Abstract—The task of tracking extended objects or (partly) unresolvable group targets raises new challenges for both data association and track maintenance. Extended objects may give rise to more than one detection per opportunity where the scattering centers may vary from scan to scan. On the other end, group targets (i.e., a number of closely spaced targets moving in a coordinated fashion) often will not cause as many detections as there are individual targets in the group due to limited sensor resolution capabilities. In both cases, tracking and data association under the one target–one detection assumption are no longer applicable. This paper deals with the problem of maintaining a track for an extended object or group target with varying number of detections. Herein, object extension is represented by a random symmetric positive definite matrix. A recently published Bayesian approach to tackling this problem is analyzed and discussed. From there, a new approach is derived that is expected to overcome some of the weaknesses the Bayesian approach suffers from in certain applications.

Keywords: Target tracking, extended targets, group targets, formations, sensor resolution, random matrices, matrix-variate analysis.

I. INTRODUCTION

In many tracking applications, the objects to be tracked are considered as point sources, i.e., their extension is assumed to be neglectable in comparison with sensor resolution and error. With ever increasing sensor resolution capabilities however, this assumption is no longer valid, e.g., in short-range applications or for maritime surveillance where different scattering centers of the objects under consideration may give rise to several distinct detections varying, from scan to scan, in both number as well as relative origin location. From the associated data—assuming that the related association problem has been solved—one cannot only estimate the kinematic state of the object but also its extension (honoring the spread of the data in comparison with the expected statistical sensor error). But, more than these quantities cannot safely be estimated as well in the (opposite) case where limited sensor resolution causes a fluctuating number of detections for a group of closely spaced targets and thus prevents a successful tracking of (all of) the individual targets.

Several suggestions for dealing with this problem can be found in literature. For an early work, see [1], for an overview of existing work up to 2004, refer to [2]. In particular, the probability hypothesis density (PHD) filter [3]–[5] has been suggested as one possibility to track an unknown number of possibly closely spaced targets while simultaneously solving the association problem in an approximate yet very efficient manner, especially in its Gaussian mixture implementation [6], [7]. By basically estimating the expected number of targets per unit volume however, this filter inherently does not differ between what part of the (hypothesis) density spread is due to estimation uncertainty and what amount is due to an actual physical extension of the extended object or group. But, exactly the discrimination between kinematic state (a random vector) on the one hand and physical extension (represented by a random matrix) on the other is the distinctive feature of the Bayesian approach suggested in [8], [9]. Under a couple of assumptions to be discussed later on, this approach provides a closed form solution to the complete Bayesian estimation problem and also delivers quantities to perform measurement gating. It does not, however, completely solve the association problem which is also beyond the scope of this paper. We rather concentrate on the track maintenance as well and try to circumvent some of the problems one may face when applying the Bayesian group tracking approach under circumstances where the aforementioned assumptions do not hold.

The paper is organized as follows: We start with summarizing known facts about the Kalman filter and recall the concept of using conjugate prior densities for tracking. Subsequently, we sketch how an application of the very same concept leads to the filter equations of the Bayesian group tracking algorithm. We then take a closer look on these equations as well as the assumptions under which they have been derived and thus motivate the need for improvement. Finally, we propose a new approach that we feel to be promising especially when it comes to tracking of formations.

II. KALMAN FILTER

Consider a discrete-time linear dynamic system of the form

$$x_{k+1} = Fx_k + v_k, \quad y_k = Hx_k + w_k$$

where the random vector $x_k$ denotes the state to be estimated (for us, position and velocity in two or three spatial dimensions, i.e., $x^T = [r^T, \dot{r}^T]$), but one may add acceleration $\ddot{r}^T$ here, of course) and $y_k$ the actual measurement (in the following, position only, i.e., $H = [I_d, 0_d]$ with $d = 2, 3$).
The additional noises $v_k$ and $w_k$ are assumed to be zero mean normally distributed random vectors with process noise variance $Q$ and measurement noise variance $R$, respectively. Furthermore, assume that there is no correlation between $x_k$, $v_k$, and $w_k$ as well as between any $v_k$ or $w_k$ and $v_l$ or $w_l$ with $k \neq l$. It is well-known that, under these assumptions, the Kalman filter constitutes the recursive minimum mean square error (MMSE) estimator for the state $x_k$, i.e., the estimate $x_k|\ell$ for time $k$ based on all measurements up to and including time $\ell \leq k$ minimizes the mean square error with corresponding estimation error variance $P_k|\ell$ for an initial estimate $x_{0|0}$ with variance $P_{0|0}$ if one uses
\begin{align}
x_k|k-1 &= Fx_{k-1|k-1} \\
P_{k|k-1} &= FP_{k-1|k-1}F^T + Q \tag{2}
\end{align}
for prediction and
\begin{align}
x_k|k &= x_k|k-1 + K_k|k-1 [y_k - Hx_k|k-1] \\
P_k|k &= P_k|k-1 - K_k|k-1 P_{k-1|k-1} K_k|k-1^T \tag{4}
\end{align}
with the innovation $y_k - Hx_k|k-1$ and the corresponding covariance
\begin{equation}
S_k|k-1 = HP_k|k-1 H^T + R \tag{6}
\end{equation}
for update (see, e.g., [10]). Optimality in the MSE sense follows from the fact that the estimate $x_k|\ell$ is the conditional mean of the state $x_k$ given the sequence $Y_{\ell} := \{y_j\}_{j=0}^{\ell}$, e.g.,
\begin{equation}
x_k|k = E[x_k|Y_{\ell}] = \int x_k p(x_k|Y_k) \, dx_k \tag{8}
\end{equation}
One possible known way of deriving the Kalman filter equations consists of applying the concept of conjugate prior densities. To this end, one starts with obtaining from eq. (1) the measurement likelihood
\begin{equation}
p(y_k|x_k) = \mathcal{N}(y_k; Hx_k, R) \tag{9}
\end{equation}
where
\begin{equation}
\mathcal{N}(x; \mu, \Sigma) = \frac{\exp\left(-\frac{1}{2}[x - \mu]^T \Sigma^{-1} [x - \mu]/2\right)}{\sqrt{2\pi} |\Sigma|} \tag{10}
\end{equation}
denotes the normal density with mean $\mu$ and variance $\Sigma$. Now, one selects a (conditional) prior density $p(x_k|Y_{k-1})$ in such a way that the posterior density $p(x_k|Y_k)$ represents the same kind of density as the prior (with updated parameters). One easily confirms that such a conjugate prior to the normal density (9) is again a normal density: Choosing the prior $p(x_k|Y_{k-1}) = \mathcal{N}(x_k; x_k|k-1, P_k|k-1)$ and evaluating Bayes’ rule $p(x_k|Y_k) = p(x_k|y_k, Y_{k-1}) \propto p(y_k|x_k) p(x_k|Y_{k-1})$, a completion of squares of $x_k$ in the exponent in combination with the matrix inversion lemma readily shows
\begin{equation}
\mathcal{N}(y_k; Hx_k, R) \mathcal{N}(x_k; x_k|k-1, P_k|k-1) = \mathcal{N}(y_k; Hx_k|k-1, S_k|k-1) \mathcal{N}(x_k; x_k|k, P_k|k) \tag{11}
\end{equation}
and thus $p(x_k|Y_k) = \mathcal{N}(x_k; x_k|k, P_k|k)$ with parameters as given in eqs. (4) to (7). In a similar manner, it can be shown that with $p(x_{k-1}|Y_{k-1})$ the prediction $p(x_k|Y_{k-1})$ is normal as well with predicted parameters (2) and (3). To this end, one evaluates the Chapman-Kolmogorov theorem $p(x_k|Y_{k-1}) = \int p(x_k|x_{k-1}) p(x_{k-1}|Y_{k-1}) \, dx_{k-1}$ in combination with, see eq. (1) again, $p(x_k|x_{k-1}) = \mathcal{N}(x_k; FX_k-1, Q)$

III. BAYESIAN EXTENDED OBJECT TRACKING

As has been stated in the introduction, the Bayesian approach to tracking extended objects and group targets in [8], [9] adds to the kinematic state of the centroid described by the random vector $x_k$ the physical extension represented by a symmetric positive definite (SPD) random matrix $X_k$ thus considering some ellipsoidal shape. It is assumed that in each scan $k$ there are $n_k$ independent position measurements
\begin{equation}
y_k^j = Hx_k + w_k^j \tag{12}
\end{equation}
in analogy to eq. (1) where we will use the abbreviations $Y_k := \{y_k^j\}_{j=1}^{n_k}$ and $y_k := \{y_n\}_{n=0}^{n_k}$ to denote the set of the $n_k$ measurements in a particular scan and for the sequence of what is measured scan by scan, respectively. Now, one decisive assumption in [8], [9] is that the statistical error of each individual measurement $y_k^j$ is small and thus neglectable in comparison with the object extension that hence dominates the spread of the measurements (an assumption that certainly needs discussion later on). In detail, the measurement noise $w_k^j$ is assumed to be a zero mean normally distributed random vector with variance $X_k$. With this, the likelihood to measure the set $Y_k$ given both kinematic state and extension as well as the number of measurements, reads
\begin{equation}
p(Y_k|n_k, x_k, X_k) = \prod_{j=1}^{n_k} \mathcal{N}(y_k^j; Hx_k, X_k) \tag{13}
\end{equation}
For the sake of simplicity, it is assumed in the following that this expression is directly proportional to the conditional likelihood of what is measured in scan $k$,
\begin{equation}
p(Y_k, n_k|x_k, X_k) = p(Y_k|n_k, x_k, X_k) p(n_k|x_k, X_k) \tag{14}
\end{equation}
i.e., all numbers $n_k$ of measurements are assumed to be equally likely. A refined version even including the dependence of the number of measurements on the extension can be found in [8], [9]. Introducing the mean measurement and the measurement spread
\begin{equation}
\overline{y}_k = \frac{1}{n_k} \sum_{j=1}^{n_k} y_k^j, \quad \underline{y}_k = \left(\sum_{j=1}^{n_k} (y_k^j - \overline{y}_k)(y_k^j - \overline{y}_k)^T\right) \tag{15}
\end{equation}
and using the identities
\begin{equation}
\begin{split}
\sum_{j=1}^{n_k} (y_k^j - Hx_k)^T X_k^{-1} (y_k^j - Hx_k) &= n_k \langle \overline{y}_k - Hx_k \rangle^T X_k^{-1} (\overline{y}_k - Hx_k) \\
+ \left(\sum_{j=1}^{n_k} (y_k^j - \overline{y}_k)^T X_k^{-1} (y_k^j - \overline{y}_k)\right) &= n_k \left(\overline{y}_k - Hx_k\right)^T \left(\overline{y}_k - Hx_k\right) + \langle \overline{y}_k \rangle X_k^{-1}
\end{split} \tag{16}
\end{equation}
one shows that eq. (13) can be written as
\[ p(Y_k|n_k, x_k, X_k) \propto \mathcal{N}(\bar{y}_k; Hx_k, \Sigma_k) \times \mathcal{W}(X_k; n_k - 1, X_k) \] (17)
where
\[ \mathcal{W}(X; m, C) \propto |C|^{-\frac{m}{2}} |X|^{-\frac{m+d+1}{2}} \exp(-\frac{1}{2}XC^{-1}) \] (18)
with \( m \geq d \) denotes the Wishart density [11] of a \( d \)-dimensional SPD random matrix \( X \) with expected SPD matrix \( mC \) and \( \exptr(\cdot) \) is an abbreviation for \( \exp(\text{tr}(\cdot)) \).

For constructing a suitable conjugate prior, the product form of eq. (17) suggests to use a factorized approach as well, i.e., to write the prior according to
\[ p(x_k|y_{k-1}) = p(x_k|x_k, y_{k-1}) p(x_k|y_{k-1}) \] (19)
(and the posterior in an analogue way) where all densities are to share the common right-hand factor \( X_k^{-1} \) in the argument of \( \exptr(\cdot) \) as suggested by eq. (16). For the matrix variate density,
\[ p(x_k|y_{k-1}) = \mathcal{IW}(X_k; \nu_k|k-1, \bar{X}_{k|k-1}) \] (20)
is an obvious choice where the inverse Wishart density [11] with parameterization
\[ \mathcal{IW}(X; m, C) \propto |C|^m |X|^{-\frac{m+d+1}{2}} \exptr(-\frac{1}{2}CX^{-1}) \] (21)
possesses the expected SPD matrix \( C/(m - d - 1) \) for \( m - d - 1 > 0 \). For the vector variate density, the approach
\[ p(x_k|y_{k-1}) = \mathcal{N}(x_k; x_{k|k-1}, \bar{P}_{k|k-1} \otimes X_k) \] (22)
as proposed in [8], [9] provides the (not quite as obvious) solution. Herein, \( \otimes \) denotes the Kronecker product [12] that, e.g., yields
\[ \begin{bmatrix} \tilde{p}_{11} \\ \tilde{p}_{21} \\ \tilde{p}_{22} \end{bmatrix} \otimes X_k = \begin{bmatrix} \tilde{p}_{11}X_k \\ \tilde{p}_{21}X_k \\ \tilde{p}_{22}X_k \end{bmatrix} \] (23)

The derivation of the update equations begins with multiplying the two normal distributions of eqs. (17) and (22). The result can be immediately taken from eq. (11) and thus from eqs. (4) to (7) if one makes the following formal replacements:
\[ R = \frac{X_k}{n_k}, \quad P_{k|k-1} = \bar{P}_{k|k-1} \otimes X_k \] (24)
However, the quantities in eqs. (4) to (7) now partly depend on \( X_k \). But application of various identities holding for the Kronecker product in combination with the fact that one has
\[ H = [I_d, 0_d] = \bar{H} \otimes I_d \quad \text{with} \quad \bar{H} = [1, 0] \] (25)
yields (intermediate results omitted)
\[ S_{k|k-1} = \tilde{S}_{k|k-1} X_k, \quad S_{k|k-1} = \bar{H} \tilde{P}_{k|k-1} \bar{H}^T + \frac{1}{n_k} \] (26)
and hence a scalar innovation variance \( \tilde{S}_{k|k-1} \) as well as
\[ K_{k|k} = \tilde{K}_{k|k-1} \otimes I_d, \quad \tilde{K}_{k|k} = \bar{P}_{k|k-1} \bar{H}^T \tilde{S}_{k|k-1} \] (27)
and thus a gain that is actually independent of \( X_k \). As one can also show \( P_{k|k} = (\bar{P}_{k|k-1} - \tilde{K}_{k|k-1} \tilde{S}_{k|k-1} \bar{P}_{k|k-1}) \otimes X_k \) to hold, it is thus confirmed that the vector variant density of the posterior can be written in analogy to eq. (22) with updated parameters
\[ x_{k|k} = x_{k|k-1} + (\tilde{K}_{k|k-1} \otimes I_d)(y_k - (\bar{H} \otimes I_d)x_{k|k-1}) \]
\[ P_{k|k} = P_{k|k-1} - \tilde{K}_{k|k-1} \tilde{S}_{k|k-1} \bar{P}_{k|k-1} \] (28)
The update equations for the matrix variate density of the posterior finally follow directly from comparing the exponents of \( X_k \) on the one hand and those arguments of \( \exptr(\cdot) \) that are independent of the kinematic state on the other. The former immediately implies
\[ \nu_{k|k} = \nu_{k|k-1} + n_k \] (29)
while the latter leads to
\[ \bar{X}_{k|k} = \bar{X}_{k|k-1} + \tilde{S}_{k|k-1}^{-1} N_{k|k-1} + \bar{Y}_k \] (30)
with
\[ N_{k|k-1} = (\bar{y}_k - \bar{H}x_{k|k-1})(\bar{y}_k - \bar{H}x_{k|k-1})^T \] (31)
where, in addition to the prior and the contribution from the Wishart density in eq. (17), the contribution of the term \( \mathcal{N}(y_k; \bar{H}x_{k|k-1}, \tilde{S}_{k|k-1} x_{k|k} \bar{X}_k) \) stemming from eqs. (11) and (26) has to be honored. Eqs. (26) to (31) build up the complete set of update equations of the Bayesian extended object tracking approach.

At this point, it should be noted that the parameters of the density \( p(x_k, x_k, Y_k) \) do not directly constitute all sought quantities as, e.g., the extension estimate or the error variance of the kinematics estimate. The former can be immediately deduced from \( p(X_k|Y_k) = \mathcal{IW}(X_k; \nu_k|k, \bar{X}_{k|k}) \) as
\[ X_{k|k} := E[X_k|Y_k] = \frac{\bar{X}_{k|k}}{\nu_k|k - d - 1} \] (32)
The latter (being neither \( P_{k|k} \) nor \( \bar{P}_{k|k} \)) has to be obtained from the marginal density
\[ p(x_k|Y_k) = \int p(x_k, x_k, Y_k) dX_k \] (33)
that, after some tedious computations, turns out to be
\[ p(x_k|Y_k) = \mathcal{T}(x_k; \nu_k|k + s - sd, x_{k|k}, \bar{P}_{k|k} \otimes \bar{X}_{k|k}) \] (34)
where \( s \) is either 2 if the state consists of position and velocity alone or 3 if one also considers acceleration therein and
\[ \mathcal{T}(x; m, \mu, C) \propto (1 + (x - \mu)^T C^{-1}(x - \mu))^{-\frac{m}{2}} \] (35)
is the Student-\( t \) density [11] of a \( p \)-dimensional random vector \( x \) with mean \( \mu \) and variance \( \frac{1}{m-2} C \) for \( m \geq 2 \) and a \( p \)-dimensional SPD matrix \( C \) (with \( p = sd \)). With this, one confirms that \( x_{k|k} \) is indeed the kinematics estimate (8) and that it leads to the error variance
\[ E[(x_k - x_{k|k})(x_k - x_{k|k})^T|Y_k] = \frac{1}{\nu_k|k + s - sd - 2} (\bar{P}_{k|k} \otimes \bar{X}_{k|k}) \] (36)
Now, for a complete Bayesian tracking cycle, it is necessary that the kind of the posterior respectively the prior also is preserved in the prediction step. Under fairly general assumptions to be found in [8], [9], in particular with an evolution of the extension that does not depend on the kinematics, the Chapman-Kolmogorov theorem, integrating over both $x_k$ and $\mathbf{X}_k$, decouples into two separated integrals. Assuming in $p(x_k|\mathbf{x}_{k-1}, \mathbf{Y}_{k-1}) = \int p(x_k|\mathbf{x}_{k-1}, \mathbf{X}_{k-1}) p(\mathbf{x}_{k-1}|\mathbf{X}_{k-1}) d\mathbf{x}_{k-1}$ a normal density $p(x_k|\mathbf{x}_{k-1}, \mathbf{X}_{k-1})$, one immediately deduces from eqs. (2) and (3) the parameters of $p(x_k|\mathbf{x}_{k-1}, \mathbf{Y}_{k-1})$. From there, it becomes clear that the kind of conditional density for the kinematic state (i.e., normal with the special form $\mathbf{P}_{k|k-1} \otimes \mathbf{X}_k$ of the covariance) is only preserved if there holds

$$F = \overline{F} \otimes I_d, \quad Q = \overline{Q} \otimes \mathbf{X}_k$$  \hspace{1cm} (37)

where $\overline{F}$ and $\overline{Q}$ are state transition matrix and process noise variance, respectively, of a corresponding movement in one spatial dimension. With this, the prediction equations for the kinematic state read

$$\mathbf{x}_{k|k-1} = (\overline{F} \otimes I_d) \mathbf{x}_{k-1|k-1}$$

$$\mathbf{P}_{k|k-1} = \overline{F} \mathbf{P}_{k-1|k-1} \overline{F}^T + \overline{Q}$$  \hspace{1cm} (38)

For $p(\mathbf{X}_k|\mathbf{Y}_{k-1}) = \int p(\mathbf{X}_k|\mathbf{X}_{k-1}) p(\mathbf{X}_{k-1}|\mathbf{Y}_{k-1}) d\mathbf{X}_{k-1}$, the density $p(\mathbf{X}_k|\mathbf{X}_{k-1})$ to make $p(\mathbf{X}_k|\mathbf{Y}_{k-1})$ distributed according to an inverse Wishart density again is unknown. In [8], [9], a heuristic approximation has been suggested (as well as a slightly more sophisticated variant that we do not consider here). Noting from eq. (29) that an increased number of measurements and thus a higher certainty about the object extension. Whether this assumption can safely be made certainly depends on the specific application.

In order to investigate the performance of the Bayesian extended object tracking algorithm, we have simulated a formation of 5 individual targets flying with constant speed $v = 300 \text{ m/s}$ in the $(x, y)$-plane. The targets were arranged in a line with $500 \text{ m}$ distance between neighboring targets where the formation first went through a $45^\circ$ and two $90^\circ$ turns (with radial accelerations $2g$, $2g$, and $g$, respectively) before performing a split-off maneuver. This formation was observed by a (fictitious) sensor with scan time $T = 10\text{s}$ delivering uncorrelated noisy $x$- and $y$-measurements with standard deviations $\sigma_x$ and $\sigma_y$, respectively, where we assumed a probability of detection $P_d = 80\%$ for each individual target (not considering the problem of limited sensor resolution here). With this, the true measurement likelihood is a Gaussian mixture and the assumption (13) can only be a (good or bad) approximation thereof.

Fig. 1 shows the tracking results of the algorithm for sensors with two different sets of sensor accuracies (taking identical sequences of unit variance normally distributed random values to generate the measurement errors). We have used a constant velocity dynamic model where the process noise term $Q$ was chosen according to a white acceleration model [10] and tuned in such a way that the estimator was (just) able to follow the $2g$ turns when using the better sensor. Here, the estimator shows satisfactory results during straight flight phases and turns. Yet, during the split-off maneuver, we note an increasing centroid estimation error, being proportional to the increasing (estimated) group size, that becomes particularly prominent when missed detections occur. There, the estimates of the centroid’s position to a larger extent follow the mean measurements $\overline{y}_k$. This disadvantage of the coupling between centroid estimation error and estimated group size becomes even more obvious for larger sensor errors when the algorithm, with unchanged $Q$, significantly overestimates the object extension (by in fact basically estimating group size plus the ignored sensor error) and thus produces even more increased centroid estimation errors while the orientation change of the formation is hidden by the large error in $x$-direction.

IV. A NEW APPROACH

With the observations of the previous section, we seek a new approach that allows reliable tracking of extended objects and
Figure 1. Group tracking scenario with 5 targets showing, for each time $t_k$, the true target positions ($\triangle$), the true centroid ($\square$), the measurements (+), the (updated) estimated centroid ($\times$) with the corresponding 90%-confidence ellipse (–), as well as the (updated) estimated group extension (–) for the Bayesian extended object tracking algorithm. Sensor errors were $\sigma_x = 100$ m, $\sigma_y = 50$ m (top) and $\sigma_x = 1000$ m, $\sigma_y = 100$ m (bottom).
group targets in cases where sensor errors cannot be ignored when compared with object or group extension. In other words, we now honor that both sensor error and extension contribute to the measurement spread. As an approximation to the true behavior, we use eq. (12) with measurement errors being normally distributed with variance $X_k + R$ which leads to

$$p(Y_k|\hat{X}_k, X_k) = \prod_{j=1}^{n_k} \mathcal{N}(y'_k; Hx_k, X_k + R)$$

(42)

It appears that, for this likelihood, no conjugate prior can be found that is both independent of $R$ and analytically traceable. And, it is no feasible option to merely apply the Bayesian algorithm of the previous section without modification and try to honor the sensor error afterwards by subtracting $R$ from the estimates (32). Apart from the increased estimation error that comes with increasing sensor errors as we have seen before, one cannot ensure that $X_k|k - R$ always remains positive definite. Without a rigorous solution to the problem, we use some approximations in order to derive our new approach.

If the object extension $X_k$ were non-random and known and thus not part of the estimation problem, and if one assumed, in addition, a normal prior $p(x_k|Y_{k-1}) = \mathcal{N}(x_k; x_{k|k-1}, P_{k|k-1})$, the updated estimate of the centroid kinematics could immediately be determined by using the standard Kalman filter update equations of section II with the single measurement $y_k$ replaced by the mean measurement $\bar{y}_k$ and the measurement variance $R$ substituted by the mean measurement’s error variance then being $\frac{X_k + R}{n_k}$. In the following, we will assume that the predicted extension $X_{k|k-1}$ is not too far away from truth, i.e., that one (roughly) has $X_{k|k-1} \approx X_k$. Consequently, we propose to perform the update, herein merely ignoring the uncertainty coming with the predicted estimate, according to

$$X_{k|k} = X_{k|k-1} + K_{k|k-1} (\bar{y}_k - HX_{k|k-1})$$

(43)

$$P_{k|k} = P_{k|k-1} - K_{k|k-1} S_{k|k-1} K_{k|k-1}$$

with

$$S_{k|k-1} = HP_{k|k-1} H^T + \frac{X_{k|k-1} + R}{n_k}$$

(44)

being an approximation of the true innovation covariance and

$$K_{k|k-1} = P_{k|k-1} H^T S_{k|k-1}^{-1}$$

(45)

denoting the corresponding gain. This proposal may be interpreted as approximating the posterior of the kinematic state conditioned on the extension by the respective (in fact unknown) marginalized density that in turn is assumed to be close to a normal density again, i.e.,

$$p(x_k|X_k, Y_k) \approx p(x_k|Y_k) \approx \mathcal{N}(x_k; x_{k|k}, P_{k|k})$$

(46)

Note that eq. (44) may be seen as a variant of eq. (26), but now having a kinematic state prediction error with a full set of parameters and an additional contribution to $S_{k|k-1}$ induced by the sensor error.

For the marginalized prior density of the extension, we will continue to assume an inverse Wishart density as in eq. (20) and that the corresponding (as well in fact unknown) posterior is again of the same form. Here, eq. (30) gives a hint on how the update can (approximately) be performed where we directly look at the update of the estimate $X_{k|k}$ rather than the parameter $\hat{X}_{k|k}$. Just like the Bayesian extended object tracking approach does, we will use a weighted sum of the predicted extension $X_{k|k-1}$, the term $N_{k|k-1}$ of eq. (31) and the measurement spread $Y_k$ of eq. (15). But, we will rather weight two of these terms with matrix-valued factors. To this end, we again consider the limiting case of a non-random $X_k$ and a prediction $X_{k|k-1}$ being (about) the same. More precisely, we note that the quantity $N_{k|k-1}$ obeys

$$E[N_{k|k-1}|\mathbf{X}_{k-1}, \mathbf{X}_k = X_{k|k-1}] = S_{k|k-1}$$

(47)

with $S_{k|k-1}$ as given in eq. (44) and, similarly,

$$E[\mathbf{Y}_k|\mathbf{Y}_{k-1}, \mathbf{X}_k = X_{k|k-1}] = (n_k - 1) Y_{k|k-1}$$

(48)

with the predicted variance of a single measurement,

$$Y_{k|k-1} = X_{k|k-1} + R$$

(49)

By an appropriate (matrix-valued) scaling, we generate some quantities $\hat{N}_{k|k-1}$ and $\hat{Y}_{k|k-1}$ that both yield conditional expected matrices being proportional to $X_k = X_{k|k-1}$ while preserving the symmetric positive (semi-)definite structure. This is done by computing some square roots (e.g., via Cholesky factorization) of the matrices $X_{k|k-1}$, $S_{k|k-1}$, and $Y_{k|k-1}$ that obey

$$X_{k|k-1} = X_{k|k-1}^{1/2} (X_{k|k-1}^{1/2})^T$$

(50)

etc. and by setting, in view of eqs. (47) and (48) above,

$$\hat{N}_{k|k-1} = X_{k|k-1}^{1/2} S_{k|k-1}^{-1/2} X_{k|k-1}^{1/2} (X_{k|k-1}^{1/2})^T$$

(51)

$$\hat{Y}_{k|k-1} = X_{k|k-1}^{1/2} Y_{k|k-1} (Y_{k|k-1})^{1/2} (X_{k|k-1}^{1/2})^T$$

Hence, the updated extension estimate

$$X_{k|k} = \frac{1}{\alpha_{k|k}} \left( \alpha_{k|k} X_{k|k-1} + \hat{N}_{k|k-1} + \hat{Y}_{k|k-1} \right)$$

(52)

yields the conditional expected matrix $X_k = X_{k|k-1}$ again, if the parameters $\alpha_{k|k-1}$ and $\alpha_{k|k}$ are related via

$$\alpha_{k|k} = \alpha_{k|k-1} + n_k$$

(53)

We are thus now able to update the extension, too. Note the similarity between the choice (52) and eq. (30) as well as between eqs. (53) and (29).

With the assumed (approximate) independence between the estimates for centroid kinematics and extension expressed in eq. (45) and further assuming independent kinematic models for both of them, prediction becomes easy. For the kinematics, the standard Kalman filter prediction equations of section II can be applied while, for the extension, one may use the same heuristics as in the previous section:

$$X_{k|k-1} = X_{k-1|k-1}$$

$$\alpha_{k|k-1} = \exp \left( -T/\tau \right) \alpha_{k-1|k-1}$$

(54)
In order to wrap up our presentations, we will sketch how our model can be integrated into the well-known interacting multiple model (IMM) approach. Details about the derivation and the different processing steps of an IMM for point targets can be found at various places in literature, e.g., in [10]. Denoting by $\pi_k^{-1/k-1}$ the probability of model $i$ (out of $R$) having been true at a time $k-1$, the processing cycle starts with the computation of the predicted model probabilities

$$ \pi_k^{-1/k-1} = \frac{\pi_k^{-1/k-1} \pi_k^{-1/k-1}}{\sum_{i=1}^{R} \pi_k^{-1/k-1} \pi_k^{-1/k-1}} \text{ where } \pi_k^{-1/k-1} \text{ is the probability of transitioning from model } i \text{ to model } j. $$

Next, the mixing probabilities $\pi_k^{-1/k-1} = \pi_k^{-1/k-1} / \sum_{i=1}^{R} \pi_k^{-1/k-1} / \pi_k^{-1/k-1}$ are computed. With these, we perform the standard interaction step

$$ x_k^{0j} = \sum_{i=1}^{R} \pi_k^{-1/k-1} x_k^{0i} \quad \text{ and } \quad P_k^{0j} = \sum_{i=1}^{R} \pi_k^{-1/k-1} \left\{ P_k^{0i} - x_k^{0i} x_k^{0i}^T \right\} $$

for the centroid kinematics estimate. This step can be motivated by replacing the true resulting (in our case marginal) Gaussian mixture densities by normal densities with the same first and second order moments. A corresponding matching of the second order moments cannot be fully done for the inverse Wishart densities that describe the extension. In view of the fact that the uncertainty of the estimate decreases with increasing $\alpha_k^{-1/k-1}$ (see [9] for a more detailed discussion of a scalar variance measure), we propose to use

$$ x_k^{0j} = \sum_{i=1}^{R} \pi_k^{-1/k-1} x_k^{0i} \quad \text{ and } \quad P_k^{0j} = \sum_{i=1}^{R} \pi_k^{-1/k-1} \left\{ P_k^{0i} / \alpha_k^{-1/k-1} \right\} $$

With these initial values, each model is now individually activated by replacing the true resulting (in our case marginal) for the centroid kinematics estimate. This step can be motivated by replacing the true resulting (in our case marginal) Gaussian mixture densities by normal densities with the same first and second order moments. A corresponding matching of the second order moments cannot be fully done for the inverse Wishart densities that describe the extension.

$$ \alpha_k^{-1/k-1} = \frac{1}{\left\{ x_k^{0j} / \alpha_k^{-1/k-1} \right\}^2} = \sum_{i=1}^{R} \pi_k^{-1/k-1} / \alpha_k^{-1/k-1} $$

With these initial values, each model is now individually updated with the measurements. In order to proceed further, the model-dependent measurement likelihoods $\Lambda_k^{ij/k-1}$ have to be computed next. As a simple ad-hoc approach, one may use the (standard) term

$$ \Lambda_k^{ij/k-1} = N(y_k^T, H x_k^{ij} / \alpha_k^{ij/k-1}) $$

with $S_k^{ij/k-1}$ as in (44) instead of the unknown innovation likelihood. Of course, more stringent approaches would require to honor the measurement spread at this point, too. Now, one obtains the updated model likelihoods according to

$$ \pi_k^{ij/k-1} = \Lambda_k^{ij/k-1} \pi_k^{-1/k-1} / \sum_{i=1}^{R} \Lambda_k^{ij/k-1} \pi_k^{-1/k-1} $$

before finally computing, for output purposes only, the mixed estimates from the updated model estimates in analogy to eqs. (55) and (56) with $\pi_k^{ij/k-1}$ replaced by $\pi_k^{-1/k-1}$.

In order to demonstrate the potential capabilities of the newly derived approach, we have applied an IMM variant to the same simulated data that we have used before. The IMM consisted of two white acceleration models for the kinematics where we have combined a low kinematic process noise with a low extension agility and a high kinematic process noise with some very high extension agility accounting for possibly rapid changes of shape, size, and (absolute) orientation during maneuvers that may be initiated by some lead target.

Fig. 2 clearly shows a significant improvement in comparison with Fig. 1. The new algorithm adopts much faster to the orientation changes after the first two maneuvers while showing smoother centroid position estimates in the non-maneuvering phases of the flight. During split-off, it reacts by far less sensitive to the missed detections. These advantages become especially apparent when comparing the results obtained with the worse sensor. Here, one sees that the new approach can indeed still compensate the sensor error to a large extent and thus, although compensation is not complete, is able to detect the orientation change. Finally, one notes that the position estimation error no longer equals the estimated extension up to a scalar factor.

V. CONCLUSION

A recently published Bayesian approach to tracking of extended objects and group targets has been modified and adopted to yield improved performance for cases where sensor errors, when compared with object extension, cannot be neglected any more. Apart from more intensive performance studies, future work will include the estimation of target numbers within a formation (also accounting for limited sensor resolution by numbers of measurements that depend on object extension) plus, surely the most challenging one, the derivation of sophisticated data association techniques.

REFERENCES


Figure 2. Tracking results of the new algorithm for the same sensor data as in Fig. 1.