for examples). We denote the spot market price by $p_S$, which is defined as follows:

$$p_S(\epsilon_M, \epsilon_L^\theta) = \bar{p}_S + (1 - \rho) \Delta_S + \rho \Delta_E(\epsilon_M, \epsilon_L^\theta), \quad (2)$$

where $\bar{p}_S$ is the expected spot market price in the absence of any endogenous randomness, $0 < \rho < 1$ measures how strongly the spot market price is correlated with the availability at the two suppliers, while $\Delta_S$ and $\Delta_E(\epsilon_M, \epsilon_L^\theta)$ represent the randomness due to exogenous and endogenous factors, respectively\(^5\). We assume that $\Delta_S$ has a distribution function $F_S$ and, without loss of generality, $E[\Delta_S] = 0$. In order to capture the inverse relationship between availability and the spot market price in an analytically tractable way, we assume that $\Delta_E(\epsilon_M, \epsilon_L^\theta)$ has the following functional form:

$$\Delta_E(\epsilon_M, \epsilon_L^\theta) = \begin{cases} +\Delta & \text{if } \epsilon_M + \epsilon_L^\theta = 0 \text{ (i.e., with prob. } \alpha_M \alpha_L^\theta) \\ 0 & \text{if } \epsilon_M + \epsilon_L^\theta = Q \text{ (i.e., with prob. } \alpha_M(1 - \alpha_L^\theta) + (1 - \alpha_M)\alpha_L^\theta) \\ -\Delta & \text{if } \epsilon_M + \epsilon_L^\theta = 2Q \text{ (i.e., with prob. } (1 - \alpha_M)(1 - \alpha_L^\theta)). \end{cases} \quad (3)$$

where $\theta \in \{h,l\}$ and $\Delta > 0$ is a measure of the extent by which the spot market price changes depending on the capacity availability. We can think of $\rho \Delta$ as an overall measure of the degree of dependence of the spot market on the amount of available capacity. In order to deter arbitrage

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\(^3\) We model supply uncertainty in the form of randomness in capacity availability (see Ciarallo et al. 1994). An alternative would be to model this uncertainty in the form of random production yield in which case the marginal costs for the two suppliers ($c_L$ and $c_M$) will be incurred for all units produced (rather than those delivered). We refer the readers to Wang et al. (2010) and Gurnani et al. (2010) for modeling and managerial implications of the latter framework. However, none of these papers consider the use of guarantees by the supplier to signal her reliability.

\(^4\) We do not allow trading in any type of financial derivatives in our model. Indeed, for certain products, e.g., memory, there does not exist any market to directly deal in such derivatives contracts (see Tevelson et al. 2007). However, we believe that the analysis of a framework incorporating such contracts is a worthwhile future research direction.

\(^5\) If the two suppliers are relatively big players in a concentrated industry, $\rho$ would likely be large. On the other hand, if all the firms in the industry are relatively small, then $\rho$ would also be small.
opportunities, we assume that there exists a positive spread ($\delta > 0$) between selling and buying prices in the spot market - the buying price is $\delta$ amount more expensive for all realizations of $\Delta_S$ and $\Delta_E$ (refer to Kazaz and Webster 2010, for more details). Lastly, in order to rule out trivial solutions, we assume that $\bar{\rho}_S$ is relatively high - specifically, $c_L < c_M < \bar{\rho}_S$. Two remarks are in order here. First, even though $\bar{\rho}_S$ is high, depending on the values of $\alpha_M$, $\alpha_L^h$ and $\Delta$, the expected spot market price and its specific realizations can either be higher or lower than the marginal supply costs of the suppliers. Second, since there is information asymmetry about $\alpha_L^h$, it implies that there is also information asymmetry between the chain partners about the spot market price.

Now that we have defined the characteristics of the chain partners and the spot market, we focus on the details of the contracting and allocation game. The basic contract terms for the two suppliers consist of per unit prices that they charge to the buyer. We also assume that supplier $S_L$ may provide a further P&Q guarantee as part of her contract terms in order to make it attractive to the buyer$^6$ (given the characteristics of $S_M$, such guarantees are not provided by her). The timing of decisions and events in our game is described as follows (also refer to Figure 1):

- Nature reveals the exact type of supply uncertainty - $h$ or $l$ - to supplier $S_L$.

- The two suppliers submit their contract terms simultaneously to the buyer. Note that $S_L$’s terms might be a function of $\theta$, her type.

- Based on the contracts, the buyer then (if possible) updates his a-priori belief about the level of risk facing $S_L$ and the spot market price. Suppose that the random variable representing the updated belief is given by $\hat{\theta}$, which is defined as follows: $S_L$ is of $h$-type with probability $\hat{\rho}^h$ and of $l$-type with probability $\hat{\rho}^l$, where $\hat{\rho}^h + \hat{\rho}^l = 1$. Note that $S_M$ cannot update her belief since she submits her contract simultaneously with $S_L$.

- The buyer makes the order allocation decision between the two suppliers ($q_L$ and $q_M$).

- The supply disruption uncertainties facing suppliers $S_L$ and $S_M$ as well as the uncertainty about the spot market price resolve, i.e., $\epsilon_L^h, \epsilon_L^l, \theta \in \{h,l\}$, and $p_S(\epsilon_M, \epsilon_L^\theta)$ realize.

- Suppliers $S_L$ and $S_M$ deliver to the buyer and are paid - the quantity delivered and the payment amount depend on the contract type as discussed below. Should $S_L$ offer a guarantee and have a shortfall, she would have to satisfy it by purchasing from the spot market at $p_S(\epsilon_M, \epsilon_L^\theta)$ per unit. If either supplier has capacity in excess of her requirement, she can sell it (if profitable) in the spot market at a price $p_S(\epsilon_M, \epsilon_L^\theta) - \delta$ per unit$^7$.

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$^6$We assume that the rule of law ensures $S_L$’s compliance with the guarantee contract. This is known as the “specific performance” in contract law, which is becoming increasingly common in practice (Plambeck and Taylor 2007).

$^7$We assume that both suppliers require significantly long delivery times, but the spot market can supply at a short notice. So, the buyer must order from the suppliers earlier and may use the spot market as and when needed.
• The buyer satisfies the end-customer demand $Q$. The buyer needs to satisfy shortfalls from the spot market at $p_S(\epsilon_M, \epsilon^L_S)$ per unit. Similarly, he can sell any quantity in excess of $Q$ that he receives from the suppliers in the spot market at $p_S(\epsilon_M, \epsilon^L_S) - \delta$ per unit.

Based on the above framework, we then develop two specific models that differ only in terms of whether or not supplier $S_L$ offers P&Q guarantee. Note that both the models account for asymmetric information. Their counterparts for the symmetric information setup (i.e., nature reveals $S_L$’s type to all chain partners at the beginning of the game) will be discussed in §4.

**Figure 1** Timing of events.

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**No-Guarantee Model:** The contract term for each supplier in this case only involves the per-unit price $- p^L_S$ for $S_L$ of type $\theta$ and $p_M$ for $S_M$, based on which the buyer updates his belief (if possible) about the risk associated with $S_L$, and then makes his order allocation decision. Each supplier will supply the minimum of the capacity available to her once the supply uncertainty resolves and the quantity ordered to her at price $p^L_S$ or $p_M$, for suppliers $S_L$ and $S_M$, respectively. The buyer is solely responsible for satisfying any shortfall relative to $Q$ from the spot market.

**Supply-Guarantee Model:** While $S_M$’s contract still consists of only the per-unit price $p_M$, the contract for $S_L$ now includes a P&Q guarantee. That is, the contract for type-$\theta$ $S_L$ now comprises three elements: a guaranteed quantity (say, $q^G_L \in [0, Q]$), a guaranteed price (say, $p^G_L$ per unit) and a per-unit price for any non-guaranteed amount $(p^L_S)$. To be more specific, $S_L$ guarantees supply of up to $q^G_L$ units at price $p^G_L$ per unit to the buyer, regardless of the realization of $\epsilon^L_S$; however, she charges $p^L_S$ per unit for units ordered exceeding $q^G_L$ (also, she does not guarantee supply in excess of $q^G$). Based on the two contracts, the buyer then updates his belief (if possible) about the level of supply risk facing $S_L$ and makes the order allocation decision.

The buyer’s payment to $S_L$ will now depend on his order quantity - if $q_L$ is less (resp., greater) than $q^G_L$, then he pays $p^G_L$ per unit (resp., $p^L_S$ per unit for the first $q^G_L$ units and $p^L_S$ per unit for the rest). The buyer satisfies any shortfall from the spot market. Lastly, in contrast to the no-guarantee model, if the realized capacity available for use is less than $\min\{q^G_L, q_L\}$, then $S_L$ procures $[\min\{q^G_L, q_L\} - \epsilon^L_S]^+$ units from the spot market in order to satisfy the guarantee.
Before we proceed to a detailed analysis of the asymmetric information models, it is worthwhile understanding the effectiveness of P&Q guarantees in dealing with supply risk by analyzing a symmetric information setting in the presence of supply uncertainty.

4. Role of Supply Guarantees in Symmetric Information Setting

In this section, we analyze the no-guarantee and supply-guarantee models of § 3 under the assumption that when nature reveals the type of uncertainty \( \theta \) confronted by supplier \( S_L \), it does so to all three parties; all other details remain the same. This means that \( \theta \) can be \( h \) or \( l \), but this information is common knowledge. Obviously, this precludes any need on the part of the buyer to update his belief about whether \( \theta \) is \( h \) or \( l \). For this information setting, we first characterize the equilibria for no-guarantee and supply-guarantee models, and then compare them to show how a P&Q guarantee from \( S_L \) affects the decisions/performance of the supply chain partners.

Equilibria Characterization for No-Guarantee and Supply-Guarantee Models

As regards the no-guarantee model, recall that the buyer bears the disruption risk and is responsible for satisfying any shortfall, relative to \( Q \), from the spot market. We solve the problem using backward induction, starting from the buyer’s optimal (cost-minimizing) allocation decision. Once the optimal allocation decision is characterized, we substitute it into \( S_L \) and \( S_M \)’s profit functions, and then solve the simultaneous price game between the two suppliers. The overall equilibrium characterization is provided in Proposition 1.

In the supply-guarantee model, for an order quantity of less than or equal to \( q^\theta_G (\in [0, Q]) \) for type \( \theta \) to \( S_L \), the risk for that part is borne solely by \( S_L \); but, if it is more than \( q^\theta_G \), the risk is shared by the buyer and \( S_L \), even for the part ordered to \( S_L \). We again solve the problem using backward induction approach, and obtain the equilibrium characterization as shown in Proposition 1 below. Note that the detailed proofs for all propositions are provided in the Appendix.

Proposition 1. The unique equilibrium allocation, contract terms and expected costs/profits for the channel partners in the no-guarantee and supply-guarantee scenarios under symmetric information are presented in Figure 2 and in Table 2.\(^8\)

When both suppliers are quite reliable (i.e., Region \( S_\theta \)), they can charge low contract prices for both no-guarantee and supply-guarantee contracts, and \( S_L \) can offer full guarantee. However, this scenario also indicates to the buyer that the probability is quite high that the total capacity available in the market will be large (=2\( Q \)), and, hence, the expected spot market price will be low. Consequently, the buyer actually opts to buy only from the spot market: we term this a spot-sourcing scenario. As the two suppliers become more risky, the expected price in the spot market

\(^8\)Tables 2-5 are provided in the Appendix.
increases, and then the buyer orders from the suppliers rather than the spot market. The buyer opts for sole-sourcing from $S_M$ when she is relatively more reliable than $S_L$, but not too much more expensive (Region $M_θ$). In this region, for both contracts, $S_M$ sets her price such that the expected cost for the buyer in buying from $S_M$ is infinitesimally less than his expected cost of buying from $S_L$. Obviously, as the difference in the risk levels between the two suppliers decreases (i.e., $α_M$ increases) and/or the cost premium for $S_M$ increases (i.e., $c_M - c_L$ increases), the buyer opts to sole-source from $S_L$ (Region $L_θ$). Lastly, when both suppliers are quite risky (Region $LM_θ$), i) $S_L$ does not offer any guarantee; and ii) the buyer, keeping in mind the increased supply risk, decides to dual-source by ordering $Q$ units from both suppliers.

Note that, in Figure 2, we present all the possible regions. Obviously, the presence and size of these regions depend crucially on the system parameters. For example:

- As the spot market price becomes less dependent on the available capacity and more dependent on exogenous factors (i.e., as $ρΔ$ decreases), then the spot-sourcing region decreases. For sufficiently low $ρΔ$, this region totally vanishes.

- When the spread in the spot market price is high (i.e., high $δ$), the dual-sourcing region is small (since the buyer does not want to sell the excess $Q$ units back to the spot market and incur extra costs), while the two sole-sourcing regions are relatively large, and vice versa.

- As the marginal cost for $S_M$ (resp., $S_L$) increases, it results in less dual-sourcing and more spot-sourcing as well as more sole-sourcing from $S_L$ (resp., $S_M$).

**Effects of Supply Guarantees**

Table 2 clearly reveals that, while the equilibrium contract parameters might be different under supply-guarantee and no-guarantee scenarios, the equilibrium order allocation and expected costs/profits for the channel partners are exactly the same for both cases. So, in that respect, when

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9 Throughout the paper, we use increase/decrease and higher/lower in the weak sense, unless otherwise specified.
there is no information asymmetry about the disruption risks facing the suppliers, P&Q guarantees do not play a significant role in our setting. However, if we consider the uncertainties associated with supplies/costs of the buyer, then the two contracts may be different. For example, the buyer’s supply as well as his costs are uncertain for all the regions under a no-guarantee contract, while under a guarantee contract, his costs and supply become risk-free in the $L_\theta$ region.

5. Supply Guarantees in Asymmetric Information Setting

We now focus on analyzing the two asymmetric information models developed in § 3. So, in this section, nature reveals whether the supply risk $\theta$ is of type $h$ or $l$ only to $S_L$. This private information is not known to the buyer or $S_M$, both of whom only have a-priori probabilistic beliefs about $\theta$, given by $r^\theta$ ($r^h + r^l = 1$). Note that, since the exact distribution representing the randomness of the spot market price depends on $S_L$’s type, only she can determine the exact distribution; the other two chain members face information asymmetry about it. We analyze the no-guarantee and supply-guarantee models in § 5.1 and § 5.2, respectively, and compare them in § 6.

Recall that the random variable $\epsilon_\theta^L$ representing supply risk for $\theta$-type $S_L$ is defined as follows: $\epsilon^0_L = 0$ with probability $\alpha^0_L$ and $\epsilon^Q_L = Q$ with probability $(1 - \alpha^0_L)$, for $\theta \in \{h,l\}$ ($r^\theta$ and the distribution of $\epsilon^\theta_L$ are common knowledge). Moreover, the supply risk for $S_M$ is $\epsilon^Q_M = 0$ with probability $\alpha^Q_M$ and $\epsilon^Q_M = Q$ with probability $(1 - \alpha^Q_M)$. Lastly, $\alpha^h_L > \alpha^l_L > \alpha^Q_M$. For the buyer and $S_M$, the “expected” default risk for $S_L$ is given by $\bar{\alpha}_L = r^h \alpha^h_L + r^l \alpha^l_L$. As discussed in § 3, $S_L$’s contract terms may help the buyer update his belief about $\theta$. The updated beliefs about the probability of supplier $S_L$ being $h$ or $l$ are $\hat{r}^h$ and $\hat{r}^l$, respectively (where $\hat{r}^h + \hat{r}^l = 1$).

5.1 No-Guarantee Model

In this case, both suppliers submit their unit contract prices, $p_\theta^L$ and $p_M$, based on which the buyer makes the allocation decision. In the next proposition, we present the equilibrium contract decisions and expected costs/profits of the channel partners for this model\(^{10}\).

**Proposition 2.** In the no-guarantee, asymmetric information scenario, the unique (pure-strategy) equilibrium is of the pooling type where both $h$- and $l$-type supplier $S_L$ charge the same price. That is, $p^h_{L^*} = p^l_{L^*}$. The unique equilibrium allocation, contract parameters and expected costs/profits for the channel partners are shown in Figure 3 and Table 3.

Interestingly, the above proposition implies that when the risk of supply disruption is private information for $S_L$, she cannot credibly signal her type (i.e., $h$ or $l$) to the buyer in the no-guarantee

\(^{10}\)The strategies and beliefs constitute an equilibrium in an asymmetric information setting only if i) for each $\theta$-type, the contract terms solve the Nash game between the two suppliers, given the buyer’s allocation strategy; ii) buyer’s updated beliefs can be derived from suppliers’ equilibrium strategy using Bayesian updating; and iii) the (overall) order allocation decision minimizes the buyer’s expected cost, given his updated beliefs and suppliers’ contracts.
scenario. Consequently, the buyer does not have visibility into the severity of supply risk that $S_L$ is facing and his allocation decision does not differentiate between $h$ and $l$ types. Note that, in this model setting, $S_L$ does not face any risk in terms of marginal supply cost though she faces availability risk. Consequently, a separating equilibrium cannot be on the equilibrium path, since $h$-type can increase her profit by mimicking the $l$-type if they charge different prices.

As noted before, part of the supply risk in the no-guarantee setting is always borne by the buyer who needs to use the spot market if $S_L$ and/or $S_M$ are unable to deliver the order. Since the buyer cannot separate the two $S_L$ types, the allocation decision is driven by the expected default risk of $S_L$ (i.e., $\hat{\alpha}$). The buyer uses $\hat{\alpha}$ to compare $S_L$’s reliability and cost to those of $S_M$ as well as to calculate the expected spot market price. Assuming $\hat{\alpha}$ as the reliability of an average-type $S_L$, the equilibrium contract and allocation can be determined by comparing the buyer’s expected cost of buying from the two suppliers - “average” $S_L$ and $S_M$ - and the spot market. Qualitatively speaking, the equilibrium allocation is quite similar to the one in § 4. For example, when both suppliers are relatively more reliable, the optimal allocation strategy is spot-sourcing (Region 1a), when both are very risky, it is dual-sourcing (Region 3b), and in other sole-sourcing regions (Regions 1b, 2a, 2b, and 3a), it depends on the relative values of the reliabilities and costs of the available options.

### 5.2 Supply-Guarantee Model

In this section, we focus on understanding how a P&Q guarantee from $S_L$ affects the equilibrium characterization and whether it is able to deal with information asymmetry in the chain. The analysis turns out to be quite complicated since there are two issues involved - whether $S_L$ should offer guarantee (if so, how much) and whether the equilibrium will be a pooling or a separating one.
Figure 4  Equilibrium regions for supply-guarantee scenario when there is information asymmetry.

\[(A) \alpha_M < \gamma_M \quad \text{(B)} \gamma_M \leq \alpha_M \leq \bar{\gamma}_M \quad \text{(C)} \alpha_M > \bar{\gamma}_M\]

The equilibrium contract parameters, order allocation and expected costs/profits for each area in (A), (B) and (C) can be seen from the corresponding column in Tables 4, 5 and 3, respectively. Note that for $\alpha_M \geq \bar{\gamma}_M$, equilibrium no-guarantee and guarantee cases are the same and so we refer to Table 3 for equilibrium values.

However, we are able to exactly characterize the equilibrium decisions and expected costs/profits of the channel partners associated with different parameter ranges as shown below.

**Proposition 3.** The type of equilibrium and the equilibrium level of the guarantee for different parameter ranges are characterized below.

- **It is possible to have either a pooling or a separating equilibrium. Specifically:**
  - When $\alpha_M$ is low ($\alpha_M < \gamma_M$), the unique equilibrium is of the separating type.
  - When $\alpha_M$ is medium ($\gamma_M \leq \alpha_M \leq \bar{\gamma}_M$), the unique equilibrium is of the pooling type if $h$- and $l$-type suppliers’ default risks - $\alpha^h_L$ and $\alpha^l_L$, respectively - are close to each other and both are not very different from $\alpha_M$; otherwise, it is of the separating type if either $\alpha^h_L$ and $\alpha^l_L$ are substantially different from each other or if they are close to each other but both are quite different from $\alpha_M$.
  - Lastly, when $\alpha_M$ is high ($\alpha_M \geq \bar{\gamma}_M$), the unique equilibrium is always of the pooling type.

- **As for the level of guarantee, both $h$- and $l$-type suppliers offer full guarantee (i.e., $Q$) on the equilibrium as long as $\alpha_M$ is low or medium, and offer no guarantee when $\alpha_M$ is high.**

The unique equilibrium allocation, contract terms and expected costs/profits for the channel partners for the above three scenarios in the supply-guarantee case are shown in Figures 4(A), 4(B), and 4(C) and Tables 4, 5 and 3, depending on whether $\alpha_M$ is low, medium or high, respectively.\(^{11}\)

The main insight of the above proposition, which reveals an interesting feature of a P&Q guarantee in an asymmetric information context, is the following:

\(^{11}\) In the separating case, both pure- (Regions 2c and 2d) and mixed-strategy (Region 2b) equilibria are possible. In the pooling case, the equilibria are always in pure strategy.
• P&Q guarantees may enable SL to eliminate informational asymmetry by credibly signaling private information about the type of supply risk - high (h) or low (l) - she is facing to the buyer. This contrasts sharply with the no-guarantee scenario of § 5.1 where SL is not able to signal her true level of risk (refer to Proposition 2).

The main reason why guarantees provide signaling opportunities is that they create cost differential between h- and l-type suppliers. By offering a guarantee, SL internalizes the cost associated with disruption risk. Since she is responsible for satisfying the order for the guaranteed units, the expected cost per unit for l-type is then less than that for h-type. The cost difference manifests itself in the form of different guaranteed prices quoted by the two types. This enables the low-risk l-type to separate herself from the high-risk h-type, and the buyer to differentiate between them.

Although SM is not privy to SL’s price signal, she knows that the buyer might be able to use it to differentiate between h- and l-types. This affects SM’s contract price compared to the no-guarantee setting. In the latter scenario, SM is competing with the buyer’s cost of buying from an “average” SL (= ̅kL) or from a spot market based on an “average” SL (= ̅kS). However, because of the buyer’s ability to differentiate between SL types in the guarantee scenario, SM needs to decide whether to compete against an l-type supplier, i.e., kL, or against an h-type one, i.e., kh (or against corresponding spot markets, kS and kH, respectively). The main trade-off is that undercutting a l-type SL will surely win SM the order allocation but at the cost of a lower margin (volume strategy), while undercutting a h-type one will earn her significant margins when she wins the order, but she loses the allocation when SL is of l-type (margin strategy).

Based on the above discussion, we can then better understand the equilibrium signaling, guarantee and order allocation strategies in the different regions of Proposition 3.

• Low-risk supplier SM (αM < γM - Figure 4(A), Table 4): Since SM is highly reliable, SL provides full guarantee in order to compete for allocation in this region. In spite of this, the buyer never procures from SL - allocation goes either to SM or to the spot market. But, guarantee offers result in a signaling equilibrium, and the buyer uses the price signal to determine SL’s type and the associated expected spot market price that he is going to face. When reliabilities of both SL types are comparable to SM (Region 1a), the buyer opts for spot-sourcing (since the spot market price would most probably be low). For SM, as discussed above, she needs to decide which strategy - volume or margin - to adopt. When she thinks that SL is most probably of h-type and l-type is quite reliable (high rh and low αl, Region 1b), SM chooses the margin strategy. But, if rl and αl are high (Region 1c), it makes more sense for SM to use the volume strategy. The point to note in this region is that the signal from SL primarily affects the contract price for SM by conveying information about the expected spot market price to the buyer, but it never enables even l-type SL to get any allocation (due to her higher price compared to SM or the spot market).
- **Medium-risk supplier** $S_M$ ($\gamma_M \leq \alpha_M \leq \bar{\gamma}_M$ - Figure 4(B), Table 5): $S_L$ again provides full guarantee in order to compete with $S_M$. When both $S_L$ types are quite reliable (Region 2a), in spite of their cost difference due to guarantee offers, both $S_L$ types can charge a price such that the buyer’s cost in buying from them is infinitesimally lower than buying from $S_M$. This results in a pooling equilibrium in Region 2a, and only $S_L$ wins the order. But in other regions, guarantee-induced difference in supply costs results in signaling equilibria. When the two $S_L$ types are quite unreliable (Region 2d), $S_M$ can always win the order by undercutting $l$-type $S_L$. However, Regions 2b and 2c are more interesting. In contrast to the scenario in Region 2d, lower price of $l$-type now enables $S_L$ to compete with $S_M$ for allocation. When $l$-type is relatively less reliable (Region 2b), she and $S_M$ employ a mixture of volume and margin strategies. Specifically, the prices are set such as to eliminate $h$-type from the buyer’s allocation consideration. When $S_L$ is of $h$-type, $S_M$ gets the allocation; otherwise, the allocation goes to whoever ($l$-type or $S_M$) quotes the price that incurs lower costs for the buyer. But, as $l$-type becomes more reliable, the buyer can use the signal about $S_L$’s type to infer that the expected spot market price will be quite low. This reduces the maximum price that $l$-type and $S_M$ can charge in the mixed strategy. In fact, when $l$-type is sufficiently reliable (Region 2c), $S_M$ may decide to no longer use the mixed strategy. Rather, she adopts a pure margin strategy by setting her price to ensure allocation when $S_L$ is of $h$-type, and concedes allocation to $S_L$ when $S_L$ is of $l$-type. So, for medium-risk supplier $S_M$, the signal not only reveals the expected spot market price to the buyer, but it also allows $l$-type $S_L$ to compete with $S_M$ and get order allocation.

- **High-risk supplier** $S_M$ ($\alpha_M > \bar{\gamma}_M$ - Figure 4(C), Table 3): Since $S_M$ is not very reliable in this scenario, $S_L$ does not provide any guarantee. Consequently, the effective supply costs for both $h$- and $l$-types are the same, and they charge the same price in equilibrium (i.e., pooling equilibrium). The buyer then makes the allocation decision by comparing an “average” $S_L$ and $S_M$. Note that, since both suppliers are quite risky, the expected spot market price is high and so the buyer never uses it. The equilibrium decisions and costs/profits are then exactly the same as in the $\alpha_M > \bar{\gamma}_M$ scenario of the no-guarantee model (Figure 3(C), Table 3).

In summary, when there is information asymmetry, $S_L$ can, under certain scenarios, effectively use P&Q guarantees to credibly signal her type ($h$ or $l$) to the buyer and eliminate informational asymmetry. The larger the spread in the spot market ($\delta$) and/or the dependence of the spot market price on the amount of supply available ($\rho \Delta$) and/or the marginal supply cost of $S_L$ ($c_L$), the greater the potential regions where such signals will be provided. In contrast, as $S_M$ becomes quite risky, such signaling becomes less likely. Moreover, how the signal is used also depends on the system parameters. When $S_M$ is quite reliable, it is mainly used by the buyer to infer the expected
spot market price. However, when $S_M$’s risk is somewhat higher and the risk difference between the two $S_L$ types is also quite high, the signal is also used by the $l$-type $S_L$ to garner order allocation.

6. Effects of Supply Guarantee on Chain Performance

In this section, we compare the equilibrium expected profits/costs of the three chain partners and the total supply chain efficiency under no-guarantee model in § 5.1 with those of the supply-guarantee model in § 5.2 in order to understand the effects of a P&Q guarantee when there is information asymmetry. For this, we need to compare the different regions in § 5.1 with those in § 5.2. Since some of the regions in the two do not match when we overlap them, we end up with four comparison regions for $\alpha_M < \bar{\gamma}M$ and six regions for $\bar{\gamma}M \leq \alpha_M \leq \check{\gamma}M$. Note that, for $\alpha_M > \check{\gamma}M$, the equilibrium no-guarantee and guarantee models are equivalent; so supply guarantees do not have any effect in that range (refer to § 5). Our detailed analysis results in the following.

**Proposition 4.** Effects of supply guarantee on supply chain partners’ profits and costs as well as on total supply chain efficiency are fully characterized in Figure 5.

The main takeaway from Proposition 4 is that, in contrast to the symmetric case (discussed in §4), a P&Q guarantee has a significant impact on the expected costs/profits when there is information asymmetry about $S_L$’s risk level. Moreover, somewhat counter-intuitively, we show that the buyer and the total chain can be worse off and $S_M$ better off with a guarantee provision under certain conditions. In order to better understand the underlying reason behind this behavior, we discuss the above proposition in more detail below starting with the effects on individual parties.

- **Low-risk supplier $S_M$ ($\alpha_M < \bar{\gamma}M$):** In this scenario, $S_L$ does not get an order allocation in either the no-guarantee or the guarantee model; so, her profits are zero under both cases. $S_M$ also does not obtain an allocation under either model in Region (1a, 1a). But, in other regions, the two models result in different profits for $S_M$. Recall that, in a no-guarantee setting, $S_M$ is always competing against an “average” $S_L$ based spot market ($\bar{k}_S$). However, because of the price signal, in a guarantee setting the buyer can exactly know $S_L$’s type. This forces $S_M$ to decide whether to adopt a margin strategy against a $h$-type based spot market ($k^h_S > \bar{k}_S$) or to adopt a volume strategy against a $l$-type based spot market ($k^l_S < \bar{k}_S$). In Regions (1a, 1b) and (1b, 1b), $S_M$ opts for the former (i.e., the margin strategy) since she is very much convinced that $S_L$ is of type $h$. This results in guarantee contracts generally enhancing $S_M$’s profit because of the richer margins and/or higher chances of order allocation (compared to the no-guarantee model). But, she is worse off in high $\alpha_L^l$ parts of Region (1b, 1b) because of loss of allocation. On the other hand, in Region (1b, 1c), $S_M$ decides to follow the volume strategy because she then strongly believes that $S_L$ is of
Figure 5  Effects of supply guarantee on supply chain partners’ profit/costs when there is information asymmetry.

<table>
<thead>
<tr>
<th>Regions</th>
<th>h-type $S_L$’s profit</th>
<th>l-type $S_L$’s profit</th>
<th>Supplier $S_M$’s profit</th>
<th>Buyer’s cost</th>
<th>Total SC Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1a,1a)</td>
<td>$NG = G$</td>
<td>$NG = G$</td>
<td>$NG = G$</td>
<td>$NG = G$</td>
<td>$NG = G$</td>
</tr>
<tr>
<td>(1a,1b)</td>
<td>$NG = G$</td>
<td>$NG = G$</td>
<td>$NG &lt; G$</td>
<td>$NG = G$</td>
<td>$NG &lt; G$</td>
</tr>
<tr>
<td>(1b,1b)</td>
<td>$NG = G$</td>
<td>$NG = G$</td>
<td>$NG \leq G$ if $\alpha_L \leq \gamma$</td>
<td>$NG = G$</td>
<td>$NG \leq G$ if $\alpha^* \leq \gamma$</td>
</tr>
<tr>
<td>(1b,1c)</td>
<td>$NG = G$</td>
<td>$NG = G$</td>
<td>$NG &gt; G$</td>
<td>$NG = G$</td>
<td>$NG &gt; G$</td>
</tr>
</tbody>
</table>

In (A) and (B), NG and G refer to no-guarantee and supply-guarantee models, respectively, and the first element in the “Regions” column refers to the regions in Table 3 and the second element refers to the regions in Tables 4 and 5. The expressions for $\Delta^B$, $\Delta_L$, and $\Delta_{TC}$ are provided in the Appendix. Since equilibrium no-guarantee and supply-guarantee models are equivalent for $\alpha_M > \gamma_M$, that range is not included in the above figure.
type $l$. This ensures that she always gets the allocation. But, $S_M$ then prefers a no-guarantee scenario since it allows her to charge a relatively high price as well as always win the allocation.

For the buyer, the expected costs remain the same under both models for all regions, except $(1b, 1c)$. In Region $(1b, 1c)$, the buyer actually prefers the guarantee contract since it forces $S_M$ to compete with a $l$-type spot market price ($k_S^l$) rather than an “average”-type spot market price ($= \bar{k}_S > k_S^l$), resulting in lower prices for the buyer.

- Medium-risk supplier $S_M$ ($\gamma_M \leq \alpha_M \leq \bar{\gamma}_M$): As expected, the performance of $h$-type $S_L$ deteriorates with guarantee offers since she cannot anymore pose as an “average” supplier. By the same token, a guarantee offer normally improves the performance of an $l$-type. In Regions $(2b, 2b)$ and $(2b, 2c)$, it enables her to signal the true type and get order allocations, where she did not get any in the no-guarantee scenario. As discussed in § 5.2, a guarantee can also be used to deter $h$-type from getting any allocation. This reduces the degree of competition at the upstream level, which in turn, enables $l$-type (and also $S_M$) to charge a premium price (e.g., in upper part of Region $(2a,2b)$). However, when $l$-type is quite reliable, the guarantee signal may indeed hurt her (e.g., in lower part of Region $(2a, 2c)$). In that case, the buyer correctly infers that there will probably be plenty of available capacity, and, hence, that the expected spot market price will be low. This forces $l$-type to charge a very low price to gain allocation resulting in her profit under a guarantee contract being lower than the no-guarantee model.

Interestingly, the signaling ability of a guarantee contract may actually hurt the buyer and help supplier $S_M$. Specifically, when the signal is used to deter $h$-type and allows both suppliers to charge a reliability premium, $S_M$ can actually benefit from a guarantee contract to the detriment of the buyer (e.g., Regions $(2a,2b)$, $(2a,2c)$). However, when the signal provided by a P&Q guarantee is used by the buyer to deduce the spot market price but the reliability premium is not very high (compared to the no-guarantee case) or is eliminated completely by $S_M$’s volume strategy, guarantees then help the buyer and decrease $S_M$’s profit (e.g. Region $(2b,2d)$).

To summarize, the buyer is better off with guarantees when $S_M$ is quite reliable and/or when both $S_L$ types are known to be quite risky. On the other hand, he favors a no-guarantee setting when $S_M$’s risk level is medium and there is considerable uncertainty about the risk level of $S_L$ (and vice versa for $S_M$). Moreover, as the spread in the spot market price ($\delta$) and/or the dependence of the spot market price on market supply ($\rho \Delta$) increases, the area where the buyer prefers a guarantee contract also increases. Lastly, while guarantees may increase the buyer’s expected cost, by making the buyer’s supply risk-free, it (at least weakly) reduces his cost uncertainty.

Another issue of interest is how a P&Q guarantee affects the total supply chain efficiency (i.e., sum of all chain partners’ expected profits). Given that, in our context, the buyer’s revenue from
selling to end consumers is fixed irrespective of the model setting, this implies investigating the effect of guarantees on the sum of supply costs for \( S_L \) and \( S_M \) and any spot market costs for the buyer and \( S_L \) (the higher the total cost, the lower the efficiency is). As is evident from the rightmost panel of Figure 5, total supply chain efficiency may deteriorate with a P&Q guarantee (although mostly it improves). This result is also driven by the fact that in a no-guarantee setting, \( S_M \) always competes with an “average” \( S_L \), while in a guarantee model, \( S_M \) needs to decide whether to follow a volume strategy or a margin strategy. In the areas where P&Q guarantees reduce efficiency, ideally (from a supply chain efficiency viewpoint), \( S_M \) should select the volume strategy. However, \( S_M \)’s own profit-maximizing strategy is to charge a relatively higher price even at the cost of allocation, i.e., to select the margin strategy. This incentive misalignment due to decentralized decision-making increases the supply chain cost, thus reducing its efficiency.

7. Conclusions and Implications

Various forms of supplier guarantees are used in procurement contracts. We focus on studying a particular one where the guarantee is in terms of price and quantity (P&Q guarantee). Our analysis provide the following insights into the causes and effects of P&Q guarantees:

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Summary of results</th>
</tr>
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<tbody>
<tr>
<td><strong>What is the role of the guarantee?</strong></td>
<td>To provide supply assurance to the buyer (symmetric and asymmetric info), and to signal the default risk of ( S_L ) (only asymmetric info)</td>
</tr>
<tr>
<td><strong>When is the guarantee offered?</strong></td>
<td>( S_M )’s default risk is low or medium (symmetric and asymmetric info)</td>
</tr>
<tr>
<td><strong>How is the information conveyed through the guarantee used by the buyer?</strong></td>
<td>To deduce the expected spot market price and to make allocation decision (asymmetric info)</td>
</tr>
</tbody>
</table>
| **What is the impact of the guarantee on expected costs/profits?** | - No impact on the expected costs and profits (symmetric info)  
- In general, buyer and \( l \)-type are better off while \( S_M \) and \( h \)-type are worse off; but, the buyer can be worse off, while \( S_M \) can be better off due to reduced upstream competition (asymmetric info) |

Guarantees do not play a significant role in the symmetric information case as the allocation decision for the buyer and the expected performance of the chain partners are unaffected. The equilibrium allocation can be sole-, dual- or spot-sourcing depending on the relative costs and reliabilities of \( S_L \) and \( S_M \) and the spot market price. Note that \( S_L \) does not offer a guarantee only when the default risks for both suppliers are quite high. In this case, it is likely that the spot market price would be high, which increases the costs associated with offering a guarantee. Moreover, guarantees are then not a competitive necessity for \( S_L \). The main effect of guarantee in the symmetric information case is its role in providing supply and cost assurances to the buyer.

On the other hand, information asymmetry regarding \( S_L \)’s default risk brings to light more interesting facets of the guarantee contract. First, in the no-guarantee scenario, \( S_L \) is not able
to credibly signal her default risk to the buyer. A guarantee contract may allow her to do so by differentiating between the two risk levels through the guaranteed price. The buyer can then make the allocation decision based on perfect visibility into the supply system. So, P&Q contracts then act both as a *supply assurance as well as a signaling device* (but only when $S_L$ is able to signal her type). A signaling guarantee becomes more likely when the spread in the spot market, and/or, dependence of spot market price on the supply available, and/or marginal supply cost of supplier $S_L$ is high. Conversely, it is less likely when both suppliers have high default risks.

One interesting feature of the guarantee offer is on how the resulting signal is used. When $S_M$ is quite reliable, the signal is used to communicate the expected spot market price to the buyer. In that case, guarantees generate lower costs for the buyer and lower profits for $S_M$. But, when $S_M$ is somewhat risky and the two $S_L$ types have quite different default risks, the $l$-type $S_L$ uses the guarantee to get allocation from the buyer. Guarantees might then actually reduce the competitive intensity between the two suppliers. Consequently, both of them can charge the buyer a premium. So, interestingly, guarantees can then increase costs for the buyer, and be beneficial for $S_M$. Finally, even the total supply chain efficiency may worsen under a guarantee contract.

Our paper is among the first in the literature to consider supplier-led initiatives to signal their capability to buyers in order to gain market share. In this context, the results have a number of managerial implications. First, it shows that when $S_M$ is somewhat risky, higher level of information asymmetry between $S_L$ and the buyer (i.e., higher $(\alpha^h - \alpha^l)$) provides more incentive to $S_L$ to provide visibility through supply guarantees. This suggests that supply guarantees can be used as an effective tool in the early stages of procurement relations when visibility into supplier reliability is likely to be low.

Our results also reveal that buyers may need to be wary about supply guarantees due to the cost associated with information rent to gain visibility via such guarantees. The amount of rent depends crucially on the level and/or likelihood of maximum risk associated with $S_L$ (i.e., $\alpha^h$ and/or $r^h$); the higher these are, the more the buyer is likely to pay to gain visibility. This does not necessarily mean that buyers should not accept guarantee offers. Indeed, such offers can help reduce the buyer’s risk as $S_L$ picks up some of it. Hence, while guarantees in the asymmetric case can be costly for the buyer in the short term, they may pay off in the long run if they help him obtain visibility more quickly than using other means. From $S_M$’s perspective, the above trade-off is just the opposite. Specifically, $S_M$ normally benefits from $S_L$’s guarantee in the short term, but loses its advantage once the buyer gains visibility over time. This suggests that, in the long run, the only option left for $S_M$ to preserve her position is to reduce her costs.
The extra transaction costs associated with selling in the spot market ($\delta$) and the volatility in the spot prices ($\rho\Delta$) also play critical roles in the signaling ability of $S_L$ and its consequences on the buyer’s cost. Specifically, for products where the above cost is high and/or the spot price is more volatile (as has been the case in recent times for a number of commodity products), low-risk $S_L$ can more easily differentiate from the high-risk type through guarantee offers. However, high values of $\delta$ and/or $\rho\Delta$ also create more opportunities for the two suppliers to increase their prices, which, in turn, increases the buyer’s procurement cost.

Next, we briefly discuss the implications of relaxing some of our assumptions. First, we assume that all parties are risk-neutral players. However, as noted in the paper, P&Q guarantees can sometimes reduce buyer’s cost variability (e.g., Region 2a in Table 5). This suggests that if we consider implications on both the mean and the variability of buyer’s costs, it becomes more likely that a risk-averse buyer would benefit from a guarantee contract. Second, we assume that $S_L$ incurs marginal costs only for the units that are actually delivered to the buyer. If she incurs marginal costs for all units produced, this would increase the cost differential between $l$ and $h$ types, which in turn, would increase the likelihood of separating equilibria. Finally, we assume that the capacity uncertainty is of “all-or-nothing” type. Adding intermediate levels into the capacity uncertainty structure may allow $l$-type $S_L$ to offer partial guarantees, which again can be used to differentiate from $h$-type. It is our expectation that this research spurs work on the above extensions so as to provide guidance to procurement managers regarding how supply guarantee contracts can address two of their main concerns - improving visibility and dealing with disruption risk.

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**Appendix: Proofs of Propositions**

*Proof of Proposition 1.* Let us first start by characterizing the equilibrium under no-guarantee (NG) contract. We solve the game using backward induction starting from the buyer’s order allocation decision. Let $TC^S_B$ be the buyer’s expected total cost under symmetric information scenario. Since it is linear in $q_L$ and $q_M$, depending on the sign of the coefficients for $q_L$ and $q_M$, optimal $q_L$ and $q_M$ can only be the endpoints, i.e., $q^L_0 \in \{0, Q\}$ and $q^M_0 \in \{0, Q\}$. The buyer’s expected total cost under symmetric information scenario for the four possible combinations are as follows: $TC^S_B(q_L, q_M) = k^S_B(p_L) = (1 - \alpha^S_L)p_L + \alpha^S_L \bar{p}_S + \rho \Delta \alpha_M \alpha^S_L$ if $q_L = Q$ and $q_M = 0$; $TC^S_B(q_L, q_M) = k^S_M(p_M) = (1 - \alpha_M)p_M + \alpha_M \bar{p}_S + \rho \Delta \alpha_M \alpha^S_L$ if $q_L = 0$ and $q_M = Q$; $TC^S_B(q_L, q_M) = k^S_{LM}(p_L, p_M) = (1 - \alpha^S_L)p_L + (1 - \alpha_M)p_M + \alpha^S_L \alpha_M (\bar{p}_S + \rho \Delta) - (1 - \alpha_M)(1 - \alpha^S_L)(\bar{p}_S - \rho \Delta - \delta)$ if $q_L = Q$ and $q_M = Q$; and finally, $TC^S_B(q_L, q_M) = k^S_S = \bar{p}_S + \rho \Delta \alpha_M \alpha^S_N - \rho \Delta (1 - \alpha_M)(1 - \alpha^S_S)$ if $q_L = 0$ and $q_M = 0$. Minimizing $TC^S_B$ with respect to $q_L \in \{0, Q\}$ and $q_M \in \{0, Q\}$, we obtain the following allocation rule: $q_L(p_M, p_L) = Q$ iff $\min(k^S_L(p_L), k^S_{LM}(p_L, p_M)) < \min(k^S_M(p_M), k^S_S)$. Similarly, $q_M(p_M, p_L) = Q$ iff $\min(k^S_M(p_M), k^S_{LM}(p_L, p_M)) < \min(k^S_S(p_L), k^S_S)$. After substituting $q_L$ and $q_M$ into supplier $S_L$ and supplier $S_M$’s profit functions, respectively, we can then solve the (Nash) price competition game between the two suppliers. We show that the price competition between two suppliers leads to three sourcing equilibria: (i) No-sourcing, (ii) Sole-sourcing, and (iii) Dual-sourcing equilibria.

*No-sourcing and sole-sourcing equilibria:* In the no-sourcing and sole-sourcing equilibria, the competition between $S_L$ and $S_M$ can be analyzed as a pure Bertrand price competition with different marginal costs. We know that in Bertrand competition, each supplier will undercut her competitor’s price as long as that
price is strictly greater than her marginal cost. Let \( k_M^S \) and \( k_M^L \) be the expected cost of buyer when supplier \( S_M \) and \( S_L \) offer their break-even prices, respectively, \( c_M \) and \( c_L \), and let \( k_S^S \) be the expected cost buyer if she buys in the spot market. So, the undercutting argument ensures that in equilibrium there will always be incentive for the party whose expected cost to the buyer is the lowest to undercut the other parties if these parties charge more than their break-even price. So, this argument implies that the only equilibrium that is sustainable against all deviations is the one in which all the losing parties charge their break-even prices and the winning party charges a price that makes the expected cost of buyer infinitesimal lower than the expected cost from the best losing party. Throughout the appendix, we use \([x]^{(-)}\) to denote a number that is infinitesimally less than \( x \). So, from buyer’s perspective, if his marginal cost of buying from spot market is less than buying from \( S_L \) and \( S_M \) when both suppliers charge their break-even prices, i.e., \( k_S^S < k_M^S \) and \( k_S^S < k_M^L \), or equivalently, \( \alpha_L^S \leq \gamma_L \) and \( \alpha_M \leq \gamma_M \), where \( \gamma_M = 1 - \frac{p_M - c_M}{\rho \Delta + \delta} \) and \( \gamma_L = 1 - \frac{p_L - c_L}{\rho \Delta + \delta} \), then, in equilibrium, both suppliers charge their marginal costs, i.e., \( p_M^* = c_M \), and \( p_L^* = c_L \). On the other hand, if buying from supplier \( S_M \) is less than buying from supplier \( S_L \) and spot market, i.e., \( k_M^S < k_M^L \) and \( k_M^S < k_M^S \), or equivalently, \( \alpha_L^S \geq \gamma(\alpha_M) \) and \( \alpha_L^S \geq \gamma_S \), where \( \gamma(\alpha_M) = \frac{c_M - p_M}{p_M - c_M} + \alpha_M \frac{c_M - p_L}{p_L - c_M} \), then, in equilibrium, \( S_L \) can at best offer \( p_M^* = c_L \), and \( S_M \) undercut \( S_L \) by setting his price such that the buyer’s marginal cost of buying from \( S_M \) is equal to \([k_M^S]^{(-)}\). By equating \( k_M^S(p_M) = [k_M^S]^{(-)} \), and solving for \( p_M \), we obtain equilibrium \( p_M^* = [c_M + \frac{k_M^S - k_S^S}{1 - \alpha_M}]^{(-)} \). Otherwise, if buying from supplier \( S_L \) is less than buying from supplier \( S_M \) and spot market, i.e., if \( \alpha_L^S < \gamma(\alpha_M) \), and and \( k_M^S < k_M^S \), \( S_M \) sets \( p_M^* = c_M \) and \( S_L \) sets her price such that the buyer’s marginal cost of buying from \( S_L \) is equal to \([k_M^S]^{(-)}\). By equating \( k_M^S(p_L) = [k_M^S]^{(-)} \), and solving for \( p_L \), we obtain equilibrium \( p_L^* = [c_L + \frac{k_M^S - k_S^S}{1 - \alpha_L^S}]^{(-)} \).

**Dual-sourcing equilibrium:** Finally, when both supplier \( S_M \) and supplier \( S_L \) become quite unreliable, then it is optimal for the buyer to procure from both to reduce the total expected cost. In order for dual-sourcing equilibrium prices to be sustainable, they should satisfy the following conditions: \( k_M^L(p_L, p_M) < \min (k_S^L(p_L), k_M^S(p_M), k_M^S) \). Since all the three functions \((k_M^L, p_M), k_M^S(p_L), k_M^S(p_M))\) are increasing in \( p_L \) and \( p_M \), in order for suppliers’ prices to satisfy the above condition in equilibrium, we need to find \( p_L^* \), and \( p_M^* \) that solve \( k_M^L(p_L, p_M) = [k_M^L]^{(-)} = [k_M^L(p_M)]^{(-)} \), and check that \( k_M^L(p_L, p_M) < k_S^S \). The solution of the above equation gives us the following equilibrium prices: \( p_L^* = [p_S - (1 - \alpha_M)(\rho \Delta + \delta)]^{(-)} \) and \( p_M^* = [p_S - (1 - \alpha_S)(\rho \Delta + \delta)]^{(-)} \). Evaluating \( k_M^L(p_L, p_M) \) at the equilibrium prices \( p_L = p_L^* \), and \( p_M = p_M^* \), we can show that \( k_M^L(p_L^*, p_M^*) = p_S + \rho \Delta \alpha_M \alpha_L^S - (\rho \Delta + \delta)(1 - \alpha_M)(1 - \alpha_L^S) < k_S^S = p_S + \rho \Delta \alpha_M \alpha_L^S - \rho \Delta (1 - \alpha_M)(1 - \alpha_L^S) \), because \( \delta > 0 \). Now, we verify that there is no profitable deviation from \( p_L^* \), and \( p_M^* \) for supplier \( S_L \) and supplier \( S_M \), respectively. Suppose that, for a given \( p_M^* \), supplier \( S_L \) increases his price from \( p_L^* \). This increases both \( k_M^S(p_L^*) \) and \( k_M^L(p_L^*, p_M^*) \) and make both of them greater than \( k_M^S(p_L^*, p_M^*) \), which implies that supplier \( S_L \) loses the order allocation from the buyer. Obviously, supplier \( S_L \)’s profit decreases as she lowers her price from \( p_L^* \), hence there is no profitable deviation for \( S_L \) in either direction. The same argument also works for supplier \( S_M \). As a final check, we need to make sure that both \( p_L^S \geq c_L \) and \( p_M^S \geq c_M \). This results in the following conditions: \( p_L^S \geq c_L \) and \( p_M^S \geq c_M \) if and only if \( \alpha_L^S \geq \gamma_L \) and \( \alpha_M \geq \gamma_M \), where \( \gamma_M = 1 - \frac{p_M - c_M}{\rho \Delta + \delta} \) and \( \gamma_L = 1 - \frac{p_L - c_L}{\rho \Delta + \delta} \).
To summarize, the resulting equilibrium allocation can be characterized in two stages. In the first stage, either none of the suppliers get the order allocation if \( \alpha_L^b \leq \underline{\gamma}_L \) and \( \alpha_M \leq \underline{\gamma}_M \) or both of them get the order allocation if \( \alpha_L^b \geq \bar{\gamma}_L \) and \( \alpha_M \geq \bar{\gamma}_M \). In the second stage, if neither of the above conditions are satisfied, then, the equilibrium is of sole-sourcing type, i.e., either supplier \( S_L \) obtains the full allocation if \( \alpha_L^b \leq \gamma(\alpha_M) \) or supplier \( S_M \) obtains the full allocation if \( \alpha_L^b > \gamma(\alpha_M) \). The resulting equilibrium profits and costs can then be derived from \( p_L^* \) and \( p_M^* \). Finally, we need to check whether supplier \( S_L \) and \( S_M \) benefit from selling directly to the spot market or not. Suppose that \( S_L \) decides to sell directly to the spot market. \( S_L \)'s expected revenue conditional on his realized capacity being equal to \( Q \) is \( \alpha_M(\bar{p}_S - \delta) + (1 - \alpha_M)(\bar{p}_S - \rho \Delta - \delta) = \bar{p}_S - \alpha_M \delta - (1 - \alpha_M)(\rho \Delta + \delta) \). Recall that \( \bar{p}_S - (1 - \alpha_M)(\rho \Delta + \delta) \leq c_L \) if and only if \( \alpha_M \leq \bar{\gamma}_M \). But since \( S_L \) earns at minimum an expected revenue of \( \bar{p}_S - (1 - \alpha_M)(\rho \Delta + \delta) \) when \( \alpha_M > \bar{\gamma}_M \), this implies that on the equilibrium, \( S_L \) never chooses to sell his capacity directly to the spot market. The same argument applies for supplier \( S_M \) showing that \( S_M \) is worse off by selling directly to the spot market.

As regards the supply-guarantee (G) contract setting, note that there are two extra decision variables for \( S_L \) in that case - guaranteed quantity \( (q_G) \) and price \( (p_G) \). We can show that except for when both suppliers are very risky (i.e., region \( LM_b \)), \( q_G^* = Q \). The equilibrium price for \( S_M \) is the same as the no-guarantee case. The guaranteed price \( p_G^* \) can be obtained by using Bertrand price competition argument as in the no-guarantee case above. The detailed expressions for equilibrium decisions and profits/costs are provided in Table 2.

**Proof of Proposition 2.** We prove this proposition for the no-guarantee (NG) case in 2 steps:

1. We first show that in equilibrium both \( h \) and \( l \) type \( S_L \) charge the same price. Note that \( \theta \)-type supplier \( L \)'s profit function in NG case is as follows: \( \Pi^\theta_L = (1 - \alpha^\theta_L)(p^\theta_L - c_L)q_L \). Based on above, it is trivial to show that in equilibrium if \( p^\theta_L \neq p^\theta_L^* \), either \( l \) or \( h \)-type can increase her profit by mimicking the other type. Hence, it implies that \( p^\theta_L \neq p^\theta_L^* \) can not be on the equilibrium path. This suggests that the buyer can not identify the correct type of supply uncertainty, and hence uses a-priori beliefs (i.e., \( \bar{\theta}_L = r_h \bar{\alpha}_L^h + r_l \bar{\alpha}_L^l \)) to decide on quantity allocations.

2. Replacing \( \bar{\alpha}_L \) with \( \alpha_L^b \) in the proof of Proposition 1, we can obtain equilibrium characterization in terms of \( \bar{\alpha}_L \) and \( \alpha_M \). Also, equilibrium profits and costs can be derived from \( p_L^* \) and \( p_M^* \).

**Proof of Proposition 3.** For the supply-guarantee (G) scenario, we will characterize the equilibrium strategies for low, medium and high \( \alpha_M \) cases, separately.

**Case-(i) \( \alpha_M \leq \underline{\gamma}_M \):** Since \( \alpha_M \leq \underline{\gamma}_M \), \( k^b_M \leq k^b_L \) for all \( \alpha_L^b \geq 0 \). This implies that buyer never procures from supplier \( S_L \). Both \( h \) and \( l \) types offer full guarantee and signal their types by offering different per-unit guaranteed prices, i.e., \( p^h_L^* = k^h_L \) and \( p^l_L^* = k^l_L \). So, depending on \( p_M^* \), buyer makes an allocation decision between \( S_M \) and spot market as follows: \( q_M = Q \) if \( k^b_M(p_M) \leq k^b_S \); otherwise, \( q_M = 0 \). As shown in Figure 4(A) and Table 4, we have 3 regions depending on \( \alpha_L^b \) and \( \alpha_M^b \). Below, we analyze each region separately:

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- **Region (1a):** Note that if \( \alpha_L^b \leq \underline{\gamma}_L \), \( k^h_M(p_M) \leq k^b_S \) for all \( \theta \in \{h, l\} \), hence, buyer procures only from spot market. Therefore, on the equilibrium \( S_M \) offers \( p_M^* = c_M \) and gets zero allocation, i.e., \( q_M^* = 0 \).

- **Regions (1b) and (1c):** Otherwise, if \( \alpha_L^b > \underline{\gamma}_L \), in order to get allocation, supplier \( S_M \) needs to offer a \( p_M \) that makes \( k^b_M(p_M) \) epsilon below \( k^b_S \). This implies that \( p_M^* = [\bar{p}_S - \rho \Delta(1 - \alpha_L^b)]^{-1} \). Note that since
\( \alpha^h_L > \alpha^l_L \) it implies that \( p^l_M < p^h_M \). Therefore, \( S_M \) can choose between two options: (i) She charges \( p^h_M = [\bar{p}_S - \rho \Delta (1 - \alpha^h_L)]^(-) \), gets full allocation only when \( \theta = h \) and, hence, earns an expected profit of \( r^h_k(k^h_S - k^h_L) \); or (ii) she charges \( p^l_M = [\bar{p}_S - \rho \Delta (1 - \alpha^l_L)] (-) \), gets full allocation under both \( \theta = h \) and \( \theta = l \), earns expected profit of \( r^l_c(k^l_S - k^l_L) \). The equilibrium pricing decision for \( S_M \) depends on her a-priori beliefs as follows: \( p^l_M = p^h_M = [\bar{p}_S - \rho \Delta (1 - \alpha^h_L)](-) \) if \( r^h > \frac{k^h_S - k^h_L}{\bar{p}_S - \rho \Delta} \), otherwise, \( p^l_M = [\bar{p}_S - \rho \Delta (1 - \alpha^l_L)](-) \).

**Case (ii) \( \gamma_M < \alpha_M \leq \gamma_M \):** In this case, we have four regions as shown in Figure 4(B) and Table 5. Below, we analyze each region separately:

— Region 2a: When \( \alpha^h_L < \gamma(\alpha_M) \), there exists only pooling equilibrium, where both \( l \) and \( h \) types receive the full order. In this case, both \( l \) and \( h \)-types can undercut \( S_M \), hence, \( p^l_M \neq p^h_M \) can not be on the equilibrium path. Given that \( p_M = c_M \), both \( l \) and \( h \)-type supplier \( S_L \) can get the full allocation as long as \( p^l_M \leq \bar{k}_M \). Since profit function of supplier \( S_L \) increases in \( p^l_M \), on the equilibrium, both types of supplier \( S_L \) charge infinitesimally less than \( \bar{k}_M \). Now, given that \( p^h_M \) is infinitesimally less than \( \bar{k}_M \), supplier \( S_M \)'s profit will be zero if \( S_M \) charges more than \( c_M \) and negative if she charges less than \( c_M \). Hence, it implies that \( p_M = c_M \), \( q^l_G = Q \) and \( p^l_M \) that is infinitesimally less than \( \bar{k}_M \) for \( \theta = \{h, l\} \) form a Nash equilibrium. We can also establish that this is a unique Nash equilibrium by noting that any contract terms other than the one specified above would give incentive to either \( S_M \) or \( S_L \) to undercut her opponent, and hence are not sustainable on the equilibrium.

— Regions 2b and 2c: In these regions, \( \gamma(\alpha_M) < \alpha^l_L \) and \( r^h > \frac{k^l_S - k^l_M}{k^l_L - k^l_M} \). The first condition implies that \( h \)-type \( S_L \) can not make a positive profit against supplier \( S_M \), even if she offers full guarantee. This is because she can set \( p^h_M \) at minimum \( k^h_L \), but \( S_M \) can still undercut her offer by charging \( c_M + \frac{k^h_L - k^h_M}{1 - \alpha_M} \). However, \( l \)-type \( S_L \) can still compete with \( S_M \) by offering a price \( p^l_M \) which is less than \( k^l_M(p_M) \). On the equilibrium, we can show that under certain conditions, there is no pure strategy equilibrium. First, we construct a mixed equilibrium and derive the conditions in order for it to be sustainable for all the parties. Note that \( S_L \) offers guaranteed price \( p^l_M \), whereas \( S_M \) offers a non-guaranteed price \( p_M \). Therefore, their prices are defined over different price ranges. Let \( \mathcal{P}_L = \{p_L, \bar{p}_L\} \) and \( \mathcal{P}_M = \{p_M, \bar{p}_M\} \) be the ranges of mixing equilibria for supplier \( S_L \) and \( S_M \), respectively. Any pair of prices in \( p_G \in \mathcal{P}_L \) and \( p_M \in \mathcal{P}_M \) can be matched with each other by using the following transformations: \( p_G = k^l_M(p_M) \) and \( p_M = h_M(p_G) = c_M + \frac{p_G - k^l_M}{1 - \alpha_M} \). So, based on the above transformations, \( S_M \) (resp., \( S_L \)) gets full allocation if and only if \( S_M \) (resp., \( S_L \)) charges less than \( h_M(p_G) = c_M + \frac{p_G - k^l_M}{1 - \alpha_M} \) (resp., \( k^l_M(p_M) \)). Let \( F^l_L \) and \( F^l_M \) be the pricing distributions for \( S_L \) and \( S_M \), respectively, that are defined over the above ranges. The upper bound for the mixed strategy interval for \( l \)-type supplier \( S_L \) must be set just at either the marginal cost of \( h \)-type \( S_L \) or \( l \)-type spot market price, whichever is smaller, i.e., \( \bar{p}_L = \min(k^l_L - (k^h_M - k^l_M), k^l_S) \). The upper bound for supplier \( S_M \) can be found by mapping \( \bar{p}_L \) to \( \mathcal{P}_M \) as follows: \( \bar{p}_M = c_M + \frac{\bar{p}_L - k^l_M}{1 - \alpha_M} \leq c_M + \frac{k^h_L - k^l_M}{1 - \alpha_M} \). Note that \( \bar{p}_M \) is less than \( c_M + \frac{k^h_L - k^l_M}{1 - \alpha_M} \) since \( S_M \) always undercut \( h \)-type supplier \( S_L \). Mixed strategy probability distribution for \( S_M \) can be derived by the condition that if \( S_M \) mixes continuously with \( F_M \) over the interval \( [p_M, \bar{p}_M] \), \( l \)-type is indifferent between charging any price over the interval \( [p_L, \bar{p}_L] \). Similarly, \( l \)-type also mixes continuously over the interval \( [p_M, \bar{p}_M] \) to make sure that \( S_M \) is indifferent between undercutting both \( l \) and \( h \) types and undercutting only \( h \)-type. To express these conditions, we need to write down each firm’s expected payoff.
first: \( \Pi'_L(p_G) = \left[ 1 - F_M \left( \frac{p_G - k^L_L}{c_M + \frac{p_G - k^L_L}{1 - \alpha_M}} \right) \right] (p_G - k^L_L) \) and \( \Pi_M(p_M) = r^h(1 - \alpha_M)(p_M - c_M) + r^l(1 - \alpha_M)(p_M - c_M)(1 - F'_L(k^M_M(p_M))) \). Recall that supplier \( S_M \)'s profit needs to be equal to \( r^h(1 - \alpha_M)(\bar{p}_M - c_M) \) for all \( p \) in the support, i.e., \( \Pi_M(p_M) = (1 - r^L_F(k^M_M(p_M)))(p_M - c_M) = r^h(\bar{p}_M - c_M) \). Note that the right-hand side of the above equation represents supplier \( S_M \)'s expected profit at the upper limit of her range \( (\bar{p}_M) \), i.e., \( r^h(\bar{p}_M - c_M) \). Using the transformation between supplier \( S_M \)'s price range and supplier \( S_L \)'s price range, we can rewrite \( S_M \)'s expected profit at \( \bar{p}_M \) as \( r^h(k^L_L - k^L_H) \). So, inverting the above profit equation for supplier \( S_M \) and applying variable transformation between \( p_G \) and \( p_M \), we obtain mixing distribution function for \( L \)-type supplier \( S_L \): \( F^L_L(p_G) = \frac{1}{h} \left( 1 - r^h \left( \frac{k^L_L - k^H_L}{p_G - k^M_M} \right) \right) \). Also note that \( \lim_{p_G \to \bar{p}_L} F^L_L(p_G) = 1 \) if \( \bar{p}_L < k^L_H \) but \( \lim_{p_G \to \bar{p}_L} F^L_L(p_G) = 1 - \frac{1}{h} \left( 1 - r^h \left( \frac{k^L_L - k^H_L}{p_G - k^M_M} \right) \right) < 1 \) if \( \bar{p}_L = k^L_H \). Hence, there is a probability mass at \( \bar{p}_L \) if \( \bar{p}_L = k^L_H \).

From \( F^L_L(p_G) \), we obtain lower bound for the support, \( \bar{p}_L = k^M_L + r^h(k^H_L - k^H_H) \). Also we need to make sure that \( \bar{p}_L \) must be greater than \( k^L_H \). Otherwise, by charging infinitesimally below \( k^L_H \), \( S_M \) can always undercut both \( l \) and \( h \) types and makes more profit. This condition implies that \( \bar{p}_L = k^M_L + r^h(k^H_L - k^H_H) \) is greater than \( k^L_H \), but this is automatically satisfied because of the second condition of the regions (2b) and (2c), i.e., \( r^h > \frac{k^L_L}{k^L_H - k^M_M} \). Since \( L \)-type can not charge more than \( k^L_H \), we need also to make sure that \( \bar{p}_L \) must be greater than \( k^L_H \). Otherwise, by charging infinitesimally below \( k^L_H \), \( S_M \) can undercut only \( h \) type and makes more profit. This condition implies that \( \bar{p}_L = k^M_L + r^h(k^H_H - k^H_H) \) if and only if \( r^h < \frac{k^L_L - k^H_H}{k^M_M} \), but this is automatically satisfied in Region (2b), whereas the opposite is satisfied in Region (2c).

First, we assume that \( r^h < \frac{k^L_L - k^H_H}{k^M_M} \), i.e., \( \bar{p}_L < k^L_H \). Note that in this case, \( \lim_{p_G \to \bar{p}_L} F^L_L(p) = 0 \). If \( k^L_H \leq k^L_H \), then \( \bar{p}_L = k^L_H \), which implies that \( \lim_{p_G \to \bar{p}_L} F^L_L(p) = 1 \). Hence, there is no mass between \( p_G \) and \( \bar{p}_L \). However, if \( k^L_H > k^L_H \), then \( \bar{p}_L = k^L_H \), which implies that \( \lim_{p_G \to \bar{p}_L} F^L_L(p) = 1 \). Hence, there is a mass \( 1 - F^L_L(k^L_H) \) at \( \bar{p}_L = k^L_H \).

Now, we derive the mixing distribution for \( S_M \) by using the fact that \( L \)-type’s profit needs to be equal to \( \bar{p}_L - k^L_H \) for all \( p \) in the support, i.e., \( \Pi'_L(p_G) = \left[ 1 - F_M \left( \frac{p_G - k^L_H}{c_M + \frac{p_G - k^L_H}{1 - \alpha_M}} \right) \right] (p_G - k^L_H) = \bar{p}_L - k^L_H \). Inverting profit equation for \( L \)-type supplier \( S_L \) and applying the variable transformation between \( p_M \) and \( p_G \), we obtain mixing distribution function for supplier \( S_M \), \( F_M \), as a function of \( p_M \): \( F_M(p_M) = 1 - \frac{p_G - k^L_H}{k^M_M(p_M) - k^L_H} \). Note that \( \lim_{p_M \to \bar{p}_M} F_M(p_M) = 0 \) but \( \lim_{p_M \to \bar{p}_M} F_M(p_M) = 1 - \frac{k^L_L - k^H_L}{k^M_M(p_M) - k^L_H} < 1 \). Hence, there is a mass at \( \bar{p}_M \) given by \( \bar{p}_M = \frac{k^L_L - k^L_H}{k^M_M(p_M) - k^L_H} \). Also, in this region, we can establish that this mixed Nash equilibrium is unique. First, consider \( h \)-type supplier \( S_L \). Suppose that she sets \( p^h_G \) more than \( k^L_H \). In this case, we can construct a mixed strategy equilibrium for both \( L \)-type and \( S_M \). But \( h \)-type can increase her profit by decreasing \( p^h_G \). Hence, \( p^h_G > k^H_H \) is not sustainable on the equilibrium. Similarly, \( p^h_G < k^L_H \) would imply a negative profit for \( h \)-type, hence it can not be on the equilibrium path. Therefore, it must be that \( p^h_G = k^H_H \) on the equilibrium. Finally, given that \( p^h_G = k^L_H \), by construction, the only mixed strategy for \( L \) type and \( S_M \) that is sustainable on the equilibrium path is the one specified above.

Next, we assume that \( r^h \geq \frac{k^L_L - k^H_H}{k^L_H - k^M_M} \), i.e., \( \bar{p}_L \geq k^L_H \). In this case, \( L \)-type supplier \( S_L \) offers epsilon below \( k^L_H \), supplier \( S_M \) offers epsilon below \( c_M - \frac{k^H_H - k^L_H}{1 - \alpha_M} \), and finally, \( h \)-type supplier \( S_L \) offers \( k^L_H \). On the equilibrium, buyer always procures from supplier \( S_L \) if \( p^*_G = k^L_H \); otherwise, he procures from supplier \( S_M \).

— Region (2d): In this region, \( \gamma(\alpha_M) < a^L_H \) and \( r^h > \frac{k^L_L - k^H_H}{k^M_M(p_M)} \). These two conditions imply that supplier \( S_M \) has an incentive to undercut both \( L \)- and \( h \)-types. Given that \( p^*_G = k^L_H \), supplier \( S_M \) can get the full allocation
as long as $p_M < c_M - \frac{k_L^h - k_L^h}{1 - \alpha_M}$. Since profit function of supplier $S_M$ increases in $p_M$, on the equilibrium, she charges infinitesimally less than $c_M - \frac{k_L^h - k_L^h}{1 - \alpha_M}$. Now, given that $p_M$ is infinitesimally less than $c_M - \frac{k_L^h - k_L^h}{1 - \alpha_M}$, $l$-type supplier $S_L$’s profit will be zero if she charges more than $k_L^h$ and negative if she charges less than $k_L^h$.

Hence, it implies that $p_L^h = k_L^h$ and $p_M$ that is infinitesimally less than $c_M - \frac{k_L^h - k_L^h}{1 - \alpha_M}$ form a Nash equilibrium. Finally, uniqueness of this equilibrium comes from the same fact that any strategy other than the above one would lead to a profitable deviation for either $S_M$, or $l$-type $S_L$, or $h$-type $S_L$.

**Case-(iii) $\alpha_M > \tilde{\gamma}_M$:** We can analyze this case in two regions. First of all, when $\alpha_M^h < \tilde{\gamma}_L$, there exists only pooling equilibrium, where both $l$ and $h$ types receive the full order since in this case, both $l$ and $h$-types can undercut $S_M$. Next, we can show that if $\alpha_M^h \geq \tilde{\gamma}_L$, there is always a profitable deviation for $l$-type if equilibrium is of separating type. The reason is that when $\alpha_M^h \geq \tilde{\gamma}_L$ and true type is $h$, both supplier $S_L$ and $S_M$ can increase their prices and charge $p_L^h = [\bar{p}_S - (1 - \alpha_M)(\rho \Delta + \delta)]^{(\cdot)}$ and $p_M^h = [\bar{p}_S - (1 - \alpha_M^h)(\rho \Delta + \delta)]^{(\cdot)}$, respectively. However, when true type is $l$, the highest price $l$-type can charge is $k_L^h$, which leads to a profitable deviation for $l$-type supplier $S_L$ and makes her mimic $h$ type. Therefore, separating equilibrium can not be sustainable on the equilibrium. This implies that buyer can not update his belief on the type of supplier $S_L$, hence she uses $\tilde{\alpha}$ to decide whether only supplier $S_L$ gets the full allocation (as in Region (3a) of Figure 4(C)) or both $S_L$ and $S_M$ get the allocation (as in Region (3b) of Figure 4(C)).

Finally, profit and cost expressions can be easily derived by evaluating $\Pi^*_M$, $\Pi_M$ and $TC_B$ at the equilibrium strategies defined in Tables 4, 5 and 3. Due to space constraints, we only derive profit and cost expressions for Region (2b) here. The other derivations are available from the authors. In Region (2b), $h$-type always loses the order to supplier $S_M$. Therefore, $h$’s profit is equal to zero. Either supplier $S_M$ or $l$-type wins the full allocation depending on who offers the lowest price (from buyer’s viewpoint). From the above analysis, we know that both $S_M$ and $l$-type mix their prices in such a way that on the equilibrium their profits are the same on any point within their respective supports $[\bar{p}_M^h, \bar{p}_M^l]$, and $[\bar{p}_L^h, \bar{p}_L^l]$. So, by construction, $S_M$’s profit in $[\bar{p}_M^h, \bar{p}_M^l]$ is equal to $r^h(k_L^h - k_L^h)Q$, and $l$-type $S_L$’s profit is equal to $(\bar{p}_L^h - k_L^h)$. Finally, the buyer’s cost is equal to $QE_{p^G, p^G} \min(k_L(p_M^h), p_L^h)$, where the expectation is taken with respect to $F_M$ and $F_L$. □

**Proof of Proposition 4.** In this Proposition, we compare the buyer’s cost and suppliers’ profits under NG and G by using the results of Proposition 2 and Proposition 3, respectively. Since we show that when $\alpha_M \geq \tilde{\gamma}_M$, supply chain partners’ profit/costs are unaffected by the guarantee contract, we skip $\alpha_M \geq \tilde{\gamma}_M$ and compare profits and costs under NG and G only when (i) $\alpha_M \leq \gamma_M$ and (ii) $\gamma_M < \alpha_M \leq \tilde{\gamma}_M$:

**Case-(i) $\alpha_M \leq \gamma_M$:** Both $l$-type and $h$-type $S_L$ do not receive order allocation when supplier $S_M$’s default risk is low. Therefore, guarantee does not have any impact on their profits. Therefore, we consider only $S_M$ and buyer.

- $\alpha_M < \gamma_M$: i.e., Region (1a,1a): Note that $S_M$’s profit is zero and buyer’s cost is $\bar{k}_S$ under both NG and G when $\alpha_M^h < \tilde{\gamma}_L$. So, guarantee contract has no impact on $S_M$’s profit and buyer’s cost.

- $\alpha_M^h \geq \tilde{\gamma}_L$: In this case, the equilibrium cost and profit under G depend on $S_M$’s a-priori belief.

* $r^h \geq \frac{k_L^h - k_M^h}{k_S^h - k_M^h}$: i.e., Regions (1a,1b) and (1b,1b): In these regions, under G contract, $S_M$ would only undercut $h$-type spot market, and generate an expected profit of $r^h(k_S^h - k_M^h)Q$. Under NG contract, her profit is equal to $\bar{k}_S = r^h(k_S^h - k_M^h)Q + r^l(k_L^h - k_L^h)Q$. Comparing these two expressions, we can show that $S_M$
would prefer $G$ over NG if and only if $r^h(k^h_L - k^h_M)Q \geq r^h(k^h_S - k^h_M)Q + r^l(k^l_S - k^l_M)Q \iff k^l_S \geq k^l_M \iff \alpha^l \geq \gamma_L$.

From buyer’s perspective, his expected cost is same (equal to $\bar{k}_S$) under both G and NG.

* $r^h < \frac{k^h_M - k^h_M}{k^h_M - k^h_M}$, i.e., Region (1b,1c): In this region, under G contract, $S_M$ would undercut l-type (hence also h-type) spot market, and generate an expected profit of $(k^l_M - (r^h k^h_M + r^l k^l_M))Q$. However, $S_M$’s profit under NG in this region is $(r^h k^h_S + r^l k^l_S - (r^h k^h_M + r^l k^l_M))Q$. Note that since $k^h_S \geq k^h_M$, it implies that $S_M$ is always worse off with the G contract in Region (1b,1c). From buyer’s perspective, his cost under G is $r^l k^l_M + r^h(k^h_S - k^h_M)$, whereas his cost under NG is $r^h k^h_S + r^l k^l_S$. Taking the difference between these two costs, we obtain $r^h(k^h_S - k^h_M) - r^h(k^h_M - k^h_M)$. This implies that if $k^h_S - k^h_M \geq k^h_S - k^h_M$, buyer loses under NG. Rewriting $k^h_S - k^h_M = (1 - \alpha_M)(\bar{p}_S - c_M - \rho \Delta + \rho \Delta \alpha^h)$, we can show that $k^h_S - k^h_M$ increases in $\alpha^h$. Since $\alpha^h \geq \alpha_L$, we can show that $k^h_S - k^h_M \geq k^l_M$, i.e., buyer prefers G.

**Case-(ii) $\gamma_M < \alpha_M \leq \gamma_M$:** As we did in case (i) above, we consider each region separately from the perspectives of l-type $S_L$, h-type $S_L$, $S_M$ and the buyer.

- Region (2a,2a): Recall that in this region, under both G and NG, the equilibrium is of pooling type and, therefore, both l-type and h-type undercut an average $S_M$ or spot market, whichever is minimum. Comparing $\theta$-type $S_L$’s profits under NG and G and arranging terms, we can show she prefers NG if and only if the following holds true: $\frac{1 - \alpha^L}{1 - \alpha^L} (\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L) \geq (\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L) \iff \frac{1 - \alpha^L}{1 - \alpha^L} \geq \min(k^h_S, k^h_M) - k^l_M$. By multiplying both denominator and numerator with $\bar{p}_S + \rho \Delta \alpha_M - \epsilon_L$, we can rewrite $\frac{1 - \alpha^L}{1 - \alpha^L}$ as follows: $\frac{\bar{p}_S + \rho \Delta \alpha_M - \epsilon_L}{\bar{p}_S + \rho \Delta \alpha_M - \epsilon_L} \times \frac{1 - \alpha^L}{1 - \alpha^L} = \frac{\bar{p}_S + \rho \Delta \alpha_M - \epsilon_L}{\bar{p}_S + \rho \Delta \alpha_M - \epsilon_L}$. Now, since $\bar{p}_S + \rho \Delta \alpha_M \geq \min(\bar{k}_S, \bar{k}_M)$, and $k^h_L \geq \bar{k}_L \geq k^l_L$, we can show that $\frac{1 - \alpha^L}{1 - \alpha^L} = \frac{\min(k^h_S, k^h_M) - k^l_M}{\min(k^h_S, k^h_M) - k^l_M}$ and $\frac{\min(k^h_S, k^h_M) - k^l_M}{\min(k^h_S, k^h_M) - k^l_M} \geq \frac{\min(k^h_S, k^h_M) - k^l_M}{\min(k^h_S, k^h_M) - k^l_M} = \frac{1 - \alpha^L}{1 - \alpha^L}$, which implies that h-type and l-type prefer NG and G, respectively. For $S_M$ and the buyer, their profits and costs are same under G and NG. Hence, G is equal to NG contract for them in Region (2a,2a).

- Region (2a,2b): In this case, h-type’s profit is zero under G, whereas her profit is positive under NG since h-type $S_L$ can hide behind l-type $S_L$ due to the pooling equilibrium, and get allocation. Similarly, $S_M$ gets nothing under NG but may receive non-zero allocation under G. Now, from the buyer’s perspective, buyer’s expected cost under NG is equal to $\min(k^h_M, \bar{k}_S)$. However, under G, supplier $S_M$ charges a price in the range of $[\bar{p}_M, \bar{p}_M]$, where $\bar{p}_M \geq c_M$. This implies that buyer would incur a cost more than $k^h_M$ (with probability $r^h$) and more than $k^l_M$ (with probability $r^l$), both of which are greater than what he would incur under NG. Therefore, buyer is worse off with the guarantee contract in this region. Finally, in order to evaluate the impact of guarantee from l-type’s perspective, we need to compare her profits under NG and G. Under NG and G, l-type $S_L$’s profits are $\frac{1 - \alpha^L}{1 - \alpha^L} (\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L)$ and $\bar{p}_L - k^l_L$, respectively. However, depending on the actual values of the parameters, the comparison can be go either way. Since we have the closed-form profit expressions, we can state the necessary and sufficient conditions (denoted by $\Delta^l_1$) under which l-type $S_L$ benefits from the guarantee contract as follows: $\Delta^l_1 = \frac{\bar{p}_L - k^l_M}{1 - \alpha^L} - \frac{\min(k^h_S, k^h_M) - k^l_M}{1 - \alpha^L} \geq 0$ where $\bar{p}_L = k^l_M + r^h(k^h_M - k^h_M)$.

- Region (2a,2c): Similar to region (2a,2b), in this region also, guarantee enables l-type supplier $S_L$ to differentiate herself from h-type $S_L$. Therefore, h-type does not get any allocation from the buyer under G. However, under NG, $S_M$ can hide behind l-type $S_L$ due to the pooling equilibrium and get allocation. So, h-type $S_L$ loses under G contract. Similarly, $S_M$ gets nothing under NG but may get non-zero allocation
under G, which implies that she benefits from G contract. From the buyer’s perspective, using Proposition 2, his expected cost is \(\min(\bar{k}_M, \bar{k}_S)\). This implies that his cost is not more than \(\bar{k}_M = r^h k^h_M + r^l k^l_M\). Under G contract, in this region, his expected cost is \(r^h k^h_L + r^l k^l_S\). Using the conditions that define the region (2a,2c), we can show that both \(k^h_L \geq k^h_M\) and \(k^l_S \geq k^l_M\). This implies that \(\min(\bar{k}_M, \bar{k}_S) \leq r^h k^h_L + r^l k^l_S\). Therefore, similar to region (2a,2b), the buyer loses from the guarantee contract. Finally, in order to evaluate the impact of G on l-type supplier \(S_L\), we need to compare his profits under NG and G. Under NG and G, l-type \(S_L\)’s profits are \(\frac{1-r^l}{\bar{\alpha}_L} \min(\bar{k}_S, \bar{k}_M) - \bar{k}_L\) and \(k^l_S - k^l_L\), respectively. However, depending on the actual values of the parameters, the comparison can go either way. Since we have the closed-form profit expressions, we can state the necessary and sufficient conditions (denoted by \(\Delta^1\)) under which l-type \(S_L\) benefits from the guarantee contract as follows: \(\Delta^1 = \frac{k^l_L - k^l_M}{1-\bar{\alpha}_L} - \frac{\min(\bar{k}_S, \bar{k}_M) - \bar{k}_L}{\bar{\alpha}_L} \geq 0\). A sufficient condition for \(\Delta^1\) to be negative can be derived by comparing \(\bar{k}_S\) to \(\bar{k}_M\). If the former is less than the latter, we can show that \(\Delta^1 \leq 0\) due to the fact that \(\alpha^L_L \leq \bar{\alpha}_L\), and \(k^l_S - k^l_M \leq k^h_L - k^h_M\).

- **Region (2b,2h):** Note that h-type \(S_L\)’s profit is zero under both NG and G, whereas l-type \(S_L\)’s profit is zero and strictly positive under NG, and G, respectively. From Proposition 2, \(S_M\)’s profit under NG is equal to \(\min(\bar{k}_S, \bar{k}_M) - \bar{k}_M\) (\(= \bar{k}_L - \bar{k}_M\) since \(\alpha_M \geq \bar{\alpha}_M\)). From Proposition 3, her profit under G is equal to \(r^h (k^h_L - k^h_M)\). Comparing \(\bar{k}_L - \bar{k}_M\) to \(r^h (k^h_L - k^h_M)\), we can show that \(S_M\) is better off with the guarantee contract if and only \(\alpha^h_L \leq \gamma(\alpha_M)\) as follows: \(\bar{k}_L - \bar{k}_M = r^h (k^h_L - k^h_M) + r^l (k^l_L - k^l_M) \leq r^h (k^h_L - k^h_M)\) if and only if \(k^l_L \leq k^l_M \Leftrightarrow \alpha^h_L \leq \gamma(\alpha_M)\). Finally, from Proposition 2, buyer’s cost is equal to \(\min(\bar{k}_L, \bar{k}_S)\). However, from Proposition 3, the calculation of the buyer’s expected cost under G requires double integration (expectation) with respect to \(F_M(p_M)\) and \(F_L(p_L)\) in the ranges of \([\bar{p}_M, \bar{p}_M]\) and \([\bar{p}_L, \bar{p}_L]\), respectively. Therefore, we only provide the condition (denoted by \(\Delta^0\)) under which the buyer is better off with the guarantee as follows: \(\Delta^0 = \min(\bar{k}_L, \bar{k}_S) - \left(E\bar{k}_M(p^*_M) - r^l E[k^l_M(p^*_M) - p^*_L]^+\right) \geq 0\).

- **Region (2b,2c):** Similar to the region (2b,2h), h-type \(S_L\) does not get any order allocation under both NG and G. Also, l-type \(S_L\) does not get any allocation under NG, however, guarantee contract enables her to signal her type, and get order allocation from the buyer. This implies that l-type strictly increases her profit with guarantee. From Proposition 2, \(S_M\)’s profit under NG is equal to \(\min(\bar{k}_S, \bar{k}_M) - \bar{k}_M\) (\(= \bar{k}_L - \bar{k}_M\)) since \(\alpha_M \geq \bar{\alpha}_M\)). From Proposition 3, her profit under G is equal to \(r^h (k^h_L - k^h_M)\). Comparing \(\bar{k}_L - \bar{k}_M\) to \(r^h (k^h_L - k^h_M)\), we can show that \(S_M\) is better off with the guarantee contract if and only \(\alpha^h_L \leq \gamma(\alpha_M)\) as follows: \(\bar{k}_L - \bar{k}_M = r^h (k^h_L - k^h_M) + r^l (k^l_L - k^l_M) \leq r^h (k^h_L - k^h_M)\) if and only if \(k^l_L \leq k^l_M \Leftrightarrow \alpha^h_L \leq \gamma(\alpha_M)\). This implies that in the lower portion of Region (2b,2c) (i.e., where \(\alpha^h_L \leq \gamma(\alpha_M)\)), \(S_M\) is better off with the guarantee contract, whereas in the upper portion of Region (2b,2c) (i.e., where \(\alpha^h_L \geq \gamma(\alpha_M)\)), she is worse off. Finally, from Proposition 2, buyer’s cost is equal to \(\min(\bar{k}_L, \bar{k}_S)\) (\(= \bar{k}_L = r^h k^h_L + r^l k^l_L\)). However, from Proposition 3, the buyer’s cost under G is equal to \(r^h k^h_L + r^l k^l_L\). Note that since \(\alpha_M \geq \bar{\alpha}_M\), \(k^l_L \leq k^l_M\), which implies that the buyer’s cost increases under guarantee contract, i.e., the buyer prefers NG.

- **Region (2b,2d):** Under both NG and G, both types of supplier \(S_L\) do not get any allocation, implying that their profits are unaffected by the guarantee contract. Supplier \(S_M\)’s profits under NG and G are \(\bar{k}_L - \bar{k}_M = r^l (k^l_L - k^l_M)\) and \(k^l_S - k^l_M\), respectively. Note that since \(k^l_L - k^l_M \geq k^h_L - k^h_M\), we can show that \(\bar{k}_L - \bar{k}_M \geq k^l_L - k^l_M\) (i.e., \(S_M\) prefers NG). Finally, from the buyer’s perspective, the above argument can be used in the opposite direction to show that the buyer benefits from the guarantee contract.
Finally, we briefly analyze the impact of P&Q guarantees on total supply chain cost. Let us first define the total supply chain cost:

\[
E_{\epsilon_L, \epsilon_M} \{ c_L \min(q_L, \epsilon_L^0) + c_M \min(q_M, \epsilon_M) + p_S(\epsilon_L^0, \epsilon_M) [Q - \min(q_L, \epsilon_L^0) - \min(q_M, \epsilon_M)] \},
\]

where the first and second terms account for the marginal costs incurred by supplier \( S_L \) and supplier \( S_M \), respectively, and the third term is for the spot market cost. Note that the total supply chain cost is only determined by the order allocation decision of the buyer. So, it implies that total supply chain cost decreases with \( G \) if and only if the allocation strategy associated with \( G \) assigns the order to less costly source than the one associated with \( \text{NG} \). Due to space constraints, we only analyze Region (2a,2b) when \( \gamma_M < \alpha_M \leq \bar{\gamma}_M \).

Under \( \text{NG} \), the order is always assigned to \( S_L \), whereas under \( G \), it is assigned to supplier \( S_M \) if \( \theta \) is \( h \)-type, otherwise it is assigned to \( S_M \) or \( S_L \) depending on whether \( k^1_M(p^*_M) \leq p^*_G \) or not. Even though the guarantee contract decreases the total cost when \( \theta = h \) by switching the order from \( h \)-type \( S_L \) to \( S_M \), it also increases the total cost when \( \theta = l \) by assigning the order to \( S_M \) if \( k^1_M(p^*_M) \leq p^*_G \). Therefore, the net impact of the guarantee contract on the total cost depends on the comparison between the two. This leads to the following condition. The total supply chain cost is reduced by the guarantee contract if and only of \( \Delta^{TC} = r^h(k^h_L - k^h_M) - r^l E_{p^*_M, p^*_G} [I(k^1_M(p^*_M) \geq p^*_G)(k^1_M - k^1_L)] \geq 0 \). Note that the analysis for other regions can be done in a similar fashion (details are available from the authors upon request).
Table 2  Equilibrium order allocation, contracts and costs/profits for the no-guarantee and supply-guarantee scenarios under symmetric information ($\theta \in \{h,l\}$).

<table>
<thead>
<tr>
<th>Regions</th>
<th>Allocation</th>
<th>$q_M = 0, qL = 0$</th>
<th>$q_M = Q, qL = 0$</th>
<th>$q_M = 0, qL = Q$</th>
<th>$q_M = Q, qL = Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NG $p_M^*$</td>
<td>$c_M$</td>
<td>$c_M$</td>
<td>$\left[c_M + \min\left(\frac{\theta l^g}{1-\alpha_M}\right)\right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_L^*$</td>
<td>$c_L$</td>
<td>$\left[c_L + \min\left(\frac{\theta l^g}{1-\alpha_M}\right)\right]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G $p_M^*$</td>
<td>$c_M$</td>
<td>$c_M$</td>
<td></td>
<td>$\left[p_S - (1 - \alpha)^{(\rho \Delta + \delta)}\right]$</td>
<td>$\left[p_S - (1 - \alpha)^{(\rho \Delta + \delta)}\right]$</td>
</tr>
<tr>
<td>$p_L^*$</td>
<td>$c_L$</td>
<td>$\left[p_S - (1 - \alpha)^{(\rho \Delta + \delta)}\right]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_G^*$</td>
<td>$k_M^*$</td>
<td>$k_M^*$</td>
<td>$\left[k_M^*\right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_G^*$</td>
<td>$Q$</td>
<td>$Q$</td>
<td>$\left[k_M^*\right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Pi^*_L$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Pi^*_M$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$TC^*_M$</td>
<td>$k_M^*Q$</td>
<td>$\left[k_M^*Q\right]$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\gamma(\alpha) = \frac{\gamma M - c_L}{\gamma L - c_L} + m \frac{\theta l^g - c_L}{\theta l^g - c_L}; \gamma M = 1 - \frac{\theta l^g - c_L}{\theta l^g - c_L}; \sum M = 1 - \frac{\theta l^g - c_L}{\theta l^g - c_L}; \gamma L = 1 - \frac{\theta l^g - c_L}{\theta l^g - c_L}; \gamma M = 1 - \frac{\theta l^g - c_L}{\theta l^g - c_L}; NA - Not applicable$

$k_M^* = (1 - \alpha_M)c_M + \alpha_M\tilde{p}_S + \rho \Delta \alpha M \alpha^*_L; k_M^* = (1 - \alpha_M)c_M + \alpha_M\tilde{p}_S + \rho \Delta \alpha M \alpha^*_L; k_M^* = \tilde{p}_S + \rho \Delta \alpha M \alpha^*_L - \rho \Delta (1 - \alpha_M)(1 - \alpha_L)$

$k_{LM}^* = \tilde{p}_S + \rho \Delta \alpha M \alpha^*_L - (\rho \Delta + \delta)(1 - \alpha_M)(1 - \alpha_L)$; In all tables, “$q_M = 0, qL = 0$” implies “spot-sourcing”.

Table 3  Equilibrium order allocation, contract terms and costs/profits for the no-guarantee, asymmetric information scenario.

<table>
<thead>
<tr>
<th>Regions</th>
<th>Allocation</th>
<th>$q_M = 0, qL = 0$</th>
<th>$q_M = Q, qL = 0$</th>
<th>$q_M = 0, qL = Q$</th>
<th>$q_M = Q, qL = Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NG $p_M^*$</td>
<td>$c_M$</td>
<td>$c_M$</td>
<td>$\left[c_M + \min\left(\frac{\theta l^g}{1-\alpha_M}\right)\right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_L^*$</td>
<td>$c_L$</td>
<td>$\left[c_L + \min\left(\frac{\theta l^g}{1-\alpha_M}\right)\right]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G $p_M^*$</td>
<td>$c_M$</td>
<td>$c_M$</td>
<td></td>
<td>$\left[p_S - (1 - \alpha)^{(\rho \Delta + \delta)}\right]$</td>
<td>$\left[p_S - (1 - \alpha)^{(\rho \Delta + \delta)}\right]$</td>
</tr>
<tr>
<td>$p_L^*$</td>
<td>$c_L$</td>
<td>$\left[p_S - (1 - \alpha)^{(\rho \Delta + \delta)}\right]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_G^*$</td>
<td>$k_M^*$</td>
<td>$k_M^*$</td>
<td>$\left[k_M^*\right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_G^*$</td>
<td>$Q$</td>
<td>$Q$</td>
<td>$\left[k_M^*\right]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Pi^*_L$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Pi^*_M$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$TC^*_M$</td>
<td>$k_M^*Q$</td>
<td>$\left[k_M^*Q\right]$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$k_M = (1 - \alpha_M)c_M + \alpha_M\tilde{p}_S + \rho \Delta \alpha M \alpha^*_L; k_M = (1 - \alpha_L)c_M + \alpha_L\tilde{p}_S + \rho \Delta \alpha M \alpha^*_L; k_S = \tilde{p}_S + \rho \Delta \alpha M \alpha^*_L - \rho \Delta (1 - \alpha_M)(1 - \alpha_L)$

Note that Regions (3a) and (3b) of the above Table are also applicable for the supply guarantee case under asymmetric information when $\alpha_M \geq \gamma M$ (see Figure 4(C)).
Table 4  Equilibrium order allocation, contract terms and costs/profits for the supply-guarantee, asymmetric information scenario. \((\alpha_M \leq \gamma_M)\)

<table>
<thead>
<tr>
<th>Regions</th>
<th>(\alpha^h &lt; \gamma_L)</th>
<th>(\alpha^h \geq \gamma_L)</th>
<th>(\alpha^h \geq \gamma_L)</th>
<th>(r^h \geq \frac{k_S^l - k_M^l}{k_S^l - k_M^l})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier (S_L)</td>
<td>((p_L^L, p_L^h))</td>
<td>((q_L^L, q_L^h))</td>
<td>((p_G^L, p_G^h))</td>
<td>((k_S^l, k_S^h))</td>
</tr>
<tr>
<td>Beliefs</td>
<td>(\mu = (\bar{r}_L, r^h))</td>
<td>(\mu = (\bar{r}_L, r^h))</td>
<td>(\mu = (1, 0)) if (p_M^l &gt; p_M^h)</td>
<td></td>
</tr>
<tr>
<td>Allocation</td>
<td>((q_L, q_M))</td>
<td>(q_L = q_M = 0) if (p_M^l &gt; p_M^h)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profits</td>
<td>(l)-type (S_L)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cost</td>
<td>(B)</td>
<td>(\bar{k}_S Q)</td>
<td>(\bar{k}_S Q)</td>
<td>(\bar{k}_S Q)</td>
</tr>
</tbody>
</table>

Table 5  Equilibrium order allocation, contract terms and costs/profits for the supply-guarantee, asymmetric information scenario. \((\gamma_M < \alpha_M \leq \gamma_M)\)

<table>
<thead>
<tr>
<th>Regions</th>
<th>(\alpha^h &lt; \gamma(\alpha_M))</th>
<th>(\alpha^h \geq \gamma(\alpha_M))</th>
<th>(\alpha^h \geq \gamma(\alpha_M))</th>
<th>(r^h \geq \frac{k_S^l - k_M^l}{k_S^l - k_M^l})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier (S_L)</td>
<td>((p_L^L, p_L^h))</td>
<td>((q_L^L, q_L^h))</td>
<td>((h_L^L, h_L^h))</td>
<td>((k_S^l, k_S^h))</td>
</tr>
<tr>
<td>Beliefs</td>
<td>(\mu = (\bar{r}_L, r^h))</td>
<td>(\mu = (\bar{r}_L, r^h))</td>
<td>(\mu = (1, 0)) if (p_M^l &gt; p_M^h)</td>
<td></td>
</tr>
<tr>
<td>Allocation</td>
<td>((q_L, q_M))</td>
<td>(q_L = q_M = 0) if (p_M^l &gt; p_M^h)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Profits</td>
<td>(l)-type (S_L)</td>
<td>(\bar{p}_L - k_L^l)</td>
<td>(k_L^l)</td>
<td>0</td>
</tr>
<tr>
<td>Cost</td>
<td>(B)</td>
<td>(\bar{k}_S Q)</td>
<td>(\bar{k}_S Q)</td>
<td>(\bar{k}_S Q)</td>
</tr>
</tbody>
</table>

\[F_M(p_G) = \frac{1}{\alpha_M} \left(1 - \frac{\alpha_M}{p_G} \right); \quad F_M(p_M) = 1 - \frac{\alpha_M}{p_M + \alpha_M}; \quad p_L = k_L^M + r^h(k_L^h - k_M^h); \quad p_M = \min(k_L^M - \alpha_M, k_S^l) + \bar{k}_S; \quad p_M = h_M(p_L) + h_M(p_G); \quad h_M(p_G) = c_M + \frac{p_G - k_M^l}{\alpha_M}; \quad k_M(p_M) = (1 - \alpha_M)p_M + \alpha_M\bar{p}_S + \rho \Delta \alpha_M; \quad k_M(p_M) = (1 - \alpha_M)p_M + \alpha_M\bar{p}_S + \rho \Delta \alpha_M\]