The double competition multigraph of a digraph

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The competition graph of a digraph is defined to be the intersection graph of the family of the out-neighborhoods of the vertices of the digraph (see [6] for intersection graphs). A digraph $D$ is a pair $(V(D), A(D))$ of a set $V(D)$ of vertices and a set $A(D)$ of ordered pairs of vertices, called arcs. An arc of the form $(v, v)$ is called a loop. For a vertex $x$ in a digraph $D$, we denote the out-neighborhood of $x$ in $D$ by $N^+_D(x)$ and the in-neighborhood of $x$ in $D$ by $N^-_D(x)$, i.e., $N^+_D(x) := \{v \in V(D) \mid (x, v) \in A(D)\}$ and $N^-_D(x) := \{v \in V(D) \mid (v, x) \in A(D)\}$. A graph is a pair $(V(G), E(G))$ of a set $V(G)$ of vertices and a set $E(G)$ of unordered pairs of vertices, called edges. The competition graph of a digraph $D$ is the graph which has the same vertex set as $D$ and has an edge between two distinct vertices $x$ and $y$ if and only if both $N^+_D(x) \cap N^+_D(y) \neq \emptyset$. R. D. Dutton and R. C. Brigham [3] and F. S. Roberts and J. E. Steif [8] gave characterizations of competition graphs by using edge clique covers of graphs. The notion of competition graphs was introduced by J. E. Cohen [2] in 1968 in connection with a problem in ecology, and several variants and generalizations of competition graphs have been studied.

In 1987, D. D. Scott [11] introduced the notion of double competition graphs as a variant of the notion of competition graphs. The double competition graph (or the competition-common enemy graph or the CCE graph) of a digraph $D$ is the graph which has the same vertex set as $D$ and has an edge between two distinct vertices $x$ and $y$ if and only if both $N^+_D(x) \cap N^+_D(y) \neq \emptyset$ and $N^-_D(x) \cap N^-_D(y) \neq \emptyset$ hold. See [4, 5, 10, 12] for recent results on double competition graphs.

A multigraph $M$ is a pair $(V(M), E(M))$ of a set $V(M)$ of vertices and a multiset $E(M)$ of unordered pairs of vertices, called edges. Note that, in our definition, multigraphs have no loops. We may consider a multigraph $M$ as the pair $(V(M), m_M)$ of the vertex set $V(M)$ and the nonnegative integer-valued function $m_M : \binom{V}{2} \to \mathbb{Z}_{\geq 0}$ on the set $\binom{V}{2}$ of all unordered pairs of $V$ where $m_M(\{x, y\})$ is defined to be the number of multiple edges between the vertices $x$ and $y$ in $M$. The notion of competition multigraphs was introduced by C. A. Anderson, K. F. Jones, J. R. Lundgren, and T. A. McKee [1] in 1990 as a variant of the notion of competition graphs. The competition multigraph of a digraph $D$ is the multigraph which has the same vertex set as $D$ and has $m_{xy}$ multiple edges between two distinct vertices $x$ and $y$, where $m_{xy}$ is the nonnegative integer defined by $m_{xy} = |N^+_D(x) \cap N^+_D(y)|$. See [9, 13] for recent results on competition multigraphs.

In this talk, we introduce the notion of the double competition multigraph of a

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digraph as follows. Let $D$ be a digraph. The \textit{double competition multigraph} of $D$ is the multigraph which has the same vertex set as $D$ and has $m_{xy}$ multiple edges between two distinct vertices $x$ and $y$, where $m_{xy}$ is the nonnegative integer defined by

$$m_{xy} = |N^+_D(x) \cap N^+_D(y)| \cdot |N^-_D(x) \cap N^-_D(y)|,$$

i.e., the multigraph $M$ defined by $V(M) = V(D)$ and $m_M(\{x, y\}) = m_{xy}$.

We give characterizations of the double competition multigraphs of arbitrary digraphs, loopless digraphs, reflexive digraphs, and acyclic digraphs in terms of edge clique partitions of the multigraphs.

(This is joint work with Yoshio SANO, and this talk is based on [7].)

References


