LINEAR PRECODING AIDED BLIND EQUALIZATION WITH INDEPENDENT COMPONENT ANALYSIS IN MIMO OFDM SYSTEMS

Jingbo Gao, Xu Zhu, and Asoke K. Nandi

Department of Electrical Engineering and Electronics
The University of Liverpool, Brownlow Hill, Liverpool, L69 3GJ, U.K.
Email: {jgao, xuzhu, a.nandi}@liverpool.ac.uk

ABSTRACT

We propose a novel blind equalization structure based on independent component analysis (ICA) for multiple input multiple output (MIMO) orthogonal frequency division multiplexing (OFDM) systems. A linear precoding is exploited to resolve completely the phase and permutation ambiguity of the ICA model, by using the correlation between the source data and the reference data. Furthermore, iterative channel interpolation is employed to improve the channel estimation accuracy of ICA, and is incorporated with minimum mean square error (MMSE) based equalization and layered space-frequency equalization (LSFE) to enhance the system performance. Simulation results show that the proposed blind OFDM receivers significantly outperform the existing subspace method, and provide a performance close to the case with perfect channel state information (CSI).

1. INTRODUCTION

Future wireless communication systems must reliably provide very high data rate over frequency selective fading channels. Orthogonal frequency division multiplexing (OFDM) [1] and multiple-input multiple-output (MIMO) [1] are two promising candidates to meet this demand. In order to combat inter-symbol interference and co-channel interference, equalization [2] is employed in MIMO OFDM systems. Blind equalization, which obtains equalizer coefficients directly from the received signals by utilizing the structure and statistics of the signal, significantly improves the spectral efficiency compared to training based methods.

Independent component analysis (ICA) [3, 4] is an attractive higher order statistics (HOS) based approach applied to blind equalization. In [5], ICA was incorporated with blind receivers for MIMO OFDM system, which applies ICA to all subcarriers and proposes a reordering method to deal with the phase and permutation ambiguity of the ICA estimates. However, it does not completely resolve the ambiguity. Furthermore, there is significant performance degradation for higher order modulation methods. In [6], a sequence approach was employed to resolve the ambiguity. But error propagation significantly degrades system performance. Ding [7] proposes a semi-blind spatial equalization to resolve ambiguity by using training data. However, training sequences consume precious bandwidth. Moreover, this method can only be used in flat fading channel.

In this paper, we propose a novel ICA based blind equalization structure for MIMO OFDM systems. Our work is different in that our proposed structure employs a simple linear precoding between the source data and the reference data at the transmitter, which allows the phase and permutation ambiguity in the equalized signal to be eliminated completely by cross-correlation operation at the receiver. It also avoids error propagation. In order to eliminate extraction errors of the source data streams in a few subcarriers, iterative channel interpolation, which is employed to refine the channel estimation of ICA, is incorporated with minimum mean square error (MMSE) based equalization. Furthermore, layered space frequency equalization (LSFE) [8] is used to improve further the system performance by using interference cancellation. Simulation results show the proposed blind structure provides significant performance improvement over the subspace method in [9]. It also provides the performance close to the case with perfect channel state information (CSI).

The paper is organized as follows. The system model is given in Section 2. In the first part of Section 3, we explain the ambiguity of the ICA model and propose a linear precoding on the source data. Then, we describe the proposed blind equalization approach. ICA-MMSE and ICA-LSFE with channel interpolation is presented in Section 4 to eliminate extraction errors and to improve the system performance. Section 5 gives the results of Monte Carlo simulation for the demonstration of the proposed approach. It also contains the discussion about the simulation results. Section 6 is the conclusion.

2. SYSTEM MODEL

We consider a MIMO OFDM spatial multiplexing system with $N_s$ subcarriers, $N_t$ transmit and $N_r$ receive antennas, as shown in Fig. 1. At the transmitter, a frame of $N_s$ OFDM symbols, which are obtained from the inverse fast Fourier transform (IFFT) of modulated symbols, are multiplexed to different subcarriers and transmitted antennas. A cyclic prefix (CP) of length $L_{CP}$ is prepended at the beginning of each OFDM symbol before transmission and removed at the receiver. Inter-symbol interference can be avoided when $L_{CP} \geq (L_c - 1)$ with $L_c$ denoting the maximum channel memory [1]. The channel is assumed to be quasi-static block fading, while the entries of fading channel matrix $H(k)$ for each subcarrier $k$ remain constant for the duration of a frame of OFDM symbols. Therefore, the input-output relation within OFDM symbol $i$ on subcarrier $k$ can be written as

$$x(k, i) = H(k)s(k, i) + n(k, i)$$

(1)

where $s(k, i) = [s_0(k, i), s_1(k, i), \ldots, s_{N_t-1}(k, i)]^T$ and $x(k, i) = [x_0(k, i), x_1(k, i), \ldots, x_{N_r-1}(k, i)]^T$ are complex valued input and output baseband signals with unit energy, and $H(k)$ is the frequency domain MIMO channel response.

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The aim of equalization is to obtain the separating matrix of size \((N_c \times N_s)\) to recover transmitted streams on each subcarrier. From the channel model (1), the received signal is instantaneous linear mixture of the transmit signal \(s(k,i)\). Thereby, ICA can be directly applied to a frame of received signal \(x(k,i)\) \(i = 0,1,…,N_t - 1\) to obtain the separating matrix by maximizing non-Gaussianity of the equalized signal \(\hat{s}(k,i)\). However, because of the phase and permutation ambiguity of the ICA model, the elements of the equalized signal \(\hat{s}(k,i)\) and the columns of the estimated MIMO channel matrix \(\hat{H}(k)\) may have different permutation and phase shift compared with the true source stream \(s(k,i)\) and channel matrix \(H(k)\). Therefore, further processing is needed to reorder and scale \(\hat{s}(k,i)\).

In this paper, we propose a linear precoding similar to [5, 10] on the source data stream before the transmission. It transforms source data stream \(d(k,i)\) according to

\[
s(k,i) = \frac{1}{\sqrt{1 + a^2}} [d(k,i) + ad_{\text{ref}}(k,i)]
\]

where \(d_{\text{ref}}(k,i)\) is the corresponding reference signal which is known in advance at the receiver before the transmission. The reference signal \(d_{\text{ref}}(k,i)\) is generated randomly with the same size and structure of source data, while the entries of \(d_{\text{ref}}(k,i)\) are independent with each other. Both the source data stream \(d(k,i)\) and the reference signal \(d_{\text{ref}}(k,i)\) have unit variance. The positive real number \(a\) is the predefined precoding constant with \(a \leq 1\). It gives tradeoff of transmit power allocation between the source data stream and the reference signal. Notice that no redundancy is introduced in the precoding and the transmit power is also preserved. The target of precoding is to introduce the correlation between the source data and the reference data. Therefore, the receiver can utilize this correlation to eliminate the ambiguity in the equalized signal.

### 3. ICA ONLY EQUALIZATION

#### 3.1 Linear Precoding

The aim of equalization is to obtain the separating matrix of size \((N_c \times N_s)\) to recover transmitted streams on each subcarrier. From the channel model (1), the received signal is instantaneous linear mixture of the transmit signal \(s(k,i)\). Thereby, ICA can be directly applied to a frame of received signal \(x(k,i)\) \(i = 0,1,…,N_t - 1\) to obtain the separating matrix by maximizing non-Gaussianity of the equalized signal \(\hat{s}(k,i)\). However, because of the phase and permutation ambiguity of the ICA model, the elements of the equalized signal \(\hat{s}(k,i)\) and the columns of the estimated MIMO channel matrix \(\hat{H}(k)\) may have different permutation and phase shift compared with the true source stream \(s(k,i)\) and channel matrix \(H(k)\). Therefore, further processing is needed to reorder and scale \(\hat{s}(k,i)\).

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#### 3.2 ICA

As a common strategy of ICA, the received signal should firstly be whitened using principal component analysis (PCA) [3]. For each subcarrier \(k\), the spatially uncorrelated signals can be obtained as

\[
z(k,i) = W(k)x(k,i)
\]

such that

\[
E\{z(k,i)z^H(k,i)\} = I_{N_t}
\]

where the size \((N_c \times N_s)\) whitening matrix \(W(k) = \Lambda^{-1/2}(k)U^H(k)\) is commonly obtained from eigenvalue decomposition of the autocorrelation matrix of the received signal

\[
R_{xx}(k) = E\{x(k,i)x^H(k,i)\} = U(k)\Lambda(k)U^H(k)
\]

JADE is one of the well-established ICA methods [4,11]. It efficiently exploits HOS of the received signal \(z(k,i)\)
and requires short data sequences than other ICA methods. Therefore, JADE is employed in this paper to obtain the size $(N_t \times N_r)$ unitary matrix $V(k)$ to suppress co-channel interference and recover the transmitted streams. The output signal within the $i$-th OFDM symbol on subcarrier $k$ of the spatial equalizer is

$$
\hat{s}(k,i) = V(k)W(k)x(k,i)
$$

(6)

However, even under the noiseless assumption, the phase and permutation ambiguity still exist in the equalized signal. The relation between the equalized signal $\hat{s}(k,i)$ and the source signal $s(k,i)$ in noiseless case can be written as [5]

$$
\hat{s}(k,i) = D(k)P(k)s(k,i)
$$

(7)

where diagonal matrix $D(k)$ accounts for phase ambiguity and permutation matrix $P(k)$ accounts for permutation ambiguity. Since $D(k)$ and $P(k)$ are generally unknown, further processing of phase shifting and reordering will be employed to resolve the phase and permutation ambiguity.

### 3.3 Phase Shifting

The phase ambiguity can be resolved by de-rotation of the equalized signal $\hat{s}(k,i)$

$$
\tilde{s}_i(k,i) = \frac{\alpha_i(k)}{|\alpha_i(k)|} \hat{s}_i(k,i)
$$

(8)

where $i = 1, \ldots, N_t$ denotes the source stream index. The estimated phase shift $\alpha_i(k)$ can be obtained as [5]

$$
\alpha_i(k) = \frac{E\{\hat{s}_i(k,i)^*\}}{|\hat{s}_i(k,i)^*|} - \frac{1}{4} e^{i\pi/4}
$$

(9)

for QPSK modulation.

However, equation (9) introduces quadrant ambiguity to the output signal $\tilde{s}_i(k,i)$, which has $2^l$ ($l = 0,1,2,3$) phase rotation for QPSK modulation. This ambiguity will be resolved together with permutation ambiguity by the following reordering operation.

### 3.4 Reordering

From the preceding (2), the transmitted symbols are correlated with the reference symbols. The reordering is achieved by finding the permutation $\pi_k$ of $N_t$ source data streams, which maximize the absolute value of the estimated cross correlation $\rho(t,k,\pi)$ between the detected streams and the reference signal.

Under the assumption of perfect phase shifting in (8), the cross correlation $\rho(t,k,\pi)$ can be defined as

$$
\rho(t,k,\pi) = \frac{E\{|\hat{d}_{\pi(t)}(k,i) [d_{\text{ref},t}(k,i)]^*\}|}{\sqrt{1 + a^2} E\{|\hat{d}_{\pi(t)}(k,i) [d_{\text{ref},t}(k,i)]^*\}}
$$

(10)

where $\hat{d}_{\text{ref},t}(k,i) = [\hat{D}(k)]^{-1}P(k)d_{\text{ref},t}(k,i)$ is the permuted reference signal up to quadrant ambiguity. The diagonal matrix $\hat{D}(k)$ accounts for the quadrant ambiguity introduced by phase shifting. The diagonal entries of $\hat{D}(k)$ have unit absolute value and will be obtained in (17). Due to the independence between the source data and reference data, and also the independence between the reference signal in different streams, we can obtain

$$
E\{|\hat{d}_{\pi(t)}(k,i) [d_{\text{ref},t}(k,i)]^*\} = 0
$$

(11)

$$
E\{|\hat{d}_{\text{ref},t}(k,i) [d_{\text{ref},t}(k,i)]^*\} = \begin{cases} 1 & \text{if } \pi(t) = t \\ 0 & \text{otherwise} \end{cases}
$$

(12)

From (2), absolute value of the cross correlation of (10) under correct permutation is

$$
|\rho(t,k,\pi)| = \frac{a}{\sqrt{1 + a^2}}
$$

(13)

otherwise

$$
|\rho(t,k,\pi)| = 0
$$

(14)

Therefore, the permutation corresponding to the largest $|\rho(t,k,\pi)|$ is the correct permutation. That is

$$
\pi_k = \arg \min_{\pi} \sum_{t=0}^{N_t-1} |\rho(t,k,\pi)|
$$

(15)

The estimated data $\hat{d}(k,i)$ after recoding is obtained as

$$
\hat{d}(k,i) = D(k)\left[\hat{d}_{\text{ref},t}(k,i), \ldots, \hat{d}_{\text{ref},t}(N_t-1)(k,i)\right]^T
$$

(16)

where the diagonal matrix $\hat{D}(k)$ is given by

$$
\hat{D}(k) = \text{diag}\left[je^{-i\pi/4} \text{sign} \left(\frac{\rho(t,k,\pi_k)}{|\rho(t,k,\pi_k)|} e^{i\pi/4}\right)\right]
$$

(17)

for QPSK data with $\text{sign}(\cdot)$ denoting the signum function. Here, $\hat{D}(k)$ is introduced to resolve the quadrant ambiguity from the phase shifting.

Finally, the soft estimates of the source data is obtained by decoding

$$
\hat{d}_k(i) = \sqrt{1 + a^2} \tilde{s}_k(i) - ad_{\text{ref},t}(k,i)
$$

(18)

which is then passed into the decision device to obtain the hard estimates $\hat{d}_k(i)$ of the source data $d_k(i)$.

We name the proposed receiver structure without further post processing to beICA only receiver, as shown in Fig. 1.

### 4. ICA-MMSE/LSFE WITH CHANNEL INTERPOLATION

Even though the phase shifting and reordering are employed to eliminate ambiguity inICA only receiver, imperfect source stream extraction has been observed occasionally in some subcarriers, which severely degrades the system performance. To correct such erroneous extraction, channel estimation obtained by ICA can be improved by channel interpolation, which can be combined with linear MMSE equalization, and nonlinear LSFE [8] with interference cancellation, to enhance further the system performance. This processing can be performed iteratively, as depicted in Fig. 1.
4.1 Channel Interpolation

For each subcarrier \( k \), the initial least-square channel estimates can be obtained as

\[
\hat{H}(k) = X(k) \left[ \hat{S}(k) \right]^+ \tag{19}
\]

where \( X(k) = [s(k,0), s(k,1), \ldots, s(k,N_t - 1)] \) is the received signal on subcarrier \( k \), and \( \hat{S}(k) = [\hat{s}(k,0), \hat{s}(k,1), \ldots, \hat{s}(0,N_t - 1)] \) is the re-encoded signal from the hard estimated data with

\[
\hat{s}(k,i) = \frac{1}{\sqrt{1 + \alpha^2}} [d(k,i) + a \hat{d}_{\text{ref}}(k,i)] \tag{20}
\]

and the operator \([\ast]^+\) denotes pseudo-inverse.

Then channel interpolation is employed to refine the channel estimation even when initial channel estimates on all subcarrier are available. The idea behind channel interpolation is to use the correlation between neighboring subcarriers to correct wrong initial channel estimates in (19) in a frequency domain. The pseudo-inverse interpolation is to use the correlation between neighbor sub-carriers to correct wrong initial channel estimates in (19) in a frequency domain. The pseudo-inverse interpolation is to use the correlation between neighbor sub-carriers to correct wrong initial channel estimates in (19) in a frequency domain.

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After the channel estimation (23), the equalization matrix \( \hat{H}_{\text{I}}(k) \) is obtained as

\[
\hat{H}_{\text{I}}(k) = F^+ \hat{H}_{\text{ref}} \tag{21}
\]

where \( \hat{H}_{\text{I}}(k) \) is the channel estimates between transmit antenna \( t \) and receiver antenna \( r \) in frequency domain from (19), \( F \) is the \((N \times L_c)\) discrete Fourier transform (DFT) matrix given as

\[
F^{(l,m)} = N^{-1/2} e^{-j 2 \pi l m / N} \tag{22}
\]

for \( l = 0, 1, \ldots, N - 1 \) and \( m = 0, 1, \ldots, L_c - 1 \), and \( F^+ \) is the pseudo-inverse of \( F \). Since during the computation of \( \hat{H}_{\text{I}}(k) \), the channel information in all subcarriers is used, a few errors in \( \hat{H}_{\text{I}}(k) \) do not have significant influence on the estimate of \( \hat{H}_{\text{I}}(k) \).

To perform the post-processing of the received signal, the refined channel estimates in frequency domain can be obtained

\[
\hat{H}_{\text{I}}(k) = F \hat{H}_{\text{I}}(k) \tag{23}
\]

4.2 ICA-MMSE

After the channel estimation (23), the equalization matrix \( g(k) \) in each subcarrier \( k \) can be obtained from the classical MMSE criterion,

\[
g(k) = \hat{H}(k) \left[ \hat{H}^H(k) \hat{H}(k) + \sigma^2 I_{N_t} \right]^{-1} \tag{24}
\]

Then, the output signal from MMSE equalization is

\[
\tilde{s}(k,i) = g^H(k) x(k,i) \tag{25}
\]

Finally, decoding and making hard decision for source data streams in subcarrier \( k \)

\[
d(k,i) = Q \left[ \sqrt{1 + \alpha^2} \tilde{s}(k,i) - a \hat{d}_{\text{ref}}(k,i) \right] \tag{26}
\]

4.3 ICA-LSFE

Rather than perform MMSE equalization to extract all source streams simultaneously, we can detect source data streams layer by layer to improve further the system performance. The layer with the lowest MSE is selected for detection [8]

\[
t_m = \arg \min_t \left\{ 1 - \left[ \hat{H}^{(t)}(k) \right]^H [R(k)]^{-1} \hat{H}^{(t)}(k) \right\} \tag{27}
\]

where \( \hat{H}^{(t)}(k) \) is the \( t \) column of the estimated channel matrix \( \hat{H}(k) \), and

\[
R(k) = \left\{ \sum_t \hat{H}^{(t)}(k) \left[ \hat{H}^{(t)}(k) \right]^H \right\} + \sigma^2 I_{N_t} \tag{28}
\]

with the summation over all the columns corresponding to the undetected layers. Then we obtain LSFE output signal [5]

\[
\hat{s}_{\text{ref}}(k+iN) = g^H_{\text{ref}}(k) x(k,i) \tag{29}
\]

where the equalizer vector \( g_{\text{ref}}(k) \) is given by [5]

\[
g_{\text{ref}}(k) = \left[ \hat{R}(k) \right]^{-1} \hat{H}^{(-m)}(k) \tag{30}
\]

After this, we do decoding and make hard decision for stream \( t_m \)

\[
d_{\text{ref}}(k,i) = Q \left[ \sqrt{1 + \alpha^2} \hat{s}_{\text{ref}}(k,i) - a \hat{d}_{\text{ref}}(k,i) \right] \tag{31}
\]

The output data stream will be canceled from the received signals, and we select another layer to detect until data streams in all layers have been detected.

5. SIMULATION RESULTS

In this section, the performance of the proposed blind ICA based equalization structure is evaluated by simulations. The MIMO OFDM system has \( N = 64 \) subcarriers, \( N_t = 4 \) transmit and \( N_r = 4 \) receive antennas. The Clarke’s block fading channel model [1] is employed, which remains constant during a frame consisting of \( N_{\text{frames}} = 200 \) OFDM symbols. The channel memory length is \( L_c = 5 \) with the exponential power delay profile. The root mean square (RMS) delay spread is \( T_{\text{RMS}} = 1.4 \), which is normalized to the sampling time interval. In order to make the performance evaluation accurate, 1000 frames have been simulated by Monte Carlo method.

The BER vs. SNR performance of the proposed structure is shown in Fig. 2. We first illustrate the performance of the proposed ICA only receiver. Then we investigate the proposed ICA-MMSE receiver. After this, we demonstrate the performance of the proposed ICA-LSFE receiver. For the ICA-LSFE receiver, we can perform the channel interpolation and LSFE processing (from (19) to (31)) iteratively to further improve the performance. In addition, the performances of subspace method [9], MMSE equalization [1] and LSFE [5] under perfect CSI are also shown in Fig. 2 as the benchmark. From the simulation results of the ICA only receiver and the ICA-MMSE receiver, it shows the performance gap between the proposed blind ICA-MMSE receiver and the classical MMSE receiver with perfect CSI is only 1 dB at BER = 10^-2. It proves that the proposed structure can effectively eliminate phase and permutation ambiguity of the ICA model. It also shows that iteration is unnecessary for ICA-MMSE receiver because the performance of ICA-MMSE receiver is close to the MMSE equalization with perfect CSI. Benefiting from channel interpolation which efficiently eliminates wrong source stream extraction in a few
At BER, the performance gap between the two-iteration receiver and LSFE with perfect CSI is only 1 dB. This shows that LSFE can significantly improve the system performance, while the performance gap between the two-iteration receiver and 2 dB. It means that the interference cancellation. The two-iteration receiver outperforms the one-iteration receiver by 2 dB. It means that iteration is helpful for ICA-LSFE for system performance. But the system improvement from more than two iterations can only be obtained from simulation results.

### 6. CONCLUSIONS

We have proposed a new ICA based blind equalization structure for MIMO OFDM systems. At the transmit side, a simple linear precoding is employed to introduce the correlation between the source data and the reference data. This correlation structure is used to resolve completely phase and permutation ambiguity of the ICA model. Due to the benefit from the channel interpolation, ICA-MMSE receiver provides performance close to the MMSE equalization with perfect CSI. The iterative LSFE receiver further enhances the system performance, while two-iteration ICA-LSFE receiver offers performance close to the LSFE with perfect CSI. All these receivers significantly outperform the subspace method.

### REFERENCES


