Elasto-Plastic Constitutive Model for Finite Element Simulation of structural Adhesives

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Abstract: Considered by the automotive industry as a good solution to bond car body parts together, structural adhesives must be integrated in the early design process of vehicles, hence in the finite element analysis. A first difficulty when modeling structural adhesives comes from their complex mechanical pressure-dependent behaviour as well as their load-dependent strength. These aspects make them impossible to describe by classical material constitutive models such as von Mises or exponent Drucker-Prager, in the general case (Jousset, 2008). Additionally meshing the very thin adhesive layer is problematic since the use of several solid elements in the thickness would lead to large and unrealistic computation time for any industrial finite element simulation. Fortunately, Abaqus offers to the user the possibility to overcome these difficulties. Accurate material models (Mahnken, 2005) can be implemented by the user into user subroutines, and computational efficiency can be reached using interface or cohesive elements. This opens the door to a good compromise between an accurate material description and acceptable computation time for the finite element simulation of structural bonded joints.

Keywords: Structural Adhesive, SikaPower, Bonding, Constitutive Model, Plasticity, Damage, Vumat, Optimization, Python Script, Cohesive Elements

1. Introduction

In the past few years structural adhesives have become more and more used in bonding processes, especially in the automotive industry. These adhesives are generally applied in very thin layers and play an important role in the way energy is distributed through the surrounding parts when the car body deforms. In order to predict the behavior of such adhesives from the early phase of design of a car, an accurate material constitutive model is required to describe their behavior in finite element (FE) simulations. Structural adhesives considered in this paper are products of the type SikaPower-490. Their chemical structure consists in an epoxy matrix containing a given amount of soft inclusions. These inclusions bring ductility to the matrix and the whole adhesive behaves under general multi-axial loading as a typical elasto-plastic material and consequently elasto-plastic constitutive models can be used to predict their non-linear behavior. As already shown in (Jousset, 2008), “classical” constitutive models based on the well-known von Mises or exponent Drucker-Prager yield functions fail to describe specific effects involved by the deformation of structural adhesives such as their yield stress dependency on the level of hydrostatic pressure and their strength dependency on multi-axial stress state. As a consequence the implementation of an extended elasto-plastic constitutive model is required to take these effects into account in FE simulation. A class of continuum constitutive models has been investigated in (Fosta, 2008), leading to a rather good approximation of the structural adhesive behavior under multi-axial loading. These models are based on the formalism developed by Schlimmer (Schlimmer 1982). Based on these observations the material model proposed in this project has also been derived from this formalism and has first been implemented in an implicit user material subroutine (Umat) in Abaqus/standard and restricted to quasi static FE simulations. This Umat has then been extended to an Abaqus/explicit user subroutine (Vumat) to allow its utilization in dynamic and crash applications. The Vumat is originally capable to predict the non-linear behavior of the adhesive until the point of maximal strength has been reached. Originally developed and calibrated with continuum solid elements the model has been finally tested with cohesive elements enabling a simplified design of adhesive mesh and enhancing computational speed. Both of these aspects are essential requirements for industrial FE computations.

The following report presents the implementation of the elasto-plastic Vumat into Abaqus/explicit for the FE simulation of structural adhesives. The identification of model parameters has been addressed and the Vumat has been tested with cohesive elements. Finally further development of the Vumat to take damage and strain-rate dependencies into account have been introduced.

2. Elasto-plastic Material Constitutive Model

2.1 Model's description

A strain-stress curve of a SikaPower-490 adhesive dog-bone specimen is sketched in Figure 1.

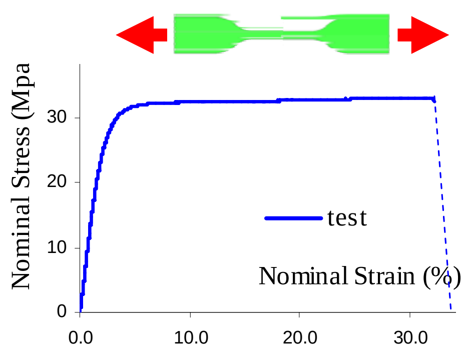

Figure 1. Strain-Stress curve of a SikaPower-490 dog bone specimen.

As recalled in the introduction, this curve is typical from an elasto-plastic material. It shows a linear-elastic behaviour for small reversible deformations, followed by a non linear plastic domain where deformations become irreversible and ends with a damage/softening zone where the material looses its load-carrying capacity until total failure (dot line).

Consequently, the material constitutive model implemented in this project is based on the elasto-plasticity theory which is quickly recalled here:

- Strains are additively decomposed between an elastic and a plastic part

$$\varepsilon = \varepsilon^e + \varepsilon^p \ , \qquad (1)$$

where ε is the total strain, ε^e is the elastic strain and ε^p is the plastic strain

- The elastic strain is computed from the elastic constitutive equation

$$d\sigma = D : d\varepsilon^e \ , \qquad (2)$$

where σ is the Cauchy stress and D the fourth order elastic constitutive tensor

- The limit of the elastic domain is reached when the yield function, f goes to zero

$$f\left(\sigma, \bar{\sigma}\left(\bar{\varepsilon}^p\right)\right) \leq 0 \ , \qquad (3)$$

where $\bar{\sigma}$ denotes the yielding stress and $\bar{\varepsilon}^p$ denotes the equivalent plastic strain
The yield function chosen in this project, is inspired from the work of Mahnken and Schlimmer (Mahnken, 2005) and has been slightly modified under the form

$$f = \sqrt{3J_2 + a_1 Y_0 I_1 + a_2 I_1^2} - Y \leq 0 \ , \qquad (4)$$

where I_1 is he first stress invariant, J_2 the second deviatoric stress invariant, a_1 and a_2 are material parameters to determined, Y is the actual yield stress and Y_0 is the initial yield stress of the material before any plasticity has developed.
f depends on the hydrostatic stress, since f is a function of I_1 and I_1 gives a measure of the hydrostatic stress or pressure reigning in the material. This is an important feature for structural adhesives already discussed in the introduction.
The yield stress Y depends on the equivalent or effective plastic strain e_v and takes hardening effects into account

$$Y = Y_0 + q(1 - e^{-be_v}) + He_v \ , \quad (5)$$

where q, H and b are material parameters to determine

- The flow rule governing the evolution of the plastic flow and defining its direction is expressed as

$$d\,\varepsilon^p = d\lambda \frac{\partial g}{\partial \sigma} \ , \quad (6)$$

Where g is he flow potential and $d\lambda$ denotes the plastic multiplier. When f is equal to g the flow is said "associated". Associated plasticity is generally only an approximation of reality. A non-associated flow rule has been selected in this project in order to stay in a general framework allowing describing the specific behaviour of structural adhesives

$$g = \sqrt{3J_2 + a_2^* I_1^2} \ , \quad (7)$$

Where a_2^* denotes a material parameter to determine.

In order to formulate strains evolution equations the principle of equivalence of dissipated power referred as "Case 1" in (Mahnken 2005) has been used

$$\dot{e}_v Y = \sigma : \dot{\varepsilon}^{pl} \quad (8)$$

2.2 Model implementation

The consistent integration into a Vumat of the constitutive Equations introduced in paragraph 2-1 has been performed by following the method proposed by Cvitanic et al. in (Cvitanic 2008). This method is based on an implicit radial return mapping integration scheme (Simo, 2000) divided in two steps:

- An elastic predictor step where the strain increment is assumed to be elastic resulting in a trial elastic stress tensor σ^t

$$\sigma^t = \sigma_n + D : \Delta\varepsilon \ , \quad (9)$$

where D denotes the tensor of elastic moduli, σ_n is the Cauchy stress tensor at the end of the increment n and $\Delta\varepsilon$ defines the deformation rate tensor.

- A plastic corrector step: In the case where the trial states violates the yield condition (Equation 3), assuming the trial state as initial condition, a plastic correction step must be run to find the exact final stress state as well as the correct ratio of elastic and plastic strain in the material.

The incremental form of the Equation is described by means of a system of four non linear algebraic Equations:

$$\begin{cases} f(\sigma_{n+1}) - Y_{n+1} & = & 0 \\ D^{-1} : (\sigma_{n+1} - \sigma^{t}) + \Delta\lambda \left(\dfrac{\partial g(\sigma)}{\partial \sigma} \right)_{n+1} & = & 0 \\ Y_{n+1} - \left(Y_0 + q(1 - e^{-b(e_v + \Delta e_v)}) + H(e_v + \Delta e_v) \right) & = & 0 \\ f\ (\sigma_{n+1}) \Delta e_v - g(\sigma_{n+1}) \Delta\lambda & = & 0 \end{cases} \tag{10}$$

where D denotes the tensor of elastic moduli, σ denotes the Cauchy stress tensor, σ^{t} is the trial stress tensor evaluated in the elastic predictor stress according to the implicit return mapping procedure (Equation 9), $\Delta\lambda$ is the plastic multiplier, Δe_v is the increment of the effective plastic strain and other symbols have the meaning given in paragraph (2-1).

The system of Equations 10 is resolved incrementally using the Newton-Raphson method and the following quantities are updated at the end of each increment k

- The variation of the equivalent plastic strain

$$\Delta e_v^{k+1} = \Delta e_v^{k} + \delta \Delta e_v^{k} \tag{11}$$

- The variation of the plastic multiplier increment

$$\Delta\lambda^{k+1} = \Delta\lambda^{k} + \delta\Delta\lambda^{k} \tag{12}$$

- The stress tensor

$$\sigma_{n+1}^{k+1} = \sigma_{n+1}^{k} + \Delta\sigma_{n+1}^{k} \tag{13}$$

- The yield stress

$$Y_{n+1}^{k+1} = Y_{n+1}^{k} + \Delta Y_{n+1}^{k} \tag{14}$$

Finally the plastic strain tensor can be found from

$$\varepsilon_{n+1}^{k+1\,p} = \varepsilon_{n+1}^{k\ \ p} + \Delta\lambda^{k+1} \frac{\partial g(\sigma)}{\partial \sigma}, \tag{15}$$

and the elastic strain tensor is deduced from

$$\varepsilon_{n+1}^{k+1^{el}} = \varepsilon_{n+1}^{k+1} - \varepsilon_{n+1}^{k+1^{p}} , \tag{16}$$

Where ε_{n+1}^{k+1} is the total strain applied at the beginning of the increment.

3. Parameters Identification

The elasto-plastic material constitutive model introduced in paragraph 2 contains 6 parameters a_1, a_2, a_2^*, q, H and b plus the elastic constants E (Young modulus) and v (Poisson Ratio).

These parameters must be identified to predict accurately the adhesive behaviour in FE Simulation

3.1 Strategy for parameters identification

As recalled in the introduction, the behavior of structural adhesive in terms of strength depends strongly on the loading and more especially on the ratio of shear and normal stress in the adhesive layer. As a consequence, experiments used to identify the parameters of material constitutive model must present different ratio of shear and normal stress. Such tests exist in the literature among which the butt bonded hollow cylinder, the combined tension shear specimen (Fosta 2008), and diverse Arcan tests. In the field of this project, the butt bonded hollow cylinder has been used and experimental tests have been carried out in the frame of a research project (Fosta 2005). The experimental sample and pending meshing used in this project have been represented in Figure 2. The adhesive layer is 0.2mm thick and has been modeled with continuum hexahedral elements c3d8r. Steel substrates surround the SikaPower-490 adhesive layer.

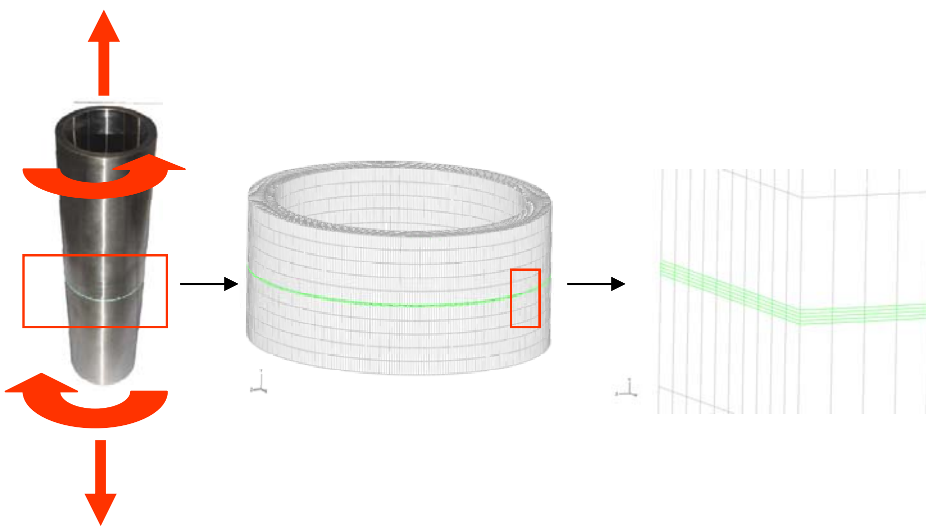

Figure 2. Butt bonded hollow cylinder: experimental sample and numerical model.

The cylinder has been proportionally loaded under different ratios of traction and torsion (or shear) loading. Defining α as the ratio of shear strain divided by two times the normal strain

$$\alpha = \frac{\gamma}{2.\varepsilon}, \tag{17}$$

where γ is the shear strain and ε is the normal strain.

The cylinder has been loaded in the configurations $\alpha = 0$ (cylinder under pure traction), $\alpha = +\infty$ (cylinder under pure shear), $\alpha = 0.5$ and $\alpha = 2$ (combination of traction and shear). Material parameters correctly identified from these different tests together should be able to reproduce the behavior of the adhesive layer under any multi-axial loading.

3.2 Identification by means of inverse method

On the one hand elastic constants *E* and *v*, can be directly identified from a dog-bone specimen of bulk material represented in Figure 1, but on the other hand material parameters in Equations 4, 5 and 7 can be directly identified if and only if the stress state is known in the adhesive layer and stays relatively homogeneous. However, as shown in (Jousset, 2008), even for the apparently simple case of a butt bonded hollow cylinder under pure traction loading, the stress state in the adhesive layer is much more complicated than a pure traction stress state, and no trivial conclusion can be derived for the sake of material parameter identification.

Alternatively, a method has been developed in the frame of this project based on inverse identification. This inverse identification method consists in finding a best set of material parameters fitting a response calculated by means of FE simulation with its relative experimental response. For example in the case of the butt bonded hollow cylinder loaded under pure traction the inverse methods consists in making the measured (normal displacement-force) response coincide with the same response deduced from numerical simulation (Figure 3). This is an optimisation problem, where the cost function to minimise with respect to the material parameters is a least square function of the form

$$f(x) = \sum_{i=1}^{n} [m_i(x) - c_i(x)]^2, \tag{18}$$

where $m_i(x)$ and $c_i(x)$ are the measured and the computed response corresponding to the same displacement. The problem is solved iteratively using local methods such as gradient algorithms or global methods such as genetic algorithms. In Figure 3, f has been minimized when the experimental and the numerical curves are superimposed (at the end of 25th iteration) and the corresponding set of parameter is the optimum solution. However this set of parameters will not necessarily be good enough to predict the behavior of the adhesive under multi-axial loading. Such a set of parameters would be deduced from inverse identification including the butt bonded hollow cylinder under the different loadings $\alpha = 0$, $\alpha = +\infty$, $\alpha = 0.5$ and $\alpha = 2$.

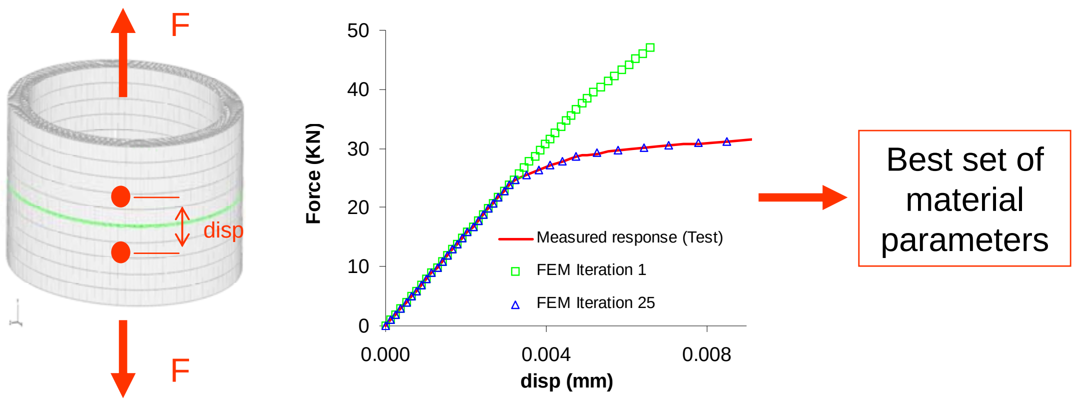

Figure 3. Butt bonded cylinder under traction and inverse identification of material parameters.

3.3 Implementation of the inverse identification method

A fully automated procedure has been developed to compute and to minimize f and to find the best set of material parameters. The procedure links Abaqus to the software Altair Hyper/Study through a Python script and has been summarized in Figure 4.

The procedure has the possibility to include several tests under different multi-axial loadings. Additionally, for each test, several measures can be included for the evaluation of f. For example considering the butt bonded hollow cylinder under multi-axial loading $\alpha = 0.5$ and $\alpha = 2$, it is possible to include the normal displacement-force curve and the tangential displacement-moment curve in the calculation of f to find the set of parameters giving the best prediction of these two curves simultaneously.

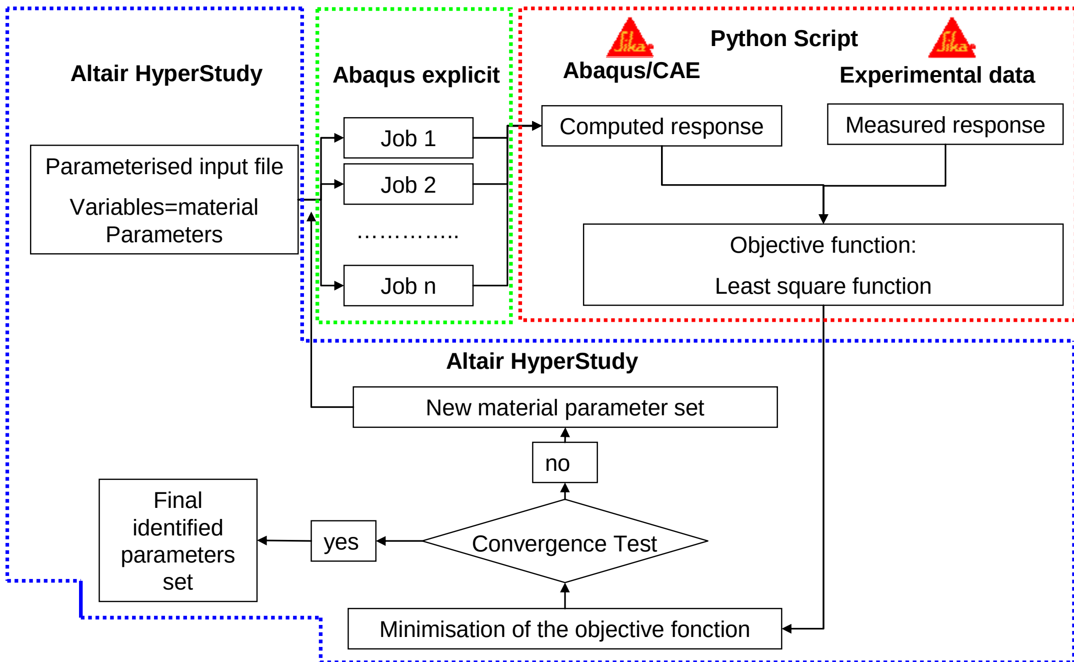

Figure 4. Inverse identification procedure developed for parameters identification.

4. Results

4.1 Results with 3D continuum elements

The inverse identification method has been used with all the four butt bonded hollow cylinder configurations $\alpha = 0$, $\alpha = +\infty$, $\alpha = 0.5$ and $\alpha = 2$ available, using both a gradient and a genetic algorithm in the optimization step, to prevent the solution to stick to a set of material parameters corresponding to a local minimum of the optimization problem.

Elasto-plastic parameters have been identified and the best set is given in table 1.

Table 1: Best set of elasto-plastic material parameters identified for the Vumat

E (MPa)	V(-)	Y_0 (Mpa)	H(Mpa)	q(Mpa)	b (-)	a_1 (-)	a_2 (-)	a_2^* (-)
2120.0	0.36	29.6	34.483	17.371	177.153	0.186	0.300	0.128

Results of the parameter identification using parameters of Table 1 are presented on Figure 5.

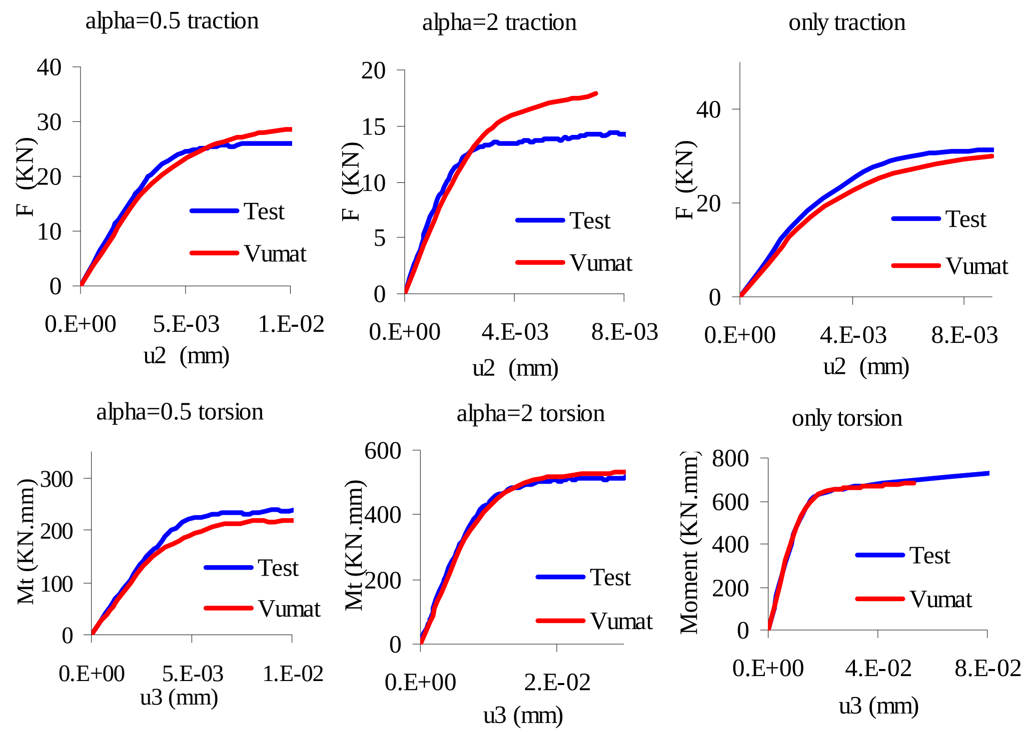

Figure 5. Results of the parameter identification tests and numerical results with the best identified set of parameters.

Figure 5 shows a very good average correlation between the experimental and the numerical behavior of the structural adhesive SikaPower-490. Using the Vumat presented in this project strongly improve the results found using other more classical constitutive models (Jousset 2008).

4.2 Results with cohesive elements

Parameters identification has been carried out using an adhesive layer meshed with continuum solid hexahedral elements, as represented in Figure 2. Several elements are included through the thickness of the adhesive. The utilization of such a mesh is justified to get a realistic approximation of stress and strains in the adhesive layer from which an objective set of material parameters can be deduced. However when modeling an adhesively bonded line in an industrial case study such as a large car-body structure, the use of several continuum solid elements in the adhesive thickness is not realistic for two reasons:

- Practically meshing such an adhesive layer between two random surfaces would take a very long time.

- The critical time step in an explicit analysis is proportional to the size of the smallest element found in the mesh. The use of several elements in an adhesive layer of less than one millimeter would results in unacceptable computation times, especially in the automotive industry.

As a solution, the adhesive layer has been modeled using cohesive elements together with the Vumat, through the Abaqus keywords

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*COHESIVE SECTION, RESPONSE=CONTINUUM
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In that case, the cohesive adhesive layer is assumed to be subjected only to the through thickness strain and to two shear strain components. The two other direct strain components and membrane shear strain components are assumed to be zero, which is realistic for such a thin confined adhesive layer. The layer includes only one element through its thickness in that case. A comparison of the results obtained with the butt bonded hollow cylinder with solid and cohesive elements is presented in Figure 6

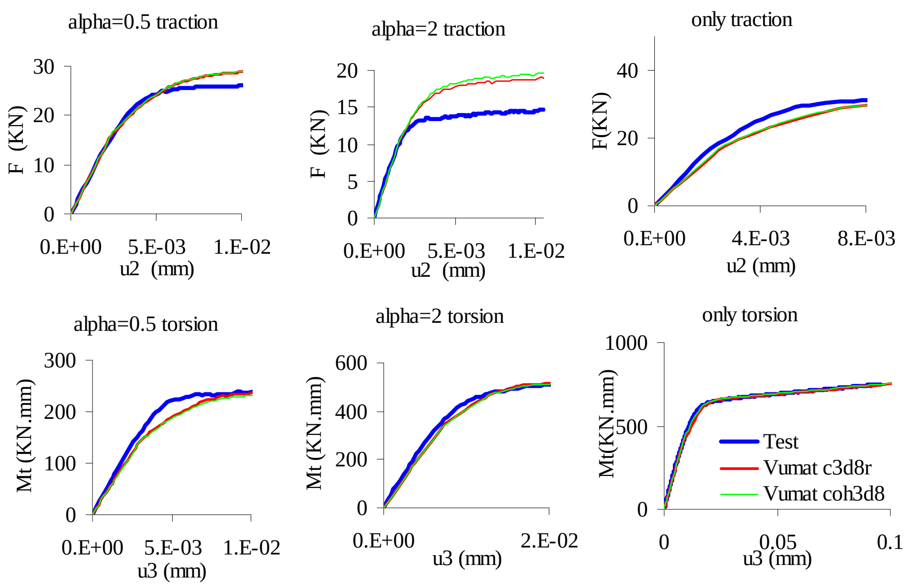

Figure 6. Comparison of results using solid and cohesive elements in the butt bonded hollow cylinder.

Results obtained with cohesive elements (using the set of parameter identified with solid elements) fit nearly perfectly with those obtain with solid elements, validating the use of cohesive zone with the Vumat for the FE simulation of future industrial case studies.

5. Further model developments

The Vumat presented before is limited to the elasto-plastic behavior of structural adhesives and can be extended to take additional effects into account such as strain rate dependency and damage allowing the FE simulation of interesting phenomenons such as failure or crash.

5.1 Strain rate dependency

Different strain rate applied to a same structural adhesive result in different strain-stress history and the yield stress of the adhesive grows with growing strain rate. Hence the expression of the yield stress in the Vumat should be updated to take stress dependency into account. This can be done by simply multiplying the varying part of the yield stress (Equation 5) by a strain rate dependent term

$$Y = Y_0 + \left(q(1 - e^{-be_v}) + He_v\right)\left(\frac{\dot{\bar{e}}_v^p}{\dot{\bar{e}}_{v0}^p}\right)^m \qquad (19)$$

Where $\dot{\bar{e}}_v^p$ is the equivalent plastic strain rate, $\dot{\bar{e}}_{v0}^p$ is a reference equivalent plastic strain rate and m is a material parameter to identify. This rate dependent model is simple and has the advantage to rule the strain rate dependency with only two parameters m and $\dot{\bar{e}}_{v0}^p$. These two parameters can be identified analytically using experimental data loaded under different speeds.

5.2 Numerical example

Explicit FE simulations of the previous structural adhesive have been started under three different speeds in order to show the potential of the strain rate-dependent Vumat to take strain-rate effects into account. For the sake of simplicity, the Vumat has been simplified to a strain rate J2 plasticity model by setting the parameters a_1, a_2, a_2^* to zero and a simple dog-bone test has been simulated under three different speeds where Speed 1< Speed 2< Speed 3. Results have been presented in Figure 7.

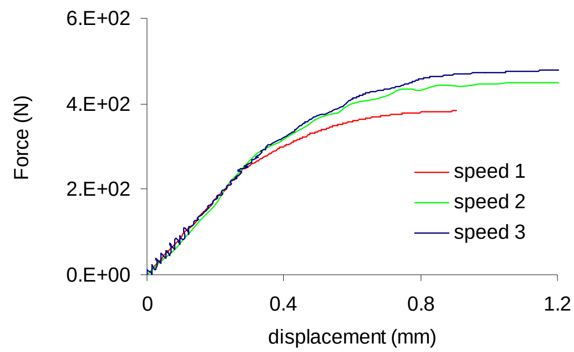

Figure 7. FE Simulation of a dog-bone specimen loaded under different speeds.

Figure 7 shows clearly the capability of the model to take the strain-rate dependency effect into account. Material parameters $\dot{\bar{e}}_{v0}^{p}$ and m must be identified accurately from experimental data.

5.3 Damage model

Damage model can be included in the Vumat using the method proposed to describe damage in ductile metals in the Abaqus Analysis user manual. The model is based on the principle of strain equivalence and on the concept of effective stress (Lemaitre 2004).

- Damage initiation

The first step consists in predicting damage initiation due to nucleation and coalescence of growth and voids. Damage initiation occurs when the following condition is fulfilled

$$\omega_D = \int \frac{d\bar{\varepsilon}^{pl}}{\bar{\varepsilon}_D^{pl}(\eta, \dot{\bar{\varepsilon}}^{pl})} = 1, \tag{20}$$

Where $\bar{\varepsilon}_D^{pl}(\eta, \dot{\bar{\varepsilon}}^{pl})$ is the equivalent plastic strain when damage initiates. This is a function of the stress triaxiality η and of the equivalent plastic strain rate $\dot{\bar{\varepsilon}}^{pl}$. The equivalent plastic strain when damage initiates, $\bar{\varepsilon}_D^{pl}$ can be computed using the criterion defined by Johnson and Cook and written in the case of quasi-static computation

$$\bar{\varepsilon}_D^{pl} = [d_1 + d_2 \exp(-d_3 \eta)] \tag{21}$$

where materials parameters d1, d2, d3 are material parameters to identify. Going back to Equation 20, $\Delta\bar{\varepsilon}^{pl}$ is usually, but not compulsory, equal to the increment of equivalent plastic strain used in the elasto-plastic model. In the frame of this project $\bar{\varepsilon}^{pl}$ would be equal to the quantity e_v appearing in the Equation 5. In Figure 8, a set of parameters d1, d2, d3 has been found giving a good correlation of the damage initiation points for the butt bonded hollow cylinder under pure traction and pure torsion. In that case the complete failure of the adhesive has been assumed as soon as damage has initiated, explaining the brutal drop of the simulation curves.

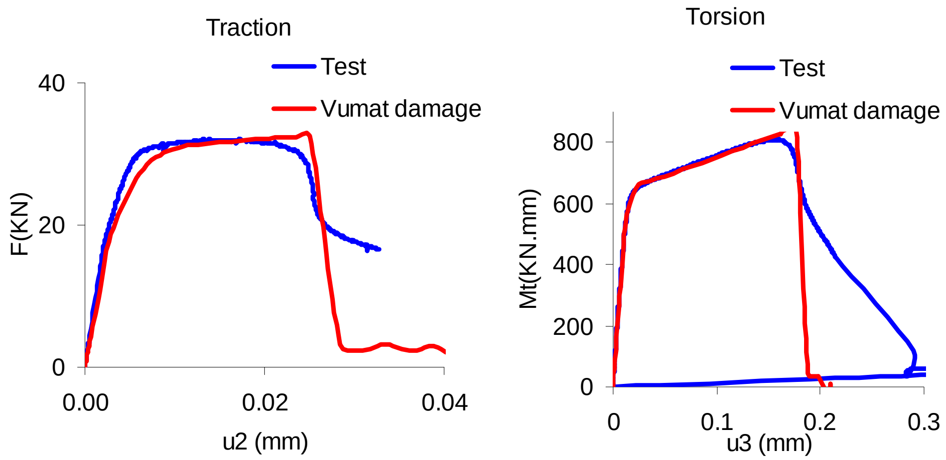

Figure 8. Description of damage initiation in the hollow butt bonded cylinder under pure traction and pure torsion.

- Damage evolution

After damage has initiated, the evolution of damage must be described by a constitutive evolution law, enabling to describe more accurately the softening parts of the curves in Figure 8.

The implementation of damage in the model must be completed to describe accurately initiation and evolution of damage in the adhesive loaded and failing on any multi-axial load case

6. Conclusion

An elasto-plastic model has been developed to simulate the behavior of structural adhesive under multi-axial loading up to their maximum strength. The implementation has been carried out in a user material subroutine (Vumat) for Abaqus/explicit. An inverse identification procedure has been developed linking Abaqus to HyperStudy through a Python script to identify material parameters efficiently. A set of parameters has been identifyed using experimental data with the adhesive SikaPower-490 giving an excellent correlation between tests and simulations of the butt bonded hollow cylinder under different multi axial loadings. The use of cohesive elements with the Vumat has then been validated enabling its future utilization for large industrial FE analysis. Finally the model has been extended to take strain rate dependency and damage of the adhesive into account, which is of prime interest for crash simulations. These two points must be further investigated and validated in the future.

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