Power Control for Space-time Coded MIMO Systems with Imperfect Feedback over Joint Transmit-receive Correlated Channel

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Abstract—In this paper, power adaptation strategies for minimizing bit error rate (BER) subject to an average power constraint for orthogonal space-time coded multiple-input multiple-output (MIMO) systems with mean and covariance feedback over joint transmit-receive correlated channels are presented. We first develop an optimal spatial-only power control (PC) based on an approximated BER bound, which can include the mean feedback only and covariance feedback only as special cases. Using a newly defined fuzzy signal-to-noise ratio (FSNR) criterion, a suboptimal spatial PC scheme with a closed-form expression is derived. This FSNR-based power adaptation not only has the computational complexity less than but also obtain the BER performance close to that of the optimal spatial-only PC scheme. Second, the degrees of freedom in temporal and spatial domains are exploited to derive a joint spatio-temporal PC with the use of FSNR strategy to minimize the average BER subject to a long term average power constraint with mean and covariance feedback. Simulation results show that the proposed two spatial-only PC schemes can provide BER lower than that of the equal power allocation, and obtain almost the same performance as the existing spatial-only optimal scheme but have lower computational complexity. Moreover, they outperform the PC schemes with mean or covariance feedback only. Most importantly, the new joint spatio-temporal PC scheme substantially outperforms all the spatial-only PC schemes.

Index Terms—Space-time coding, Spatial-temporal power control, Mean feedback, Covariance feedback, Spatial correlation, MIMO

I. INTRODUCTION

The ever increasing demand of information has stimulated much interest in multiple-input multiple-output (MIMO) communications [1] for the development of reliable high data rate transmission over fading channels. It has been shown that MIMO systems can significantly increase spectral efficiency and/or reliability by exploiting channel state information (CSI) [1, 2]. However, the system performance will be degraded when the CSI is imperfect [3].

Without any knowledge of CSI at the transmitter, space-time block coding (STBC) is commonly used for MIMO systems because it can provide effective diversity for combating fading effects [4, 5]. If perfect CSI is available, the transmitter can select the largest eigen-mode of the channel for transmission so that the received signal-to-noise ratio (SNR) is maximized. This scheme is known as maximum ratio transmission or one-directional (1D) beamforming (BF) [6]. In practical systems, the CSI at the receiver can be accurate enough to be assumed perfect, but the CSI at the transmitter will be far from perfect due to feedback delay, channel estimation error, or quantization [7]. Hence, the system design should take the imperfect CSI into account for practical consideration, otherwise, the system performance will degrade significantly. To improve robustness, different precoding designs for STBC-MIMO systems with mean feedback have been developed in the literature [7]-[15]. Based on the mean feedback, adaptive modulation schemes are designed to improve the spectrum efficiency of MIMO systems and beamforming MIMO systems under imperfect CSI in [16,17]. Basically, the precoder consists of adaptive power allocation and multiple eigen-beamforming. Based on channel covariance feedback, different precoders or power control schemes are designed for MIMO systems in [18]-[22]. With the channel mean feedback and covariance feedback, the precoding scheme is investigated in transmit correlated MIMO channel to optimize the system capacity [23]-[27], but the closed-form power control is not yet given. Based on a Kronecker channel model, the precoder is designed for space-time coded MIMO (STC-MIMO) systems over joint transmit-receive correlated Rayleigh channel in [28, 29] of which the mean feedback is not taken into consideration, but the preceding schemes need numerical search and iterative calculation. The study is extended to STC-MIMO systems over Rician MIMO channels with arbitrary correlation model in [30,31], where SER criteria are considered. The scheme in [31] can avoid the need for exact channel information at the transmitter due to the use of differential STBC. However, these optimal precoders are obtained by the fixed-point iteration method that suffers from high computational complexity and slow convergence. Moreover, they cannot guarantee convergence to the global optimum, especially at high SNR [28]. Precoding schemes are also applied to cooperative systems and
distributed antenna systems over Rician channels in [32] and [33] respectively. These precoders are designed by iterative procedures and no closed-form solutions are derived.

Overall speaking, most of the above-mentioned power allocation algorithms or precoding schemes require numerical search for the value of the Lagrange multiplier of the constrained optimization and iterative calculations to determine the number of eigen-beams of positive power allocation. Thus, the computational complexity is high. It is noteworthy that the above strategies are spatial-only power allocation schemes, while the temporal dimension is not being exploited. So the system performance is limited. In [34], a temporal power allocation based on the power gain of the largest eigen-beam is introduced, and then the spatial power allocation in [8] is used to distribute the calculated temporal power among the eigen-beams, but the largest power gain is treated as perfectly known. In [13]-[15], we report different joint spatio-temporal power allocation schemes, but these schemes are designed with mean feedback only over spatially uncorrelated channels.

In this paper, we develop two spatial-only power control schemes and a joint spatio-temporal power control scheme for STBC-MIMO systems over joint transmit-receive correlated MIMO channels based on a Kronecker model, where the mean and covariance feedback are both considered. The first spatial-only PC scheme is designed to minimize an average bit error rate (BER) subject to a power constraint, for which a closed-form power allocation is obtained and a simple procedure for determining the number of eigen-beams of positive power allocation is developed. The second spatial-only PC scheme is derived from a newly defined Fuzzy SNR (FSNR) criterion, where a fuzzy factor can be used to minimize the BER. This scheme provides direct computation of power allocation without any iteration. With this closed-form power control, we employ the FSNR strategy to develop a joint spatio-temporal power allocation in minimizing the average BER subject to the average transmit power constraint. Based on the asymptotic analysis at high SNR, the diversity order and coding gain are derived. The main contributions of this paper are summarized as follows:

1) Based on the Kronecker correlation model, a closed-form spatial-only PC scheme is derived by minimizing the average BER subject to a fixed power constraint. This scheme includes the mean-feedback PC and covariance-feedback PC as special cases, and outperforms the latter two schemes because more CSI is available.

2) The BER performance of the system at high SNR is analyzed. The analysis shows that the coding gain is affected by the numbers of transmit and receive antennas, code rate, modulation mode, and mean and covariance information of the channel. The diversity order is shown to be the product of the numbers of transmit and receive antennas. Thus, full diversity order can be achieved.

3) We develop another spatial-only PC scheme based on a newly defined FSNR criterion. Unlike the first scheme, the second scheme does not need a procedure to pre-determine the number of eigen-modes of positive power allocation. This scheme can achieve similar performance as the optimal scheme while it is more computationally efficient. Moreover, the complexity analysis shows that the proposed second scheme has lower complexity than the first scheme and the existing spatial-only optimal scheme [30] since it has a closed-form formula with no iteration to compute power control.

4) Based on the FSNR power adaptation, we derive a joint spatio-temporal power allocation method to minimize the average BER subject to the average transmit power constraint. This method can provide a large BER performance gain over spatial-only schemes by utilizing the degrees of freedom of power adaptation both in space and time. Numerical results show that STBC-MIMO systems using the joint spatio-temporal power adaptation scheme have BER much lower than those of spatial-only methods.

The notations throughout this paper are as follows. Bold upper case and lower case letters denote matrices and column vectors, respectively. The superscripts $(\cdot)^H$, $(\cdot)^T$ and $(\cdot)^*$ denote the Hermitian transposition, transposition and complex conjugation, respectively. $[A]_{ij}$ means the $(i,j)$-th element of matrix $A$. The abbreviation $'CN'$ represents the complex Gaussian distribution. $[A]_{ij} \sim CN(0, \sigma^2)$ means the entry is a complex Gaussian random variable of zero mean and variance $\sigma^2$. $E\{\cdot\}$, $tr(\cdot)$, $vec(\cdot)$, $|\cdot|$ and $\otimes$ denote the expectation operator, trace, matrix vectorization operator, absolute value, and Kronecker product, respectively. $A^{1/2}$ denotes the square root of Hermitian matrix $A$, $I_N$ and $\Phi_N$ denote the $N \times N$ identity matrix and $N \times N$ zero matrix respectively. $\|B\|_F$ represents the Frobenius norm of matrix $B$.

II. SYSTEM MODEL

In this paper, we consider a wireless multi-antenna communication system with $N$ transmit antennas and $K$ receive antennas operating over a correlated MIMO flat Rayleigh fading channel represented by a $K \times N$ channel matrix $H = \{h_{kn}\}$ as depicted in Fig.1. The complex element $h_{kn}$ denotes the channel gain from the $n$-th transmit antenna to the $k$-th receive antenna, which is assumed to be constant over a frame of $L$ symbols and varies from one frame to another. It is assumed that the spatial correlation of the channel is described by the well-known Kronecker correlation model with the channel matrix modeled as $H = R_1^{1/2}H_0R_0^{1/2}$, where the entries of $H_0$ are independent, identically distributed (i.i.d.) complex Gaussian random variables (r.v.s.) with zero mean and unit variance, $N \times N$ matrix $R_0$ and $K \times K$ matrix $R_1$ are the transmit and receive covariance matrices to characterize the spatial correlation among the transmit antennas and receive antennas, respectively. Thus, the channel covariance can be expressed as a $NK \times NK$ matrix $R = E\{vec(H)vec^H(\Phi)\} = R_0^{1/2} \otimes R_1$, $[35,36,28]$. Considering $R_0$ and $R_1$ as positive semi-definite Hermitian matrices (a Hermitian matrix is said to be positive semi-definite if all its eigenvalues are non-negative), $R_0^{1/2}$ and $R_1^{1/2}$ are the square-root matrices of $R_0$ and $R_1$ respectively.

As shown in Fig.1, the transmitter is composed of an orthogonal space-time block encoder, power control and a set of $N$ beamformers. The STBC exploits the space-time diversity of MIMO, while the beamforming utilizes the eigen-property of the CSI. This STBC and beamforming combination is robust to channel fading and provides an efficient way...
to exploit the available CSI. A complex orthogonal STBC (OSTBC), which is represented by an \( N \times T \) transmission matrix \( \mathbf{D} \), is used to encode \( L \) input symbols into an \( N \)-dimensional vector sequence of \( T \) time slots. The matrix \( \mathbf{D} \) is a linear combination of \( L \) symbols satisfying the complex orthogonality: \( \mathbf{D} \mathbf{D}^H = \varepsilon [d_1]^2 + \cdots + [d_L]^2] \mathbf{I}_N \), where \( \{d_i\} \) are the \( L \) input symbols, and \( \varepsilon \) is a constant which depends on the STBC transmission matrix [4]. Hence, the transmission rate of the STBC is \( r = L/T \).

In this paper, we assume that CSI is perfectly known at the receiver, and the transmitter has the knowledge of \( \mathbf{R}_t \) and \( \mathbf{R}_r \) and imperfect (time-delayed) mean feedback from the receiver. The channel matrix \( \hat{\mathbf{H}}_w \) in \( \mathbf{H} \) can be related to its \( \tau \) time-delayed version \( \hat{\mathbf{H}}_f \) at the transmitter as [34]

\[
\hat{\mathbf{H}}_w = \rho \hat{\mathbf{H}}_f + \mathbf{E}_w = \hat{\mathbf{H}}_w + \mathbf{E}_w, \tag{1}
\]

where \( \mathbf{E}_w \) is a \( K \times N \) channel error matrix independent of \( \hat{\mathbf{H}}_f \). The entries of the \( K \times N \) matrix \( \hat{\mathbf{H}}_f \) are i.i.d. zero mean complex Gaussian r.v.s of unit variance, while the entries of \( \mathbf{E}_w \) are i.i.d. complex Gaussian r.v.s with zero mean and variance \( \sigma_e^2 = 1 - \rho^2 \), where \( \rho \) is the correlation coefficient given by \( \rho = J_0(2 \pi f_d \Delta f) \), \( J_0(\cdot) \) is the zero-order Bessel function of the first kind [37], and \( f_d \) is the maximum Doppler frequency [38]. The correlation between the entries of \( \hat{\mathbf{H}}_f \) and the entries of \( \hat{\mathbf{H}}_w \) is given by the correlation coefficient \( \rho \). The entries of \( \hat{\mathbf{H}}_w \) are \( \rho \hat{\mathbf{H}}_f + \mathbf{N}_w \sim C N(0, \sigma_n^2) \). Based on the Kronecker relationship, the channel matrix \( \hat{\mathbf{H}}_w \) is expressed as \( \hat{\mathbf{H}}_w = \hat{\mathbf{R}}_{w,K} \hat{\mathbf{H}}_f \mathbf{R}_w^{-1/2} = \hat{\mathbf{H}} + \mathbf{E}_w \), where \( K \times N \) matrix \( \hat{\mathbf{H}} = \hat{\mathbf{R}}_{w,K} \hat{\mathbf{H}}_f \mathbf{R}_w^{-1/2} \) is the mean channel feedback [7, 26], and \( \mathbf{E}_w = \mathbf{R}_w^{-1/2} \mathbf{E}_w \mathbf{R}_w^{-1/2} \). The correlation matrix of \( \mathbf{H} \) conditioned on \( \hat{\mathbf{H}}_w \) is given by

\[
\mathbf{R}_w = E(\hat{\mathbf{H}}^H \hat{\mathbf{H}} | \hat{\mathbf{H}}_w) = \mathbf{R}_w + E(\mathbf{R}^H \mathbf{E}_w) \tag{2}
\]

where \( \mathbf{R}_w \) is a \( N \times N \) matrix. Utilizing \( \mathbf{E}_w = \mathbf{R}_w^{-1/2} \mathbf{E}_w \mathbf{R}_w^{-1/2}, E(\mathbf{R}^H \mathbf{E}_w) \) in (2) can be expressed as

\[
E(\mathbf{R}^H \mathbf{E}_w) = \{ \mathbf{R}_w^{-1/2} \mathbf{E}^H_w \mathbf{R}_w^{-1/2} \mathbf{R}_w^{-1/2} \mathbf{E}_w \mathbf{R}_w^{-1/2} \} \tag{3}
\]

where \( \{\zeta_{r,k} \} \) are the eigenvalues of \( \mathbf{R}_r \). The last equality in (3) can be obtained from the following Lemma.

**Lemma:** If \( \mathbf{E}_w \) is a complex Gaussian random matrix whose entries are i.i.d. with zero-mean and variance \( \sigma_e^2 \), then \( E(\mathbf{R}_w^{-1/2} \mathbf{E}_w \mathbf{R}_w^{-1/2}) = \sigma_e^2 \sum_{k=1}^{K} \zeta_{r,k} \mathbf{I}_N \), where \( \{\zeta_{r,k} \} \) are the eigenvalues of \( \mathbf{R}_r \).

**Proof:** Let \( \mathbf{R}_r = \mathbf{V}_r \Lambda_r \mathbf{V}_r^H \) be the eigenvalue decomposition (EVD) of \( \mathbf{R}_r \), where \( \Lambda_r = \text{diag}(\zeta_{1,r}, \ldots, \zeta_{K,r}) \), then we have:

\[
E(\mathbf{R}_w \mathbf{R}_r \mathbf{E}_w) = E(\mathbf{V}_r^H \mathbf{E}_w \Lambda_r \mathbf{V}_r \mathbf{E}_w) = \mathbf{E}_w^H \Lambda_r \mathbf{E}_w = \sum_{k=1}^{K} \zeta_{r,k} \mathbf{e}_w^H \mathbf{e}_w \tag{4}
\]

where the \( K \times N \) matrix \( \mathbf{E}_w = \mathbf{V}_r^H \mathbf{E}_w \), and \( \{\mathbf{e}_w,k\} \) are the \( N \times 1 \) column vectors of \( \mathbf{E}_w \). Since \( \mathbf{V}_r \) is a unitary matrix, \( \mathbf{E}_w \) and \( \mathbf{E}_w \) have the same statistical distribution. Thus the entries of \( \mathbf{E}_w \) are also i.i.d. complex Gaussian r.v.s with zero mean and variance \( \sigma_e^2 = 1 - \rho^2 \). Hence,

\[
E(\mathbf{R}_w \mathbf{R}_r \mathbf{E}_w) = \sum_{k=1}^{K} \zeta_{r,k} E(\mathbf{e}_w,k \mathbf{e}_w,k^H) = \sigma_e^2 \sum_{k=1}^{K} \zeta_{r,k} \mathbf{I}_N. \tag{5}
\]

Substituting (3) into (2) gives

\[
\mathbf{R}_w = 1 \mathbf{H}^H \mathbf{H} + \sigma_e^2 \sum_{k=1}^{K} \zeta_{r,k} \mathbf{R}_r \tag{6}
\]

Using EVD, \( \mathbf{R}_w \) can be expressed as

\[
\mathbf{R}_w = \mathbf{V}_h \Lambda_h \mathbf{V}_h^H \tag{7}
\]

where the \( N \times N \) unitary matrix \( \mathbf{V}_h \) contains the \( N \) eigenvectors of \( \mathbf{R}_w \), \( \Lambda_h = \text{diag}(\alpha_1, \ldots, \alpha_N) \) is the diagonal matrix containing the nonnegative eigenvalues \( \{\alpha_n\} \) sorted in a decreasing order.

With beamforming \( \mathbf{U} \) and power control \( \mathbf{P} \), the transmitted signal matrix can be expressed as

\[
\mathbf{X} = \mathbf{U} \mathbf{P} \tag{8}
\]

where the \( N \times N \) unitary matrix \( \mathbf{U} \) (whose elements are \( u_{ij}, i,j = 1, \ldots, N \)) is set equal to the orthogonal eigenvector matrix \( \mathbf{V}_h \) of \( \mathbf{R}_w \) [26], \( \mathbf{P} = \text{diag}(\sqrt{P_1}, \sqrt{P_2}, \ldots, \sqrt{P_N}) \) is a diagonal \( N \times N \) matrix, where \( \{P_1,n \} \) is the power control to the \( N \) eigen-beams with the following power constraint.

\[
\sum_{n=1}^{N} P_n = 1, P_n \geq 0. \tag{9}
\]

Thus, the received signal matrix can be written as

\[
\mathbf{Y} = \mathbf{H} \mathbf{X} + \mathbf{Z} = \text{HUPD} + \mathbf{Z} \tag{10}
\]

where \( \mathbf{Y} \) is a \( K \times T \) received signal matrix, \( \mathbf{Z} \) is a \( K \times T \) noise matrix with i.i.d. entries modeled as complex Gaussian r.v.s with zero-mean and variance \( \sigma_n^2 \). The equivalent received signal vector after the space-time decoding of the receiver can be expressed as [4, 5]

\[
\mathbf{y} = \varepsilon ||\mathbf{HUP}||_F \mathbf{d} + \tilde{\mathbf{z}} \tag{11}
\]

where \( \mathbf{y} \) is an \( L \times 1 \) received signal vector, \( \mathbf{d} = [d_1, \ldots, d_L]^T \) is the \( L \times 1 \) input data symbol vector. Each symbol in the transmission matrix has an average power \( T \mathbf{S}/(\varepsilon L) \), where \( \mathbf{S} \) is the average transmitted power radiated from \( N \) transmit antennas, \( ||\mathbf{HUP}||_F = \sum_{n=1}^{N} P_n \sum_{k=1}^{K} \sum_{i=1}^{N} h_{k,n}^{(i)} \mathbf{u}_n^H \mathbf{u}_n \), and \( \tilde{\mathbf{z}} \) is an \( L \times 1 \) noise vector whose elements are i.i.d. complex Gaussian r.v.s with zero-mean and variance \( \varepsilon ||\mathbf{HUP}||_F^2 \sigma_n^2 \).
The instantaneous received SNR per symbol after STBC decoding is expressed as
\[
\gamma = \frac{\bar{\gamma}}{r} \sum_{n=1}^{N} P_n \sum_{k=1}^{K} \left| \sum_{i=1}^{N} h_{ki} u_{i,n} \right|^2 = \frac{\bar{\gamma}}{r} \sum_{n=1}^{N} P_n \xi_n.
\]  
(12)
where \(\xi_n = \sum_{k=1}^{K} \left| \sum_{i=1}^{N} h_{ki} u_{i,n} \right|^2\), and \(\bar{\gamma} = \bar{S}/\sigma^2\) is the average SNR. Given \(\bar{H}\), we can whiten \(H\) to matrix \(\bar{H}_w\) as \(H = \bar{H}_w R_h^{1/2} K^{-1/2}\) with \([\bar{H}_w]_{ij} \sim CN(0,1)\). Using the transformed variables, (7) and \(U = V_h\), for given \(\bar{H}\), the received SNR in (12) can be expressed as
\[
\gamma = tr(\bar{H} \mathbf{P}^{H} U^{H} \bar{H}^{H}) / \gamma / r
\]
(13)
where \{\(\alpha_n\)\} are the eigenvalues of \(\mathbf{A}_h\) determined from the mean and covariance feedback, \(\bar{H} = \bar{H}_w V_h\) is a \(K \times N\) matrix, which has the same distribution as \(\bar{H}_w\) due to the unitary transformation. Based on this, \(\beta_n = \sum_{k=1}^{K} |h_{kn}|^2\) is a central Chi-squared distributed random variable with \(2K\) degrees of freedom. Thus, the probability density function (pdf) of \(\beta_n\) can be expressed as
\[
f(\beta_n) = \beta_n^{K-1} \exp(-\beta_n)/\Gamma(K)
\]  
(14)
where \(\Gamma(K) = (K-1)!\) is the Gamma function.

The assumptions adopted in this paper are listed as follows.

A1) The channel is flat Rayleigh fading and remains constant over a frame of \(L\) symbols, but varies from one frame to another.

A2) The spatial correlation structure of the channel follows the well-known Kronecker model.

A3) Channel state information is perfectly known at the receiver and the transmitter has the knowledge of \(\bar{R}_t\) and \(\bar{R}_r\) as well as the mean feedback from the receiver.

A4) Covariance matrices \(\bar{R}_t\) and \(\bar{R}_r\) are positive semi-definite Hermitian matrices.

III. POWER CONTROL SCHEME FOR IMPERFECT FEEDBACK

In this section, we will derive a power control scheme for space-time coded MIMO system with mean and covariance feedback to minimize an average BER. The PC scheme is referred to as BER-PC.

According to [38], the BER of MQAM or MPSK modulation of size \(Q\) with Gray code and received SNR \(\gamma\) over an additive white Gaussian noise (AWGN) channel is approximately given by
\[
BER_{\gamma} = 0.2 \exp \left(-\frac{g \gamma}{g}\right)
\]  
(15)
The exact BER is tightly bounded from above by this approximated BER with \(g = 1.6/(Q-1)\) for square MQAM and \(g = \sin^2(\pi/Q)\) for MPSK. With (13) and (15), the approximated average BER with power control \(\{P_n\}\) can be obtained by taking expectation of \(BER_{\gamma}\) in (15) with respect to the r.v.s. \{\(\beta_n\)\} as
\[
P_0 = \int_0^\infty \ldots \int_0^\infty 0.2 \exp \left( -g \sum_{n=1}^{N} P_n \alpha_n \beta_n \gamma / (Kr) \right) / \gamma / r / \sum_{n=1}^{N} P_n \alpha_n \beta_n \gamma / (Kr)
\]  
(16)
Utilizing (14) and the independence of \{\(\beta_n\)\}, (16) can be expressed as
\[
P_0 = 0.2 \int_0^\infty \ldots \int_0^\infty 0.2 \exp \left( -g \sum_{n=1}^{N} P_n \alpha_n \beta_n \gamma / (Kr) \right) / \gamma / r / \sum_{n=1}^{N} P_n \alpha_n \beta_n \gamma / (Kr)
\]  
(17)
where \(\lambda = \gamma / (rK)\) is referred to as effective average SNR.

We now derive the power adaptation to minimize the logarithm of the average BER in (17) subject to the power constraint in (9). This constrained optimization problem can be solved by the Lagrange multiplier method. An auxiliary objective function is defined as follows
\[
L(P_1, \ldots, P_N) = -K \sum_{n=1}^{N} \left[ \log(1 + \lambda P_n \alpha_n) \right] + \eta_1 \left( \sum_{n=1}^{N} P_n - 1 \right)
\]  
(18)
where \(\eta_1 > 0\) is the Lagrange multiplier. Taking partial derivatives of the objective function with respect to \{\(P_n\)\} and equating the partial derivatives to zero gives
\[
-K \lambda \alpha_n / (1 + \lambda P_n \alpha_n) + \eta_1 = 0, n = 1, \ldots, N.
\]  
(19)
From (19), we can obtain:
\[
P_n = K / \eta_1 - 1 / (\lambda \alpha_n), n = 1, \ldots, N.
\]  
(20)
Substituting (20) into (9) yields
\[
\eta_1 = K N / [1 + \sum_{n=1}^{N} (\lambda \alpha_n)^{-1}]
\]  
(21)
Then substituting (21) into (20) gives
\[
P_n = 1 / \lambda \left[ \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\alpha_i} - \frac{1}{\alpha_n} \right] + 1 / N
\]  
(22)
Equation (20) shows that the power control to individual eigen-beams is a waterfilling solution to fill up the beams to reach the constant level of \(K / \eta_1\). As shown in (22), the calculated power control could be negative for small \(\alpha_n\). Under this circumstance, no power will be allocated to such beams so that these beams are abandoned, and the power control is re-optimized to the remaining beams until the power allocation all are positive. Hence, \(P_n\) can be re-expressed as:
\[
P_n = \left\{ \left[ 1 / N + \frac{r K}{g \gamma} \left( \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\alpha_i} - \frac{1}{\alpha_n} \right) \right] ^+ \right\} / \lambda N_p + \frac{r K}{g \gamma} \left( \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\alpha_i} - \frac{1}{\alpha_n} \right), n = 1, \ldots, N_p
\]  
(23)
where \(\{x\}^+ = \max(x,0)\) , and \(N_p\) is the number of eigen-beams of positive power allocation. Based on the second equality of (22) and (23), we define a function \(Q_{N_p} = \lambda N_p P_{N_p} = S_{N_p} + \lambda - N_p / \alpha_{N_p}\), where \(S_{N_p} = \sum_{i=1}^{N_p} 1 / \alpha_i\), to
pre-determine the value of $N_p$. The number of eigen-beams of positive power allocation can be obtained by finding the largest $N_p$ that gives $Q_{N_p} > 0$ starting from $N_p = N$ to 1 with $S_{N_p-1} = S_{N_p} - 1/a_{N_p}$.

With (23), when the average SNR $\bar{\gamma}$ is very large (correspondingly, $\lambda$ is very large), $N_p = N$, and $P_n$ is equal to $1/N$. Hence, for high SNR, the power adaptation scheme is equivalent to the equal power scheme. From (23), it is observed that the larger the $a_n$, the bigger the power control $P_n$ is. Thus, more power will be allocated to the eigen-beams with large $a_n$ at low SNR. Based on this, when the SNR is small enough, the power control is positive only when $N_p = 1$. Hence, all the power will be allocated to the eigen-beam corresponding to $a_1$ like the optimal eigen-mode-selection-based power control scheme (i.e. 1D BF scheme) for perfect CSI.

The power control scheme in (23) includes covariance feedback only and mean feedback only as special cases. For the first case, the transmit covariance matrix $\mathbf{R}_t$ and receive covariance matrix $\mathbf{R}_r$ are available, but not the mean value. With such information, $\mathbf{R}_h = \sum_{k=1}^{K} \zeta_k \mathbf{R}_t$ and $\{a_n\}$ are the eigenvalues of $\mathbf{R}_h$ that give $P_n = \left\{ \lambda^{-1} \left( N_p^{-1} \sum_{i=1}^{N_p} a_i^{-1} - a_n^{-1} \right) + N_p^{-1} \right\},$ where $\lambda = g\bar{\gamma}/(rK)$. For the second case, only the mean is fed back to the transmitter, so the correlation matrix $\mathbf{R}_h = \mathbf{H}^{H} + \sigma_e^2 K \mathbf{I}_N$, where $C = \mathbf{I}_N$ when the channel is spatially uncorrelated at both links (transmitter and receiver) and set equal to $0_N$ when no spatial correlation information is fed back [26, 13]. Using the above $\mathbf{R}_h$, $\{P_n\}$ are computed by (23). Because only partial CSI is available for these two special cases, the corresponding system performance are worse than that with both mean and covariance feedback.

In addition, the power control scheme in (23) and beam-forming matrix $\mathbf{U} = \mathbf{V}_h$ can also be applied to the precoder design for the following two special cases: (1) mean feedback in transmit-correlated channel, and (2) mean and covariance feedback in receive-correlated channel. For the first case, the transmit covariance matrix $\mathbf{R}_t$ is known and the mean is fed back with $\mathbf{R}_r = \mathbf{I}_K$ to the transmitter. So we have $\mathbf{R}_h = \mathbf{H}^{H} + \sigma_e^2 K \mathbf{I}_N$. Setting $\{a_n\}$ equal to the eigenvalues of $\mathbf{R}_h$, $\{P_n\}$ are computed by (23). For the second case, $\mathbf{R}_t = \mathbf{I}_N$, and the receive covariance matrix $\mathbf{R}_r$ and the mean are fed back to the transmitter. We have $\mathbf{R}_h = \mathbf{H}^{H} + \sigma_e^2 \sum_{k=1}^{K} \zeta_k \mathbf{I}_N$. Setting $\{a_n\}$ equal to the eigenvalues of $\mathbf{R}_h$, $\{P_n\}$ are obtained from (23).

We notice that [28] gives a power control scheme for space-time coded MIMO systems over joint transmit-receive correlated channel. However, [28] does not consider the mean feedback information, and the approach in our paper is also different from that of [28]. By minimizing the symbol error rate, [28] employs the Karush-Kuhn-Tucker (KKT) conditions and a multi-dimensional gradient descent method to find the solution of power allocation iteratively. But for our paper, we use the Lagrange multiplier method to solve the constrained minimization for the power control. Unlike [28] using iterative calculations to obtain the power allocation, our BER-PC scheme has a closed-form power allocation and a simple procedure to determine the number of eigen-beams of positive power allocation. As a result, the computational complexity of the proposed BER-PC is lower than that of [28].

IV. DIVERSITY ORDER AND CODING GAIN

In this section, we analyze the asymptotic performance of the system using the proposed BER-PC scheme at high SNR. With (17) and $\lambda = g\bar{\gamma}/(rK)$, we have:

$$P_b \approx \frac{0.2}{N_{\alpha}} \left( \frac{1}{N_{\alpha}} \prod_{n=1}^{N_{\alpha}} \left( 1 + g\bar{\gamma}/(rK) a_n \right)^{-K} \right)$$  \hspace{1cm} (24)

When the SNR $\bar{\gamma}$ is very large, $\{P_n\}$ becomes $1/N$ according to the asymptotic analysis in Section III, and thus, $g\bar{\gamma}/(rK) a_n >> 1$. Hence, (24) can be approximated as

$$P_b \approx \frac{0.2(\bar{\gamma})^{-NK[K]} \left( g/(rK) a_n \right)^{-NK} \prod_{n=1}^{N} a_n^{-K}}{N}$$  \hspace{1cm} (25)

Based on (25), we can evaluate the system diversity order $G_d$ and coding gain $G_c$, which are two important indicators of error performance. $G_d$ and $G_c$ are respectively defined as the slope and offset of BER curve as SNR approaches infinity [5], that is,

$$G_d = \lim_{\gamma \rightarrow \infty} \frac{\log_2 P_b}{\log_2 \bar{\gamma}} = NK$$  \hspace{1cm} (26)

and

$$G_c = \lim_{\gamma \rightarrow \infty} \left( \frac{P_b}{\gamma} \right)^{-1/G_d} = \frac{1}{(NK)^{G_c}} \frac{q}{K r} \left( \prod_{n=1}^{N} a_n \right)^{\frac{1}{K}}$$  \hspace{1cm} (27)

According to (26) and (27), the diversity order equals the product of the numbers of transmit and receive antennas, i.e., $NK$, while the coding gain is affected by the code rate $r$, antenna numbers ($N$ and $K$), modulation mode (related to $g$), and mean and covariance feedback (related to the eigenvalues $\{a_n\}$).

Based on the asymptotic analysis in Section III, when the SNR is very low, all the power will be allocated to the largest eigen-beam corresponding to $a_1$, i.e., $P_1 = 1$, and $P_n = 0$ for $n = 2, \ldots, N$. Based on this result, (24) becomes $P_b \approx 0.2[1 + g\bar{\gamma}/(rK) a_1]^{-K}$. From this expression, it is shown that the BER performance at low SNR is affected by the code rate $r$, receive antenna number $K$, modulation mode (related to $g$), and maximum eigenvalue $a_1$.

As a summary, the transmit antenna number and receive antenna number, modulation mode, code rate, and mean and covariance feedback information (related to eigenvalues $\{a_n\}$) can affect the error performance.

V. POWER CONTROL SCHEME BASED ON FUZZY SNR

In this section, we will present a suboptimal PC strategy derived from the maximization of a newly defined fuzzy SNR (FSNR) that does not need to pre-determine the number of eigen-beams. It is well-known that the maximization of the received SNR defined in (12) will lead to 1D-BF irrespective of the certainty of the power gains [6, 13, 26]. As channel errors may bring about uncertainty onto the power gains of individual beams, the received SNR given in (12) is not a suitable criterion for coping with imperfect CSI. Hence, we propose to use a power law to introduce a fuzzy measure...
on the power gains in the average received SNR to take uncertainty into account. The exponent of the power law called as fuzziness $v$ has a value between zero and one reflecting the degree of certainty: from uncertainty to certainty. Based on (12), the new average received SNR, called as fuzzy SNR, is defined as

$$\gamma_v = E\left\{\frac{1}{r} \sum_{n=1}^{N} (P_n \xi_n)^v \right\} \quad \text{for } 0 \leq v \leq 1$$  \hspace{1cm} (28)

Substituting (13) into (28) gives

$$\gamma_v = \frac{1}{rK^v} \sum_{n=1}^{N} (P_n a_n)^v E\{\beta_n^v | \hat{H}\}$$  \hspace{1cm} (29)

From (14), we can obtain:

$$E\{\beta_n^v | \hat{H}\} = \int_{0}^{\infty} \beta_n^v \beta_n^{K-1} \exp(-\beta_n) / \Gamma(K) d\beta_n$$  \hspace{1cm} (30)

Substituting (30) into (29) yields:

$$\gamma_v = \frac{1}{rK^v} \sum_{n=1}^{N} (P_n a_n)^v \Gamma(K + v) / \Gamma(K)$$  \hspace{1cm} (31)

For perfect CSI, $v$ can be set equal to 1 to make FSNR in (31) become the average received SNR to give the optimal 1D-BF solution. But for imperfect CSI, $v$ is set less than 1 so that the power law in (31) is employed to adjust the power gains in a nonlinear (fuzzy) manner, where the power law is adapted in accordant with mean and covariance feedback, SNR, modulation and antenna configuration to minimize the BER. The FSNR can lead to a closed-form power control mechanism by maximizing $\gamma_v$ in (31) subject to the power constraint in (9).

Removing the coefficient unrelated to power allocation \( \{P_n\} \) from the FSNR in (31), we define an auxiliary objective function by integrating the power constraint with the FSNR as

$$L_v(P_1, ..., P_N) = \sum_{n=1}^{N} (P_n a_n)^v + \eta_2(1 - \sum_{n=1}^{N} P_n)$$  \hspace{1cm} (32)

where $\eta_2 > 0$ is a Lagrange multiplier. Taking partial derivatives of the objective function with respect to \( \{P_n, n = 1, ..., N\} \) and equating the partial derivatives to zero gives:

$$a_n^v P_n^{v-1} - \eta_2 = 0$$  \hspace{1cm} (33)

From (33), we can obtain:

$$P_n = \eta_2^{-1} [\nu a_n]^{1/v}, \quad n = 1, 2, ..., N.$$  \hspace{1cm} (34)

Substituting (34) into (9) yields

$$\left(\eta_2 / v\right)^{1/v - 1} = 1 / \sum_{n=1}^{N} a_n^{v/(1-v)}.$$  \hspace{1cm} (35)

Substituting (35) into (34) gives

$$P_n = \frac{a_n}{\sum_{n=1}^{N} a_n^{v/(1-v)}} = a_n^{\mu} a_1^{1/v(1-v)}, \quad n = 1, ..., N$$  \hspace{1cm} (36)

where $\mu = v/(1 - v)$ is called as fuzzy factor. With $0 \leq v < 1, \quad \mu \in [0, +\infty)$. It is noted that $P_n$ in (36) is an analytical function of $\mu \in [0, +\infty)$ and $a_n \in [0, \infty)$ expressed in a form of $a_n^\mu$ normalized by the sum of $a_n^\mu$. Hence, $P_n$ is always nonnegative, automatically satisfying the requirement of positive power allocation and power constraint. This FSNR scheme gives a closed-form expression of power allocation and does not need to perform pre-determination. The power law in (36) make the FSNR scheme become a general scheme to cover from equal power allocation ($\mu = 0$) to 1D-BF ($\mu \to \infty$). With the analytical property, we can solve for an optimal $\mu$ to minimize the BER at high SNR.

Based on the logarithm of the BER formula in (17) and using (36), we define an objective function of fuzzy factor, $L(\mu)$, as

$$L(\mu) = -\sum_{n=1}^{N} \log(1 + \lambda a_n^{\mu+1} / \sum_{l=1}^{N} a_l^{\mu})$$  \hspace{1cm} (37)

According to the asymptotic analysis of the optimal power control scheme in Section III, more power will be allocated to the eigen-beam of gain $a_1$ for low SNR. It means that $\mu$ is large for low SNR. On the other hand, the power allocations to all eigen-beams for very high SNR are almost the same. It means that the $\mu$ in $P_n$ of (36) is close to zero. Based on this asymptotic analysis result for high SNR, we can use Taylor’s series expansion to approximate $L(\mu)$ in (37) at $\mu = 0$ as

$$L(\mu) \approx -\sum_{n=1}^{N} \log (1 + \lambda a_n / N) - \sum_{n=1}^{N} \lambda a_n b_n / (N + \lambda a_n)$$  \hspace{1cm} (38)

where $b_n = \log a_n - \sum_{l=1}^{N} \log a_l / N$. By setting $\partial L(\mu) / \partial \mu = 0$, we have:

$$- \sum_{n=1}^{N} \lambda a_n b_n / N + \lambda a_n + \sum_{n=1}^{N} \lambda^2 a_n^2 b_n^2 / (N + \lambda a_n)^2 = 0$$  \hspace{1cm} (39)

With (39), $\mu$ is given by

$$\mu = \frac{\sum_{n=1}^{N} a_n b_n / (N + \lambda a_n)}{\lambda \sum_{n=1}^{N} a_n^2 b_n^2 / (N + \lambda a_n)^2}$$  \hspace{1cm} (40)

It is shown that the fuzzy factor $\mu$ calculated by (40) is always positive. For small $\lambda, \lambda a_n \ll N, \quad \mu \approx -\lambda^{-1} N \sum_{n=1}^{N} a_n b_n / \sum_{n=1}^{N} a_n^2 b_n^2$. This result shows that (40) can yield a large $\mu$ for small $\lambda$. With large $\mu$ for small $\lambda$ (equivalent to low SNR), the FSNR scheme gives most of the power to the largest eigen-beam like the 1D-BF (which is optimal at low SNR). For high SNR (large $\lambda$), we have:

$$\lim_{\lambda \to \infty} \mu = \lim_{\lambda \to \infty} \frac{\sum_{n=1}^{N} a_n b_n / (N + \lambda a_n)}{\lambda \sum_{n=1}^{N} a_n^2 b_n^2 / (N + \lambda a_n)^2} = \frac{\sum_{n=1}^{N} a_n b_n / a_n}{\sum_{n=1}^{N} a_n^2 b_n^2 / a_n} = 0$$  \hspace{1cm} (41)

where $\sum_{n=1}^{N} b_n = \sum_{n=1}^{N} \log a_n - \sum_{n=1}^{N} (\sum_{l=1}^{N} \log a_l / N) = 0$. With $\mu = 0$, the FSNR-PC scheme equally allocates power to all the eigen-beams, which is an optimum scheme for high SNR.
Substituting (37) and (40) into (17), the logarithm of $P_b$ at high SNR can be approximated as

$$
\log(P_b) = -K \sum_{n=1}^{N} \log(1 + \frac{\lambda a_n}{N}) + K \sum_{n=1}^{N} \log(\frac{a_n b_n}{(N + \lambda a_n)^2}) + \log(0.2) \quad (42)
$$

The first term in (42) is equivalent to the BER of equal power allocation. For high SNR (large $\lambda$), the second and third terms in (42) can be neglected, and thus $P_b$ in (42) is basically identical to (25) with equal power allocation giving the same diversity order and coding gain.

Besides, we notice that [30] designs an optimal precoder for STBC-MIMO system with mean feedback in joint transmit-receive correlated channels by minimizing the exact symbol error rate (SER), and derives an iterative numerical optimization algorithm to search the precoding matrix based on a fixed-point iteration method. However, the design method needs expensive computational complexity and cannot guarantee convergence to the global optimum, especially at high SNR [28, 39]. Moreover, closed-form expressions of power allocation for precoding design are not provided. On the contrary, our two proposed schemes can provide closed-form expressions of power allocation with low complexity. Regarding the computational complexity, the design method in [30] needs to compute the determinant and inverse of an $NK \times NK$ matrix, $N^2$ integrals, and other matrix multiplications in each iteration. The computational complexity is roughly $O(NK^3 + N^2)$ integrals. For our methods, they require to compute the EVD of $N \times N$ matrix $R_{\tilde{a}}$. The computational complexity is mainly dominated by the EVD with the complexity of $O(N^3)$. Overall speaking, the computational complexity of the optimal precoder design in [30] is large, while our proposed schemes have much lower computational complexity. The BER-PC scheme needs to pre-determine the number of eigen-beams for positive power allocation and then calculate the power allocation. The proposed FSNR-PC scheme requires neither a mechanism to determine the number of eigen-beams nor numerical root finding for computing the power. Thus the FSNR-PC scheme is more computationally efficient. Furthermore, they can obtain the performances close to that of the optimal scheme in [30] as shown by computer simulation. Besides, in [13], we propose a compressed SNR based PC scheme (CSNR-PC) for space-time coded MIMO system. However, this scheme is only suitable for mean feedback and spatially independent channels, and has relatively higher computational complexity than the FSNR-PC scheme because the calculation of the compressed factor in the CSNR-PC scheme is more complicated. Briefly, the complexity of the above methods is summarized in Table I.

VI. SPATIO-TEMPORAL POWER CONTROL SCHEME FOR IMPERFECT CSI

The BER-PC and FSNR-PC schemes are both spatial-only and neglect the degree of freedom in the temporal domain. For this reason, in this section, we develop a new joint spatio-temporal (S-T) power control scheme for STBC-MIMO systems with mean and covariance feedback by utilizing the degrees of freedom of power adaptation both in space and time. Basically, in the S-T power allocation, the temporal power varies from STBC symbol to STBC symbol according to the CSI feedback subject to a long term (time) average constraint, while the spatial power allocation follows our proposed spatial-only FSNR scheme. By computer simulation, this proposed S-T power allocation can provide significant reduction in BER.

With temporal power control, (10) is modified as

$$
Y = \sqrt{S_a H_{\text{UPD}}} + Z \quad (43)
$$

where $S_a$ is the temporal transmit power depending on the eigenvalues $a = \{a_1, a_2, \ldots, a_N\}^T$. The approximate average BER of (17) now becomes

$$
P_b \cong 0.2 \prod_{n=1}^{N} (1 + \lambda_a a_n)^{-K} \quad (44)
$$

where the effective average SNR in (17) is replaced by $\lambda_a = g S_a \gamma/(r K S)$ with the temporal transmit power adapted in accordant with the CSI feedback subject to an average power constraint. The constrained optimization of minimizing the BER in (44) can be formulated as

$$
\min_{\mu_a, \lambda_a} E_a[P_b] \quad \text{s.t.} \quad E_a[S_a] = \tilde{S} \quad (45)
$$

where $E_a[P_b] = \int P_b f_a(a) da$, $E_a[S_a] = \int S_a f_a(a) da$, $f_a(a)$ is the joint pdf of $a$, and $\mu_a$ is the fuzzy factor depending on $a$. For the sake of simplicity of algorithms, the FSNR-PC scheme is adopted for the spatial power allocation in (44). Based on the expression of $\lambda_a$ below (44), the constraint in (45) can be rewritten in terms of $\lambda_a$ as $E_a[\lambda_a] = g \gamma/(K r)$. Hence, the constrained optimization problem (45) can be rewritten as

$$
\min_{\mu_a, \lambda_a} E_a[P_b] \quad \text{s.t.} \quad E_a[\lambda_a] = g \gamma/(Kr) \quad (46)
$$

where the effective average SNR $\bar{\lambda} = g \gamma/(Kr)$. This is a calculus of variations problem with an isoperimetric constraint [40]. Applying the Lagrange multiplier method to the constrained problem (46) with a Lagrange multiplier $\theta$ gives the following unconstrained problem.

$$
\min_{\mu_a, \lambda_a} \int F(\mu_a, \lambda_a) da \quad (47)
$$

with

$$
F(\mu_a, \lambda_a) \Delta [P_b + \theta(\lambda_a - \bar{\lambda})] f_a(a) \quad (48)
$$

According to the optimality condition, the optimal $\mu_a$ and $\lambda_a$ should satisfy the following Euler-Lagrange equations [40],

$$
\frac{\partial F}{\partial \mu_a} = 0 \quad \text{and} \quad \frac{\partial F}{\partial \lambda_a} = 0 \quad (49)
$$

For given $\lambda_a$, solving $\frac{\partial F}{\partial \mu_a} = 0$ is equivalent to minimizing $P_b$ with respect to $\mu_a$, which has been done in Section V, resulting in the closed-form expression (40).

Substituting (17) into (48), $\frac{\partial F}{\partial \lambda_a}$ gives

$$
\frac{\partial F}{\partial \lambda_a} = [-0.2 \sum_{n=1}^{N} P_n a_n K/(1 + \lambda_a P_n a_n) 
\times \prod_{i=1}^{N} (1 + \lambda_a P_i a_i)^{-K} + \theta] f_a(a) \quad (49)
$$
The optimal value of $\lambda_n$ is the positive root of $\partial F/\partial \lambda_n = 0$. It is noted that solving $\partial F/\partial \lambda_n = 0$ does not involve $f_\alpha(a)$. Hence, we define $G(\lambda_n)$ from (49) without $f_\alpha(a)$ as

$$G(\lambda_n) = [-0.2 \sum_{n=1}^{N} P_n a_n K/(1 + \lambda_n P_n a_n) \prod_{i=1}^{N} (1 + \lambda_n P_i a_i)^{-K + \theta}]$$

From (50), it is shown that the Lagrange multiplier $\theta$ is positive. Taking the derivative of $G(\lambda_n)$ in (50) with respect to $\lambda_n$ yields

$$G'(\lambda_n) = 0.2 \sum_{n=1}^{N} \prod_{i=1}^{N} \left[ \left( \sum_{n=1}^{N} \frac{K P_n a_n}{1 + \lambda_n P_n a_n} \right)^2 + \sum_{n=1}^{N} \frac{K P_n^2 a_n^2}{(1 + \lambda_n P_n a_n)^{2}} \right] > 0$$

Equation (51) shows that $G(\lambda_n)$ is a monotonically increasing function of $\lambda_n$. According to (40), $\mu = \infty$ for $\lambda_n = 0$. For this case, the power allocation in (36) sets the power $P_1 = 1$ and $P_n = 0$ for $n = 2, ..., N$. Substituting $\lambda_n = 0$ and the corresponding power control into (50), $G(0) = -0.2 K a_1 + \theta$. For $\lambda_n \to \infty$, $G(\infty) = \theta > 0$. Since $G(0)$ is negative, that is, $\theta < 0.2 K a_1$. If the condition of existence is satisfied, we can find the optimal value of $\lambda_n$ by using Newton’s method (which has the quadratic convergence rate) to solve for the root of the monotonic function $G(\lambda_n)$ for given Lagrange multiplier $\theta$.

In summary, the online operations of the joint S-T power control (S-T-PC) based on the FSNR-PC scheme are summarized as follows.

1. Use (6) to calculate $\hat{R}_h$ and compute its eigen decomposition to obtain eigenvalues $\{a_n\}$.
2. Based on the obtained $a_1$, if $-0.2 K a_1 + \theta \geq 0$, then set $\lambda_n = 0$, otherwise go to the next step.
3. Use Newton’s method to solve $G(\lambda_n) = 0$ in (50) for the root $\lambda_n$ and calculate the temporal power as $S_n = r K S_n/(\sigma_n^2)$.
4. Use (40) and (36) to compute the spatial power control $\{P_n\}$.

Regarding the value of the Lagrange multiplier $\theta$ for the above online procedure, it can be obtained by an off-line procedure to satisfy the average power constraint in (46). In this paper, we use the Monte-Carlo method to calculate $E_{\alpha}\{\lambda_n\}$ in (46). We generate $10^5$ channel realizations of $H_i$ for producing $\hat{H} = R_{ij}^{1/2} \hat{H}_{ij} R_{ij}^{1/2}$. For each realization of $\hat{H}$, we use the on-line procedure of the joint S-T-PC scheme to obtain $\lambda_n$. Then, $E_{\alpha}\{\lambda_n\}$ is estimated as the average of $\{\lambda_n\}$. The bisection method can be used to find the value of $\theta$ by comparing the computed $E_{\alpha}\{\lambda_n\}$ with $\lambda$ in the constraint (46).

VII. NUMERICAL RESULTS

In this section, we use computer simulation to evaluate the performances of the BER-PC and FSNR-PC schemes as well as the joint spatio-temporal power control scheme (S-T-PC) in joint transmit-receive correlated Rayleigh fading channels. In the simulation, the channel is assumed to be quasi-static flat fading, that is, the fading gains remain constant over each space-time block. Gray code is employed to map the data bits into symbol constellations. Matlab is the programming tool, and the Monte-Carlo method is employed for simulation. Different orthogonal space-time block codes, such as G2, G3, H2, G4, and H4, are adopted for evaluation and comparison. The details of the codes can be found in [4]. The exponential correlation model [22, 30] is considered, that is, $[R_{ij}]_{ij} = \rho_{ij}^{1/2}$ and $[R_{ij}]_{ij} = \rho_{ij}^{1/2}$, where $\rho_{ij}$ and $\rho_{ij}$ are the transmit and receive spatial correlation coefficients respectively. The average transmit power $\bar{S}$ is set equal to one and the average SNR $\bar{\gamma} = 1/\sigma_n^2$. In the following figures, the equal power control and 1D beamforming schemes are denoted as EQ-PC and 1DBF-PC respectively. The PC schemes based on covariance feedback only and mean feedback only are referred to as CF-PC and MF-PC respectively.

Fig. 2 shows the average BER performance of a 1 x 4 system with G4 code using different spatial-only PC schemes (BER-PC, EQ-PC, 1DBF-PC, CF-PC and MF-PC) and two constellation sizes (16QAM and 64QAM) under imperfect CSI, where the transmit correlation coefficient is 0.6, time correlation coefficient is 0.95. It is found that our BER-PC scheme has better performance than the EQ-PC and 1DBF-PC schemes. At low SNR, the performance of the 1DBF-PC scheme is close to that of BER-PC but noticeable performance degradation is observed at high SNR, while the performance of the equal power scheme is only close to that of BER-PC at high SNR. The results accord with the asymptotic analysis in Section III. Moreover, the proposed scheme is superior to 'CF-PC' scheme, but it is only slightly superior to the MF-PC scheme. This is because the mean feedback is reliable under this imperfect CSI case. Clearly, the performance of the above five schemes degrades as the modulation size increases. From this figure, it may be observed that the BER-PC scheme tends to be the equal power scheme for average SNR ≥ 32.5dB under 16QAM, and for average SNR ≥ 37.5dB under 64QAM.
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In Fig.3, we plot the average BER performance of 3 × 4 system with H2 code for using different spatial-only PC schemes (BER-PC, EQ-PC, 1DBF-PC, CF-PC and MF-PC) and two constellation sizes (16QAM and 64QAM), where transmit correlation coefficient ρt = 0.9, receive correlation coefficient ρr = 0.7, and ρ = 0.6. The CSNR-PC scheme in [13] is also included for comparison. As shown in Fig.3, the results similar to Fig.2 are obtained. Namely, the developed BER-PC scheme still outperforms EQ-PC and 1DBF-PC schemes, and the performance of the system with large modulation size is worse than that with small modulation size. Moreover, under this imperfect CSI, the quality of mean-feedback is low, and the covariance-feedback is more effective than the mean-feedback. Under this case, our scheme is only slightly superior to the CF-PC scheme. Besides, since MF-PC and CSNR-PC schemes only utilize the mean feedback information, they have almost the same performance, but their performances are worse than that of the BER-PC scheme because they do not take the covariance information into account.

In Fig.4, we evaluate the performance of 2 × 3 system with the proposed BER-PC under imperfect CSI, where ρt = 0.4, 0.8, ρr = 0.4, 0.8, and ρ = 0.6, 1. The H2 code is used for space-time coding, and 64QAM is employed for modulation. The proposed BER-PC is used for performance evaluation. It is found that the BER decreases as either spatial correlation (ρt or ρr) decreases for fixed ρ. Also the performance decreases as ρ decreases for fixed ρt and ρr. The results accord with the existing knowledge. The above results show that the performance evaluation of the BER-PC scheme is valid.

Fig.5 shows the behavior of the average BER versus the correlation coefficient ρ for 3 × 3 system with H2 code, 16QAM, ρt = 0.6, and ρr = 0.6 under imperfect CSI. The average SNR is set equal to 5dB, 10dB, and 15dB for evaluation. The proposed BER-PC and FSNR-PC schemes are compared. From Fig.5, we can see that the BERs of the two schemes decrease as the correlation coefficient increases for fixed average SNR, especially for large ρ. Moreover, the BER decreases as average SNR increases. The results are consistent with the existing knowledge. Besides, the FSNR-PC scheme has almost the same performance as BER-PC scheme showing that the FSNR-PC with the fuzzy factor obtained by (40) is a good suboptimal solution, while the former has lower computational complexity. The above results show that the FSNR-PC scheme is a practical method in considering performance and complexity.

Fig.6 gives the performance comparison of the joint spatio-temporal PC scheme (S-T-PC) and spatial-only PC schemes for 2 × 2 system with G2 code. The six power control schemes: EQ-PC, BER-PC, FSNR-PC, CSNR-PC, S-T-PC and the existing optimal spatial-only PC scheme in [30] (referred to as HG-PC) are compared. In the simulation, 4QAM and 16QAM are used for modulation, and the correlation coefficients are ρt = 0.4, ρr = 0.6 and ρ = 0.9. From Fig.6, we can see that the FSNR-PC scheme has almost the same
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Fig. 6. Average BER of $2 \times 2$ system for different power control schemes and different modulations under imperfect CSI ($\rho_t = 0.4$, $\rho_r = 0.6$, $\rho = 0.9$).

performance as BER-PC scheme. The two proposed spatial-only PC schemes are superior to the EQ-PC scheme and have the BER performance close to that of the optimal HG-PC scheme. Moreover, the previous CSNR-PC scheme can obtain the performance very close to the BER-PC scheme since the mean-feedback is reliable for this case. At high SNR, it is observed that the HG-PC schemes suffers from convergence and has performance degradation in comparing with the two proposed spatial-only schemes. Furthermore, our schemes have lower computational complexity than that of HG-PC. More importantly, the performance of the three spatial-only PC schemes is worse than that of the joint spatio-temporal PC scheme for 4QAM and 16QAM. Especially in the medium and high SNR regions, the performance difference is obviously large. Thus, by designing power control jointly in space and time, the system performance is enhanced significantly. In Table II, we list the average fuzzy factor $\mu$ computed by (40) for different average SNRs for the FSNR-PC scheme. It is found that fuzzy factor $\mu$ decreases as the average SNR increases, which accords with the theoretical analysis in Section V.

The above results show that the proposed joint spatio-temporal PC scheme and the joint spatio-temporal PC scheme in [14] respectively. The optimal HG-PC scheme and the existing optimal spatial-only PC scheme with covariance feedback only in [28] (denoted by PVT-PC) are also included for comparison. In the simulation, $\rho_t = 0.6$, $\rho_r = 0.9$, and 64QAM for modulation are considered. For the $2 \times 3$ system, $\rho_r = 0.4$. As shown in Fig.7, the proposed BER-PC1 scheme with covariance feedback only has the BER performance very close to the optimal PVT-PC scheme. However, our scheme has lower computational complexity since it has the closed-form expression of power allocation, while the PVT-PC scheme has no closed-form solution. Interestingly, their BERs are the same for the $1 \times 3$ system. This is because the proposed PC scheme is the same as the PVT-PC scheme for transmit correlation only. Due to covariance feedback only, their performances are worse than the schemes with both mean and covariance feedback (i.e., BER-PC2 and HG-PC schemes). Besides, it is found that the proposed spatial-only BER-PC2 scheme obtain similar BER performance as the optimal HG-PC scheme. However, at high SNR, our scheme is superior to the optimal HG-PC scheme because the latter cannot always converge to the optimum. As a result, significant performance degradation can be observed for the HG-PC scheme at high SNR, which can also be seen in Fig.6. Moreover, the four spatial-only PC schemes all perform worse than the proposed S-T-PC1 scheme because they do not take the degree of freedom in the temporal domain into consideration.

Furthermore, it is observed that the proposed S-T-PC1 scheme has better performance that the S-T-PC2 scheme in [14]. This is because the latter only utilizes the mean feedback information, while the former takes both mean and covariance feedback information into account. Besides, under this imperfect CSI, the mean-feedback is more effective than the covariance-feedback, and thus their performance difference is not that large. Even so, their performance difference for the $2 \times 3$ system is larger than that for the $1 \times 3$ system. The reason is that the $2 \times 3$ system has both transmit and receive correlations, while the $1 \times 3$ system only has transmit correlation. Thus, the impact of spatial correlation on the performance of the $2 \times 3$ system is larger than that of the $1 \times 3$ system. It is shown that the $2 \times 3$ system outperforms the $1 \times 3$ system since the former has greater diversity than the latter. The above results show that the proposed joint spatio-temporal PC scheme can effectively make use of the degrees of freedom in space and time to enhance the system performance.

VIII. CONCLUSION

This paper presents two spatial-only power adaptation schemes and a spatio-temporal power adaptation scheme for
orthogonal space-time coded MIMO systems with imperfect CSI in joint transmit-receive spatially correlated Rayleigh fading channels. Based on mean and covariance feedback, the first spatial-only BER-PC scheme is derived by minimizing a BER upper bound, which includes the PC scheme with mean feedback only or covariance feedback only. It also includes the PC scheme with mean and transmit covariance feedback only or with mean and receive covariance feedback only as special cases. The second spatial-only scheme is FSNSR-PC, which is derived from a newly defined fuzzy SNR. This suboptimal scheme has a closed-form power allocation with a closed-form expression of computing the fuzzy factor for minimizing the BER. The closed-form solution provides not only a direct computation of the power control and also BER performance close to that of the BER-PC and the existing optimal PC scheme.

Based on the FSNSR power allocation scheme, a new joint spatio-temporal PC scheme is derived subject to an average power constraint with mean and covariance feedback. This joint scheme can outperform the spatial-only PC schemes substantially. The online spatio-temporal power control scheme can be done simply by solving the root of a monotonic function via Newton’s method. The Lagrange multiplier for the constraint optimization can be obtained offline through the bisection method. Simulation results show that the proposed BER-PC and FSNSR-PC schemes not only can achieve better performance than the equal-power PC and the PC scheme with mean feedback only or covariance feedback only, but also have the performance close to that of the existing optimal scheme and low computational complexity. Most importantly, the joint spatio-temporal PC scheme can obtain a significant BER performance gain over the spatial-only PC schemes. As a summary, the new power allocation schemes can provide low-complexity precoders to enhance the BER of MIMO systems with delayed feedback over joint transmit-receive spatially correlated Rayleigh fading channels. The new precoders can significantly improve the reliability of the future wireless communications.

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REFERENCES


### Table I

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<th>Complexity of different power control schemes.</th>
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<tr>
<td>Optimal spatial-only PC [29]</td>
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<tr>
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<tr>
<td>FSNR-PC spatial-only</td>
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<td>CSNR-PC spatial-only</td>
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<tr>
<td>$O((NK)^3/N^2)$</td>
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<tr>
<td>$O(N^4)$</td>
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### Table II

<table>
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<th>Average fuzzy factor for different average SNRs.</th>
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<td>Average SNR 0dB 5dB 10dB 15dB 20dB 25dB 30dB</td>
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<tr>
<td>Average $\mu$ 83.432 26.678 8.706 2.961 1.041 0.354 0.114</td>
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