Closed Timelike Curves

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1 Introduction

In this paper, we explore the possibility that closed timelike curves might be allowed by the laws of physics. A closed timelike curve is perhaps the closest thing to time travel that general relativity allows. We will begin our discussing just what closed timelike curves are, and in what kinds of contexts they were first shown to appear. We then explore how one might actually travel on a closed timelike curve, and discuss two recent no-go results which suggest that this endeavor is impossible.

2 What is a Closed Timelike Curve?

Intuitively, a Closed Timelike Curve (CTC) is a path along which a massive body could travel, such that its lifetime repeats again and again indefinitely. Some physicists seem to identify CTCs with time travel, but this is somewhat of a misnomer. Traveling along a CTC would be more like what Nietzsche called 'eternal recurrence,' in which one's life repeats again and again, than it would be like traveling in Doc's DeLorean.

Our discussion of CTCs begins with a pseudo-Riemannian spacetime $(\mathcal{M}, g_{\mu\nu})$, which for us will be a connected four dimensional manifold $\mathcal{M}$ with a non-degenerate metric $g_{\mu\nu}$ of Lorentz-signature $(+ - - -)$. This signature means that the tangent space at a point $T_p\mathcal{M}$ will have a basis set $\overset{1}{e}{}^{\mu}, \overset{2}{e}{}^{\mu}, \overset{3}{e}{}^{\mu}, \overset{4}{e}{}^{\mu}$ such that $\overset{i}{e}{}^{\mu}\overset{i}{e}_{\mu}$ is equal to 1 if $i = 1$, and -1 if $i \in \{2, 3, 4\}$. This allows us to classify a vector v in $T_p\mathcal{M}$ into one the following three (suggestively named) categories:

Spacelike:	$v^\mu v_\mu > 0$
Lightlike:	$v^\mu v_\mu = 0$
Timelike:	$v^\mu v_\mu < 0.$

The same idea extends to smooth curves in the obvious way: a curve γ is timelike (lightlike, spacelike) if every vector in its tangent field is, too. In relativistic theories, lightlike velocities are an upper bound for the motion of all massive bodies. So, one of the basic interpretive commitments of general relativity (and for us today) is that massive bodies follow timelike curves through spacetime.

Can there exist timelike curves that are *closed?* Obviously, which curves on $\mathcal{M}$ are timelike depend on the metric structure of the spacetime. In turn, the Einstein Field Equations (EFEs) tell us that metric structure depends on the distribution of matter. So, this question reduces to the following: are there any solutions to the EFEs containing closed timelike curves? This question was settled in the affirmative by the mathematician Kurt Gödel [2], who provided an exact solution in which there is a closed timelike curve through every point[1]. It is instructive to take a moment to see what this solution, as well as its closed timelike curves, are like.

The matter field that Gödel imagined was a pressure-free perfect fluid $T_{\mu\nu} = \rho u_\mu u_\nu$ with density ρ, describing universe uniformly filled with spinning dust particles. His spacetime $(\mathbb{R}^4, g_{\mu\nu})$ has a metric such that for any point $p \in \mathcal{M}$, there is a globally adapted coordinate system that one can (suggestively) call t, r, φ, y, with metric,

$$\begin{aligned} g_{\mu\nu} =& \nabla_\mu t \nabla_\nu t - \nabla_\mu r \nabla_\nu r + \nabla_\mu y \nabla_\nu y \\ & + (\sinh^4 r - \sinh^2 r)\nabla_\mu \varphi \nabla_\nu \varphi + 2^{3/2} \sinh^2 r (\nabla_{(\mu} \varphi \nabla_{\nu)} t) \end{aligned}$$

and in which $t(p) = r(p) = y(p) = 0$. Here, t and y both range from $-\infty$ to $+\infty$, r ranges

[1] In fact, an exact solution containing CTCs was produced much earlier by Von Stockum [8]. However, it was not realized until recently that this solution contains CTCs.

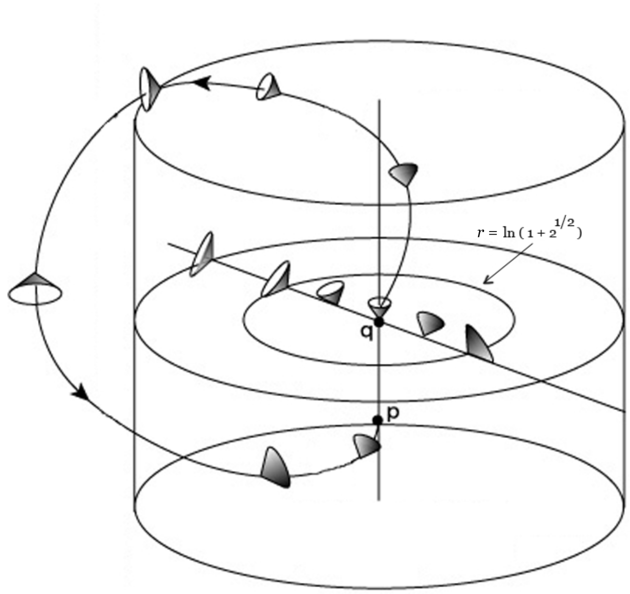

Figure 1: A closed timelike curve in Gödel spacetime.

from 0 to ∞, and $\varphi \in [0, 2\pi)$. The vector field $(\frac{\partial}{\partial t})^\mu$ is a timelike Killing field[2] of unit length, which represents the four-velocity of the dust-particles. The field $(\frac{\partial}{\partial \varphi})^\mu$ is a 'rotational' Killing field. Its integral curves are characterized by constant values of t, r, and y, some of which turn out to be CTCs. The situation is illustrated in Figure 1. To find closed timelike curves, we notice first the special fact that the $\nabla_\mu \varphi \nabla_\nu \varphi$ term in the metric vanishes when $r = ln(1 + \sqrt{2})$ (since this entails that $\sinh r = 1$). There is a closed lightlike curve passing through this radius. Moreover, any integral curve of $(\frac{\partial}{\partial \varphi})^\mu$ with radius greater than this value is a CTC. (However, it turns out that no such curve is a geodesic. For details, see Hawking and Ellis [3, 168-170].)

That the laws of physics allow for CTCs caused great tension among early general relativists, and Gödel's spacetime was generally deemed 'unphysical' for a number of reasons. For one, the existence of a CTC requires the existence of global constraints on the kinds of events that can take place on the spacetime, in order to guarantee consistency. For example, a football game on a CTC that begins with the players in some configuration at time t, *must* play out in such a way that the players return to exactly the same configuration when they come back around to time t again. This kind of constraint on football players may seem

[2] A *Killing field* is a vector field that generates a local one-parameter group of isometries.

unmotivated, but it is required if the spacetime is to be consistent. Nevertheless, it seems worth heeding the advice of Professor Newman on matters such as these:

> Certainly, if the universe does contain closed timelike curves, a revision of fundamental premises of physics, and philosophy, may be necessary. However, to dismiss this, and other forms of causality violation, out of hand is reminiscent of the dogmatism regarding singularities prior to the singularity theorems. (Newman 1989, 982.)

Let us then follow Newman's advice, and refrain from dismissing CTCs just yet. Certainly, Gödel's spacetime is not a viable source of CTCs, because the universe ostensibly does not consist of a morass of spinning dust particles. But instead of asking whether or not CTCs *currently* exist, let's instead ask: could it *ever* be possible to travel on a CTC?

3 Could Anyone Ever Travel on a CTC?

Gödel's solution shows that the laws of physics permit closed timelike curves. But if anyone is to ever travel on a CTC, it would have to be the case that the universe we *currently observe* is compatible with a CTC-inducing spacetime. This is indeed the case, although we might have to wait a long time and be very lucky. It might be the case that our universe just happens to produce CTCs, for example through something like the following.

Assume there is a region U of spacetime that is approximately Minkowski. (That is, begin with a region of spacetime that can be isometrically embedded in $(\mathbb{R}^4, \eta_{\mu\nu})$, where $\eta_{\mu\nu}$ is the Minkowski metric.) Such a region is clearly compatible with the current state of the Universe, in small regions far away from any matter source. Assume that a time orientation has been chosen for this region. The future of an observer in the region U is restricted to $\mathcal{I}^+(U)$, the union of all timelike curves that intersect U and lie in its future. But, there are at least two possibilities as to what the immediate future has in store for such an observer. $\mathcal{I}^+(U)$ might simply be Minkowski, in which case there are no CTCs. But it also be Deustch-Politzer, in which case there *are* CTCs.

Deustch-Politzer spacetime is an exact solution to the EFEs proposed in 1991 by Deustch [1] and Politzer [7]. It is formed by cutting up Minkowski spacetime and gluing

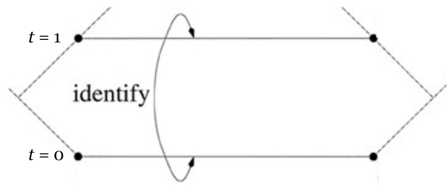

Figure 2: Deustch-Politzer spacetime in two dimensions is formed by identifying top-lip to bottom-lip, bottom-lip to top-lip.

it back together so that it has a handle. We begin with a Cartesian coordinate system t, $\mathbf{x}$ on a patch U of $(\mathbb{R}^4, \eta_{\mu\nu})$, and delete two spatial spheres: first, $t = 0$, $|\mathbf{x}| < 1$; second, $t = 1$, $|\mathbf{x}| < 1$. We then identify the upper side of the first ball with the lower side of the second ball, and vice versa. That is, for a positive infinitesimal quantity ϵ, we identify the points $t = \epsilon$, $|\mathbf{x}| \leq 1$ with the points $t = 1 - \epsilon$, $|\mathbf{x}| \leq 1$ and the points $t = 1 + \epsilon$, $|\mathbf{x}| \leq 1$ with the points $t = 1 + \epsilon$, $|\mathbf{x}| \leq 1$ with the same $\mathbf{x}$ values. Suppressing two spatial dimensions, Deustch-Politzer spacetime can be illustrated as in Figure 2. A observer that begins in U and passes into the handle will enter a region in which there are closed timelike curves, in a way that is perfectly compatible with the current state of the Universe. Both Minkowski spacetime and Deustch-Politzer spacetime are permissible vacuum solutions to the EFEs. However, from the perspective of an observer in U, there is simply no guarantee which extension of U he lies in: Minkowski, or Deustch-Politzer. So although it now seems possible that closed timelike curves *might* appear in our Universe, we have not yet seen any way to guarantee this would be the case.

4 Is it Possible to Create CTCs?

Human beings are very capable of manipulating matter; hence, by the EFEs, people are capable of manipulating spacetime geometry. Might it be that we (or some more advanced civilization) could *create* a situation akin to Gödel's universe? That is, might it be possible to create a machine that manipulates matter in such a way that CTCs are created somewhere in the future?

A suggestion in the affirmative began (perhaps appropriately) with a work of science fiction. While writing the novel *Contact*, Carl Sagan asked Kip Thorne if it were possible to create a situation one could rapidly travel large interstellar distances. Thorne's answer, which he later published with Michael Morris [6], was yes. Morris and Thorne suggested that an advanced civilization might be able to induce just the right densities of matter so as to create a handle in spacetime, across which one could travel long distances in very little time. They famously dubbed the resulting class of solutions to the EFEs *wormhole spacetimes*. But these spacetimes admitted another interesting feature. Amazingly, Morris, Thorne and Yurtsever discovered that wormhole spacetimes can quite generally be modified so as to admit CTCs.

Here's a very general picture how wormholes spacetimes can lead to CTCs. For coordinates t, l, θ, φ, the line element of a typical (static, spherical) wormhole spacetime has the form,

$$ds^2 = e^{2\Phi}dt^2 - dl^2 - r^2(d\theta^2 - sin^2\theta d\varphi^2),$$

where Φ and r are both functions of proper radial distance l. Like Schwartzschild spacetime (to which this construction is very similar), wormhole spacetime is asymptotically flat. The 'throat' of the wormhole has the value $l = 0$, so that $l < 0$ on one side of the throat, and $l > 0$ on the other side. From this, one readily computes $R^{\alpha}{}_{\beta\gamma\delta}$ (by first computing the Christoffel symbols), which contracts to give the Ricci and scalar curvature tensors needed on the left-hand side of the EFEs. On the right-hand side, we adopt

$$T_{\mu\nu} = \mathrm{diag}\left[\rho(r), -\tau(r), p(r), p(r)\right],$$

where ρ is mass-energy density, τ is tension per unit area (in the radial direction), and p is the pressure in directions orthogonal to the radial. The most severe constraint that the curvature tensors impose on this matter field (via the EFEs) is an enormous tension τ at each mouth of the wormhole. This tension, equal to approximately $10^{37}\mathrm{dyn/cm}^2$, is of the same order as the pressure near the center of some of the largest known stars. Its difficult to imagine someone actually manipulating energy on this scale. But it is crucial to the existence of wormholes, since without this enormous pressure, the matter inside the throat would collapse.

Now, we still haven't described a situation in which CTCs are created. To create

CTCs, we now send one mouth of the wormhole (say, the right side) quickly accelerating away from the left side, stretching the wormhole out with it. This right-mouth then comes to a stop, and finally accelerates back to where it started, contracting the wormhole into its original form. This motion is described by multiplying the t-component of the above metric by a factor $(1 + alF\cos\theta)$. In this factor, $F(l)$ is a function of l that vanishes at the left mouth $l \leq 0$, and increases smoothly to 1 as one moves towards the right mouth, and $a(t)$ is a function of t characterizing the acceleration of the right mouth in its own (asymptotic) rest frame.

The construction is very similar to a well-known thought experiment in relativity theory, called the *twin problem*. Given two twins, let one twin remain at rest relative to the left-mouth, and let the other twin accelerate in accordance with the motion of the right-mouth. Then, by the time the right-mouth returns to its original position, the accelerating twin will have 'aged less.' (That is, less proper time will elapse for the accelerating twin.) Thus, suppose some observer traverses the wormhole handle from right to left towards the end of this procedure. Due to the short length of the handle, she will come out at an earlier time than when she started, and thus be capable of looping back around to the moment that she began her journey. This construction thus allows for closed timelike curves[3].

5 Can CTC-Creation Still Be Outlawed?

Clearly, the kind of scenario described above is far beyond our practical ability to manipulate energy, even before we start stretching and contracting the wormhole at relativistic speeds. Moreover, as Morris and Thorne [6] note, stress-energy tensor of this particular wormhole spacetime requires negative energy densities, which violates a well-regarded energy principle called the *weak energy condition*[4]. If we're willing to elevate this principle to a law of the Universe that must be obeyed, Morris and Thorne's wormhole spacetime seems to be impossible in principle.

[3] A similar construction can also be carried out using two wormholes that move quickly past each other, as noted in [3].

[4] The *weak energy condition* states that at any point $p \in \mathcal{M}$, and for any vector $v^\mu \in T_p\mathcal{M}$, the stress-energy tensor obeys the inequality, $0 \leq T_{\mu\nu}v^\mu v^\nu$ [3].

One might still think that some *other* clever spacetime might allow for the creation of CTCs. However, Stephen Hawking [4] has argued against this claim, famously conjecturing the following:

Chronology Protection Conjecture. *The laws of physics do not allow the appearance of closed timelike curves.*

Hawking's argument has two parts. In the first part, Hawking argues that in any spacetime that allows the creation of CTCs, an observer can *only* take advantage of the CTCs he creates if the weak energy condition is violated. In the second part, Hawking considers the possibility that the weak energy condition might actually be false, as suggested by certain effects in quantum field theory (like the Casimir effect). He argues that, even when such quantum effects are taken into account, the resulting stress-energy tensor does not allow for the creation of CTCs. Let's take a brief look at the first (non-quantum) part of Hawking's argument.

There are two interpretive commitments that make the argument work. The first is: *if it is possible to create CTCs, then they are the result of data on a non-compact partial Cauchy surface S.* A partial Cauchy surface is a spacelike surface with no edge, which is intersected by a timelike or a lightlike curve no more than once. This claim is meant to capture the minimal requirement that we must begin our endeavor with some initial data surface; that is, our CTC-machine must have some initial surface on which we can program in the instructions for creating CTCs. Moreover, since we can only build a machine in a finite local region of spacetime, Hawking requires that S be non-compact.

The second commitment is: *if the data on S leads to the creation of a CTC, then the future Cauchy horizon $H^+(S)$ is compactly generated.* Intuitively, the future Cauchy Horizon $H^+(S)$ is the boundary that separates the future region in which S can have a determinate causal influence, from the future region in which it can't. More precisely, it is the boundary of the future domain of dependence $D^+(S)$, which is the set of all points $p \in \mathcal{M}$ such that every past-inextendible non-spacelike curve through p intersects S [3, 201]. We say that $H^+(S)$ is compactly generated if the past-directed lightlight geodesics generating $H^+(S)$, when followed back into the past, enter and remain in a compact region C.

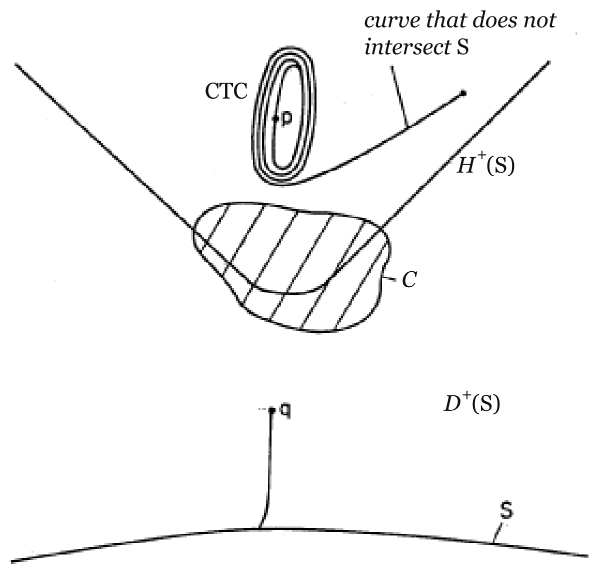

Figure 3: A compactly generated Cauchy horizon with CTCs in its future.

The idea behind this second requirement is the following. One first notices that, embarrasingly, a CTC *cannot* appear where we would like it to, in the domain of dependence $D^+(S)$ of S. For this would mean that the CTC intersects S, and would then have to do intersect it only once (since S is Cauchy), which it cannot if it is really a CTC. Therefore, there must by a nontrivial boundary $H^+(S)$ in a CTC-creating spacetime, such that any CTCs created on S are to the future of that boundary. But now we need to make sense of how it is that the CTCs are the result of *something that happened on S*. Thus, Hawking imposes the minimal requirement that the generators of S do not come in from infinity, or from a singularity – that they can be followed to a compact region, part of which will be in the domain of S. Hawking illustrates the situation as in Figure 3.

What Hawking is then able to show is that, if $H^+(S)$ is compactly generated, and if the EFEs together with the weak energy condition are true, then S must be compact, and thus not a viable region in which to build a CTC-machine. This indeed means that the the above two commitments are incompatible. However, one might relax the first requirement, and willingly consider building a time machine on a compact region S. Hawking is still able to show that, even if we thought we could build a machine on a compact region S, it is still the case that the convergence and the shear of a generator of $H^+(S)$ would vanish everywhere on $H^+(S)$. He interprets this result: "This would mean that no matter or information, and in particular no observers, could cross the Cauchy horizon into the region of closed timelike curves" [4, 606].

In spite of these strong results, it seems that Hawking's result is not conclusive. Of course, his final observation does mean that a CTC-machine operator could not *herself* enjoy the fruits of her labors. But this does not prevent another observer beginning in a different region of spacetime from exploring the CTCs. In response to this kind of suggestion, it's worth briefly reviewing one more recent no-go result about CTC-creation, which seeks to overcome this problem.

S. Krasnikov [5] construes the possibility of a CTC-machine somewhat differently. The two requirements that he proposes for such a machine are that (i) there exist no CTCs in the past $\mathcal{I}^-(U)$ of U; and (ii) every maximal extension of U contains a CTC. The first requirement is meant to avoid picking up on CTCs that were already in existence before the machine on U was programmed. (Note that U might or might not be compact.) The second requirement is a clever way to make sense of how U is actually *responsible* for the production of CTCs, even though they do not lie in its domain of dependence. Krasnikov is then able to give a constructive proof of the following:

Theorem. *An arbitary, extendable spacetime U (that satisfies any local energy conditions you like, call them C) admits a maximal extension U' (which also satisfies C) such that closed timelike or likelike curves in U' are confined to the past of U, $\mathcal{I}^-(U)$.*

Given Krasnikov's interpretation of a CTC-machine, his theorem seems to effectively rule such machines out. At the very least, it seems that we can have no guarantee that our putative CTC-machine will actually do its job.

However, we must not be too quick in this judgement, before examining what kind of extension Krasnikov actually constructs. The crucial and most interesting part of Krasnikovs result is the construction of a preliminary, non-maximal extension of U that he calls M_N, with the properties that (1) no closed timelike curves are properly contained in the causal future of M_N, and (2) this extension can be repeated, in such a way that an inductive ordering is created on the resulting family of extensions. It is then a consequence of Zorn's lemma that U admits a maximal extension with the desired property. The sticking point is that Krasnikov's key extension also admits a naked singularity. As a result, as the process of extending is repeated, the maximal extension U' acquires countably many singularities, making it a very strange maximal extension indeed.

For a physicist still hopeful about building a CTC-machine, this should be good news. One might dub such a robustly singular spacetime as 'unphysical,' and thus rule it out as impossible or unlikely. (This is admittedly a strange move when one is trying to create CTCs.) As far as I can tell, it still remains to be shown whether or not a CTC-machine might be possible once these kinds of extensions have been ruled out.

6 Conclusion

We now have a taste for what closed timelike curves involve in general relativity. As we have seen, the possibility of CTCs is tied up in a number of interrelated topics: the topology of spacetime, the local energy conditions we choose to impose, and the lightcone structure that the spacetime metric assigns. As a result, there appear to be many open questions still remaining. Are there generic constraints on the topology of CTC-machine spacetimes? Can CTC-machine spacetimes be shown to exist once an interesting class of 'unphysical' extensions has been ruled out? Unfortunately, such questions now lie outside the scope of the current paper.

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