RISK ANALYSIS WITH CONTRACTUAL DEFAULT.
DOES COVENANT BREACH MATTER?

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Abstract
Mergers and acquisitions (M&A), private equity and leveraged buyouts, securitization and project finance are characterized by the presence of contractual clauses (covenants) that trigger the technical default of the borrower even in the absence of insolvency. Therefore, borrowers may default on loans even when they have sufficient available cash to repay outstanding debt. This condition is not captured by the NPV distribution obtained through a standard Monte Carlo simulation. In this paper, we present a methodology for including the consequences of covenant breach in a Monte Carlo simulation, extending traditional risk analysis in investment planning. We introduce a conceptual framework for modelling technical and material breaches from the standpoint of both lenders and shareholders. We apply this framework to a real case study concerning the project financing of a 64-million euro biomass power plant. The simulation is carried out on the actual model developed by the financial advisors of the project and made available to the authors. Results show that both technical and material breaches have a statistically significant impact on the NPV distribution, and the impact is more relevant when leverage and cost of debt increase.

Keywords: Investment Analysis; Project Finance; Risk Analysis; Monte Carlo Simulation; Coherent Risk Measures; Covenants; Risk Management; Business Planning.
JEL codes: C61, C63, G31.

1 Introduction

Risk analysis is a central tool for supporting decision-making in investment decisions. One of the main outcomes of a risk analysis is the distribution of the valuation criterion (henceforth, risk profile). Based on the risk profile, analysts estimate all risk measures concerning a given investment.

Recent literature has raised the question of whether covenant breaches affect an investment’s risk profile. The answer is currently the subject of debate, with some authors arguing that covenants do not matter in a quantitative project risk analysis, and others contending the contrary. To the best of our knowledge, the question has not been addressed with a direct investigation; in this paper, we aim to do just this.

Covenants characterize a vast portion of financial transactions [Bengtsson (2011)] represented by private bank debts [Demioglu and James (2009), Nini et al (2009), Sudarsanam and Moir (2007)], leveraged buyouts (LBOs) [Cumming and Johan (2003), Ljungqvist et al (2008)], private equity [Achleitner et al (2011)], corporate and project bonds [Dailami and Hauswald (2003)], asset-backed securitization, syndicated loans and project finance [Corielli et al (2010), Finnerty (2007)]. A relevant part of these deals is financed on a structured basis,
meaning that the borrower is a special purpose entity, different from an already existing firm, frequently known as sponsor or originator. In this paper, we focus our analysis on project finance which, together with securitization and LBOs, is the prototypical example of a structured finance transaction [Megginson (2010)]. Creditors can rely only on the cash generated by the initiative and only on the assets of the SPV as collateral, but not on the cash flows and assets of SPV’s originators/sponsors [Esty (2001), Esty (2004), Esty and Sesia (2004), Bonetti et al (2009)]. The rationale behind including strict covenants in loan agreements is, then, to give the lender the right to intervene in the management of the initiative to avoid financial distress (Rajan and Winton, 1995). Covenants impose on the SPV either obligations to do something (positive covenants) or to refrain from doing something (negative covenants) [Smith (1993)]. Furthermore—and this is the most important implication for our work—covenants specify financial ratios that must be respected throughout the life of the loan (financial covenants).

Because loan covenants are set tightly ex-ante, i.e. before the deal is closed or the credit agreement with lenders is signed, it happens frequently that a covenant is violated even if the borrower is not in severe financial distress [Chen and Wei (1993)]. This situation is often referred to as technical default. Creditors waive a technical default only when they do not consider the breach of covenants a “material” event [SEC (1999), Novo (2007)] able to disrupt the orderly continuation of the deal. Establishing a “materiality test” requires the ex-ante definition of a range of values of the coverage ratios. At one end of this range there is a preset value that triggers technical default but allows creditors and SPV shareholders to continue managing the transaction with enforced monitoring or with a renegotiation of the loan terms; at the other end is a final value below which the breach is considered material. At this point, creditors consider the loan subject to resolution, accelerate the loan and force the deal into bankruptcy without the possibility of renegotiation. In this respect, covenant violations expose on to an underhand risk, which, if overlooked, can lead both shareholders and lenders into severe consequences. The survey of Davison et al (2010) reveals material violations in around 8% of the project finance deals analyzed in their study.

For investors analyzing a given transaction, the existence of a materiality test and the different implications of the breach for lenders and sponsors challenge the results of standard risk analysis obtained through Monte Carlo propagation. More precisely, three issues raised are: 1) how frequent one expects covenant breach to be, and 2) given a breach (either a technical default or a material breach) what consequences are sponsors and lenders incurring into and 3) Is the modelling of covenant breach causing a statistically significant impact on the risk profile and, consequently, on the risk measures.

To answer these questions requires a modification of the standard risk analysis procedures, which we address as follows.

First, we design a conceptual model of covenant breach. The model distinguishes a technical from a material breach. In the case of a technical breach, the model accounts for the change in debt schedule repayment due to the fact that lenders would demand that dividend distribution to sponsors be suspended and use all the available
cash to accelerate debt repayment (\textit{cash sweep}). Instead, in the case of a material breach, the model accounts for shareholders’ loss of the equity cash flows determined by the lenders’ enforcement of the security package. Furthermore, lenders are in the position to take over control of the project, and can continue the project with the objective of recovering the amount lent to the SPV [Davidson et al. (2010), p. 44].

Second, we translate the model into a simulation methodology that nests the modelling of covenant breaches and their different consequences for sponsors and lenders into traditional Monte Carlo Simulation.

Third, we apply the simulation procedure to a real project finance transaction involving a recent biomass power plant located in Italy worth around 64 million euro that underwent financial closing in 2009 with General Electric-Interbanca as lead arranger. For this project, access to the financial model used by sponsors and the mandated lead arranging bank in their evaluation of the deal was granted to the authors. Thanks to the availability of the financial model, we are in a favourable position to examine quantitative results relying on real data and not on fictitious or simulated deals. Results show that the effect of covenant breach on the cash flow distributions to the SPV’s sponsors is small if the violations are technical. Conversely, in the case of material breach, the cumulative distribution function of the equity NPV exhibits a fat left tail, with a considerable change in the value of risk measures.

Our contribution to the existing literature on risk analysis and debt covenants is threefold.

First, available papers on covenants have analyzed contractual clauses almost exclusively in the context of corporate finance settings, i.e. in case of bond or loan contracts between lenders and \textit{already existing firms} [Aghion and Bolton (1992), Beneish and Press (1995), Chava and Roberts (2008)]. However, much less is known about the importance and the role of covenants included in lending agreements designed for project finance, and more generally for structured deals. As we have said, the typical structured finance transaction is a fully self-contained, one-time financing event with a definite economic life-cycle [Gatti et al, (2011)]. With such a deal, the previous lending relations between lenders and the SPV shareholders are much less important than the soundness of the stand-alone deal to be financed. These characteristics are true for no other corporate financing sample, making project finance deals ideal to study the disciplining role of covenants in isolation from other borrower-specific factors.

Second, we discuss the use of covenants for a significant segment of international capital markets which is not extensively covered in previous theoretical and empirical papers. Thomson Reuters reports that syndicated loans amounted to US$3.9 trillion at the end of 2011, of which US$214.5 bn were PF loans. These figures compare with US$180 bn of private equity invested globally in 2010 and with global securitization issuance of mortgage and asset backed securities of US$799 bn at the end of 2010.

Third, we design a methodology that is useful to model the effect of technical and material breach of covenants for sponsors and lenders. Knowledge of the expected frequency of covenant violations provides analysts with a degree of confidence as to the project’s ability to sustain stresses. This information can then be used by lenders to fine-tune the loan tenor, the spread and the debt/equity ratio. Our methodology can also help
creditors assess unexpected loss and allocate equity capital in accordance with Basel II and Basel III rules [Basel Committee on Banking Supervision., 2009].

The remainder of the paper is organized as follows. Section 2 reviews the valuation criteria, covenants of credit agreements typically used by lenders and sponsors, and standard risk measures. Section 3 designs the model that includes the possibility of a covenant breach. Section 4 illustrates how to turn the model into a corresponding simulation procedure. Section 5 presents additional risk analysis insights and the Monte Carlo estimation of the frequency of covenant breach and of risk measures. Section 6 presents the case study analysis and results. Section 7 offers conclusions.

2 Valuation Criteria, Covenants, Risk Analysis and risk measures

Throughout this work, we investigate a project finance investment structured through an SPV funded with equity and debt provided by project sponsors and a pool of banks respectively. In this section, we describe the valuation criteria used by lenders and sponsors to assess project viability, the covenants included in the credit agreement and the risk measures we use in the simulations included in Section 4.

2.1 The Valuation Criteria

By the adjusted present value principle [Myers (1974)], analysts separate equity from debt cash flows. We call free cash flow of period \( t \) (FCF\(_t\)) the line of the cash flow statement at period \( t \) preceding financial items (debt service and dividends paid to sponsors).

Specifically, letting \( R_t \) denote revenues, Tax\(_t\) the cash outflow due to taxes, OE\(_t\) operating expenses, \( \Delta WC_t \) the correction of working capital changes and Capex\(_t\) the capital expenditures for period \( t \), \( (t=1,2,...,T) \), we have,

\[
FCF_t = R_t - OE_t - \text{Taxes}_t + \Delta WC_t - \text{Capex}_t
\]  
(1)

FCF\(_t\) [eq. (1)] represents the cash produced by the project in period \( t \). If the investment is financed fully with equity, then FCF\(_t\) is entirely in the hands of shareholders. If the investment is financed partly with equity and debt, then a part of FCF\(_t\) goes to lenders and a part to shareholders. More precisely, let \( P_t \) and \( I_t \) represent the principal and interest on loan to be repaid in period \( t \), respectively. Then, the quantity

\[
FCFE_t = FCF_t - P_t - I_t
\]  
(2)

represents the free cash flow to equity at period \( t \) (FCFE\(_t\)). Letting ShNPV denote the shareholder NPV and assuming no frictions in the FCFE disbursement, we obtain

\[
\text{ShNPV} = \sum_{t=0}^{T} \frac{FCFE_t}{(1 + k)^t}
\]  
(3)
Knowledge of FCFEt can also be utilized to estimate alternative shareholders’ valuation criteria as, for instance, the internal rate of return (IRR) [For an overview about he consistent utilization of IRR as valuation criterion, see among others Hazen (2003) and Hazen (2009)].

As documented in Davidson et al (2010), the perspective of lenders differs from that of shareholders in several respects. First, lenders do not have direct responsibility for the operation of the project. At the same time, for lenders it is vital for the project to stay afloat so they can be reimbursed as planned or with limited deviations from the original debt cash flow stream. This stream is determined by the flow of principal and interest and solves the equality

\[ \ell \cdot A = \sum_{t=1}^{T^L} \frac{P_t + I_t}{(1 + k_d)^t} \]  (4)

where \( \ell \cdot A \) is the percentage amount disbursed by lenders, \( P_t \) and \( I_t \) the corresponding principal and interest payments, \( T^L \) is the final repayment date of the loan and \( k_d \) is the cost of debt. By eq. (4), the loan NPV is null \( (NPV^{Loan} = 0) \) at \( k_d \).

2.2 Covenants of the credit agreement

Covenants are defined as “... supplemental obligations of the borrower in addition to the basic obligation to repay the lenders the amount due on the scheduled maturity dates [...] These supplemental obligations may be either correlated to loan repayment--as in the case the borrower doesn't take certain actions that will hamper debt repayment at the scheduled dates--or required by lenders in order to monitor their credit investment and verify that it is being managed properly” [Novo (2007); p. 254].

The evidence available for standard corporate finance settings (i.e. already existing firms with a portfolio of ongoing assets) shows that most covenant violations are actually either waived by the creditors or resolved by renegotiations of the loan or refinancing with a different lender [Chen and Wei (1993)]. Indeed, Beneish and Press (1995) and Nini et al. (2009) demonstrate that only a small fraction of violating firms experience a distressed-exit. Moreover, a covenant breach is followed by the imposition of further constraints on the borrower’s behaviour (like an increased spread on the loan [Smith (1993), Sweeney (1994), Sufi (2009)], request of additional collateral and limitations to dividend distributions or to additional capital expenditures [Nini et al (2009)]). Finally, creditor intervention in case of covenant violation is in fact good for the borrowers’ shareholders as it is associated with operating performance and share price increase.\(^1\)

While several of the findings in the existing literature can be extended to individual project finance transactions, there is one main difference. This type of transaction requires the lenders’ risk assessment of a

\[^1\] In the same fashion, Demiroglu and James (2009) show that shareholders respond more positively to announcements of loans with tighter financial covenants. However, Beneish and Press (1995) document wealth losses to shareholders following the renegotiation of loans triggered by a technical default.
specific project and not of an existing pool of real projects, like in an ongoing corporate entity. This fact has four important implications for sponsors and creditors that we address in the paper, implications which, to the best of our knowledge, have not been covered by previous studies.

First, the breach of covenants cannot always be followed by a revision or renegotiation of the contractual terms of the debt contract. Examples are large infrastructure projects where the minimum values of the coverage ratios discussed in Section 2.2 are breached and very close to 1. In these cases it becomes unfeasible for lenders to raise the level of spread on the loan tranches because operating cash flows cannot sustain an increased debt service and there are no other cash flows that can be used for loan repayment.

Second, the request for additional collateral is excluded by definition, since these deals are based on a no-recourse clause to the originators/sponsors of the SPV. Furthermore, lenders include in the credit agreement a negative pledge clause [Gatti (2007)] that obliges the borrower not to allow any other lender outside the pool of existing lenders to use the present and future assets of the SPV as collateral.

Third, project finance implies the financing of one single project, so a very detailed security package is set up before the deal closing [Finnerty (2007)]. This allows existing creditors (and no others) to take over the control rights of the shareholders when no waiver or renegotiation of the loan terms is possible. Furthermore, the security package grants lenders the option to take full control of the SPV, take possession of all project assets, expropriate shareholders’ control rights and capture the remaining enterprise value. Yescombe (2002) reports that a typical project finance security package includes: 1) a pledge on the borrowers' shares, so that banks have the right to vote in the event of default; 2) a mortgage on land rights, buildings, and other tangible and intangible fixed assets; 3) credit assignment for proceeds coming from key contracts of the borrower [Bonetti et al. (2009)]; 4) a pledge on all the bank accounts of the borrower; and 5) assignment of insurance payments in case of damages. Given such a tight package, the option to refinance the loans with other creditors—which is a possibility in standard corporate finance settings following a covenant violation—is simply not possible in project finance.

Fourth, other limitations to management behaviour that have been extensively documented in the literature do not apply in a clear-cut way to project finance deals. The limitation to dividend distributions is included from the beginning in the original credit agreement with the lenders and not added after a covenant breach. Limitations to increased capital expenditures after a breach of covenants is not possible, because the SPV is created with the sole purpose to complete one single project that requires exactly that amount of money. Should the level of capital expenditures be limited after a breach of covenants, the project would automatically be terminated and forced into bankruptcy given the impossibility to complete construction and start operations.²

² The impossibility to restrict capital expenditures upon violation of financial covenants explains why very big infrastructure projects (like the Eurotunnel) underwent several rounds of refinancing in order to complete the construction phase. See Esty and Sesia (2004).
A standard categorization of covenants is not available in the huge volumes of existing literature. However, most authors agree on the broader categories of positive, negative, and financial covenants [Rosenbaum and Pearl (2009), Smith (1993)]. Within financial covenants, Demerjian (2007) and Nini et al. (2009) distinguish the categories of coverage ratios, current ratios/liquidity ratios, leverage ratios, gearing ratios, and net worth ratios. Demiroglu and James (2010), instead, classify financial covenants as either incurrence or maintenance covenants. Incurrence covenants are designed to increase the recovery ratios of lenders in case of borrower default and to impose restrictions on borrower behaviour. Maintenance covenants, instead, must be satisfied on an ongoing basis as a wealth-increasing measure for lenders and are linked to mandatory, positive actions by the borrower.

Among the various financial covenants that can be included in a credit agreement, one is particularly relevant to our work: the debt service cover ratio (DSCR) (Gatti, 2007). DSCR, is defined as

$$DSCR_t = \frac{FCF_t}{P_t + I_t}$$

(5)

DSCR, represents the ratio between the cash available before debt repayment (FCF,) and the amount to be repaid in period t. It provides lenders as a measure of the project’s capability to repay the loan. By eq. (1),

$$DSCR_t = \frac{R_t - O_E_t - Taxes_t + \Delta WC_t}{P_t + I_t}$$

(6)

In addition to DSCR, for lenders it is of interest to know the minimum DSCR in all periods. This quantity is here denoted as

$$\min DSCR = \min_{t=1...T} DSCR_t$$

(7)

From the decision-making viewpoint, minDSCR is utilized by lenders in the loan negotiation phase to help them decide the optimal debt-to-equity ratio of the deal (Gatti, 2007). They first set the target minDSCR. Then, they run the financial model and iteratively adjust the loan amount in such a way that minDSCR is always greater than the minimum required value.

3 Demerjian (2007) indicates that in highly leveraged transactions (like project finance), higher relevance is given to coverage ratios and leverage.

4 A second cover ratio that is frequently used in conjunction with DSCR is the loan life cover ratio (LLCR). LLCR measures project repayment capacity as the present value of operating cash flows plus the existing reserve accounts. The LLCR provides the investor with a measure of the number of times the unlevered free cash flows can be used to repay the outstanding debt balance (Ot) over the scheduled life of the loan.

5 It is important to remember that the numerator of eq. (6) does not include Capex, as indicated in eq. (1). The reason is that in all project finance transactions, capital expenditures are concentrated only during the construction phase. Instead, the operational phase is dedicated to managing the project and no additional investments are required. See, among others, Corielli et al. (2010) and Yescombe (2002).
A typical project finance loan contract usually includes two sets of values for this financial covenant. The first indicates the minimum threshold below which lenders can require an accelerated repayment of the loan; the second, which is lower than the first, triggers the resolution of the credit agreement and represents the material breach of covenant. The joint consideration of the two sets of values generates a range of values for DSCR associated with a technical default that is frequently waived by means of a renegotiation of the loan repayment terms. This renegotiation is always available until DSCR reaches the lower bound when the breach of covenants becomes material, forcing the project into bankruptcy.6

The actual levels of coverage ratios depend on the deal. For instance, Mariano and Tribo (2010) report an average DSCR of 1.53x for a large sample of 1,352 lending non-project finance agreements. Gatti (2007) reports average DSCR values between 2x and 2.25x for merchant power plants, and between 1.2x and 1.3x for water and sanitation projects. The material breach of covenants is frequently associated with DSCR close to 1.03x-1.05x.

Eqs. (1)-(7) represent the basic equations of financial modelling. In the practice, $R_t$, $OE_{it}$, $Taxes_{it}$, $P_t$ and $I_t$ are, in turn, expressed as functions of exogenous variables (e.g., macroeconomic variables, escalation indices, etc.) here represented by vector $\omega$.

The functional dependence of the lenders and sponsors valuation criteria on $\omega$ is, in the professional practice, the result of complex financial calculations that take into account technical, operational, fiscal, and accounting aspects of the investment project. These calculations are implemented in a financial model validated through expert opinion. As the valuation of the investment progresses from the due diligence to the negotiation phases, the model becomes the central tool for negotiations between shareholders and lenders. A detailed modelling of the investment project characteristics is required and the financial model tends to become a complex mapping, usually known only numerically (Borgonovo et al., 2010). Let us write the relationship between the decision-support criterion of interest and $\omega$ as $Z(\omega)$.

$Z(\omega)$ is determined if $\omega$ is known. However, this is seldom the case in practice, as $\omega$ usually consists of several uncertain random variables. Analysts, when performing data analysis, request forecasts from specialized institutions and information from consultants (Gatti et al., 2007). This allows them to assess distributions for $\omega$. By propagating the uncertainty in $\omega$ through the Monte Carlo simulation, we obtain the distribution of $Z(\omega)$. The cumulative distribution function (CDF) of $Z(\omega)$ is called the risk profile [Smith (1998)]. This is the first step in risk analysis.

6 In a recent PF loan in the renewable energy sector, the group of lenders set a minimum debt service cover ratio of 1.45x. If in any year, the actual DSCR was between 1.45x and 1.05x lenders forced sponsors to use the excess cash to repay the loan (i.e. cash sweep), considering repaid the farthest instalments. Below 1.05x, no renegotiation of the loan terms was allowed and the loan agreement was subject to resolution.
2.3 Risk Analysis and Risk Measures

The term “risk analysis” originates in the seminal works of Hillier (1963), Hertz (1964), Hillier (1965), Van Horne (1966), Wagle (1967). These authors propose the use of the Monte Carlo simulation to obtain the distribution of an NPV or IRR. This has become best practice in the analysis of industrial investments and business planning [see Carmichael and Balatbat (2008)] More recently, also the term “probabilistic sensitivity” is used as a synonym of risk analysis [Hazan and Huang (2006)].

Formally, given a probability space \((\Omega, \mathcal{B}(\Omega), P)\), we let the application \(Z: \Omega \rightarrow \mathbb{R}\) represent a generic loss or profit function. The function \(F_Z(z) = \Pr(Z < z)\) is the cumulative distribution function of \(Z\), also called the risk profile. From \(F_Z(z)\) an analyst obtains a wide range of insights about the investment viability and infers risk measures [Rockafellar and Uryasev (2002)]. Generally speaking, a risk measure is a function of \(Z\), \(\rho: Z \rightarrow \mathbb{R}; \rho\) becomes a coherent measure of risk if it satisfies the axioms of translational invariance (Axiom T), subadditivity (Axiom S), positive homogeneity (Axiom PH), monotonicity (Axiom M), and relevance (Axiom R) of Artzner et al. (1999). The most widely utilized risk measures in the practice of industrial investment, and consequently our main interest in this work, are the value at risk (VaR) and conditional value at risk (CVaR).

\(VaR_\alpha\) is defined as follows. Given \(\beta \in (0, 1)\) [Gourieroux et al (2000), Rockafellar and Uryasev (2002)]:

\[
VaR_\alpha := \inf \{ z \in \mathbb{R} : \Pr(Z > z) \leq 1 - \alpha \}
\]  

In probabilistic terms, \(VaR_\alpha\) is the quantile corresponding to the upper bound of the \(\alpha\)-tail of the distribution of \(Z\). Alternatively, \(VaR_\alpha\) can be seen as the maximum potential loss, which is exceeded only in \(\alpha\)% of the cases.

\(VaR_\alpha\) is widely used as a risk management tool by corporate treasurers, dealers, fund managers, financial institutions, and regulators [Alexander and Baptista (2004)]. According to Berkowitz et al (2009) [p.2-3], \(VaR_\alpha\) is associated with three major fields of application: portfolio choice, risk controls and regulatory uses. It is probably in these last two areas, however, where some limitations of \(VaR_\alpha\) became apparent [see Artzner et al (1999)].

To address these limitations, authors have pursued two lines of research. One approach addresses changes in axiomatizations and conditioning that make \(VaR_\alpha\) a coherent measure of risk with no need to modify its definition [see, among others, Garcia et al (2007)]. The other approach modifies the definition of \(VaR_\alpha\), transforming it into a coherent measure of risk. For instance, in Natarajan et al (2008), \(VaR_\alpha\) is extended to Asymmetry-Robust \(VaR_\alpha\), which results in a coherent risk measure. One of the first and most studied modifications of \(VaR_\alpha\) - and the most relevant for this work - is CVaR, introduced by [Rockafellar and Uryasev (2002)] as follows:
\( CVaR_\alpha := \mathbb{E}[Z \mid Z > VaR_\alpha] \) 

\( CVaR_\alpha \) is both a coherent risk measure and a natural risk statistic [Hong and Liu (2009), p. 281]. Rockafellar and Uryasev (2002) also assert that \( CVaR_\alpha \) coincides with expected shortfall and is closely related to mean excess loss, which are also coherent measures of risk. Since its introduction, CVaR has been intensively studied, especially in comparison to VaR. A comprehensive review goes beyond the scope of this paper. However, we wish to highlight the following works. Benati (2003) addresses portfolio selection with coherent risk measure constraints. Alexander and Baptista (2004) compare results for portfolio optimization in the presence of VaR and CVaR constraints. [Szego (2004)] offers a comprehensive overview of risk measures, presenting a critical comparison of the conceptual aspects of VaR and CVaR. Optimization in portfolio selection with coherent risk measures is discussed in [Miller and Ruszczynski (2008)]. Robust optimization in CVaR portfolio selection is presented in [Huang et al (2010)]. CVaR is also utilized as a risk measure in fields outside finance. Ahmed et al (2006) and Gotoh (2006) detail applications in inventory management.

We are interested in studying how/whether these risk measures are affected by a covenant breach (technical or material). The dual perspective of sponsors and lenders is considered in the next section.

3 A Conceptual Model of Covenant Breach

We consider an investment project that develops over \( T \) periods. For simplicity of notation in the next equations, the construction phase is assumed to be fully contained in period \( t=0 \), while operation takes place from periods 1 to \( T \). We let \( A_0 \) denote the investment costs. The investment is financed through equity and debt, with \( \ell \) denoting the debt proportion to the total investment costs. We let \( I_{dc} \) represent the interest capitalized during construction, \( I_t \) and \( P_t \) the interest and the principal repaid at time \( t \). Loan repayment goes from period 1 to \( T^L \), with \( T^L \leq T \).

Letting \( \tau \) be the income tax rate, and \( \delta \) be the depreciation rate, the expression of FCF, is:

\[
FCF_t = (1 - \tau)(R_t - OE_t) + \tau \cdot [ \delta \cdot (A_0 + I_{dc}) + \chi_t I_t ] \quad t = 1, 2, ..., T
\]

where \( \chi_t = \begin{cases} 1 & \text{if } t \leq T^L \\ 0 & \text{if } t > T^L \end{cases} \). Correspondingly, the free cash flow to equity at period \( t \) (FCFE, ) is:

\[
FCFE_t = (1 - \tau)(R_t - OE_t) + \tau \cdot [ \delta \cdot (A_0 + I_{dc}) + \chi_t I_t ] - \chi_t (I_t + P_t)
\]

Assuming no frictions in the equity repayment to shareholders, ShNPV becomes:
\[
ShNPV = -(1-\ell)A_0 + \sum_{t=1}^T \frac{(1-\tau)(R_t - OE_t) + \tau[\delta(A_0 + I_{dc}) + \chi_t I_t] - \chi_t(P_t + I_t)}{(1+k_p)^t} \tag{12}
\]

We assume that \( \mathbf{R}, \mathbf{OE}, A_0, I_{dc} \) cannot be determined with certainty (bolded fonts indicate vectors). Thus, we have \( \omega=(\mathbf{R}, \mathbf{OE}, A_0, I_{dc}) \), and \( ShNPV = Z_1(\omega) \). Then, the decision criterion in eq. (12) becomes \( \mathbb{E}[ShNPV] > 0 \), where the expectation is taken over the joint distribution of \( \omega \).

The loan agreement includes a standard clause on minDSCR. For our reference investment model, we have:

\[
DSCR_t = \frac{(1-\tau)(R_t - OE_t) + \tau[\delta(A_0 + I_{dc}) + I_t]}{P_t + I_t} \quad \text{if } 1 < t < T^L \tag{13}
\]

and

\[
\min DSCR = \min_{t=1, \ldots, T^L} \frac{(1-\tau)(R_t - OE_t) + \tau[\delta(A_0 + I_{dc}) + I_t]}{P_t + I_t} \tag{14}
\]

Thus, \( \min DSCR = Z_2(\omega) \) is a random variable. As we will see, assessing the distribution of minDSCR provides analysts with a range of important insights concerning project risk, both for shareholders and lenders. In this respect, we observe that financial models equip analysts with all indicators concerning the breach of financial covenants. However, in professional practice, such indicators are usually not included as part of a standard risk analysis (In other words, analysts do not look at the distribution of these indicators.) Instead, they are assessed at base case first and then subjected to stress testing that produces the values of the indicators in correspondence with pre-determined scenarios. Yet, scenarios do not provide any information about the probability of covenant breach.

Furthermore, no additional modelling of the consequences of default accompanies the traditional risk analysis. Thus one cannot quantitatively assess the effects of covenant breach. The modelling of these effects from the dual perspective of shareholders and lenders is discussed in the following two sections.

### 3.1 The Shareholders’ Perspective

Following covenant breach, shareholders and lenders are exposed to different consequences depending on the type of default (technical or material) and the contractual clauses. Let us start with shareholders. If a covenant breach is deemed technical by lenders, they call for the suspension of remissions of cash flows to shareholders (i.e. dividend payments) in the period of default, and measures like cash sweeps come into force. Let \( t_{TB} \) indicate the period of technical breach occurrence. Then, the debt cash flows are equal to the originally scheduled ones for \( t = 1, 2, \ldots, t_{TB} - 1 \). For \( t = t_{TB}, \ldots, T^L \) the cash flows will be those corresponding to a cash sweep repayment of the loan. Let us denote them by \( P'_t \) and \( I'_t \).
Then ShNPV [eq. (3)] in the presence of technical breach becomes:

\[
ShNPV_{TB} = -(1 - \ell) A_0 + \sum_{t=1}^{t_{TB}-1} \frac{FCFE_t}{(1 + k_E)^t} + 0 + \sum_{t=t_{TB}}^{T} \frac{FCFE_t}{(1 + k_E)^t}
\]  

(15)

where the 0 evidences the presence of a null repayment at time \( t_{TB} \), and \( FCFE_t \) are the new free cash flows to equity after debt rescheduling. Because at least one equity cash flow repayment is suspended and, after \( t_{TB} \), cash is redirected earlier towards lenders, shareholders will experience a loss equal to \( ShNPV_{TB} - ShNPV \).

According to eqs. (11) and (15), we have:

\[
ShNPV_{TB} = -(1 - \ell) A_0 + \sum_{t=1}^{t_{TB}-1} \frac{(1 - \tau)(R_t - O_E_t) + \tau[\delta(A_0 + I_{dc}) + \max(0, T^L - t)I_t] - (P_t + I_t) \cdot \max(0, T^L - t)}{(1 + k_E)^t} + \sum_{t=t_{TB}}^{T} \frac{(1 - \tau)(R_t - O_E_t) + \tau[\delta(A_0 + I_{dc}) + \max(0, T^L - t)I_t] - (P_t + I_t) \cdot \max(0, T^L - t)}{(1 + k_E)^t}
\]  

(16)

Consequences are more severe if the materiality test is positive. A material covenant violation triggers project default. The security package implies that the pledged shares of the SPV fall into the hands of lenders. Then, the present value of equity cash flows from the moment of default to the project end is taken away from sponsors and redirected to lenders. Let \( t_{MB} \) represent the time at which a material breach happens. Because shareholders receive cash flows up to the period preceding \( t_{MB} \), ShNPV [eq. (3)] becomes

\[
ShNPV_{MB} = -(1 - \ell) A_0 + \sum_{t=1}^{t_{MB}-1} \frac{FCFE_t}{(1 + k_E)^t} = 
\]

\[
-(1 - \ell) A_0 + \sum_{t=1}^{t_{MB}-1} \frac{(1 - \tau)(R_t - O_E_t) + \tau[\delta(A_0 + I_{dc}) + \max(0, T^L - t)I_t] - (P_t + I_t) \cdot \max(0, T^L - t)}{(1 + k_E)^t}
\]  

(17)

Correspondingly, one can measure the loss incurred by shareholders as the present value of the cash flows from \( t_{MB} \) to the end of the project. Let

\[
NPV_{Loss} | MB = \sum_{t=t_{MB}}^{T} \frac{FCFE_t}{(1 + k_E)^t}
\]  

(18)

In eq. (18), \( NPV_{Loss} | MB \) denotes expectation conditional on material breach. \( NPV_{Loss} | MB \) represents the loss incurred by shareholders in a given scenario in which material breach happens. We consider the expectation of this breach and have:
\[ ShLoss^{MB} := -E_{MB}[NPVloss | MB] = -E_{MB}\left[ \sum_{t=1}^{T} \frac{FCFE_t}{(1 + k_E)^t} \right] \]  

In eq. (19), \( ShLoss^{MB} \) is the expected loss for shareholders conditional on material breach, with the expectation taken over all scenarios in which material breach happens.

3.2 The Lenders’ Perspective

When a covenant breach happens, lenders and shareholders (better still, their respective legal advisors) must assess the severity of the breach. In most cases, the attitude of lenders is usually to try to allow shareholders to keep going on managing the project by waiving the covenant breach [Chen and Wei (1993), Beneish and Press (1995), Nini et al (2009)]. In this case, the sole consequence is a cash sweep in favour of the lenders. As a result, the project’s entire free cash flow of the period at which the technical breach happens is redirected to lenders, the debt outstanding is correspondingly reduced and the remaining debt is repaid over the subsequent periods.

Hence, in the case of a technical breach, we have:

\[ NPV_{Loan}^{TB} = \sum_{t=1}^{t_{TB}-1} \frac{P_t + I_t}{(1 + k_d)^t} + \sum_{t\geq t_{TB}} \frac{P'_t + I'_t}{(1 + k_d)^t} \]

where \( t_{TB} \) is the period when a technical breach happens and \( P_t' \) and \( I_t' \) are the newly scheduled principal and interest repayment flows. Of course, the rescheduling is such that no financial losses are incurred by lenders.

In the case of material breach, instead, the situation for lenders is different. Shareholders are, in principle, excluded from the project and lenders now face the decision of what to do. A first possibility, denoted as a workout by Davidson et al. (2010), is to ensure the project’s operational phase continues. In fact, a material breach does not necessarily correspond to a situation of insolvency and the project is potentially capable of producing enough cash to repay the loan. Thus, lenders work out a restructuring of the SPV, which involves revising the contractual package and frequently having new shareholders step in. Of the 213 projects studied by Davidson et al (2010) that underwent a material breach, 116 went through a workout with creditors. In these cases, the ultimate recovery rate on the loan was 76%. In the remaining cases, bankruptcy was declared and lenders recovered the outstanding amount on the loan only through distressed sales. In these circumstances, the average ultimate recovery rate on the outstanding amount was only 48%.

Formally, let \( O_{MB} \) denote the value of the outstanding debt at \( t_{MB} \), with \( p_{Workout} \) the probability of a workout, and \( 1 - p_{Workout} \) the probability of a fire sale, the expected recovery for lenders equals:

\[ \mathbb{E}[W_{Lenders}] = p_{Workout} \cdot \lambda_{workout} \mathbb{E}[O_{MB}] + (1 - p_{Workout}) \cdot \lambda_{distressed} \mathbb{E}[O_{MB}] =
\]

where \( \lambda_{workout} \) and \( \lambda_{distressed} \) are the recovery rates on the loan in the case of workout and distressed sales, respectively. According to eq. (21), we can also determine the percentage of the expected recovery rate:
Eqs. (21) and (22) provide measures of expected recovery for lenders given material breach. In particular, only if \( \lambda_{\text{workout}} > \lambda_{\text{distressed}} \) and \( p_{\text{workout}} \) is strictly non-null, lenders have the possibility of recovering more through workout than what they would otherwise recover through distressed sales. As discussed in Davidson et al (2010), when a material breach occurs, in a realistic application, decision-making and negotiation aspects (i.e., the ability to lead the transaction to a successful recovery) become the key concerns of the problem.

We are now left to describe how to nest the modelling of the consequences of default described in this section in the context of the Monte Carlo simulation.

4 The Simulation Procedure

![Figure 1: Simulation steps for modeling the effects of covenant breach](image)

\[
W_{\text{Lenders}} = \frac{E[R_{\text{Lenders}}]}{E[Q_{\text{lab}}]} = P_{\text{workout}}(\lambda_{\text{workout}} - \lambda_{\text{distressed}}) + \lambda_{\text{distressed}}
\] (22)
In this section, we describe how our conceptual model can be turned into a simulation procedure. The first step consists of running a probabilistic simulation of the financial model registering the values of both lenders and shareholders’ criteria at the end of each simulation (Figure 1).

Then, the examination of covenant breach at scenarios \(n=1,2,\ldots,N\) starts. At each scenario, we analyse the value of the covenant indicators produced by the model, and check the materiality test (Step 2 in Figure 1). If no covenant breach is registered, then the value of the valuation criterion obtained in the probabilistic simulation is registered as final value of scenario \(n\). Conversely, we have to simulate the consequences of material breach with the different perspectives of lenders and shareholders. If the covenant breach is technical, we need to estimate the new \(FCF_i\) and \(FCFE_i\). With eq. (16) we obtain the value of \(ShNPV\) to be stored in scenario \(n\). If the breach is material, eq. (17) applies. Simultaneously, we can utilize eqs. (20) and (21) to assess consequences for lenders.

By comparing the NPV distributions [risk profiles in Smith (1998)] obtained with and without modelling default, we have a way to assess the consequence of covenant breaches for the given transaction. To quantitatively establish whether the effect is significant, several statistical tests are available. If the test provides a negative answer (that is, the separation is not significant), then the information on the original distribution (the one obtained without explicitly modelling the consequences of default) can be maintained in further project evaluation. Conversely, if the answer is positive, contractual default has an impact and the new risk profile has to be utilized to assess risk measures for the project.

The above procedure can be also applied if we are evaluating the model through a scenario analysis rather than a Monte Carlo simulation, observing that one Monte Carlo run can be interpreted as a possible scenario. Both techniques are well known and widely described in the literature, and it is not within the scope of this work to present a detailed description. In either case the procedures available in the literature allow us to build distributions or scenarios reflective of the decision maker's state of knowledge with respect to the uncertain quantities [in the case of distribution assignment in Monte Carlo simulation; Apostolakis (1990), Glasserman (2004)] or to future states of the world [in the case of scenario analysis; Jungermann and Thuring (1988), Tietje (2005).]

## 5 Estimating Covenant Breach Probabilities, Workout Probability and Risk Measures

In light of the simulation procedure, we can infer several additional insights about the project risk with and without inclusion of covenant breach. The first is the frequency of technical and material breaches.

**Definition:** Consider a financial transaction with covenants. Let \(CB\) be the event “any of the covenants is breached.” Then, we let

\[
\nu_{CB} = \Pr(CB)
\]

(23)

denote the probability of covenant breach.
As mentioned above, a covenant breach can be technical or material, depending on the specific contractual clauses set forth in the loan agreement. Thus, a material breach is actually associated with only a subset of the states in which a breach occurs. Then, we can define the following probabilities.

**Definition:** Consider a financial transaction with covenants. Let $TB$ and $MB$ denote the events “technical breach” and “material breach” of any covenant, respectively. Then, we let

$$v_{TB} = \Pr(TB) \quad \text{and} \quad v_{MB} = \Pr(MB)$$

(24)

denote the project probabilities of technical and material covenant breach, respectively.

Clearly, the set of the states of the world corresponding to material breach is included or equal to the set of the states of the world with technical breach. However, the two events are usually to be considered disjointed. In other words, $CB = TB \cup MB$, and therefore we have $v_{MB} + v_{TB} = v_{CB}$.

Elsinger et al (2006) (p. 1310) observe that the relative frequency of default across scenarios is then interpreted as a default probability. Let $N$ be the number of Monte Carlo runs. Then, let $n_{TB}$ and $n_{MB}$ denote the number of Monte Carlo runs at which covenant breach and contractual default are registered, respectively. Then, by constructing a Monte Carlo simulation [see, for instance, Glasserman (2004)], we have:

$$v_{TB} = \lim_{N \to \infty} \frac{n_{TB}(N)}{N}, \quad v_{MB} = \lim_{N \to \infty} \frac{n_{MB}(N)}{N}$$

(25)

Hence, at any sample size $N$ we obtain an estimate of $v_{TB}$ and $v_{MB}$, that we denote as $\hat{v}_{TB}$ and $\hat{v}_{MB}$, respectively. A procedure for readily automating the calculation of these two variables in a Monte Carlo simulation is the following.

Let us consider $TB$ for simplicity’s sake. At each Monte Carlo run, we can introduce the auxiliary Boolean indicator of technical breach $U_{TB}(n) = \left\{ \begin{array}{ll} 1 & \text{if} \ TB \\ 0 & \text{if} \ no \ TB \end{array} \right.$ Then, it is $n_{TB}(N) = \sum_{n=1}^{N} U_{TB}(n)$ . By $\sum_{n=1}^{N} U_{TB}(n) / N$, we obtain $\hat{v}_{TB}$ estimate at sample size $N$. Similar reasoning applies for obtaining estimates of $v_{MB}$.

We further observe that, from the simulation, we can estimate an endogenous probability of workout. In eq.(21), we choose the value of $p_{\text{workout}}$ that best matches the geographic location and industrial sector of the project at hand, resorting to the data published by rating agencies, for example. However, a workout probability can be endogenously generated by our Monte Carlo simulation. We consider scenario “s” in which a material default happens. We assume that lenders will be able to continue the project operations and find a new project sponsor if the following two conditions are met: a) $\varphi \leq \min\text{DSCR} < 1.051$; and b) the equity NPV of the outstanding cash flows is positive. Condition a) is about the severity of the breach. In our simulation, we impose $\varphi = 1$, which corresponds to cases where the project is not bankrupt although in material breach. However, there might be cases in which lenders attempt to rescue the project even if $\min\text{DSCR} < 1$, but not far away from unity.
Of course, $\varphi$ can then be modified in the simulation. Both conditions can be extracted from a Monte Carlo simulation. Operationally, we define the indicator

$$U_{WO}(n) = \begin{cases} 1 & \text{if workout conditions met} \\ 0 & \text{if workout conditions not met} \end{cases}$$

(26)

Which leads to the following Monte Carlo estimate of the workout probability:

$$\hat{V}_{Workout} = \frac{\sum_{n=1}^{N} U_{WO}(n)}{N}$$

(27)

It is also interesting to consider the conditional frequency of workout given material breach. This can be estimated by simply normalizing $\sum_{n=1}^{N} U_{WO}(n)$ over the scenarios in which material breach has occurred:

$$\hat{V}_{Workout,MB} = \frac{\sum_{n=1}^{N} U_{WO}(n)}{\sum_{n=1}^{N} U_{MB}(n)}$$

(28)

Finally, based on our knowledge of the risk profile, we can estimate risk measures in the presence and absence of contractual breaches. In this paper, we focus on VaR and CVaR in eqs. (8) and (9). The Monte Carlo estimation of these indicators is widely discussed in the literature [Rockafellar and Uryasev (2002), Ruszczyński and Shapiro (2005), Jin and Zhang (2006)] and, therefore, can be omitted in this work. Suffice to say that by comparing the values of VaR and CVaR in two sets of simulations (one that explicitly includes covenant breaches and one that doesn’t), lenders and sponsors are able to form a comprehensive view of the investment at hand.

6 A Case Study

6.1 Project Background and economic rationale

In mid 2007, Bonollo Distilleries – one of the most renowned high-end liquor producer in Italy – started studying the project of a biomass plant to be annexed to its already existing brewing factory. According to Bonollo Distilleries’ management, the plant was needed to reduce both energy costs and polluting emissions and to diversify the energy sources currently employed by the factory.

At that time, Bonollo Distilleries already used some biomass energy to cover approximately 30% of its heat energy needs and 40% of its electrical energy needs. The remaining part of its requirements was covered with methane gas and electrical energy from the national electricity Grid. The new plant was expected to provide the
Distillery with 10MW of clean heat and electrical energy. This represented 97% of heat energy needs and 100% of electrical energy needs. The energy surplus of 57 MWh per year would be sold to the electricity distribution network.

Another key feature of the biomass plant was that it would enable the Distillery to reduce its emissions from a total of 513,779 kg/year to less than 403,260. Furthermore, biomass combustion did not increase the level of CO₂ present in the atmosphere and therefore represented a clean and environmental-friendly energy source.

In addition to these advantages for Bonollo Distilleries, another element that prompted management to embark on the project in question was the possibility to use the byproducts of the distillation process as base feedstock for the power plant. The waste byproducts would be sold to the biomass plant by Bonollo Distilleries under a long term raw material supply agreement. Finally, the ashes deriving from combusted biomasses could be used as fertilizers in the company’s plantations, resulting in minimal waste and further costs reduction.

6.2 Contractual structure

The contractual structure underpinning the project is as follows. The biomass plant was incorporated in a new entity (SPV), Bonollo Energia Spa, granted with the rights to build and operate the biomass plant. The SPV had to be financed with a debt/equity ratio of 4/1 with equity provided in equal parts by the sponsors Bonollo Distilleries and Alerion Clean Power. Alerion is a company listed on the Milan Stock Exchange specialized in the management and operations of renewable power plants in Italy and Central and Eastern Europe.

Debt was provided by a pool of banks headed by General Electric (GE)-Interbanca. Interbanca was appointed as Mandated Lead Arranger (MLA) of the bank syndicate in July 2008 and started working with project sponsors on project contracts and loan syndication during the third and fourth quarter of 2008. The financial close took place in February 2009 with a syndicate composed by General Electric-Interbanca as MLA, GE Interbanca, MPS Capital Services, Banca Popolare dell’Etruria and Centrobanca as underwriters. GE-Interbanca also acted as hedging bank, agent bank and depository bank.

Plant construction was regulated by an EPC (engineering, procurement and construction) contract signed with STC Group as general contractor. STC was a leader in the construction of industrial plants for power generation on the Italian market. The construction contract was signed on a fixed-price turnkey basis and was supplemented by guarantees provided to the SPV by the general contractor (timing of delivery and minimum performance standards). Furthermore, the contractor was required to provide performance bonds and warranty bonds for 2 years after construction. Construction was expected to start at the beginning of 2009 and last a total of 24 months, with an additional 6-month testing and start up phase.

The two project sponsors played key roles in the contractual network of the project finance deal. Bonollo Distilleries was the seller of approximately 142,000 tons/year of base feedstock (biomass) under a long term raw
material supply agreement of 16.5 years. Furthermore, Bonollo Distilleries would purchase around 116,000 tons of steam per annum and 13,000 MWh per annum of biomass-produced energy under a long term power purchase agreement.

Alerion contributed to the project in two roles. First, a joint venture was created by the two sponsors in order to carry out O&M (operations and maintenance) services for the SPV. Second, Alerion had to enter a service agreement with the SPV for the sale of power and green certificates on the market.

Finally, as mentioned, the energy surplus produced by the plant had to be sold on the Italian electricity market based on market spot contracts. The sale of this surplus was carried out by Alerion Clean Power pursuant the service agreement with the SPV.

6.3 Financial data and covenants

The agreement between sponsors and lenders required the project to be financed with a senior debt/equity ratio of about 4/1, in line with standard market practice at the time of the deal. The total investment cost was expected to be around 64 mil euro, including among others about 48 mil euro of EPC cost, 6 mil euro capitalized interest and 9 mil of VAT under construction.

GE-Interbanca and the other banks of the syndicate organized the loan facilities in two tranches:

1. A base facility of 41 mil euro with duration of 15 years and a grace period of 3 years at the beginning of the operational phase where only interest payment was due.

2. A VAT loan of about 9 mil euro, to be repaid during the operational phase using the cash corresponding to the VAT on Energy and steam sales. Duration: construction period plus 2 year repayment.

The two facilities were supplemented by the typical project finance security package discussed in Section 2.2. Minimum DSCR was set at 1.4x with a DSCR triggering default (material breach) at 1.05x.

6.4 Risk Analysis Results

The financial model developed to value the transaction was made available to the authors by one of the sponsors of the project. The model was developed by the Mandated Lead Arranger Bank and realistically simulates the investment's financial performance over its life span reproducing the SPV income statement, cash flow statement, and balance sheet through a complex dovetail of Excel worksheets. All valuation criteria used by sponsors and lenders for decision-making are estimated by the model that also provides information about the occurrence of covenant breach.

We use the financial model and the data of the Bonollo Energia case to perform a series of numerical experiments to investigate the conditions under which covenants impact the transaction and the extent of this impact. Each experiment is a risk analysis performed in accordance with the procedure presented in Section 4,
with varying levels of percentage of debt financing $\ell$ (which is kept at $\ell = 0.8$ in the base case) and cost of debt $k_d$ (7.5% at base case) (see Table 1).

In this respect, we adopted the point of view of the project shareholders before the deal closing, trying to investigate what conditions the lenders are most likely to set down in covenants, based on the project’s financial performance.

Table 1 also reports the remaining problem parameters. Following Step 1 of the procedure, we conducted a set of 10,000 Monte Carlo simulations through the original model, to produce the unconditional risk profile and the values of the random variables in eqs. (16) – (27), namely $R$, $OE$, $A_0$, $I_{dc}$. With these values in hand, we can replicate the results of the present simulation. The dataset containing $R$, $OE$, $A_0$, $I_{dc}$ is available upon request.

Table 1: List of parameters for the case-study simulation.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>Revenues</td>
<td>Dataset</td>
</tr>
<tr>
<td>OE</td>
<td>Operating Expenses</td>
<td>Dataset</td>
</tr>
<tr>
<td>$A_0$</td>
<td>Initial Cash Outflow</td>
<td>Dataset</td>
</tr>
<tr>
<td>$I_{dc}$</td>
<td>Interest during construction</td>
<td>Dataset</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tax Rate</td>
<td>35%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation Rate</td>
<td>8%</td>
</tr>
<tr>
<td>$k_e$</td>
<td>Cost of Equity</td>
<td>10%</td>
</tr>
<tr>
<td>$k_d$</td>
<td>Cost of Debt</td>
<td>5.5%:6.5%:7.5%:8.5%:9.5%</td>
</tr>
<tr>
<td>$T$</td>
<td>Project Operational Period</td>
<td>16years</td>
</tr>
<tr>
<td>$T_L$</td>
<td>Loan Repayment Period</td>
<td>13years</td>
</tr>
<tr>
<td>$T_C$</td>
<td>Construction Period</td>
<td>2years</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Percentage of Debt Financing</td>
<td>70%:80%:90%</td>
</tr>
</tbody>
</table>

We start by presenting the results of a standard risk analysis (i.e., without simulation of covenant breaches) we performed on the project. Figure 3 presents the distributions of $R$ and $OE$.

Figure 3 shows that the project is operationally sound, with a 4/1 to 3/1 ratio between revenues and operating expenses. Such a high operating margin grants sustainability of high percentages of debt financing $\ell$, in line with the typical characteristics of project-financed initiatives (Esty, 2003). For the sake of confidentiality, in Figure 3 and in the remainder of this section, all data are rescaled by the mean value of the revenues in Year 2, which is assumed equal to 10MEUR.

---

7 Our choice of extracting $R$, $OE$, $A_0$, $I_{dc}$ from the financial model and inserting them in eqs. (16) – (27) is motivated by transparency. An alternative option would have been to directly perform the simulation creating a Visual-Basic Macro on the original model. However, we could not have made the model publicly available.
The results of the Monte Carlo simulation on the equity cash flows for the base case $\ell=0.8$ and $k_d=7.5\%$ are as follows: $\mathbb{E}[ShNPV] = 11.2\text{MEUR}$ This suggests that the investment is economically convenient for project sponsors. We also obtain $v_{NPV} = \Pr(NPV \leq 0) = 0.026$, $CVaR_{0.05} \approx -0.44$ and $VaR_{0.05} \approx 1.3\text{MEUR}$

These values indicate that the likelihood of incurring losses for shareholders is small and confirm that we are dealing with a very sound project, one that is attractive from a shareholder’s perspective, as expected from the high operating margin.

Let us next examine the distribution of minDSCR as obtained by our simulation (Figure 4). From this distribution, we are able to infer the probabilities of covenant breach, both technical and material.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2}
\caption{Distributions of R and OE over time. The values have been normalized to unity by dividing for the mean value of the revenues in period 2.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3}
\caption{Distribution of minDSCR at $\ell=0.8$ and $k_d=7.5\%$.}
\end{figure}
Figure 4 suggests that $\nu_{CB} \cong 0.72$, $\nu_{TB} = 0.68$, and $\nu_{MB} = 0.04$. In fact, minDSCR is smaller than 1.4 in 7,191 out of the 10,000 scenarios, signalling either a default or a material breach. Of these breaches, 6,821 are technical, 370 material.

An analysis of the timing of technical (left graph) and material (right graph) breaches occur, shows that around 95% of technical breaches occur at year 1, with 5% happening at year 13. For material breaches, 90% is at year 1, around 10% at year 13 and 2 events of material breach occur at year 12.

To understand the timing of default, one must examine the distributions of DSCR, as $t$ varies (Figure 6).
remaining portion of FCF₁ goes into the repayment of principal. Then, the outstanding loan amount is reduced more than forecasted by the original loan schedule. The new \( P'_t \) and \( I'_t \) are scheduled so that

\[
NPV^{\text{Loan}}_{TB}(n) = NPV^{\text{Loan}} = 0.
\]

They are lower than the original values, insuring an increase in the project’s cover ratio in the following years.

The 370 events of material breach happen either at \( t=1 \) (in 321 scenarios), \( t=12 \) (in 2 scenarios), or \( t=13 \) (47 scenarios); 260 breaches occur in scenarios in which equity NPV is negative, the remaining 110 in scenarios of positive NPV. The first case of material breach takes place in scenario 4 at \( t_{MB}=1 \). According to contractual clauses, shareholders lose the right to cash flows from \( t_{MB}=1 \) to the final year of the operational life of the project.\(^8\) The associated loss on equity NPV is a significant 6.0MEUR. Consequences of material breach for lenders are the following. The outstanding loan amount is equal to nearly the entire facility (41MEUR) when \( t_{MB}=1 \) and around 3.15MEUR when \( t_{MB}=13 \). The recovery rate \( w_{\text{Lenders}} \) when computed exogenously [eq. (22)] is equal to approximately 71%, using \( p_{\text{workout}}=0.84 \), \( \lambda_{\text{workout}}=0.76 \) and \( \lambda_{\text{distressed}}=0.48 \), in line with the data reported by Davidson et al (2010). When computed endogenously, i.e. by the simulation using eq. (27), we obtain \( w_{\text{Workout}} = 1.1\% \). The reason for this is that the recovery condition in eq. (26) is met only in 110 of the 10,000 scenarios. Thus, the conditional recovery probability, eq. (28), is approximately 1/3 in this case. This result also proves that material breaches do not necessarily occur in scenarios with negative NPV. Conversely, NPV is positive in around 1/3 of the scenarios in which material breach is registered.

Repeating the analysis scenario by scenario and simulating the effects of both technical and material breaches, we obtain the risk profiles in Figure 7.

Figure 7 shows that the effect of technical breach (first graph), material breach (second graph) and combined (third graph). When all types of breach are considered, the risk measures change as follows. The estimate of the probability of negative NPV increases to 4%, and CVaR₀.₀₅ lowers to -4.8MEUR. Conversely, VaR₀.₀₅ remains practically unaffected. The events of concern (material breach) affect the left tail of the distribution corresponding to a probability lower than 0.05. The invariance of VaR₀.₀₅ reveals, once again, the non-coherent nature of VaR. In fact, by changing the quantile from 0.05 to 0.01, we find a change in VaR₀.₀₁ which shifts from -1.6MEUR (VaR₀.₀₁ when covenant breach is not taken into account) to -7.7MEUR (VaR₀.₀₁ when covenant breach is taken into account).

---

\(^8\) This effect is due to the fact that, as explained in the introduction, upon material breach creditors force the deal into bankruptcy, expropriate shareholders’ control rights and become the real owners of the remaining cash flows generated by the project either in case of continued operations or liquidation/distressed sale.
The next question is then whether the difference in the risk profiles in the third graph is of statistical significance. We use the two-sided Kolmogorov-Smirnov (KS) test [see Massey (1951) for a review]. The null hypothesis (to be tested) is: the risk profiles with and without covenant breach come from the same distribution, with $h=0$ denoting acceptance of the null hypothesis, and $h=1$ rejection. For the risk profiles of our simulation, the result of the test is $h=1$ (rejecting the null hypothesis) with a $p$-value of $6.4590\times 10^{-5}$. Thus, the two risk profiles are significantly different from a statistical viewpoint. We then further investigate this aspect performing a series of experiments at different levels of leverage and cost of debt. Table 2 reports results.

Table 2: Statistical significance of covenant breach. * denotes statistical significance at 0.01.

<table>
<thead>
<tr>
<th>$\ell$</th>
<th>$k_d=5.5%$</th>
<th>$k_d=6.5%$</th>
<th>$k_d=7.5%$</th>
<th>$k_d=8.5%$</th>
<th>$k_d=9.5%$</th>
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<tr>
<td>70%</td>
<td>$h=0$;</td>
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<td>80%</td>
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<td>90%</td>
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<td>$h=1^*$;</td>
<td>$h=1^*$;</td>
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</tbody>
</table>
As evidenced by additional numerical experiments performed by the authors, the impact of covenant breach on the risk profile (and therefore on the risk measures and on the expected shareholder NPV) increases as $t$ and $k_d$ increase. At $t=0.7$, covenant breaches (either technical or material) are statistically insignificant for any value of $k_d$. These breaches become statistically significant at $t=0.8$ and $t=0.9$. Numerical experiments also reveal that as $t$ increases, the relevance of material breaches over technical breaches augments. We refer to Appendix A for further details.

7 Conclusions

In this paper, we investigated the consequences of the inclusion of covenant breaches in the risk analysis of large industrial projects, and provided an objective view on their effect and significance. We have analyzed both theoretical and managerial implications of the violation of financial covenants included in debt contracts. To achieve our objective, we developed both a conceptual model and a simulation procedure. This dual perspective is dictated by the fact that to the best of our knowledge, no other papers have studied the topic in a quantitative manner. The methodology we developed provides insights on project performance and consequences for shareholders and lenders in cases of covenant waivers or material breaches that trigger a distress sale or a restructuring. Our methodology allows us to answer the question of whether covenants matter when considering the risk analysis of a given transaction. Numerical experiments have shown that covenants affect the perspectives of sponsors and lenders slightly if breaches are merely technical, consistent with Smith and Warner (1979). However, if we also take into account the consequences of renegotiation and material breach, covenants might significantly impact the risk profiles. Therefore risk measures must be adjusted accordingly. Also, material breach has operating consequences for lenders, who must decide how to act if the control rights of a project are expropriated from the shareholders as a result of security package enforcement.

Acknowledgements. We would like to thank Andrea Resti, Andrea Sironi, Carlo Chiarella, Giovanni Puopolo and Alessandro Sbuelz for useful comments on earlier drafts of the paper. We also thank Carefin Bocconi Research Center for generous financial support. Responsibility for the contents remains our own.

References


Demiroglu C. and James, C. M., 2009: The information content of bank loan covenants, Review of financial Studies, 9 (10), pp. 3700-3737.


Appendix A:

Tables 3 and 4 report a series of sensitivities performed for investigating the behaviour of the risk measures with and without the modelling of contractual breach as the cost of debt ($k_d$) and leverage ($\ell$) vary.

Table 3: Sensitivity of Risk Measures to $k_d$.

<table>
<thead>
<tr>
<th>$k_d$</th>
<th>$V_{CB}$</th>
<th>$V_{TB}$</th>
<th>$V_{MB}$</th>
<th>$P(\text{ShNPV}&lt;0)$</th>
<th>$E[\text{ShNPV}]$ (MEUR)</th>
<th>VaR$_{0.05}$ (Including covenant breach)</th>
<th>CVaR$_{0.05}$ (Including covenant breach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5%</td>
<td>0.50</td>
<td>0.495</td>
<td>0.0081</td>
<td>0.009</td>
<td>13.1</td>
<td>3.0</td>
<td>0.3</td>
</tr>
<tr>
<td>6.5%</td>
<td>0.61</td>
<td>0.596</td>
<td>0.017</td>
<td>0.017</td>
<td>12.2</td>
<td>2.1</td>
<td>-1.7</td>
</tr>
<tr>
<td>7.5% (Base case)</td>
<td>0.72</td>
<td>0.68</td>
<td>0.037</td>
<td>0.040</td>
<td>11.2</td>
<td>1.23</td>
<td>-4.8</td>
</tr>
<tr>
<td>8.5%</td>
<td>0.81</td>
<td>0.74</td>
<td>0.07</td>
<td>0.070</td>
<td>10.3</td>
<td>-7.5</td>
<td>-7.7</td>
</tr>
<tr>
<td>9.5%</td>
<td>0.87</td>
<td>0.75</td>
<td>0.12</td>
<td>0.11</td>
<td>9.2</td>
<td>-7.7</td>
<td>-7.8</td>
</tr>
</tbody>
</table>

Table 3 shows that $V_{CB}$, $V_{TB}$ and $V_{MB}$ and $P(\text{ShNPV}<0)$ systematically increase with $k_d$. Conversely, $E[\text{ShNPV}]$, VaR$_{0.05}$ and CVaR$_{0.05}$ systematically decrease with $k_d$. In correspondence to a high material breach probability, we register a high negative value of the risk measures. This signals an increasing fattening of the left tail of the risk profile (see the second and third graphs in Figure 7). The same behaviour of the risk measures and $E[\text{ShNPV}]$ is registered in a sensitivity to $\ell$, while keeping $k_d=7.5\%$ (see Table 4).

Table 4: Sensitivity of Risk Measures to $\ell$.

<table>
<thead>
<tr>
<th>$\ell$</th>
<th>$V_{CB}$</th>
<th>$V_{TB}$</th>
<th>$V_{MB}$</th>
<th>$P(\text{ShNPV}&lt;0)$</th>
<th>$E[\text{ShNPV}]$ (MEUR)</th>
<th>VaR$_{0.05}$ (Including covenant breach)</th>
<th>CVaR$_{0.05}$ (Including covenant breach)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70%</td>
<td>0.3666</td>
<td>0.36</td>
<td>0.0025</td>
<td>0.0424</td>
<td>10.4</td>
<td>0.33</td>
<td>-1.6</td>
</tr>
<tr>
<td>80% (Base case)</td>
<td>0.72</td>
<td>0.68</td>
<td>0.037</td>
<td>0.040</td>
<td>11.2</td>
<td>1.23</td>
<td>-4.8</td>
</tr>
<tr>
<td>90%</td>
<td>0.92</td>
<td>0.76</td>
<td>0.16</td>
<td>0.15</td>
<td>12.00</td>
<td>-3.8</td>
<td>-3.9</td>
</tr>
</tbody>
</table>