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MIXING OF NEUTRAL B MESONS AND FACTORIZATION

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Abstract

A brief review of checking the factorization hypothesis for matrix element of $B^0 - \bar{B}^0$ mixing within operator product expansion and QCD sum rules is given. Both perturbative and power corrections are considered.

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1. Introduction

Determination of the exact pattern of CP violation is one of the most important problems of modern particle physics. The system of neutral B mesons can provide some useful experimental information on the subject. To decipher this information and convert it into some knowledge of theoretical parameters of standard model or some extended model one needs quite accurate theoretical calculation of corresponding observables within the adopted theory. At present an essential obstacle in getting precise theoretical estimates for characteristics of CP violation related processes is a necessity of computing hadronic matrix elements that is a completely nonperturbative problem. The most popular approximation for estimating such elements is the factorization, or vacuum saturation, hypothesis the justification of which is quite unclear. In the present note we very briefly review some recent results of checking the validity of the factorization hypothesis with analytical methods. We consider three point correlator for computing power corrections violating the factorization approximation and two point correlator for computing perturbative ones.

2. Power corrections: three point correlator

In order to develop a machinery of operator product expansion and QCD sum rules^{1,2,3} we use here a three point correlator of the form^{4,5}

$$\begin{aligned}\Pi_{\mu\nu}(p, p') &= i^2 \int dx dy e^{ipx - ip'y} \langle 0 | T J_\mu(x) O(0)_{\Delta B=2} J_\nu(y) | 0 \rangle \\ &= p_\mu p'_\nu \Pi_1(p^2, p'^2, q^2) + \dots = p_\mu q_\nu \Pi_2(p^2, p'^2, q^2) + \dots\end{aligned}\quad (1)$$

where $q = p' - p$, $q^2 = 0$, $J_\mu = \bar{d}\gamma_\mu\gamma_5 b$ is an interpolating current for B meson,

$$\langle 0 | J_\mu | B^0(p) \rangle = i f_B p_\mu.$$

The operator $O_{\Delta B=2}$ is chosen with a standard normalization

$$O_{\Delta B=2} = \bar{b}\gamma_\mu(1 + \gamma_5)d\bar{b}\gamma_\mu(1 + \gamma_5)d$$

and the parameter B_B is defined by the relation

$$\langle \bar{B}^0 | O_{\Delta B=2} | B^0 \rangle = \frac{8}{3} f_B^2 m_B^2 B_B.$$

Within factorization approximation $B_B = 1$. For higher reliability we take for our analysis two invariant functions $\Pi_{1,2}(p^2, p'^2, q^2)$ that appear in the expression for the three point correlator Eq. (1). The dispersion relation dictates the following representation for these functions after using saturation with the low lying resonance state

$$\Pi_i(p^2, p'^2)|_{q^2=0} = \int ds ds' \frac{\rho(s, s')}{(s - p^2)(s' - p'^2)} = \frac{8/3 f_B^4 m_B^2 B_B}{(p^2 - m_B^2)(p'^2 - m_B^2)} + \dots$$

For both theoretical and physical parts of sum rules the factorization corresponds to the following representation of the amplitudes

$$\Pi_{\mu\nu}^{fact}(p, p') = \frac{8}{3} T_{\mu\beta}(p) T_{\nu\beta}(p'),$$

where

$$T_{\mu\beta}(p) = i \int dx e^{ipx} \langle 0 | T J_\mu(x) \bar{b}(0) \gamma_\beta (1 + \gamma_5) d(0) | 0 \rangle,$$

and gives the value $B_B = 1$ for the parameter that describes the ratio of exact matrix element to the factorized one.

Leading contributions of power corrections that are not caught within the factorization approximation are given by some specific diagrams with external vacuum fields. The first one is given by the gluon condensate⁵

$$\Delta\Pi_1^G = -\frac{1}{48\pi^2} \langle GG \rangle (pp') (5r(p^2)r(p'^2) + e(p^2)e(p'^2)),$$

$$\Delta\Pi_2^G = \Delta\Pi_1^G - \frac{1}{48\pi^2} \langle GG \rangle p^2 (r(p^2)r(p'^2) + e(p^2)e(p'^2)) + \frac{1}{24\pi^2} \langle GG \rangle e(p^2)g(p'^2)$$

where

$$r(p^2) = \int_0^1 dx \frac{x}{-p^2x + m^2}, \quad e(p^2) = \int_0^1 dx \frac{x - 2x^2}{-p^2x + m^2},$$

$$g(p^2) = - \int_0^1 dx (1 - 2x) \ln(1 - xp^2/m^2),$$

m is the b quark mass, $\langle GG \rangle = \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle$.

Next contributions are due to condensates of operators with dimension five in mass units. They are

$$\Delta\Pi_1^\sigma = 0, \quad \Delta\Pi_2^\sigma = -\frac{m \langle g \bar{d} \sigma_{\mu\nu} G^{\mu\nu} d \rangle}{6\pi^2 (p^2 - m^2)} \int_0^1 dx \frac{x(x + 1/2)}{-p'^2x + m^2}.$$

Finally, four quark operators contribute the amount that is not taken into account by the factorization approximation for vacuum expectation values

$$\Delta\Pi_1^{4q}(p, p') = \Delta\Pi_2^{4q}(p, p') = -\frac{2 \langle (\bar{d} \gamma_\mu (1 + \gamma_5) d)^2 \rangle^{non-fact}}{(p^2 - m^2)(p'^2 - m^2)}.$$

As in most applications of operator product expansion we do not include operators with dimension higher than six in our analysis. Numerical results are obtained after using Borel transformation in both p^2 and p'^2 independently and putting $M^2 = M'^2$ afterwards. For both invariant functions there is a fairly wide window of stability with respect to change of the Borel parameter. The numerical results are rather stable and reveal only

small violation of factorization. Namely, if $B_B = 1 + \Delta B_B$ then for both invariant functions and for wide range of parameters ($m_0^2, \langle \bar{q}q \rangle, \dots$) we get⁵

$$-\Delta B_B = 0 \div 0.1.$$

This estimate is very conservative. The absolute value of deflection from factorization approximation is about -0.05 for the parameter B_B . Main uncertainty is due to poor knowledge of numerical value of f_B on which there is a strong dependence (to the fourth power).

3. Perturbative corrections: two point correlator

Perturbative corrections of order α_s that violate the factorization approximation are connected with genuine three loop massive diagrams. Their computation with layout for sum rules technique based on three point correlator with independent Borel transformation with regards to kinematical variables p^2 and p'^2 can not be done at present because of technical complexity. Therefore we turn to two point correlator^{6,7} and introduce a quantity⁸

$$T(x) = \langle 0 | T O_{\Delta B=2}(x) O_{\Delta B=2}(0) | 0 \rangle.$$

The leading term of α_s expansion for the above correlator has an expression in the configuration space that reads

$$\begin{aligned} T_0(x) &= 2N_c^2 \left(1 + \frac{1}{N_c}\right) 16S'(x, m)S(-x, 0)S'(x, m)S(-x, 0) \\ &= 2 \left(1 + \frac{1}{N_c}\right) \text{tr}[S(x, m)S(-x, 0)]\text{tr}[S(x, m)S(-x, 0)] = 2 \left(1 + \frac{1}{N_c}\right) \Pi_5(x)\Pi_5(x) \end{aligned} \quad (2)$$

where $S(x, m)$ is the free fermion propagator and N_c stands for the number of quark colors. The prime means taking only the part of the propagator that is proportional to a γ matrix. The function $\Pi_5(x) = \langle 0 | T j_5(x) j_5(0) | 0 \rangle$ is the two point correlator associated to the current $j_5 = \bar{b}i\gamma_5 d$. Thus one observes a complete factorization in this order.

Eq. (2) can be rewritten in the form

$$T_0(x) = 2 \left(1 + \frac{1}{N_c}\right) \Pi_{\mu\nu}(x)\Pi^{\mu\nu}(x), \quad (3)$$

where $\Pi^{\mu\nu}(x) = \langle 0 | T j_L^\mu(x) j_L^\nu(0) | 0 \rangle$ and $j_L^\mu = \bar{b}_L \gamma^\mu d_L$. The Lorentz decomposition in x -space reads

$$\Pi^{\mu\nu}(x) = (-\partial^\mu \partial^\nu + g^{\mu\nu} \partial^2) \Pi_T(x^2) - \partial^\mu \partial^\nu \Pi_L(x^2)$$

that again demonstrates an explicit factorization in the configuration space to leading order in α_s .

The dispersion representation in x -space for any two point correlator $\Pi_j(x)$ ($j = T, L, 5$) has the form

$$i\Pi_j(x^2) = \int_{s_j}^{\infty} r_j(s)D(x, s)ds \quad (4)$$

where $D(x, s)$ is a free boson propagator with the “mass” \sqrt{s} . The spectral functions r_j read to leading order in α_s

$$r_L^{(0)}(s) = \frac{N_c}{16\pi^2}z(1-z)^2, \quad r_T^{(0)}(s) = \frac{N_c}{48\pi^2}(1-z)^2(2+z), \quad r_5^{(0)}(s) = \frac{m^2 N_c}{8\pi^2} \frac{(1-z)^2}{z}$$

where $z = m^2/s$, m is the b quark mass.

The spectral function $\rho(s)$ of the full correlator $T(x)$ is defined in the same way as in Eq. (4). To first order in α_s it can be expressed in terms of the spectral functions $r_j(s)$ associated to the two-line correlators in the form

$$\rho(s) = \int r_1(s_1)r_2(s_2)\Phi(s; s_1, s_2)ds_1ds_2$$

where

$$\Phi(s; s_1, s_2) = \frac{1}{16\pi^2s} \sqrt{(s - s_1 - s_2)^2 - 4s_1s_2}$$

is the two-body phase space factor.

To leading order in $1/N_c$, one can write the correlator $T(x)$ as a product of two two-line correlators. It is worthwhile to notice that this decomposition is gauge invariant and finite, i.e. it does not require any renormalization.

The full spectral density $\rho(s)$ has been computed numerically to the first order in α_s ⁸ with a heavy use of known results for two loop massive diagrams obtained earlier (e.g.⁹) and the program of symbolic computation REDUCE. We analyze our results concentrating on presentation of the entire spectral density $\rho(s)$ as a sum of factorizable and nonfactorizable pieces

$$\rho(s) = \rho_0(s) (1 + \Delta\rho_f(s) + \Delta\rho_{nf}(s)).$$

Nonfactorizable part of the spectral density is given by one gluon exchange diagrams of a two-line correlator.

Within sum rules approach one works with moments

$$M_i(s_{th}) = \int_{4m_b^2}^{s_{th}} \rho(s)s^{-i}ds$$

that are decomposed as

$$M_i(s_{th}) = M_i^0(s_{th}) \left(1 + \Delta M_i^f(s_{th}) + \Delta M_i^{nf}(s_{th})\right)$$

according to the decomposition of the spectral density.

A set of input parameters for numerical estimates is $\Lambda_{\overline{MS}}^{(5)} = 175$ MeV, $m_b = 4.6$ GeV. The moments of the factorizable spectral density are almost independent of the power i of the weight function s^{-i} . The nonfactorizable correction does not exceed a 15% level with respect to the full factorized spectral density. Corrections of order α_s (factorizable+nonfactorizable) are large for both the spectral density itself and its moments. Depending on the energy s they can reach a magnitude of 100% with respect to the leading term. The nonfactorizable corrections measured in terms of the fully factorized (lowest order + α_s terms) spectral density are moderate. All the factorizable corrections to the correlator, however, can be absorbed into the calculation of the decay constant f_B from two point correlator with two quark lines, in such a way that the relevant corrections to the B_B parameter are only due to nonfactorizable ones. Results for spectral density itself and for its moments for different values of integration regions are collected in Tables 1,2 taken from ref.⁸.

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s/m_b^2	$\Delta\rho_f$	$\Delta\rho_{nf}$	$\Delta\rho_{nf}/(1 + \Delta\rho_f)$
5.5	1.03	0.02	0.01
6.0	0.95	0.21	0.11
6.5	0.89	0.29	0.15
7.0	0.84	0.32	0.17

Table 1: Normalized spectral densities

i	s_{th}/m_b^2	ΔM_f	ΔM_{nf}	$\Delta M_{nf}/(1 + \Delta M_f)$
0	5.5	1.07	-0.16	-0.08
	6.0	0.99	0.11	0.06
	6.5	0.93	0.23	0.12
	7.0	0.88	0.29	0.15
5	5.5	1.08	-0.21	-0.10
	6.0	1.00	0.08	0.04
	6.5	0.94	0.20	0.11
	7.0	0.89	0.27	0.14
10	5.5	1.09	-0.27	-0.13
	6.0	1.01	0.02	0.01
	6.5	0.96	0.16	0.08
	7.0	0.91	0.23	0.12

Table 2: Normalized moments of spectral densities