Modelling and optimization of active filters by hybrid „*VIV-MMC*"-graphs

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Abstract: - Novel "*VIV*" signal flow graphs are introduced. They provide a description of active filter subblocks consisting of *OpAmps* with grounded inputs. The Tow-Thomas, the *Åckerberg-Mossberg*, the *Fleischer-Tow*, and *MFB* structures are typical representatives of such a filter class. Many kinds of active filter like the *Sallen-Key* utilize a more general *OpAmp* connection with ungrounded inputs. These circuits are well modelled by the modified *Mason-Coates* („*MMC*") graphs. Finally, combined „*VIV-MMC*" graphs are appropriate for the filters that compound both the grounded and ungrounded *OpAmps*. In virtue of these graphs three propositions are framed, which enable optimizing the component ratio and dynamic range.

Key-Words: - Active filter, optimization, signal flow graph, *MC* graph, *MMC* graph, „*VIV*" graph, „*VIV-MMC*" graph.

1 Introduction

All the active filters based on *VFAs* (Voltage Feedback Amplifiers) can be divided into two groups: „*VIV*" filters (Voltage, Current, Voltage – for details see Chapter *VIV* graph), and the others. Let us call the latter „$\overline{VIV}$". The „*VIV*"-type filters are defined by the following conditions that have to be fulfilled at the same time:

1. The filter has to consist of ideal *VFA(s)* and passive components (*R*, *C*, or also *L* types). In addition, the filter may contain only finite-gain *VCVS*(s) (Voltage Controlled Voltage Sources).
2. Each *VFA* has to preserve zero voltage on its inputs. It is mostly ensured by grounding one of the inputs.
3. Each passive component may be connected only between the nodes that represent *VFA* outputs, ideal voltage source outputs, *OpAmp* inputs, or the ground.

A logical condition of proper operation is that the *OpAmp* outputs may not be connected with the outputs of other voltage sources.

Many types of active filter, for example all the filters in Fig. 1 [1], fulfill condition 1. Condition 2 ensures – among others –the fundamental DC stability of all „*VIV*" filters. For instance, the *Tow-Thomas*, *Åckerberg – Mossberg*, and *MFB* filters in Fig. 1 *a, b,* and *c* satisfy this condition. Condition 3 is satisfied for all filters in Fig. 1, except for the *MSB* type. Only the *Tow-Thomas* and *Åckerberg –Mossberg* „*VIV*" filters fulfill all three conditions at the same time. As shown below, while a general active filter can be described by the modified *Mason-Coates* (*MC*) graph [2], the „*VIV*" filter will be modelled by using a simplified „*VIV*" graph [3]. In virtue of the structure of these graphs some knowledge emerges on the dynamic range and component ratio optimization. In addition, we will show that it is possible to combine „*VIV*" and „*MMC*" graphs for such filters that do not satisfy the above condition but contain nested „*VIV*" sections. We refer to the effects on filter optimization.

2 Modified Mason-Coates graph (*MMC*)

It is well known that the classical *MC* graph can be modified in the case of circuit elements of the types of voltage source and ideal *VFA*. The first modification is common to both types of element. That is why the *OpAmp* output behaves as a voltage source: concerning the modelled element, all the outer branches in the circuit graph that are directed into the voltage node of the voltage source are omitted. That is why we do not use the equation of *Kirchhoff's* current law to define a given voltage. The next modification concerns the ideal *VCVS* with gain *A* – see Tab. 1. The corresponding graph contains a self-loop with a gain of 1 (which need not be drawn) and simply corresponds to the equation given in the Table. As shown in Tab. 1, the infinite *OpAmp* gain, which leads to zero difference voltage, can be respected by a self-loop with a gain of 0.

Practical utilization of *MMC* graphs will be illustrated in the following text.

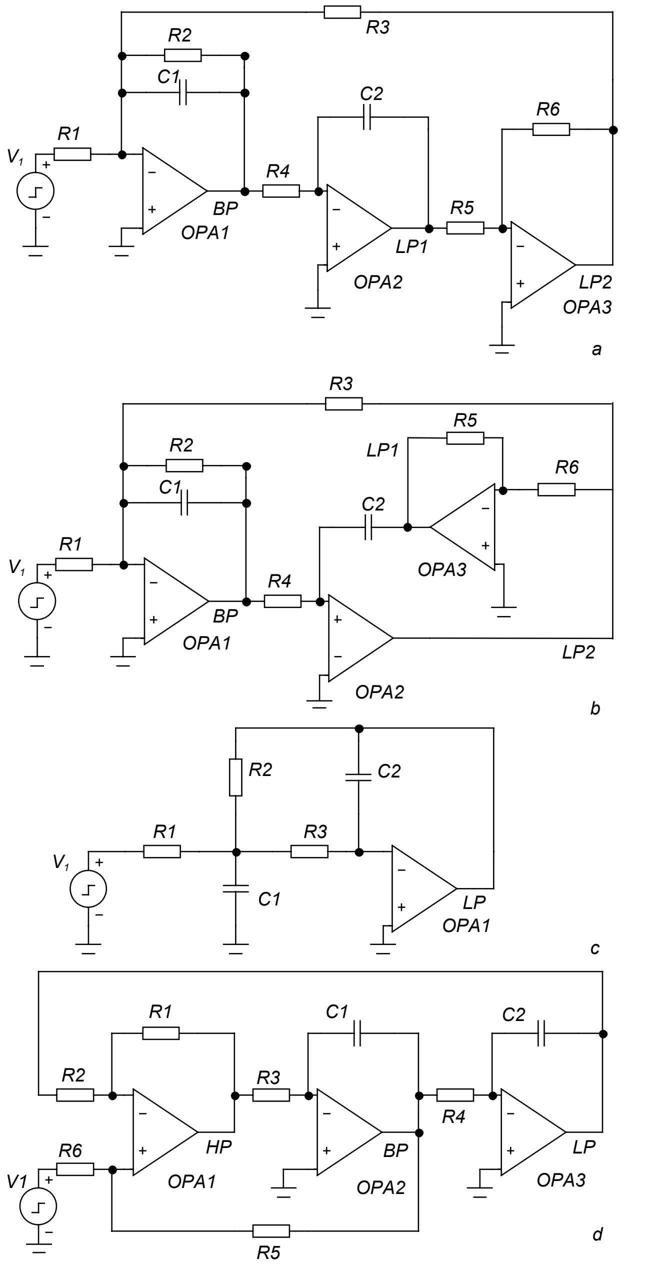


Fig. 1. Examples of well known active filters: *a* Tow-Thomas, *b* Åckerberg –Mossberg, *c* MFB, *d* KHN. Filters *a* and *b* belong to the class of „*VIV*“ filters.

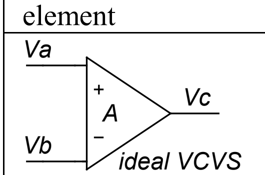
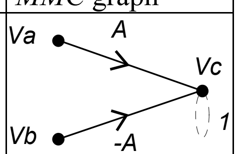
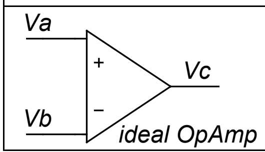
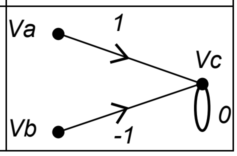
element	equation	*MMC* graph
	$1xV_c =$ $AV_a - AV_b$	
	$0xV_c =$ $V_a - V_b$	

Table 1. Modified *MC* graphs of ideal *VCVS* and ideal *OpAmp*.

3 „*VIV*“-graph

Consider the circuit in Fig. 2*a*. It can be understood as a basic building block of filters with grounded *OpAmps*. The following equation is true:

$$V_1Y_1 + V_2Y_2 + \ldots + VY = 0, \tag{1}$$

where the symbol „*Y*“ specifies admittances that correspond to impedances „*Z*“.

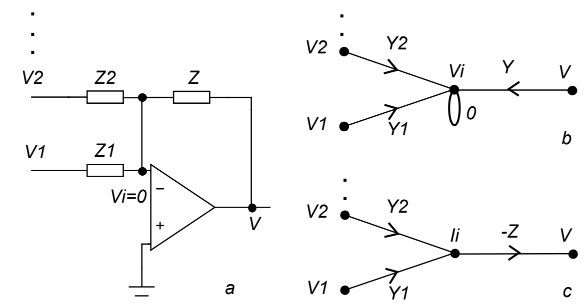


Fig. 2. *a* Block with the grounded *OpAmp*, *b* the *Mason-Coates*' graph with zero self-loop, *c* the modified *Mason's* graph with the self-loop reduction: "*VIV*"-graph.

The *Mason-Coates'* graph in Fig. 2*b* corresponds to this equation. However, equation (1) can be modified to the form

$$V = -ZI_i = -Z(V_1Y_1 + V_2Y_2 + \ldots) \tag{2}$$

where I_i is the total current flowing from individual inputs to feedback impedance *Z*. The corresponding graph is in Fig. 2*c*. Let us call it „*VIV*“-graph: input voltages are first transformed into current I_i and this in turn is converted into voltage through feedback impedance *Z*. Two simple rules of thumb are valid for the compilation of the „*VIV*“-graph:

Directed branches with gains Y_k flow from the nodes of voltage sources V_k to current node I_i. Here, Y_k are admittances of the elements that are connected between the *OpAmp* input and voltage source V_k.

The directed branch with a gain of $-Z$ leaves current node I_i and leads to the node of the *OpAmp* output voltage. Here, *Z* is the impedance of the feedback element that is connected between the *OpAmp* input and output.

In case of more feedback impedances in parallel (e.g. the block with *OPA1* in Fig. 1*a*, where C_1 and R_2 are in parallel), one can use the following procedures for graph compilation. Either all the impedances in parallel are joined to the resulting impedance *Z* and graph 2*c* is then used, or one of the impedances is chosen as *Z* and we model the others by the feedback admittance branches that lead from the *OpAmp* output to current node I_i. The second variant is used for the graph construction of the entire filter from Fig. 1*a* (see Fig. 3).

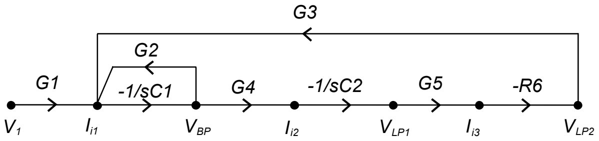


Fig. 3. „*VIV*“ graph of the *Tow-Thomas* filter from Fig. 1 *a*.

Another problem appears if no *OpAmp* feedback branch exists that contains only passive components. The *Åckerberg – Mossberg* filter in Fig. 1*b* is a typical example: there is another *OpAmp* in the *OPA2* feedback. Let us now consider an auxiliary feedback resistor $R\to\infty$. We eliminate its influence by limit transition after graph evaluation. The corresponding graph in Fig. 4 *a* offers a further solution: using the rule indicated in Fig. 2*b*, *c*, we can transform the feedback path $-sC_2R_5G_6$ to the forward-path with a gain of R_6G_5/sC_2.

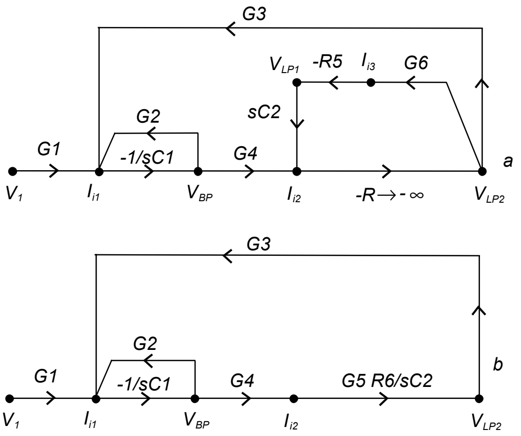


Fig. 4. „*VIV*“ graph of the *Åckerberg –Mossberg* filter from Fig. 1 *b*. Implementation of an auxiliary feedback resistor *R* (*a*), its reduction (*b*).

4 „*VIV-MMC*“-graph

An arbitrary linear circuit can be decomposed into a „*VIV*“ subcircuit and the circuit remainder „$\overline{VIV}$“. The „*VIV*“ subcircuit is described by the „*VIV*“ graph, whereas the „$\overline{VIV}$“ subcircuit by the „*MMC*“ graph. A graph interface will correspond to the region where both circuits are connected.

In the following, we will consider so-called *proper connection* of subcircuits without fundamental violation of their operation. Let us define the *input gate* of the „*VIV*“ subcircuit as a set of all nodes that correspond to the ungrounded *OpAmp* inputs. The *output gate* is then a set of all nodes that the ideal voltage sources are connected to (including *OpAmp* outputs). The term *proper connection* means that no ideal voltage sources of the „$\overline{VIV}$“ subcircuit may be connected to either the input or output gate of the „*VIV*“ subcircuit.

Connection of the input gate of „*VIV*“ subcircuit to the environment will be represented in the graph by paths directed to the input gate, i.e. to supernode I_i. Along with the sources in the outer „$\overline{VIV}$“ circuit these paths will represent currents flowing from this circuit to the *OpAmps* virtual zero. Since interface voltage is zero, the paths going out of this gate will be missing.

Connection of the output gate of „*VIV*“ subcircuit to the environment will be represented in the graph by paths directed out of the output gate, i.e. from the *OpAmp* outputs and from other voltage sources. Along with the voltage sources inside the „*VIV*“ circuit these paths will represent currents flowing to the outside „$\overline{VIV}$“circuit. Since ideal voltage sources operate on this interface, the paths directed to this gate will be missing.

In the compilation of a „*VIV-MMC*“ graph we proceed as follows: We decompose the filter into „*VIV*“ and „$\overline{VIV}$“ subcircuits. We compile their corresponding „*VIV*“ and „*MMC*“ graphs. Both graphs will be interconnected according to the rule discussed. The resulting graph will be evaluated by the classical *Mason-Coates* rule.

For illustration, „*VIV-MMC*“ graphs of the *MFB* and *KHN* filters from Fig. 1*c* and *d* are given in Fig. 5.

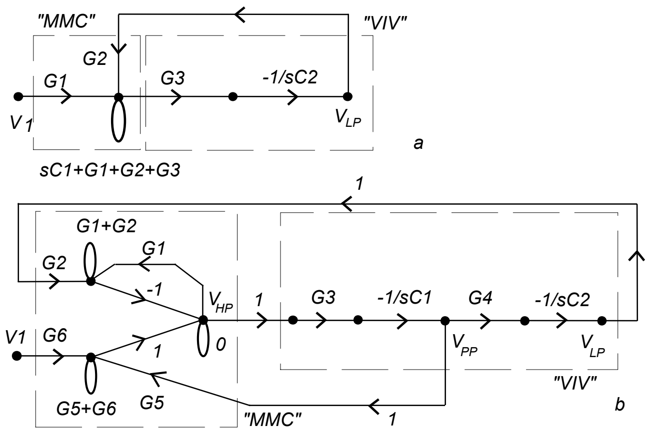


Fig. 5. „*VIV-MMC*“ graphs of the *MFB* (*a*) and *KHN* (*b*) filters.

Several interesting propositions are valid for „*VIV*“ filters. They can help us optimize the component ratios and dynamic range. These propositions result from the graph description. Similar propositions have been conceived earlier [4,5] for switched-capacitor filters.

5 Propositions resulting from the graph description

In the first step, we focus on the „*VIV*“ filters. Starting from the example of the *Tow-Thomas* filter in Fig. 1*a* and from its „*VIV*“ graph in Fig. 3, let us consider such a variation of filter elements that the transfer functions to all nodes remain unchanged. A well-known possibility is impedance scaling: impedances of all filter elements are multiplied by the same constant $a>0$. However, further and more specific possibilities result from the properties of "*VIV*" subgraphs of individual blocks.

The "*VIV*" graph of the given block in Fig. 2*a* consists of current node I_i and of voltage nodes (one of them is the node of *OpAmp* output voltage). Varying the elements leads to a change in input current I_i. As is obvious from the "*VIV*" graph, this change can be neutralized by the modification of impedance Z to leave the output voltage unchanged. To have voltages/transfer functions to all the nodes unchanged, the variation of filter elements must be performed only in such a way that both the forward-path gains from the input to individual outputs and all the loop gains remain unchanged.

There are analogous ways of changing the transfer to a concrete *OpAmp* output and preserving voltages on the outputs of the other *OpAmps*.

Proposition 1. *Within the "VIV" circuit, we multiply by a > 0 the impedances of all elements that are connected to the ungrounded OpAmp input. Then neither the OpAmp output voltage nor the other nodal voltages are changed. In other words, the transfer functions from the input to all the nodes remain unchanged.*

Proof of this proposition is easy: All forward-path gains from the input to a given output that go through the block considered comprise the product of the admittance, connected to the input, and the feedback impedance. This product is independent of the multiplication described in proposition 1. If the given block is part of feedback loops, there can be generally two types of loop: **1**. Loops of the local feedback inside the block (see the loop with a gain of $-G_2/sC_1$ in Fig. 3). They are due to more feedback impedances in parallel. Then the loop gain is given by the sum of products of impedance Z and of the admittances of the other feedback elements. These products are also unchanged. **2.** Loops of the global feedback through more blocks. The gains of these loops comprise the products of feedback impedance Z and the admittances connected between the *OpAmp* input and the output of the other amplifier in the loop. These products are also unchanged. We can conclude that neither the denominator of all the transfer functions nor the numerators are changed by the above operation.

In the case of "*VIV*" filters, this proposition can be utilized for the minimization of the ratios of filter elements.

It should be noted that **this proposition need not be valid** if the „*VIV*" circuit is connected to a „$\overline{VIV}$" circuit. This fact can be demonstrated on the graph of *MFB* filter in Fig. 5*a*. Multiplying resistance R_3 and impedance $1/sC_2$ by the same number $a>0$, both the forward-path gain $-G_1G_3/sC_2$ and the feedback loop gain $-G_2G_3/sC_2$ remain unchanged. However, the self-loop gain, which affects the graph determinant, is now changed: The original gain is $G_1+G_2+G_3+sC_1$, the gain after multiplication is $G_1+G_2+G_3/a+sC_1$. To preserve the graph determinant constant, we could try – for instance – to change G_1 to G_1' such that the following equality is valid:

$$G_1' + G_2 + G_3/a = G_1 + G_2 + G_3 \Rightarrow G_1' = G_1 + G_3(1 - 1/a).$$

If $G_1'>0$, then such transformation is realizable. However, a modification of G_1 will result in the modification of the forward-path gain, i.e. the total input-output transfer. This approach can thus be used for a defined filter gain setting without modifying the frequency response.

Proposition 2. *Within the "VIV" circuit, we multiply by a > 0 the impedances of all elements that are connected to the OpAmp output. If any VCVS is connected to this output, we divide its gain by a. Then the OpAmp output voltage will increase a-times, and the other nodal voltages will remain unchanged. In other words, the transfer function from the input to the OpAmp output will be multiplied by a, and the transfer functions to the other nodes will remain unchanged.*

It should be noted that only the feedback impedance of the given block and the input impedance of the cascading block are connected to the *OpAmp* output. In the "*VIV*" graph, these elements represent a forward path leading to the current node of the immediately following block. The gain of this path remains constant after multiplying both impedances by the same number. That is why the gains of global feedback loops that comprise the given path are preserved. The gain preservation of the prospective local feedback loop is ensured by the same mechanism. The only thing that is changed is the gain of the forward path between the current node and the output node of the given block (it is changed *a*-times). Thus proposition 2 is proved.

This proposition can be utilized to optimize of the dynamic range of the "*VIV*" filter.

Proposition 2 need not be valid if the „*VIV*" circuit is connected to a „$\overline{VIV}$" circuit. For example, consider the *KHN* filter whose „*VIV-MMC*" graph is in Fig. 5*b*. Passive components C_1, R_4, and R_5 are connected to the output of OPA2. R_5 does not belong to the „*VIV*" subcircuit and thus it will be a potential source of problems. Multiplying the impedances of all three elements by a number *a* causes a modification of the parameters to C_1/a, aR_4, aR_5. The graph analysis leads to the conclusion that all the loop gains remained unchanged and that the forward-path gain from input to *OPA2* output increased *a* times. However, the self-loop gain G_5+G_6 is now modified to $G_5/a+G_6$. To reset the original gain, we can change G_6 but then the total gain will be modified again.

Proposition 3. *Within the "VIV" circuit, we multiply by a ≠ 0 both the gain of VCVS and the impedances of all elements that are connected to the VCVS output. Then the VCVS output voltage will increase a-times, and the other nodal voltages remain unchanged. In other words, the transfer function from the input to the VCVS output will be multiplied by a, and the transfer functions to the other nodes will remain unchanged.*

As results from the definition of „*VIV*" circuit, a *VCVS* has to be controlled only by the voltage source (or by the output of *OpAmp* as a special case of voltage source). Leaving aside the singular case of amplifiers in cascade, the *VCVS* output has to be connected to a passive component. The graph determinant will be unchanged if the product of the *VCVS* gain and the admittance of cascading passive element is constant. Then the output voltage of *VCVS* will be changed.
This proposition can be used to monitor and optimize the upper bound of the dynamic range of „*VIV*" filters that are tuned by *VCVSs*.
Proposition 3 need not be valid in the case of connecting „*VIV*" and „$\overline{VIV}$" circuits. We can verify this by a set of examples, utilizing the approach demonstrated in Propositions 1 and 2.

6 Optimization examples

6.1 Example 1 – „*VIV*" filter

Let us consider the filter in Fig. 1*a*, which is designed as a bandpass filter with a maximum gain of 0 dB on the output *BP*. The preliminary design leads to the component values

R_1=R_2=R_3=24kΩ, R_4= 955Ω, R_5=R_6=1kΩ, C_1=C_2=330pF, (3)

which correspond to the following filter parameters:

f_0 =100.7kHz, Q =5.01, B =20.1kHz.

The 50MHz *OpAmp* AD826 has been selected for the realization. Simulation results (*MicroCap VI*, *Spice* models) are in Fig. 6*a*. The gain peaks measured around the frequency 100kHz are as follows: 0.3 dB for the *BP* output, 14.3 dB for outputs *LP1* and *LP2*.
It is obvious that the upper bound of the dynamic range is not optimized: *OPA2* and *OPA3* will saturate for a relatively low input signal because their gain peaks are 14 dB above the peak of *BP* output. The required lowering is thus by 14 dB, i.e. 5 times.
In the first step, we lower five times the transfer to the *LP1* output. According to proposition 2, we must increase C_2 five times and decrease R_5 five times:

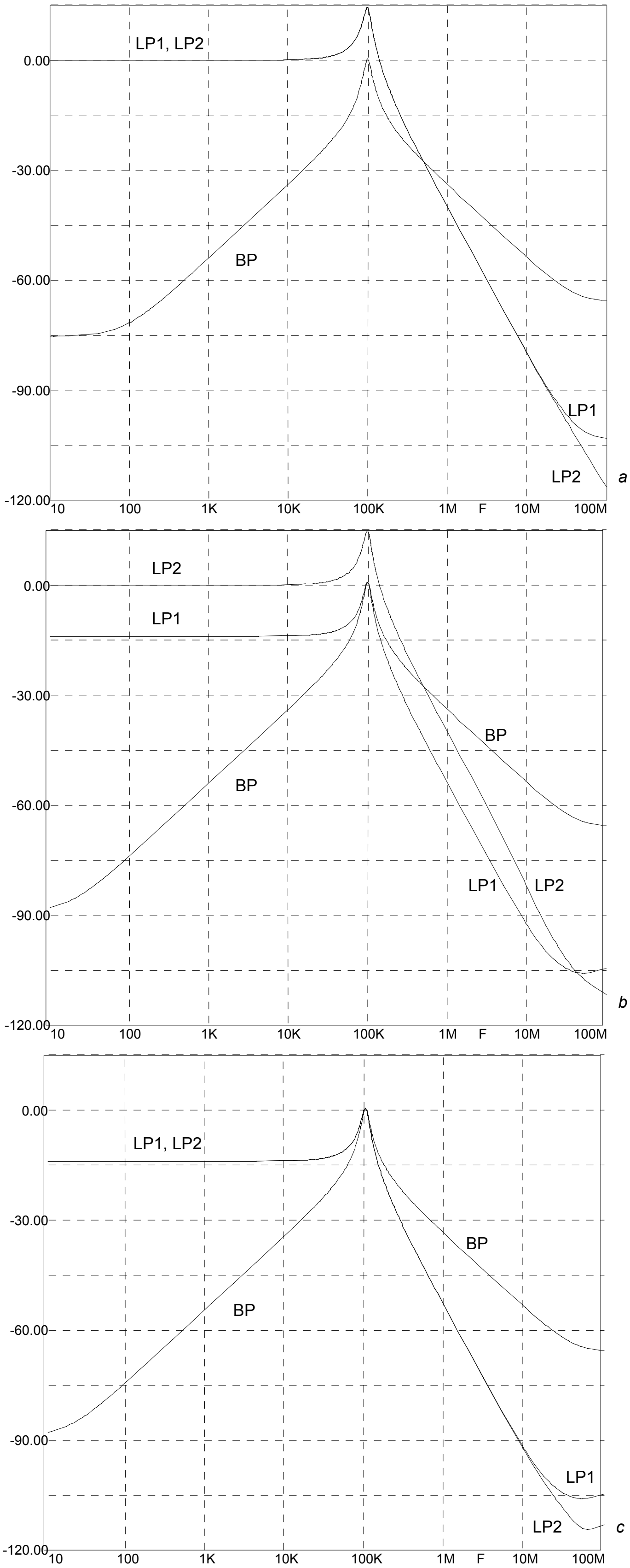


Fig. 6. Frequency responses of the filter from Fig. 1*a*, *a* element values according to (3), *b* lowering the gain peak in *LP1* output to the level of *BP* peak, *c* lowering the gain peak in *LP2* output to the level of *BP* peak.

C_2=1.65nF, R_5=200Ω. The corresponding responses are in Fig. 4b). Note the modification of the bandpass response in the low-frequency region due to the real properties of *OpAmps*.
Now we lower five times the transfer to output *LP2*, i.e. we reduce five times R_6 and R_3: R_6=200Ω, R_3=4.8 kΩ. The responses in question are in Fig. 6*c*. The upper bound of the dynamic range is now optimized.
Let us use proposition 1 for final filter optimization. Applying this proposition to the filter leads to the following:
We split the passive components into three independent sets:

R_1, R_2, R_3, C_1
R_4, C_2
R_5, R_6

The elements of set No. *i* represent the resistors and capacitors that are connected to the input of *OpAmp* No. *i*. Then if we increase resistances in an arbitrary set *a*-times and if simultaneously we decrease capacitances in the same set *a*-times, where $a>0$, the transmission properties of the filter will not be changed.
We decide to change capacitance C_2 back from 1.65nF to 330pF. C_2 belongs to set No. 2. That is why we increase R_4 from 955Ω to 4775Ω. Then we modify R_5 and R_6 from set No. 3 to original values of 1kΩ.

6.2 Example 2 – classical filter

Consider the *KHN* filter in Fig. 1*d*, which is utilized as a bandpass filter in the output *BP*. Parameters f_0 and Q are the same as in Example 1. Initial element values are as follows:

R_1=R_2=R_5=R_6=1kΩ, R_3= 21,4kΩ, R_4=965Ω,
C_1=C_2=330pF. (4)

Simulation would show that the *BP* peak value (0 dB) is 13.8 dB below the peaks of *LP* and *HP*. The reason consists in the improper Q setting by resistors R_3 and R_4 instead of R_5 and R_6.
Let us try to increase the *BP* gain by 13.5 dB, i.e. $a = 10^{13.8/20} = 4.9$ times in respect of the other *OpAmp* outputs. However, we cannot use proposition 2 directly because the filter is not of the „*VIV*“ type. An analysis of the „*VIV-MMC*“ graph in Fig. 5*b* yields the following:
The forward-path gain to the *BP* output can be increased, for example, by increasing G_3. If we decrease inversely G_4, then the forward-path gain to the *LP* output remains unchanged. At the same time, the gain of loop, which includes both integrators, is preserved. This is one of the conditions of invariance of the graph determinant.
However, increasing G_3 results in increased gain of the loop from the *BP* output to the input. This can be compensated by decreasing G_5. The self-loop G_5+G_6 indicates that modification G_5 will cause a subsequent proportional modification of transfer to all the outputs.
The above analysis leads to the procedure of dynamic range optimization: we increase G_3 *a* times and decrease G_4 *a* times simultaneously. Then we modify G_5 to bG_5 with the aim to preserve the self-loop gain discussed:

$$\frac{aG_3}{-sC_1}\frac{bG_5}{bG_5+G_6} = \frac{G_3}{-sC_1}\frac{G_5}{G_5+G_6}. \quad (5)$$

In the graph evaluation, the rule of self-loop reduction has been used.
We compute from equation (5) that

$$\frac{1}{b} = a + \frac{R_6}{R_5}(a-1) = 8.8. \quad (6)$$

That is why *R*5 must be increased 8.8 times. The new resistances are as follows: R_3=4.37k, R_4=4.73k, R_5=8.8k. Then all three peak gains will be the same: a short computation leads to the value $R_5/R_6 = 8.8 = 18.9dB$.
It should be noted that in spite of the „$VIV+\overline{VIV}$“ type of *KHN* filter, we can use Proposition 2 for its following prospective optimization.

7 Conclusion

A procedure is shown how to decompose an active filter into the „*VIV*“ and „$\overline{VIV}$“ subcircuits. The graph representations of both types of subcircuit are demonstrated as well as their association. On the basis of given graph properties three propositions are stated, which enable us to subsequently optimize the upper bound of the dynamic range and to optimize of the distribution of element values and ratios according to the designer’s requirements.

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