Abstract. The tracking and synchronization problem of uncertain chaotic system, which is considered to be applied in secure communication in the future by many researchers, is considered in this paper. A double integral sliding mode controller is adopted to cope with the uncertainties of the chaotic system. Adaptive and robust strategies, such as Nussbaum gain method, are used to solve the unmodeled dynamic problem and unknown control direction problem. Meanwhile, the stability of the whole system is guaranteed by constructing of a big Lyapunov function for the whole system. Finally, a four dimension super-chaotic system is used as an example to do the numerical simulation and it testifies the rightness and effectiveness of the proposed method.

Keywords: Synchronization, adaptive, chaos, unknown control direction

1 INTRODUCTION

As a main aspect of nonlinear science, chaos has attracted many researchers of various fields [1, 2, 3]. It also has comprehensive applications in natural science and social science.

Chaos synchronization is an important research direction of chaotic science. It has been researched by many experts since the 1990’s [4, 5, 6, 7, 8]. Much progress has also been made in its applications such as secret communication and image manipulation [9]. There are many methods proposed to solve synchronization problem
of chaotic systems [9, 10, 11, 12]. In many researches the situation that there only exist static uncertainties between driver system and response system is considered. So the unmodeled dynamics of synchronization between chaotic system with different structure were seldom considered, especially for the situation that there exist static uncertainties, unknown parameters and dynamic uncertainties simultaneously; but it is very possible for the actual system that driven systems have different structure with response systems [13], or parameters may be changed unexpectedly because of the disturbance of environment, or the system model is inevitably inaccurate because of the dynamic uncertainties. The above situations are very possible to happen when synchronization of chaotic systems is used in the application of secure communication. So it is meaningful to study the synchronization of chaotic systems with both static and dynamic uncertainties.

In this paper, four kinds of uncertainties such as unknown parameters, static uncertain functions, unmodeled dynamics and unknown control directions [14, 15, 16, 17, 18] are considered simultaneously for the synchronization of chaotic systems. Adaptive method, robust control and Nussbaum gain control strategy are integrated to handle the above complex uncertainties. Also a Lyapunov function is constructed to guarantee the stability of the whole system with a double integral sliding mode type controller. Finally, numerical simulations are done and the good performance of the controller testifies the effectiveness and rightness of our proposed method. Especially, it is worth pointing out that a novel characteristic of the Nussbaum gain function is firstly defined and used to solve the synchronization problem with unmodelled dynamics.

2 MODEL DESCRIPTION

The following typical uncertain chaotic system with nonlinear functions is considered as a response system:

\[
\begin{align*}
\dot{\xi} &= q(x_1, \xi, t) \\
\dot{x} &= f(x) + \Delta(x, \xi, t) + n(u)
\end{align*}
\]

where \(x = [x_1, \ldots, x_n]^T\), \(u = [u_1, \ldots, u_n]^T\) are vectors, \(n(u)\) are continuous nonlinear input functions. A three dimensional coordinate system is taken as an example, and it can be extended as follows:

\[
\begin{align*}
\dot{\xi} &= q(x, \xi, t) \\
\dot{x}_1 &= f_1(x_1, \ldots, x_4) + \Delta_1(x, \xi, t) + n_1(u) \\
\dot{x}_2 &= f_2(x_1, \ldots, x_4) + \Delta_2(x, \xi, t) + n_2(u) \\
\dot{x}_3 &= f_3(x_1, \ldots, x_4) + \Delta_3(x, \xi, t) + n_3(u)
\end{align*}
\]

where \(f(x)\) are known functions of the system and \(\Delta(x, \xi, t)\) are uncertain nonlinear dynamic functions of the system.
So the objective of tracking problem of chaotic system is to design a control \( u = u(x, \dot{\theta}) \), \( \dot{\theta} = g(x, \dot{\theta}) \), such that states of the system can track to the desired value. In other word, it satisfies \( x \rightarrow x^d \), where \( x^d \) is the desired value.

Without loss of generality, assume \( x^d \) is a constant value; then \( \dot{x}_i^d = 0 \). Define a new variable \( e_i = x_i - x_i^d \); then the error system can be described as

\[
\dot{e}_i = f_i(x_1, \ldots, x_4) + \Delta_i(x_1, \ldots, x_4, \xi) + b_i u_i. \tag{7}
\]

To make the following illustration and proof easy, the input nonlinearity \( n_i(u) \) is neglected in the tracking problem and it will be considered in the synchronization problem. So \( b_i \) is a known constant coefficient here. The driven system can be described as

\[
\dot{y} = f(y) + \theta f_\theta(y). \tag{8}
\]

Taking a three dimensional coordinate system as an example, it can be extended as

\[
\dot{y}_1 = f_{y1}(y_1, \ldots, y_4) + \theta_1 f_{\theta 1}(y) \tag{9}
\]

\[
\dot{y}_2 = f_{y2}(y_1, \ldots, y_4) + \theta_2 f_{\theta 2}(y) \tag{10}
\]

\[
\dot{y}_3 = f_{y3}(y_1, \ldots, y_4) + \theta_3 f_{\theta 3}(y) \tag{11}
\]

where \( \theta \) are unknown parameters, \( f_\theta(y) \) are known functions.

So the objective of the synchronization problem is to design a control \( u = u(x, \dot{\theta}, \ddot{d}) \), where \( \dot{\theta}' = g_1(x, \dot{\theta}, \ddot{d}) \) and \( \ddot{d}' = g_2(x, \dot{\theta}, \ddot{d}) \) such that the response system can track the driven system, that is to say \( y \rightarrow x \).

Define a new variable as \( z_i = y_i - x_i \).

Then the error system can be described as

\[
\dot{z}_i = f_i(x_1, \ldots, x_4) - f_{yi}(y_1, \ldots, y_4) - \theta_i f_{\theta i}(y) + \Delta_i(x, \xi, t) - n_i(u_i) \tag{13}
\]

where \( \Delta_i(\bullet) \) and \( q_i(\bullet) \) are unknown continuous Lipschitz functions, the \( \xi \) subsystem is the uncertain dynamic part of the above system, and \( \Delta_i(\bullet) \) represents the uncertain nonlinearities of the system, which satisfies the following assumption.

### 3 Assumptions

**Assumption 1.** The \( \xi \) subsystem can be viewed that it has a input as state \( x \) and there exists an input-to-state practical stability Lyapunov function \( V_0(\xi) \). That is to say there exists a smooth positive definite and canonical function \( V_0(\xi) \) such that

\[
\frac{\partial V_0(\xi)}{\partial \xi} q(x, \xi, t) \leq -\alpha_2(V_0(\xi)) + v_z(|s_i|) + d_z, \forall (x, \xi, t) \in R \times R^{n_\theta} \times R_+ \tag{14}
\]
where $\alpha_z(\bullet)$ and $v_z(\bullet)$ are $k_\infty$ type functions, $s = f(x, y)$ and $y$ are chaotic signals, so they are bounded, and $d_z$ is a nonnegative constant.

**Assumption 2.** For $1 \leq i \leq n$, there exists an unknown constant $p_i^* \leq d_i$ such that

$$|\Delta_i(X, \xi, t)| \leq p_i^* \psi_{i1}(|(x_1, \ldots, x_i)|) + p_i^* \psi_{i2}(|\xi|), \forall (X, \xi, t) \in \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}_+$$  \hspace{1cm} (15)

where $d_i$ is a known constant, $\psi_{i1}(\bullet)$ and $\psi_{i2}(\bullet)$ are known nonnegative smooth functions with $\psi_{i2}(0) = 0$.

**Remark 1.** Without loss of generality, assume that there exists constant $\varepsilon_{ci}$ big enough such that

$$\frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{ci})^2} - \alpha_z(V_0(\xi)) < 0.$$  \hspace{1cm} (16)

Similarly, there exist parameters big enough $\varepsilon_{c3i}$ such that

$$v_z(|s_i|) - \varepsilon_{c3i}s_i^2 < 0.$$  \hspace{1cm} (17)

**Definition 1.** $N(\chi)$ is a Nussbaum-type function, if it has the following characteristics

$$\lim_{s \to \infty} \sup \frac{1}{s} \int_0^s N(x)dx = +\infty$$  \hspace{1cm} (18)

$$\lim_{s \to \infty} \inf \frac{1}{s} \int_0^s N(x)dx = -\infty.$$  \hspace{1cm} (19)

Meanwhile, it is easy to prove that $N(\chi) + k_d$ also satisfies

$$\lim_{s \to \infty} \sup \frac{1}{s} \int_0^s \{N(x) + k_d\}dx = +\infty$$  \hspace{1cm} (20)

$$\lim_{s \to \infty} \inf \frac{1}{s} \int_0^s \{N(x) + k_d\}dx = -\infty.$$  \hspace{1cm} (21)

**4 TRACKING OF UNCERTAIN CHAOTIC SYSTEM**

Considering $i^{th}$ subsystem of the error system about tracking problem, it has

$$\dot{e}_i = f_i(x_1, \ldots, x_4) + \Delta_i(x_1, \ldots, x_4) + b_i u_i.$$  \hspace{1cm} (22)

Design the control $u_i$ as follows:

$$u_i = f_{2i}(x)[-f_i(x_1, \ldots, x_4) - \eta(x, z) + f_{zi}(z_i)].$$  \hspace{1cm} (23)

Remember that

$$|z_i\Delta_i(X, \xi, t)| \leq p_i^* |z_i| \psi_{i1}(|(x_1, \ldots, x_i)|) + p_i^* |z_i| \psi_{i2}(|\xi|)$$  \hspace{1cm} (24)
where \( f_{zi}(x) = b_i^{-1} \) and

\[
    f_{zi}(z_i) = -k_{i1}z_i - k_{i2} \frac{z_i}{|z_i| + \varepsilon_{i1}} - k_{i3} \frac{3}{2} z_i^{1/3} \exp(z_i^{2/3}) - k_{i4} \text{sign}(z_i). \tag{25}
\]

Then it holds

\[
    z_i \dot{z}_i = z_i[\Delta_i(x) - \eta(x, z) + f_{zi}(z_i)] \\
    \leq z_i f_{zi}(z_i) + p_i^* |z_i| \psi_{i1}([(x_1, \ldots, x_i)]) + p_i^* |z_i| \psi_{i2}(|\xi|) - z_i \eta(x, z) \\
    = z_i f_{zi}(z_i) + p_i^* |z_i| \psi_{i1}([(x_1, \ldots, x_i)]) + \varepsilon_{c3}^2 z_i^2 + \frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{c1})^2} - z_i \eta(x, z). \tag{26}
\]

Design the robust control law as

\[
    \eta(x, z) = \hat{p}_i^* |z_i| \psi_{i1}([(x_1, \ldots, x_i)]) + \hat{\varepsilon}_{c2} z_i \tag{27}
\]

where \( \hat{p}_i^* \) is defined as

\[
    \hat{p}_i^* = p_i^* - \hat{p}_i^*, \quad \hat{\varepsilon}_{c2} = \varepsilon_{c1}^2 + \varepsilon_{c3} - \hat{\varepsilon}_{c2}. \tag{28}
\]

Then it satisfies

\[
    z_i \dot{z}_i = z_i f_{zi}(z_i) + \hat{p}_i^* |z_i| \psi_{i1}([(x_1, \ldots, x_i)]) + \hat{\varepsilon}_{c2} z_i^2 + \frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{c1})^2} - \varepsilon_{c3} z_i^2. \tag{29}
\]

Design the adaptive control law as

\[
    \frac{d\hat{p}_i^*}{dt} = \text{sign}(z_i) \psi_{i1}([(x_1, \ldots, x_i)]), \quad \frac{d\hat{\varepsilon}_{c2}}{dt} = z_i^2. \tag{30}
\]

Choose a Lyapunov function as

\[
    V = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \frac{1}{2}(\hat{\varepsilon}_{c2})^2 + \frac{1}{2}(\hat{p}_i^*)^2 + V_0(\xi). \tag{31}
\]

Solve its derivative along its trajectory of differential equations; it holds

\[
    \dot{V} = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \frac{1}{2}(\hat{\varepsilon}_{c2})^2 + \frac{1}{2}(\hat{p}_i^*)^2 + V_0(\xi) \\
    \leq \sum_{i=1}^{n} z_if_{zi}(z_i) + \sum_{i=1}^{n} \frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{c1})^2} - \alpha_z(V_0(\xi)) + v_z(|z|) + \dot{z} - \varepsilon_{c3} z_i^2 \\
    \leq \sum_{i=1}^{n} z_if_{zi}(z_i) + v_z(|z|) + d_z - \varepsilon_{c3} z_i^2 \leq \sum_{i=1}^{n} z_i f_{zi}(z_i) + d_z. \tag{32}
\]

Then it is easy to prove that \( z_i \) is bounded and it can converge to a small neighborhood of zero with a proper design of \( f_{zi}(z_i) \).

Since tracking problem is easy compared with the below synchronization problem situation, the numerical simulation result and details are ignored here.
4.1 Synchronization of Uncertain Chaotic Systems

Consider the subsystem
\[ \dot{z}_i = f_{yi}(y_1, \ldots, y_4) + \theta_i f_{yi}(y) - \Delta_i(x, \xi, t) - f_i(x_1, \ldots, x_4) - n_i(u_i). \]  

Remark 2. Wanglong Li assumed the nonlinear input function \( n_i(u_i) \) is bounded by \( u_i \) in paper [13]. It yields positive constants \( c_{i1} \) and \( c_{i2} \), such that the following conditions are satisfied.
\[ c_{i1} \leq \frac{n_i(u_i)}{u_i} \leq c_{i2}, i = 1, \ldots, n. \]  

Then they have
\[ c_{i1} u_i^2 \leq u_i n_i(u_i) \leq c_{i2} u_i^2 \]  

It is still a strict condition for many real systems. In this paper, we further relax the restriction for the nonlinear input of the previous work as following Assumption 3.

Assumption 3. For \( 1 \leq i \leq n \), there exists an unknown time varying variable \( b_i(t) \) such that
\[ n_i(u_i) = b_i(t) u_i \]
and assume \( b_i(t) \) is bounded. To make it simple, write \( b_i(t) \) as \( b_i \); then \( b_i \) is an unknown bounded time-varying parameter. Especially, the sign of \( b_i \) is unknown.

It is easy to prove that Assumption 3 is more relax than the assumption in [13]. For any \( n_i(u_i) \) satisfies \( c_{i1} \leq \frac{n_i(u_i)}{u_i} \leq c_{i2} \) in paper [13], \( b_i \) can always be chosen as \( b_i = \frac{n_i(u_i)}{u_i} \); then \( c_{i1} \leq b_i \leq c_{i2} \). \( b_i \) is restricted to be positive; but in this paper, \( b_i \) can be positive or negative; what is worse, the sign of \( b_i \) is changing during a comparatively long time interval. With Assumption 3, the error system can be written as follows:

\[ \dot{z}_i = f_{yi}(y_1, \ldots, y_4) + \theta_i f_{yi}(y) - \Delta_i(x, \xi, t) - f_i(x_1, \ldots, x_4) - b_i u_i. \]  

Define a double integral sliding mode surface as
\[ s_i = z_i + a_{si} \int_0^t z_i dt + b_{si} \int_0^t \int_0^t z_i dt dt. \]

Solve the derivative as
\[ \dot{s}_i = \dot{z}_i + a_{si} z_i + b_{si} \int_0^t z_i dt = f_{yi}(y_1, \ldots, y_4) + \theta_i f_{yi}(y)
- f_i(x_1, \ldots, x_4) - \Delta_i(x, \xi, t) - b_i u_i + a_{si} z_i + b_{si} \int_0^t z_i dt \]
and design the control \( u_i \) as
\[
 u_i = f_{si}(x)u_i^d = f_{si}(x)[-f_{yi}(y) - \tilde{\theta}_i f_{\theta i}(y)] + f_i(x) + \eta_i(x, y, z_i, s_i) - a_{si}z_i - b_{si} \int_0^t z_i dt + f_{sri}(s_i)
\]
where
\[
f_{si}(x) = N(k_i)
\]
\[
f_{sri}(s) = -k_{i1} s_i - k_{i2} \frac{s_i}{|s_i| + \varepsilon_{i1}} - k_{i3} \frac{3}{2} s_i^{1/3} \exp(s_i^{2/3}) - k_{i4} \text{sign}(s_{i1}).
\]
Then
\[
s_i \dot{s}_i = s_i f_{si}(s_i) + s_i \{ \tilde{\theta}_i f_{\theta i}(y) + \eta_i(x, y, z_i, s_i) - \Delta_i(x, \xi, t) \} + s_i(-b_i N(k_i) u_i^d - u_i^d)
\]
\[
|s_i \Delta_i(x, \xi, t)| \leq p_1^* |s_i| \psi_{i1}(|(x_1, \ldots, x_i)|) + p_1^* |s_i| \psi_{i2}(|\xi|)
\]
and it also can be written as
\[
s_i \dot{s}_i = s_i f_{si}(s_i) + s_i \{ \tilde{\theta}_i f_{\theta i}(y) + \eta_i(x, y, z_i, s_i) - \Delta_i(x, \xi, t) \} + s_i(-b_i N(k_i) u_i^d - u_i^d)
\]
\[
s_i \dot{s}_i \leq s_i f_{si}(s_i) + p_1^* |s_i| \psi_{i1}(|(x_1, \ldots, x_i)|) + p_1^* |s_i| \psi_{i2}(|\xi|) + s_i \eta(x, y, z_i, s_i) + s_i \tilde{\theta}_i f_{\theta i}(y) + s_i(-b_i N(k_i) u_i^d - u_i^d).
\]
It can be arranged as follows:
\[
s_i \dot{s}_i \leq s_i f_{zi}(z_i) + p_1^* |s_i| \psi_{i1}(|(x_1, \ldots, x_i)|)
\]
\[
+ \varepsilon_{ci}^2 \varepsilon_{c1}^2 + \frac{[\psi_{i2}(|\xi|)]^2}{(2\varepsilon_{ci}^2)^2} + s_i \eta(x, y, z_i, s_i)
\]
\[
+ s_i \tilde{\theta}_i f_{\theta i}(y) + s_i(-b_i N(k_i) u_i^d - u_i^d).
\]
Design
\[
\eta(x, y, z_i, s_i) = -\bar{p}_i^* \text{sign}(s_i) \psi_{i1}(|(x_1, \ldots, x_i)|) - \bar{\varepsilon}_{c2i} s_i
\]
and define
\[
\bar{p}_i^* = p_1^* - \bar{p}_i^*
\]
\[
\bar{\varepsilon}_{c2i} = \varepsilon_{ci}^2 + \varepsilon_{c1}^2 - \bar{\varepsilon}_{c2i}
\]
\[
\frac{d \tilde{\theta}_i}{dt} = s_i f_{\theta i}(y).
\]
Then the following equation holds:
\[
s_i \dot{s}_i = s_i f_{si}(s_i) + \bar{p}_i^* |s_i| \psi_{i1}(|(x_1, \ldots, x_i)|)
\[ + \varepsilon c_2 s_i^2 + \frac{[\psi_2(|\xi|)]^2}{(2\varepsilon c_i)^2} - \varepsilon c_3 s_i^2 + s_i(-b_i N(k_i) u_i^d - u_i^d). \] (47)

Define
\[ \frac{d\tilde{p}_i^*}{dt} = |s_i| \psi_{i1}(|x_1, \ldots, x_i|), \frac{d\varepsilon c_2i}{dt} = s_i \] (48)
and choose a Lyapunov function as
\[ V_i = \frac{1}{2} \left[ s_i^2 + (\tilde{\theta}_i)^2 \right] + \frac{1}{2} (\varepsilon c_2 i)^2 + \frac{1}{2} (\tilde{p}_i^*)^2 + V_0(\xi) \] (49)
and the derivative of the Lyapunov function can be written as
\[ \dot{V}_i \leq s_i f_{zi}(s_i) + \frac{[\psi_2(|\xi|)]^2}{(2\varepsilon c_i)^2} - \alpha_z(V_0(\xi)) + v_z(|s_i|) + d_z - \varepsilon c_3 s_i^2. \] (50)

According to the assumption, it is easy to prove that
\[ \dot{V}_i \leq s_i f_{zi}(s_i) + d_z + s_i(-b_i N(k_i)) u_i^d - u_i^d. \] (51)

With the discussion of Assumption 3, it is necessary to adopt a new kind of control strategy to solve the unknown control direction of \( b_i \). Then use the Nussbaum gain method and design the Nussbaum gain regulation law as
\[ \dot{k}_i = -s_i u_i^d. \] (52)

Then,
\[ \dot{V}_i \leq d_z + \dot{k}_i (1 + b_i N(k_i)). \] (53)

With integral computation on both sides of the inequality, we have
\[ V_i(t) - V_i(0) \leq (k(t) - k(0)) + b_i \int_{k(0)}^{k(t)} (N(k_i) + d_z)dk. \] (54)

**Remark 3.** Use the apagoge method; assume that \( k(t) \) will be unstable in finite time, so when \( t \to t_n \), it has \( k(t) \to \infty \). With the help of Nussbaum gain function characteristics, it is easy to prove the above inequality is contradicting. So \( k(t) \) is bounded in finite time.

Now, it is also easy to prove that \( s_i \) is bounded and design \( f_{si}(s_i) \) such that \( s_i \) can be converged to a small enough interval near zero. Furthermore, because of the design of sliding mode coefficients, it is easy to guarantee that \( s_i \to 0 \); then it has \( z_i \to 0 \). So the system is proved to be stable.
5 EXAMPLE AND SIMULATION

Taking the three dimensional coordinate chaotic system as an example to make a numerical simulation, the model can be described as

\[
\begin{align*}
\dot{\xi} &= -5\xi + 3x_1 + 0.2x_2 + 1.4x_3 + 2.7x_1x_3 \\
\dot{x}_1 &= a(x_2 - x_1) + k_b(x_2 \cos x_2 + \xi) + \lambda_1 u_1 \\
\dot{x}_2 &= bx_1 - x_1x_3 - x_2 + k_b[(1 + \sin(x_2x_3))x_2 + 0.7\xi x_2] + \lambda_2 u_2 \\
\dot{x}_3 &= -cx_3 + x_1x_2 + k_b[(2 - \cos(x_1x_2x_3))x_1 + 3.5\xi] + \lambda_3 u_3
\end{align*}
\]

where \(a, b, c\) are unknown constants, which are set as \((a, b, c) = (10, 28, 8/3)\), and the uncertain nonlinear function obviously satisfies all assumptions of this paper. The initial state of the system can be chosen as

\[
(\xi, x_1, x_2, x_3) = (0, 1, 1, 1).
\]

The model of the driven system can be described as a Genesio system

\[
\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= y_3 \\
\dot{y}_3 &= -a_1y_1 - b_1y_2 - c_1y_3 + y_1^2
\end{align*}
\]

where the unknown parameters are chosen as

\[
(a_1, b_1, c_1) = (6, 2.92, 1.2)
\]

and the initial states are chosen as

\[
(y_1, y_2, y_3) = (1, 1, 1).
\]

The comparison between free trajectory of driven system and it of response system without control can be seen in Figures 1 and 2. It is obvious that the synchronization between the above two system can not be realized.

Assume that the unknown control direction switches twice at the time of 2.5 s and 4.5 s, respectively. Using the proposed method, the synchronization of chaotic system can be achieved (Figure 3, 4 and 5).

The curve of the error of synchronization is shown in Figures 6, 7 and 8.

The curve of Nussbaum gains can be seen in Figures 9, 10 and 11. They are converged to a new value at the time of 1 s when the input direction switches. According to the figures, a conclusion can be made that synchronization between the driven and response systems can be achieved quickly.

The curve of real control gains is given in Figures 12, 13 and 14. The figures show that the gain of control can be adapted to the change of input directions such that the chaotic systems with both input unmodeled dynamics and uncertain input can be synchronized.
6 CONCLUSIONS

The main contribution of this paper can be summarized as follows. First, to make the synchronization problem easy to be understood, a simple situation of super-chaotic system is considered and the tracking problem is investigated. Second,
Figure 3. Synchronization of $x_1$ and $y_1$

Figure 4. Synchronization of $x_2$ and $y_2$
Figure 5. Synchronization of $x_3$ and $y_3$

Figure 6. Error of synchronization $e_1$
Figure 7. Error of synchronization $e_2$

Figure 8. Error of synchronization $e_3$
Figure 9. Nussbaum gain of $k_3$

Figure 10. Nussbaum gain of $k_2$
Figure 11. Nussbaum gain of $k_3$

Figure 12. Real control gain of $u_1$
Figure 13. Real control gain of $u_2$

Figure 14. Real control gain of $u_3$
the synchronization problem is studied and a double integral sliding mode method, robust control, adaptive control strategy and Nussbaum gain method are perfectly integrated to solve complex uncertainties. Third, a novel characteristic of Nussbaum function is proposed and used to cope with dynamic uncertainties in this paper. Also, a numerical simulation is made and good performance is achieved; this testifies the rightness and effectiveness of the proposed method.

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