Numerical Investigation of Unsteady MHD Free Convective Flow Past an Oscillating Vertical Porous Plate with Oscillatory Heat Flux

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***Abstract*—**The unsteady MHD convective flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate in a porous medium with an oscillatory heat flux in the presence of a uniform transverse magnetic field is studied. The governing equations describing the flow along with the boundary conditions are solved by Finite Difference Method using the MATLAB computer generated program. A numerical investigation are performed to estimate the effects of varying Darcy number, radiation, magnetic, and suction parameters on fluid velocity and temperature on MHD free convective flow. The effects of changing these values on Temperature distribution and velocity flow are shown in the graphical and tabular forms. We have presented the non-dimensional velocity u and temperature θ for several values of magnetic parameter M^2, Radiation parameter *R*, Darcy number Da, Prandtl number Pr and Suction parameter *S*. It was discovered that the velocity u increase with decrease in Suction parameter *S* and magnetic parameter M^2 and increase with increase Darcy number. For temperature, it is also found that the temperature θ increases with decrease in Prandtl number *Pr* and increases with increase in Radiation parameter *R* and Suction parameter *S*.

***Keywords*—**Crank Nicolson Numerical Scheme; Darcy Number; Finite Difference Method; Magnetic Parameter; Prandtl Number; Radiation Parameter; Suction Parameter; The Partial Differential Equations.

***Abbreviations*—**Forward Difference Equation (FDE); Forward Difference Scheme (FDS); Magneto Hydrodynamics (MHD); Partial Differential Equation (PDE).

I. INTRODUCTION AND LITERATURE REVIEW

1.1. Introduction

MAGNETO HYDRODYNAMICS (MHD) is the study of flow of electrically conducting fluid in the presence of magnetic field. The word magneto hydrodynamic is derived from: Magneto meaning magnetic field, Hydro meaning Liquid and Dynamics which means movement. Hydrodynamics is the study of fluid flow and the forces that cause the flow in the absence of electromagnetic field. Magneto hydrodynamic free convection flow through porous media are very important particularly in the fields of petroleum technology for the flow of oil through porous rocks, in chemical engineering for the purification and filtration processes and in the cases like drug permeation through human skin. The principles of this subject are very useful in recovering the water for drinking and irrigation purposes. The knowledge of flows through porous medium is also useful to study the movement of natural gas and water through the oil reservoirs. A number of studied have appeared in the literature where the porous medium is either bounded between parallel plates. At the high temperature attained in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor. The ionized gas or plasma can be made to interact with the magnetic field and alter heat transfer and friction characteristic. Recently, it is of great interest to study the effect of magnetic field on the temperature distribution and heat transfer when the fluid is not only an electrical conductor but also when it is capable of emitting and absorbing thermal radiation. The heat transfer by thermal radiation is becoming of greater importance when we are concerned with space applications, higher operating temperatures and also power engineering. The mechanism of conduction in ionized gases in the presence of a strong magnetic field is different from that in metallic substance. The electric current in ionized gases is generally carried by electrons, which undergoes successive collisions with other charged or neutral particles. In the ionized gases, the current is not proportional to the applied potential except when the field is very weak in an ionized gas where the density is low and the magnetic field is very strong, the conductivity normal to the magnetic field is reduced due to the free spiraling of electrons and ions about the magnetic lines of force before suffering collisions and a current is induced in a direction normal to both electric and magnetic fields. This phenomenon, well known in the literature, is called the Hall Effect. The study of hydro magnetic flows with Hall currents has important engineering applications in problems of Magneto hydrodynamic generators and of Hall accelerators as well as in flight magneto hydrodynamics. It is well known that a number of astronomical bodies posses' fluid interiors and magnetic fields. It is also important in the solar physics involved in the sunspot development, the solar cycle and the structure of magnetic stars. In space technology applications and at higher operating temperatures, radiation effects can be quite significant. The radiative convective flows are frequently encountered in many scientific and environmental processes, such as astrophysical flows, water evaporation from open reservoirs, heating and cooling of chambers, and solar power technology. The unsteady hydro magnetic flow of a viscous incompressible electrically conducting fluid through a vertical channel is of considerable interest in the technical field due to its frequent occurrence in industrial and technological applications.

1.2. Literature Review

The history of MHD can be traced to Faraday [2] when he did the first quantitative observation of Magneto hydrodynamics. He did experiments with mercury as a conducting fluid flowing in glass tube placed in magnetic field and observed that voltage was induced in direction perpendicular to both the direction of flow and magnetic field. Since then a lot has been done on MHD and its related field. Among them include;

Ritcher (1932) showed that when an electric field is applied to a conducting fluid in a direction which is perpendicular to magnetic field, a force is exerted on the fluid in the direction perpendicular to both electric field and magnetic field. Hartmann (1938) discussed theoretically and experimentally the flow of a conducting fluid between two parallel plates while Stokes [9] concentrated on the flow of an incompressible and viscous fluid past impulsively started infinite flat plates. Rao & Lingaraj [5] studied the heat transfer in porous medium in the presence of transverse magnetic field. The effects of the heat source parameter and Nusselt number were analyzed. They discovered that the effect of increasing porous parameter is to increase the Nusselt number. Further, Ram et al., (1995) solved magneto hydrodynamics stokes problem of convection flow for a vertical infinite plate in a dissipative rotating fluid with Hall current. This is an analysis of the effects of various parameters on the concentration velocity and temperature profiles while Kwanza et al., [3] presented their work on MHD stokes free convection past an infinite vertical porous plate subjected to a constant heat flux with ion-slip and radiation absorption. They discussed their tabulated results on concentration, velocity profiles and temperature distributions both theoretically and graphically. Abd El-Naby et al., [1] investigated magneto hydrodynamic transient natural convection-radiation boundary layer flow with variable surface temperature, showing that velocity, temperature, and skin friction are enhanced with a rise in radiation parameter, whereas Nusselt number is reduced. Sigey et al., [8] presented an investigation on a numerical study on natural convection turbulent heat transfer in an enclosure while Chandra B.S (2005) studied a steady MHD flow of an electrically conducting fluid between two parallel infinite plates when the upper plate is made to move with constant velocity while the lower plate is stationary. Okello (2007) investigated, unsteady free convection incompressible fluid past a semi infinite vertical porous plate in the presence of a strong magnetic field inclined at angle α to the plate with Hall and ion-slip current effects. The effects of modified Grasshof number, suction velocity, the angle of inclination, time, Hall current, ion-slip current, Eckert number, Schmidt number and heat source parameter on the convectively cooled or convectively heated plate restricted to laminar boundary were studied. He found that an increase in mass diffusion parameter causes a decrease in concentration profiles, absence of suction velocity or an increase of it causes an increase in concentration profiles, an increase of Eckert number causes an increase in temperature profiles and also an increase of an angle of inclination leads to an increase in primary velocity profiles but a decrease in secondary velocity profiles. The results were presented in tables and graphs. Sigey et al., [8] carried out a study of magnetic hydrodynamic free convection flow past an infinite porous plate in an incompressible electrically conducting fluid. The investigation of the effect of viscous dissipation on the velocity profiles and temperature distribution of the fluid in the presence of a transverse magnetic field subject to a constant suction velocity was conducted. The ordinary differential equations governing the flows were analyzed using an explicit finite. The numerical results of the study showed that an increase in the viscous dissipation causes an increase in the velocity profiles and temperature distribution of the fluid. This study finally asserted that an increase in the viscous dissipation parameter or term leads to an increase in velocity and temperature profiles. This increase in the velocity profiles and temperature profile occurred at a distance away from the porous plate. Sib et al., [7] studied the effects of radiation on unsteady MHD free convective flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate embedded in a porous medium with an oscillatory heat flux in the presence of a uniform transverse magnetic field. The governing equations describing the flow were solved analytically. It was observed that increase in radiation parameter leads to decrease in fluid velocity near the plate and to increase away from the plate. The fluid velocity increases near the plate and it decreases away from the plate with an increase in suction parameter. The solution exists for both suction and blowing at the plate. The fluid velocity increases near the plate and it decreases away from the plate with an increase in Darcy number. Also the fluid temperature was seen to decrease near the plate and it increases away from the plate with an increase in either radiation parameter or Prandtl number or suction parameter. Narayana & Babu [6], did a study on MHD free convective heat and mass transfer past a vertical porous plate with a variable temperature. The investigation of the effects of the chemical reaction and radiation absorption on unsteady flow of a viscous incompressible electrically conducting fluid past an infinite hot vertical non-conducting porous plate with mass transfer was done. The results showed that the skin friction decreases when Prandtl or Hartman number is increased. Nyabuto [4], investigated MHD stokes free convection of an incompressible, electrically conducting fluid between two horizontal parallel infinite plates subjected to a constant heat flux and pressure gradient. Analysis of velocity profiles and temperature distribution were obtained and the effect of the Eckert, Prandtl and Hartmann numbers on velocity profiles and temperature distribution investigated. The resulting non-linear differential equations obtained were solved using finite difference method. The results obtained indicated that an increase in Hartmann number leads to an increase in velocity profiles and temperature distribution while an increase in values of Prandtl number results in a fall in temperature distribution.

Sankar et al., [6] studied the combined effects of Hall current and radiation on the unsteady MHD free convective flow in a vertical channel with an oscillatory wall temperature. They two different cases one being the flow due to the impulsive motion of one of the channel walls and the other was flow due to the accelerated motion of one of the channel walls. The governing equations were solved analytically and it was found that the primary velocity and the magnitude of the secondary velocity increased with an increase in Hall parameter for the impulsive as well as the accelerated motions of one of the channel walls. The fluid temperature also decreases with an increase in radiation parameter. Further, the shear stresses at the left wall reduced with an increase in either radiation parameter or frequency parameter for the impulsive as well as the accelerated motions of one of the channel wall. From the review of the pertinent literature presented above, it can be inferred that MHD free convective flow of a viscous incompressible electrically conducting fluid in a vertical channel with an oscillatory wall temperature of one of the channel walls has received considerable attention. However, there exists scope for further investigation of heat transfer and fluid flow characteristics for this kind of geometries numerically with use of Finite Difference Method.

1.3. Statement of the Problem

From the review of the pertinent literature presented above, it can be inferred that MHD free convective flow of a viscous incompressible electrically conducting fluid in a vertical channel with an oscillatory wall temperature of one of the channel walls has received considerable attention using analytical methods. It was also found that minimal work has been reported on investigation of the same using Finite Difference Method. For this reason we have addressed the effects of varying radiation parameter, magnetic parameter, and suction parameter and of Darcy number on fluid velocity and temperature distribution on MHD free convective flow of a viscous incompressible electrically conducting fluid in a vertical channel with an oscillatory heat flux in the presence of a uniform transverse magnetic field by Finite Difference Method.

1.4. Geometry of the Problem

Consider the unsteady flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate embedded in a porous medium with uniform suction or blowing at the plate. The plate oscillates in its own plane with the velocity u_0 in a given direction. We choose the x -axis along the plate, in the vertical upward direction and y -axis perpendicular to the plate. An external uniform magnetic field of strength B_0 is imposed perpendicular to the plate [See Figure 1] and the plate is taken electrically non-conducting. Thermal radiation acts as a Unidirectional flux in the y -direction. The fluid is gray and absorbing-emitting but non-scattering and the magnetic Reynolds number is assumed to be small so that induced magnetic field can be neglected. The velocity components are (u, v, 0) relative to a frame of reference. Since the plate is infinitely long, all the physical quantities will be the function of y and t only. The equation of continuity $\nabla \bullet \vec{q} = 0$ gives $\frac{\partial v}{\partial y} = 0$ which on integration yields $v = -v_0$ (constant), where $\vec{q} = (u, v, 0)$. The constant 0 v denotes the normal velocity at the plate which is positive for suction and negative for blowing. The solenoid relation $\nabla \bullet \vec{B} = 0$ gives $\vec{B}$=constant everywhere in the fluid where $B = B(0, B_y, 0)$.

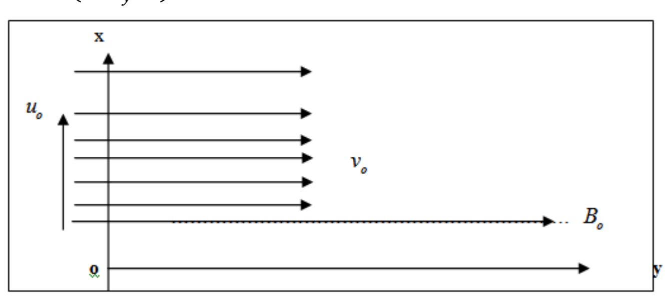

Figure 1: Geometry of the Problem

1.5. General Objective

The aim of the present paper was to study using Finite Difference Method the effects of varying various parameters on fluid velocity and temperature on MHD free convective flow.

1.6. Specific Objectives of the Study

The specific objectives of the study are to;

i) Investigate the effects of varying radiation parameter, magnetic parameter, and suction parameter and Darcy number on temperature on MHD free convective flow using finite difference method
ii) Investigate the effects of varying radiation parameter, magnetic parameter, and suction parameter and Darcy number on fluid velocity on MHD free convective flow using finite difference method.

1.7. Justification of the Study

The study of hydro magnetic flows with Hall currents has important engineering applications in problems of Magneto hydrodynamic generators and of Hall accelerators as well as in flight magneto hydrodynamics. It is well known that a number of astronomical bodies posses fluid interiors and magnetic fields. It is also important in the solar physics involved in the sunspot development, the solar cycle and the structure of magnetic stars. In space technology applications and at higher operating temperatures, radiation effects can be quite significant. The radiative convective flows are frequently encountered in many scientific and environmental processes, such as astrophysical flows, water evaporation from open reservoirs, heating and cooling of chambers, and solar power technology.

II. MATHEMATICAL FORMULATION

2.1. Assumptions

The following assumptions have been taken into account in the research problem:

i) The fluid is assumed to be incompressible and with constant density.
ii) The plate is electrically non-conducting.
iii) Fluid velocity is too low and its Reynolds number is small.
iv) There is no external applied force that is $\vec{E} = 0$.
v) Thermal conductivity electrical conductivity and co-efficient of viscosity are constant.
vi) Magnetic flux density is $\vec{B} = \mu_e \vec{H}$.
vii) The fluid does not undergo any chemical reaction.
viii) Hall current is ignored since the magnetic field applied is weak.

2.2. Governing Equations

Under usual Boussinesq approximations, the flow of a radiating gas is governed by the following set of equations

$$\frac{\partial u}{\partial t} - u_o \frac{\partial u}{\partial y} = g\beta\left(T - T_\infty\right) + v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\beta_0^2}{\rho} - \frac{\mu}{\rho\kappa}u \quad (1)$$

$$\rho C_p\left(\frac{\partial T}{\partial t} - v_o\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} - \frac{\partial^2 q_r}{\partial y} \quad (2)$$

where u is the velocity in the x -direction, g the acceleration due to gravity, β the coefficient of thermal expansion, v the kinematic viscosity, ρ the fluid density, k the thermal conductivity, C_p the specific heat at constant pressure and q_r the radiative heat flux. The boundary conditions of the problem are

$$u = u_o, \frac{\partial T}{\partial y} = -\frac{q}{\kappa} \text{ at y=0, } u \to 0,\ T \to T_\infty \text{ as } y \to \infty \quad (3)$$

where u_o is a positive constant.

The radiative heat flux can be found from Rosseland approximation (Brewster-1992) and its formula is derived from the diffusion concept of radiative heat transfer in the following way

$$q_r = \frac{4\sigma}{3k*}\frac{\partial T^4}{\partial y} \quad (4)$$

where σ is the Stefan-Boltzmann constant and $k*$ the spectral mean absorption coefficient of the medium. It should be noted that by using the Rosseland approximation, we limit our analysis to optically thick fluids. If the temperature differences within the flow are sufficiently small, then equation (4) can be linearized by expanding T^4 into the Taylor series about T_∞ and neglecting higher order terms to give:

$$T^4 - 4T_\infty^3 T - 3T_\infty^4 \quad (5)$$

It is emphasized here that the equation (5) is widely used in computational fluid dynamics involving radiation absorption problems (Chung-2002) in expressing the term T^4 as a linear function.

On the use of (4) and (5), equation (2) becomes

$$\rho C_p\left(\frac{\partial T}{\partial t} - v_o\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma * T_\infty^3}{3k*}\frac{\partial^2 T}{\partial y^2} \quad (6)$$

Introducing the non-dimensional variables,

$$u = \frac{u}{u_o}, \tau = \frac{u_o}{v}t, \eta = \frac{u_o}{v}y, \theta = \frac{(T - T_\alpha)ku_o}{qv} \quad (7)$$

Equations (1) and (2) become

$$\frac{\partial u}{\partial \tau} - S\frac{\partial u}{\partial \eta} = Gr\theta + \frac{\partial^2 u}{\partial \eta^2} - \left(M^2 + \frac{1}{Da}\right)u \quad (8)$$

$$\alpha\left(\frac{\partial \theta}{\partial \tau} - S\frac{\partial \theta}{\partial \eta}\right) = \frac{\partial^2 \theta}{\partial \eta^2} \quad (9)$$

Where $M^2 = \frac{\sigma B_0^2 v}{\rho u_0^2}$ is the magnetic parameter, $Pr = \frac{\rho C_p v}{\kappa}$ is the Prandtl number, $Gr = \frac{gBq\, v^2}{\kappa u_0^4}$ the Grashof number, $S = \frac{v_0}{u_0}$ the suction parameter, $Da \frac{u_0^2 \kappa^*}{v^2}$ the Darcy number, $n = \frac{v\omega}{u_0^2}$ is the frequency parameter and $\alpha = \frac{3RPr}{3R+4}$.

2.3. Dimensional Analysis

It is a mathematical technique that helps in analysis of fluid flow problems. It is built in the principle of homogeneity. It helps formulate fluid problems that defy analytical solutions and that must be solved experimentally. In this research dimensional analysis has been used to non dimensionalize governing equations. Non-dimensional analysis refers to partial or full removal of units from an equation involving physical quantities by suitable substitution of variables.

2.4. Non-Dimensional Parameters

These are non-dimensional numbers. The numbers are introduced into the governing equations to ensure that given solutions of natural phenomenon hold for all units. Some of the parameters used in this research area include:

2.4.1. Prandtl Number, P_r

Is the ratio of viscous force to thermal force.

It is also defined as the ratio of momentum diffusivity and thermal diffusivity. It is expressed as

$$\mathrm{Pr} = \frac{\rho C_p v}{\kappa}$$

2.4.2. Grashof Number, Gr

It is the ratio between buoyancy force and the viscous force. It is expressed as

$$Gr = \frac{gBqv^2}{\kappa u_o^4}$$

2.4.3. Magnetic Parameter M^2

It is the ratio of electromagnetic to inertial forces, which gives an estimate of the relative importance of a magnetic field of flow. It is expressed as

$$M^2 = \frac{\sigma B_o^2 v}{\rho u_o^2}$$

2.4.4. Suction Parameter, S

A force that causes a fluid to be drawn into an interior space or to adhere to a surface between external and internal pressures. It is expressed as

$$S = \frac{v_o}{u_o}$$

2.4.5. Darcy Number, Da

It represents the relative effect of the permeability of the medium versu its cross-sectional area. It is expressed as;

$$Da = \frac{u_o^2 \kappa *}{v^2}$$

III. METHOD OF SOLUTION

3.1. Introduction

Many real life problems generally do not have "analytical" solutions. Mathematics being one of the scientific research disciplines that lead to real life situations requires numerical techniques to accomplish non-analytical solutions. The part of numerical analysis which has been most changed so far, is the solution of partial differential equations by difference methods. This is owing to the fact that second-order partial differential equations govern many of the real-life physical phenomena. Such equations include Maxwell's equations, heat and momentum equations and Newton's laws of motion. A very powerful and quite a general method of dealing with most second-order (partial) differential equations is the finite difference method. In this section, the method and procedure of solving the problem will be discussed**.**

3.2. Computational Procedure

In this study a Hybrid numerical scheme shall be developed and Finite Difference Method will be used solve the governing equations. We have solved the governing equations subject to the given boundary conditions. MATLAB software was used to generate solution values in this study.

3.3. Finite Difference Method

This is a numerical method that makes use of finite difference codes/solvers that take low computational memory and is easy to program and modify, hence more advantageous to use in electrical problems. The governing equations described in the previous section are approximated by the application of finite difference techniques. The use of the finite difference techniques for the solution of partial differential equation is a three step process namely:

1) The partial differential equations are approximated by a set of linear equations relating to the values of the functions at each mesh point.
2) The set of the algebraic equations, generated in (1) must be solved and
3) An iteration procedure has to be developed which takes into account the non-linear character of the equation.

The solution of the Finite Difference Equation (FDE) requires a suitable technique to advance the transient fluid motion through time. If the transient solutions are not required then the transient terms in the equations are dropped and the problem is simplified and reduced to just determining the steady state solution. In this study the partial differential equations governing the flow are replaced by a set of difference equations which are solved by Successive Over-Relation method (SOR) to converge at each time interval. On the other hand the governing equations (8) and (9) together with initial and boundary conditions imposed (depending on the problem considered) are properly posed (i.e. their solution exists, is unique and depends on the given conditions) thus any finite difference set of equations to them which satisfies consistency conditions and is stable ensure that the method is convergent. In order to solve the system of finite difference equations, a computer program will be used for the iterative scheme.

3.4. Discretization of the Governing Equations

In this section, equations governing fluid flow and temperature distribution are discretized.

3.5. Fluid Velocity Equation

Hybrid scheme, U_x is replaced by forward difference approximation while U_{xx} and U_{yy} is replaced by central difference approximation, equation (8) becomes

$$\left[\frac{U_{i,j+1}-U_{i,j}}{\Delta\tau}\right]-1.2\left[\frac{U_{i+1,j}-U_{i,j}}{\Delta\eta}\right]=5\left[\frac{\theta_{i+1,j}+\theta_{i,j}}{\Delta\tau}\right]+\left[\frac{U_{i+1,j}-2U_{i,j}+U_{i-1,j}}{(\Delta\eta)^2}\right]-\left(5+\frac{1}{1.2}\right)\left[\frac{U_{i,j+1}+U_{i,j}}{2}\right]$$

We investigate the effect of Re, Ri on the fluid velocity. Taking $\Delta x=\Delta y=0.1$, M^2=3, 4,5, Da=0.3, 0.2, 0.1 and S=1.2,1.4,1.6, we get the scheme

$$2.049U_{i,j}-1.12U_{i+1,j}-U_{i-1,j}=1.029U_{i,j+1} \qquad (10)$$

Taking and i=1,2,3…….10 and j=1 we form the following systems of linear algebraic equations

$$2.049U_{1,1}-1.12U_{2,1}-U_{0,1}=-1.029U_{1,2}$$
$$2.049U_{2,1}-1.12U_{3,1}-U_{1,1}=-1.029U_{2,2}$$
$$2.049U_{3,1}-1.12U_{4,1}-U_{2,1}=-1.029U_{3,2}$$
$$2.049U_{4,1}-1.12U_{5,1}-U_{3,1}=-1.029U_{4,2}$$
$$2.049U_{5,1}-1.12U_{6,1}-U_{4,1}=-1.029U_{5,2}$$
$$2.049U_{6,1}-1.12U_{7,1}-U_{5,1}=-1.029U_{6,2}$$
$$2.049U_{7,1}-1.12U_{8,1}-U_{6,1}=-1.029U_{7,2}$$
$$2.049U_{8,1}-1.12U_{9,1}-U_{7,1}=-1.029U_{8,2}$$
$$2.049U_{9,1}-1.12U_{10,1}-U_{8,1}=-1.029U_{9,2}$$
$$2.049U_{10,1}-1.12U_{11,1}-U_{10,1}=-1.029U_{10,2}$$

The above algebraic equations can be written in matrix form as when

$$\begin{bmatrix} 2.049 & -1.12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2.049 & -1.12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2.049 & -1.12 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2.049 & -1.12 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2.049 & -1.12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2.049 & -1.12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2.049 & -1.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2.049 & -1.12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2.049 & -1.12 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2.049 \end{bmatrix} \begin{bmatrix} U_{11} \\ U_{21} \\ U_{31} \\ U_{41} \\ U_{51} \\ U_{61} \\ U_{71} \\ U_{81} \\ U_{91} \\ U_{101} \end{bmatrix} = \begin{bmatrix} 1.2214 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (11)$$

Solving the above matrix equation, we get the solutions for changing Re.

IV. RESULTS AND DISCUSSION

4.1. Effects of Magnetic Parameter on Velocity

Table 1: Velocity for Varying Magnetic Parameter

Plate Distance η	M^2=3	M^2=4	M^2=5
0	3.109855	2.294685	1.900172
1	4.571066	3.089069	2.385761
2	5.545134	3.588725	2.668083
3	6.013804	3.791326	2.751018
4	5.997333	3.714951	2.650673
5	5.548891	3.394673	2.393045
6	4.747193	2.878358	2.011318
7	3.688067	2.222045	1.5442987
8	2.475693	1.48527	1.027021
9	1.21417	0.7266487	0.5012301

The results in the table 1 above is represented graphically as seen in figure 2 below

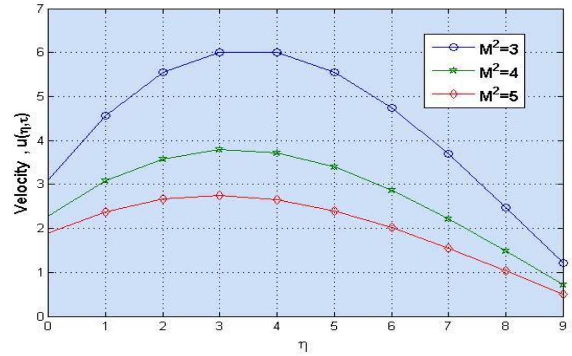

Figure 2: Velocity Graph Plate Distance when Varying the Magnetic Parameter

Figure 2 shows that the fluid velocity u decreases for 0< η>3.0 and it increases for η>3.0 with an increase in magnetic parameter M^2.It is also seen that a decrease in magnetic parameter makes the velocity of the fluid to increase. The presence of a magnetic field in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as considered in the present problem. This type of resistive force tends to slow down the flow field.

4.2. Effects of Suction Parameter on Velocity

Table 2: Vertical Velocity for Varying Suction Parameter

Plate Distance η	S=1.2	S=1.4	S=1.6
0	1.795293	1.380718	1.204647
1	2.177859	1.447767	1.14559
2	2.36193	1.428781	1.050923
3	2.355461	1.334011	0.9279268
4	2.179335	1.176739	0.7841433
5	1.864467	0.9722273	0.6270581
6	1.448494	0.736676	0.4638487
7	0.9723319	0.4862398	0.3011458
8	0.476867	0.231534	0.1448513
9	0	0	0

The results in the table 2 above is represented graphically as seen in figure 3 below

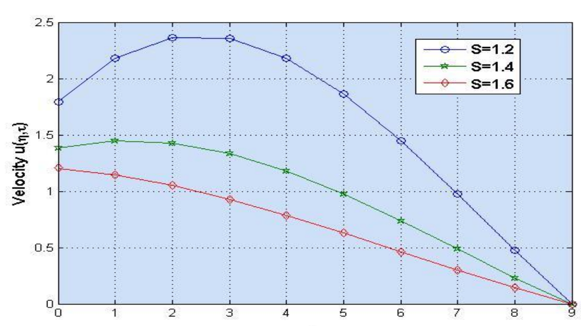

Figure 3: Velocity Graph against Plate Distance when Varying Suction Parameter

Figure 3 displays that the fluid velocity u increases for 0< η<1.0 and it decreases for η>1.0 with an increase in suction parameter S=1.6 and S=1.4. The velocity u increases for 0< η <2.0 and it decreases for η>2.0 with an increase in suction parameter S=1.2. This means that the suction at the plate have a retarding influence on the flow field.

Table 3: Velocity for Varying Darcy Number

Plate Distance η	Da=0.3	Da=0.2	Da=0.1
0	1.427011	1.270305	1.119008
1	1.402571	1.25726	1.002608
2	1.368763	1.189485	0.8763004
3	1.305612	1.075869	0.7439653
4	1.149098	0.9263972	0.6092026
5	0.9477942	0.7515829	0.4752805
6	0.7172841	0.5619458	0.3450982
7	0.4730456	0.3675402	0.221161
8	0.2296338	0.1775556	0.1055661
9	0	0	0

The results in the table 3 above is represented graphically as seen in figure 4 below

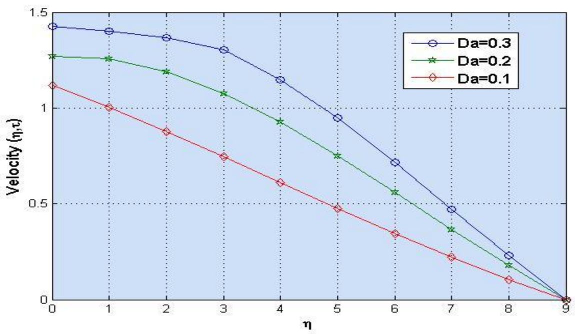

Figure 4: Velocity Graph against Plate Distance when Varying the Da Number

In figure 4, it is illustrated that the fluid velocity u increases near the plate and it decreases away from the plate with an increase in η. It is also seen that the fluid velocity u increase in Darcy number *Da*.

4.3. Effects of Suction Parameter on Temperature Distribution

Table 4: Temperature for Varying Suction Parameter

Plate Distance η	S=1.2	S=1.4	S=1.6
0	1.152561	1.141772	1.130817
1	1.245362	1.224112	1.202731
2	1.277316	1.246916	1.216584
3	1.250262	1.212875	1.175861
4	1.168594	1.12704	1.086201
5	1.038913	0.9964494	0.9550051
6	0.869621	0.8296928	0.7909816
7	0.6704419	0.6364334	0.6036712
8	0.4519178	0.426911	0.4029681
9	0.2248907	0.2114509	0.1986591

The results in the table 4 above is represented graphically as seen in figure 5 below

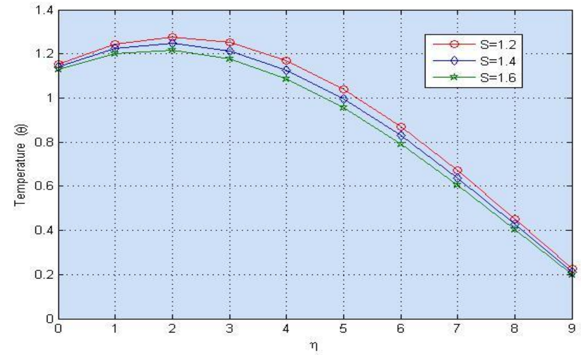

Figure 5: Temperature Graph against Plate Distance when Varying Suction Parameter

It is found from figure 5 that the fluid temperature θ increases near the plate for 0<2.0< η and it decreases away from the plate for η > 2.0. It is found that when the fluid temperature θ increases with decrease in suction parameter *S*.

4.4. Effects of Prandtl Number on Temperature Distribution

Table 5: Velocity for Varying Prandtl Number

Plate Distance η	Pr=0.6	Pr=0.8	Pr=1.0
0	1.419432	1.333985	1.302384
1	1.726065	1.570167	1.3212
2	1.909692	1.702121	1.283736
3	1.968192	1.72983	1.197562
4	1.906945	1.659094	1.071435
5	1.737986	1.500795	0.9148161
6	1.47883	1.269933	0.7374052
7	1.151058	0.9844975	0.5487162
8	0.7787585	0.664257	0.3577074
9	0.3869415	0.3295269	0.1724722

The results in the table 5 above is represented graphically as seen in figure 6 below

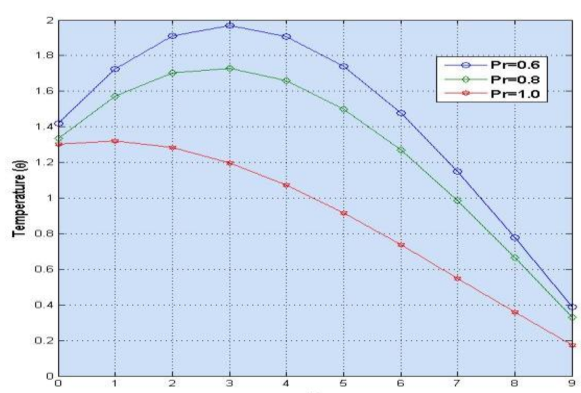

Figure 6: Temperature Graph Plate Distance when Varying the Pr Number

Figure 6 reveals that the fluid temperature θ increases near the plate and it decreases away from the plate with an increase plate distance η. The fluid temperature θ increases with decrease in Prandtl number *Pr*. Prandtl number controls the relative thickness of the momentum and thermal boundary layers. When *Pr* is of low value, heat diffusion exceeds momentum diffusion. For *Pr* <1, the thickness of the thermal boundary layer therefore exceeds the thickness of the velocity boundary layer that is, temperatures will be greater. Temperatures are seen to decrease considerably with an increase in the value of *Pr* as we progress into the boundary layer regime.

Prandtl number *Pr* encapsulates the ratio of momentum diffusivity to thermal diffusivity for a given fluid. It is also the product of dynamic viscosity and specific heat capacity divided by thermal conductivity. Higher Pr fluids will therefore posses higher viscosities (and lower thermal conductivities) implying that such fluids will flow slower than lower Pr fluids.

4.5. Effects of Radiation Parameter on Temperature Distribution

Table 6: Velocity for Varying Radiation Parameter

Plate Distance η	R=9	R=7	R=5
0	1.210862	1.186545	1.130817
1	1.349826	1.306203	1.202731
2	1.414157	1.356959	1.216584
3	1.405096	1.34033	1.175861
4	1.327424	1.26095	1.086201
5	1.189014	1.126169	0.9550051
6	1.000264	0.9455523	0.7909816
7	0.7734361	0.7302988	0.6036712
8	0.5219414	0.4926133	0.4029681
9	0.2596129	0.2450715	0.1986591

The results in the table 6 above is represented graphically as seen in figure 7 below

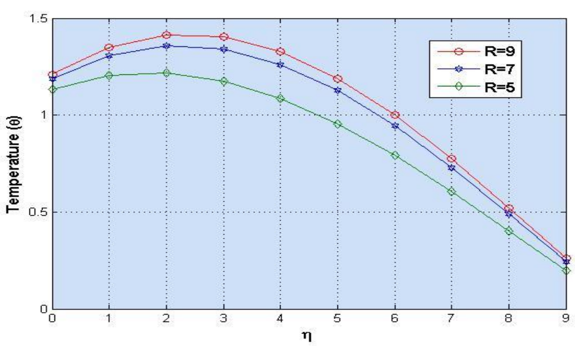

Figure 7: Temperature Graph against Plate Distance when Varying Radiation Parameter

It is found from figure 7 that the fluid temperature θ increases near the plate and it decreases away from the plate with an increase plate distance η. When radiation parameter increases, the fluid temperature θ also increases. It is observed that the velocity u increases for η > 2.0 and it decreases for η > 2.0 with an increase in radiation parameter *R*. The radiation parameter arises only in the energy equation in the thermal diffusion term and via coupling of the temperature field with the buoyancy terms in the momentum equation, the velocity is indirectly influenced by thermal radiation effects.

V. Conclusion and Recommendations

5.1. Conclusion

We have presented the non-dimensional velocity u and temperature θ for several values of magnetic parameter M^2, Radiation parameter *R*, Darcy number Da, Prandtl number Pr and Suction parameter *S*. It was seen that the velocity u increase with decrease in Suction parameter *S* and magnetic parameter M^2 and increases with increase Darcy number. For temperature, it is also found that the temperature θ increases with decrease in Prandtl number *Pr* and increases with increase in Radiation parameter *R* and Suction parameter *S*.

5.2. Recommendations

Further work is recommended to improve on the results so far obtained for velocity u and temperature θ. This may be done by;

(i) Investigating unsteady MHD convective flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate in a porous medium with Pr >2.0
(ii) Investigating unsteady MHD convective flow of a viscous incompressible electrically conducting fluid past an oscillating vertical porous plate in a porous medium a non uniform transverse magnetic field

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