Polarimetric and Interferometric SAR Image Partition Into Statistically Homogeneous Regions Based on the Minimization of the Stochastic Complexity

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Abstract—In this paper, we show that polarimetric and interferometric SAR (PolInSAR) images can be efficiently partitioned into homogeneous regions with a statistical technique based on minimization of a parameter-free criterion. This technique consists of finding a polygonal partition of the image that minimizes the stochastic complexity, assuming that the image is made of a tessellation of statistically homogeneous regions. The obtained results demonstrate that a global partition in statistically homogeneous regions of PolInSAR images can provide better results than a partition based on a single characteristic such as polarimetry or interferometry only.

Index Terms—Image segmentation, polarimetric interferometry, SAR interferometry, SAR polarimetry, synthetic aperture radar (SAR).

I. INTRODUCTION

POLARIMETRIC and interferometric synthetic aperture radar (SAR) (PolInSAR) images have been a real advance in the SAR imaging domain since they contain complete information about polarimetry and interferometry. They can reveal the nature of the wave interaction with the ground and the soil topography, respectively [1]–[3]. Major works have contributed to a better understanding of PolInSAR data. Indeed, Cloude and Papathanassiou have derived a formulation of coherent interferometry between polarized waves [4]. Forest and vegetation analysis [5] is one of the main applications; the estimation of forest height using PolInSAR data and the Random Volume over Ground model (RVoG) [6] have notably been given much attention. The RVoG describes the forest as a two-layer coherent scattering model and enables one to estimate the height and the canopy extinction of a forest.

Image-processing techniques such as image segmentation, partition, or classification also enhance our knowledge of SAR images. Many unsupervised or supervised classification methods have been applied on polarimetric image [polarimetric SAR (PolSAR)] since they provide information on the medium imaged by the radar. Indeed, Cloude and Pottier [7] proposed an unsupervised classification based on their target decomposition theorem. More statistical approaches have also been studied, notably by Lee et al. [8]–[10] who used multivariate complex Wishart distribution and integrated the estimation of the coherency matrices in the classification procedure for multilook data. The same methods have also been applied on PolInSAR data [11], [12]. Hoeckman [13], [14] proposed a supervised method that leads to interesting results with a great accuracy and without a large amount of training data, but an accurate ground truth is needed. Image segmentation is also a source of interest for the analysis of SAR image. In the case of PolSAR images, edge detection filters based on likelihood ratio and the Wishart distribution can be applied with constant false alarm rate [15]. Other approaches that estimate the coherency matrices with region-growing techniques [16] and filtering [17] have also been evaluated.

In this paper, we address image partition into statistically homogeneous regions, i.e., regions which have the same polarimetric and interferometric statistical properties. Partition into statistically homogeneous regions can be, for example, the first stage of a segmentation method that will partition an image into semantically coherent regions. We present an algorithm based on a statistical active grid that can be applied to PolInSAR, PolSAR, and InSAR images. It consists of the estimation of the number of regions, the number of nodes, and the segments of the polygonal grid that partitions the image into statistically homogeneous regions. For this purpose, we apply the stochastic complexity minimization principle [18]. The stochastic complexity depends on the grid model that has been chosen to partition the image and on a statistical description of the data. It is composed of two terms: the first one corresponds to the code length needed to describe the data once the model is known (estimated with the log likelihood), and the second term is the code length of the parameters of the model that
depends on the number of nodes, segments, and on the number of parameters of the probability density functions (PDFs). This statistical grid-based approach has already been applied on scalar [19] and on vectorial images whose channels are assumed statistically independent [20]. In this paper, we will extend it to the case of PolInSAR images modeled by correlated Gaussian circular processes. The fundamental question that appears in that case is to determine if this technique is sensitive to the curse of dimensionality which may appear when the number of unknown parameters (here, the 6 × 6 covariance matrix) increases.

We present in Section II-A the image models that we use to describe the InSAR, PolSAR, and PolInSAR data. Then, in Section II-B, we define the information-theory-based criterion that will be used to perform partition of multicomponent SAR images. Application of this principle to such high-dimensional data as PolInSAR images involves specific issues that are addressed in Section III. Namely, we address in Section III-A the PolInSAR matrix estimation and region merging in the special case of very small regions. Then, we describe in Section III-B how the polarimetric contrast and the interferometric contrast can be extracted from PolInSAR data. Section IV is devoted to the application of the statistical grid to the partition in statistically homogeneous regions of real PolInSAR images. We conclude in the last section, and we discuss the perspectives of this method for partitioning PolInSAR images in statistically homogeneous regions based on the minimization of a parameter-free criterion.

II. HOMOGENEOUS POLINSAR PARTITION

In this section, we present the model of multicomponent SAR data, then the criterion used to obtain partitions in statistically homogeneous regions, and, finally, the proposed optimization procedure.

A. PolInSAR, InSAR, and PolSAR Models

The models generally used to partition the different SAR data (PolInSAR, PolSAR, and InSAR) are quite similar since they comprise a multicomponent Gaussian circular process. Indeed, scalar interferometry (InSAR) is based upon the coherency between two complex signals \( S_1 \) and \( S_2 \) with the same polarization. A complex 2-D vector represents the back-scattered signal measured by two antennas: \( \vec{K}_{\text{interf}} = (S_1, S_2)^T \). In fully PolSAR images, the coefficients of the scattering matrix in linear horizontal and vertical basis enable us to define the target vector \( \vec{K}_{\text{pol}} \): It is defined by three parameters considering the reciprocity theorem and is represented by a 3-D vector \( \vec{K}_{\text{pol}} = (1/\sqrt{2})(S_{HH+VV}, S_{HH-VV}, 2S_{HV})^T \). PolInSAR combines the principles of polarimetry and interferometry since two fully PolSAR images \( \vec{K}_{\text{pol1}} \) and \( \vec{K}_{\text{pol2}} \) are measured. It is defined as a complex 6-D vector corresponding to the polarimetric response of both antennas: \( \vec{K}_{\text{polint}} = (\vec{K}_{\text{pol1}}, \vec{K}_{\text{pol2}})^T = (1/\sqrt{2})(S_{1HH+VV}, S_{1HH-VV}, 2S_{1HV}, S_{2HH+VV}, S_{2HH-VV}, 2S_{2HV})^T \). Classically, the vectors \( \vec{K}_{\text{polint}}, \vec{K}_{\text{interf}}, \) and \( \vec{K}_{\text{pol}} \) are measured at each pixel and are assumed circular Gaussian with zero mean and covariance matrices defined as follows:

\[
Y_{\text{pol}} = (\vec{K}_{\text{pol}} \bar{K}_{\text{pol}}^*)
\]

\[
Y_{\text{interf}} = (\vec{K}_{\text{interf}} \bar{K}_{\text{interf}}^*)
\]

\[
Y_{\text{polint}} = (\vec{K}_{\text{polint}} \bar{K}_{\text{polint}}^*)
\]

where \(*\) and \(< >\) denote complex conjugate transpose and ensemble averaging, respectively. In the case of statistically homogeneous, this ensemble averaging can be approximated by spatial averaging. It is not any more the case when the regions are not statistically homogeneous, which appears, for instance, when there is a linear variation of the phase due to a linear variation of the height. In the next section, we use the circular Gaussian-correlated model to determine the stochastic complexity associated to the partition of a multicomponent SAR image.

B. Stochastic Complexity

In order to partition a SAR image into statistically homogeneous regions, we use a recently proposed method based on the minimization of the stochastic complexity [19]. It consists of partitioning the image with a polygonal grid that is composed of nodes joined together by segments. Fig. 1 shows the definition of regions, nodes, and segments of a polygonal grid. This grid defines the different regions \( \Omega_r, r \in [1, R] \) of the image. The authors proposed minimizing the stochastic complexity \( \Delta \) of the image to determine the number of regions \( R \), the number of nodes, and their locations in three different steps: merging regions, moving nodes, and removing nodes. Figs. 2 and 3 present these three different optimization steps to partition a synthetic image. The stochastic minimization principle is a general principle which has been introduced by Rissanen in 1978 [21]. The main idea is to find the image description that has the lowest complexity. For that purpose, Rissanen proposed the stochastic complexity which corresponds to the mean number of bits needed to describe the image with an entropic code [18], [22] in the meaning of Shannon. The number of bits to describe the image is a sum of two terms. The first one corresponds to the number of bits needed to describe the image model, and it thus characterizes the complexity of this model. The second one is the number of bits needed to encode the gray levels of the image with an entropic code once the model is known. In the proposed
the data. The first term of the stochastic complexity is thus

\[ \Delta_G(w) = n \left( \log(N) + \log(p) \right) + p \left( 2 + \log(2\hat{m}_x) + \log(2\hat{m}_y) \right) \]

where \( N \) is the total number of pixels of the image, \( n \) is the minimum number of starting nodes that are necessary to draw the grid by passing one and only one time by all the segments, \( p \) is the number of segments of the grid, and \( \hat{m}_x \) (respectively, \( \hat{m}_y \)) is the horizontal (vertical) mean length of the segments of the grid measured in pixels.

2) \( \Delta_P \) is the number of bits necessary to encode the parameters \( \theta \) (in the case of PolInSAR, PolSAR, or InSAR, \( \theta \) is related to the covariance matrix) of the parametric PDF in each region of the grid. This term depends on the type of PDF and on the partition \( w \) of the image [19], [23]. An approximation of the number of bits associated to this term is [18]

\[ \Delta_P(\theta, w) = \alpha \sum_{r=1}^{B} \log(N_r) \]

where \( \alpha \) is the dimension of the parameter vector \( \theta \), and \( N_r \) is the number of pixels in the regions \( r \). In the case of PolInSAR image, \( \theta \) contains different real-valued coefficients of the covariance matrix \( \Upsilon_{\text{polint}} \). Since \( \Upsilon_{\text{polint}} \) is an Hermitian \( 6 \times 6 \) matrix, \( \theta \) is a 36-D vector, and thus, \( \alpha \) is equal to 36 (the six real coefficients from the diagonal of the matrix and the 15 real and imaginary parts from its triangular superior part). In the same way, we have \( \alpha = 9 \) for PolSAR and \( \alpha = 4 \) for InSAR.

3) \( \Delta_L \) is the number of bits needed to encode the backscattering coefficients of the image once the partition in statistically homogeneous regions (i.e., the grid) and the PDF parameters have been described. \( \Delta_L \) can be approximated [19], [23], [24] by the opposite sum of the likelihoods \( L_r \) of the data in each region

\[ \Delta_L(w) = -\sum_{r=1}^{B} L_r. \]

If the parameter \( \theta \) is estimated with the maximum-likelihood method, \( L_r \) is the profile likelihood in region \( r \). InSAR, PolSAR, and PolInSAR data are classically modeled as an \( n \)-component \( (n = 2 \) for InSAR, \( n = 3 \) for PolSAR, and \( n = 6 \) for PolInSAR) complex vector with Gaussian circular PDF with covariance matrix \( \bar{\Upsilon} \). This model of PDF enables us to determine the profile likelihood of a sample that is given by Conradsen et al. [15], Goudail and Réfrégier [25], and Chesnaud et al. [26]

\[ L_r = N_r \log \left( \det \bar{\Upsilon}_r \right) + 6N_r \left[ 1 + \log(\pi) \right] \]  

where \( N_r \) is the number of pixels in region \( \Omega_r \) and

\[ \bar{\Upsilon}_r = \frac{1}{N_r} \sum_{(x,y) \in \Omega_r} K(x,y)K(x,y)^T. \]
\( \mathbf{K}(x, y) \) denotes the measurement vector at pixel \((x, y)\). The matrix \( \mathbf{T} \) is the maximum-likelihood estimate of \( \mathbf{Y} \) in region \( \Omega_r \). Thus, \( \Delta_L \) has the following expression:

\[
\Delta_L(w) = \sum_{r=1}^{R} N_r \log \left[ \det \mathbf{T}_r \right] + 6N \left[ 1 + \log(\pi) \right].
\]

The stochastic complexity is thus equal to

\[
\Delta = \Delta_G(w) + \Delta_P(w) + \Delta_L(w). \tag{3}
\]

The optimal partition of an image is thus defined as the one that minimizes the stochastic complexity \( \Delta \). We can notice that \( \Delta \) does not depend on any parameter that has to be tuned by the user but only on the data and on the grid model. A weighted version of the stochastic complexity will be discussed in Section II-D. The statistically homogeneous regions based on stochastic complexity are thus the result of a tradeoff between the cost in bits to describe the image model and the cost to describe the image data knowing the image model. Indeed, the general purpose of this algorithm is to determine if it costs more bits to describe neighboring pixels with two statistically homogeneous regions and thus two different PDFs or with one statistically homogeneous region and one PDF. In the first case, the image data are well-characterized but it increases the number of bits to describe the image model. In the latter case, the image model is simple but the number of bits to describe the image data is important. Let us now detail the optimization strategy to find the minimum of the stochastic complexity \( \Delta \).

C. Three-Stage Optimization Process

The aim of the proposed partitioning method is to minimize the stochastic complexity \( \Delta \) that depends on the number of regions \( R \), the number of nodes, and their locations in the grid. Starting from an initial grid (generally \( 8 \times 8 \) or \( 4 \times 4 \) pixel brick wall shape), the optimization technique consists of moving, removing, and merging regions (removing segments) so that the deformations make the stochastic complexity decrease. These three operations are applied until convergence, i.e., until the stochastic complexity reaches its local minimum [19], [24]. Fig. 2 gives an example of partition in statistically homogeneous regions with the statistical active grid.

1) Merging Regions: The region-merging stage is very simple; two neighboring regions are merged if and only if their fusion leads to the decrease of the stochastic complexity of the image. Let \( w \) be the partition before the fusion of two neighboring regions \( A \) and \( B \) and \( w' \) the partition where \( A \) and \( B \) are merged to form the region \( A \cup B \). The fusion of the regions \( A \) and \( B \) is consequently accepted if \( \Delta(w') < \Delta(w) \).

If we develop this inequality with the different terms of the stochastic complexity, we obtain

\[
\Delta_G(w') + \Delta_P(w') + \Delta_L(w') < \Delta_G(w) + \Delta_P(w) + \Delta_L(w)
\]

i.e.,

\[
\Delta_L(w') - \Delta_L(w) < \Delta_G(w) + \Delta_P(w) - \Delta_G(w') - \Delta_P(w').
\]

and consequently

\[
\mathcal{L}_A + \mathcal{L}_B - \mathcal{L}_{AB} < \Delta_G(w) + \Delta_P(w) - \Delta_G(w') - \Delta_P(w'). \tag{4}
\]

We can remark that it is similar to a generalized likelihood ratio test (GLRT) compared to a threshold \( S(w, w') = \Delta_G(w) + \Delta_P(w) - \Delta_G(w') - \Delta_P(w') \). This threshold is not constant but depends on the partitions \( w \) and \( w' \). The merging stage applied with the threshold \( S(w, w') \) considerably decreases the number of regions of the partition, and since it is an irreversible process, it could lead to an underpartition of the image. Thus, to obtain a more gradual convergence, we decide, for the first merging stage, to arbitrarily choose a GLRT threshold \( S \) equal to 30. The other merging stages are performed by minimizing the stochastic complexity. Each couple of neighboring regions is merged if they satisfy the inequality in (4). The stage is ended when any fusion of two neighboring regions increases the stochastic complexity.

2) Moving Nodes: We also have to estimate the location of the nodes. Each node of the grid is successively moved, and its new location is accepted if it decreases the stochastic complexity. If it is not the case, the node is restored to its initial location. The new locations of a node are tested in eight directions with constant distance. When no moving of a node is possible, then the new location of the nodes are tested but with a smaller moving distance than before. The end of this stage is reached when node moving with a weak amplitude is not possible without increasing the criterion \( \Delta \).

3) Removing Nodes: A partition with a too large number of nodes does not provide a satisfactory description of an image. It is thus necessary to introduce a node-removing stage. We only consider in this process the nodes that have a multiplicity equal to two (nodes that are only linked to two other nodes). A node is removed if its suppression involves a decrease in the stochastic complexity. The stage is finished when no node can be removed without increasing the stochastic complexity.

Fig. 2 shows the partition of a synthetic \( 256 \times 256 \) PolInSAR image. It illustrates the different stages of the algorithm from an initial \( 8 \times 8 \) pixel brick-wall-shaped partition to the final one, when no node can be moved or removed and no region can be merged.

As in any optimization technique, the final partition depends on some initialization parameters such as the initial grid size or the value of the GLRT threshold (30 in our case). Nevertheless, it is possible to test different initializations and choose the one that minimizes the stochastic complexity. Fig. 4 shows partitions in statistically homogeneous regions of a synthetic \( 256 \times 256 \) PolInSAR image with different sizes of initial grid. The best partition is obtained with a \( 6 \times 6 \) initial grid since it leads to the lowest stochastic complexity \( \Delta \). However, it can be noticed that all the partitions obtained with different grid initializations are similar, which shows that the proposed algorithm does not strongly depend on the initialization. The same type of experiment can be performed to determine the most efficient threshold \( S \). In conclusion, it can be said that initial grids between \( 4 \times 4 \) and \( 8 \times 8 \) pixels and threshold value \( S = 30 \) are experimental parameters that often lead to
Fig. 4. Partitions in statistically homogeneous regions of a synthetic $256 \times 256$ PolInSAR image with different initial brick-wall-shape grid size. For this image, the initial grid that leads to a minimal value of the criterion is a $6 \times 6$ initial brick grid.

Fig. 5. Partitions of the synthetic $256 \times 256$ image of Fig. 4 obtained for three different values of parameter $\lambda$.

D. Stochastic Complexity Analysis

We have shown that the stochastic complexity leads to a criterion that is free from parameter adjustment by a user. In order to analyze more precisely the relevance of this approach, it is possible to introduce an adjustment parameter $\lambda$ such as classically

$$\Delta = \Delta_L(w) + \lambda \Delta_M(w)$$

where $\Delta_M(w)$ is a regularization term equal to $\Delta_G(w) + \Delta_P(w)$. The Risannen’s stochastic complexity used in the proposed algorithm corresponds to $\lambda = 1$.

In Fig. 5, we show three partitions on a $256 \times 256$ synthetic image that have been obtained for $\lambda = 0.8$, $\lambda = 1$, and $\lambda = 4$. For $\lambda = 4$, the statistical active grid does not detect all the regions of the image, and for $\lambda = 0.8$, it is not regularized enough and the number of regions is clearly too large. $\lambda = 1$ corresponds to an equal weight between the likelihood and regularization terms and seems to be the most efficient. In order to confirm this qualitative result, we have performed the simulation in Fig. 6. Knowing the true partition of a $256 \times 256$ synthetic image, we generate 20 independent realizations of this image, then partition them, and finally estimate the average number of misclassified pixels (ANMP) [23] of the partitions obtained for different values of $\lambda$. It is shown in Fig. 6 that $\lambda = 1$ leads to the minimal value of misclassified pixels. Thus, on synthetic data, the value $\lambda = 1$ given by Risannen’s theory actually leads to the best partition results in terms of ANMP. Of course, real data may not exactly correspond to the image model (varying mean reflectance over regions, etc.), and when working on thematic segmentation, semantic regions may not be statistically homogeneous. In such cases, better results may be obtained for values of $\lambda$ different from one. As shown in (5), the proposed method can be easily adapted to such supervised applications.

III. ANALYSIS OF THE PARTITIONING METHOD

The statistical active grid algorithm has been validated on scalar and vectorial images whose channels are assumed independent [19], [20]. Its application on PolInSAR raises specific issues due to the dimension of the data vector and to the fact that its components are correlated. In this section, we first focus on the partition of small regions in PolInSAR data and then analyze how the contrast in a PolInSAR image can be divided into a polarimetric and an interferometric contribution.

A. Change in Encoding Due to Covariance Matrix Estimation

The application of the grid on real PolInSAR image can lead to covariance estimation problems in the case of regions with a few pixels. To solve this problem, it is necessary to partially modify the criterion $\Delta$. The region-merging stage is an essential step in the method, since region splitting is not implemented. One term of the stochastic complexity is the opposite sum of the profile likelihoods of each region, and consequently, it
involves the estimation of a covariance matrix [see (1)]. If this estimation is not performed on a sufficient number of pixels, then the resulting matrix may be singular, and problems can occur when very small regions appear. A necessary condition for an estimated covariance matrix \( \Sigma_r \) of a region \( r \) to be invertible is that it is estimated on a sufficient number of pixels. This number of pixels \( N_r \) has to be larger than the dimension \( d \) of the covariance matrix to be estimated [27], i.e., \( d = 6 \) for PolInSAR, \( d = 3 \) for PolSAR, and \( d = 2 \) for InSAR. Even if \( N_r \) is a little larger than these limit values, the estimation of a covariance matrix may not be accurate. Indeed, the determinant \( \Sigma_r \) may tend to zero and the profile log likelihood to \( -\infty \) (1). Consequently, a small region \( r \) cannot be merged to its neighboring regions because a fusion will automatically increase the stochastic complexity. We have thus defined a new expression of the profile likelihood that depends on the size of the region. If the number \( N_r \) of pixels in the region \( r \) is greater than \( d \), the backscattering coefficients are modeled by the correlated Gaussian circular process. Otherwise, if the number of pixels \( N_r \) is smaller than \( d \), the pixels are modeled by uncorrelated Gaussian circular process, i.e., we only consider a diagonal covariance matrix for the region \( r \). The diagonal terms of the covariance matrix represent the intensities in the different channels. To summarize, the likelihood in the region \( r \) has the following expression. If \( N_r > d \)

\[
L_r = N_r \log \left( \det(\Sigma_r) \right)
\]

and if \( N_r \leq d \)

\[
L_r = N_r \log \left( \det \left( \text{diag}(\Sigma_r) \right) \right)
\]

where \( \text{diag}(\Sigma_r) \) is a diagonal matrix whose coefficients are the diagonal coefficients of \( \Sigma_r \). This choice avoids the problem due to singular matrices in the case of small regions.

**B. Polarimetric and Interferometric Partition on Synthetic Images**

Since PolInSAR combines polarimetry and interferometry, the resulting partition is able to detect both polarimetric and interferometric variations between the regions. Indeed, two neighboring regions in a PolInSAR image are partitioned if they have a different polarimetric and/or interferometric behavior. In this section, we show how we can define a polarimetric and an interferometric contrast from PolInSAR data and illustrate this result on synthetic images. For that purpose, we use the PolInSAR matrix decomposition from Cloude and Papathanassiou’s coherence optimization algorithm [28]. It is based on the selection of the projection vectors that maximizes the interferometric coherence and leads to a \( 3 \times 3 \) complex eigenvector problem. We examine here the results of this method reformulated with matrix products.

As seen in Section II-A, PolInSAR data are modeled by a 6-D complex target vector \( \vec{K} \) with \( \vec{K} = [\vec{K}_1, \vec{K}_2]^T \), where \( \vec{K}_1 \) and \( \vec{K}_2 \) correspond to the 3-D polarimetric target vectors from the two antennas. They follow the Gaussian PDF with zero mean and covariance matrix \( \Sigma \)

\[
\Sigma = \begin{pmatrix}
\langle \vec{K}_1^H \vec{K}_1 \rangle & \langle \vec{K}_1^H \vec{K}_2 \rangle \\
\langle \vec{K}_2^H \vec{K}_1 \rangle & \langle \vec{K}_2^H \vec{K}_2 \rangle
\end{pmatrix} = \begin{pmatrix}
\Gamma_1 & \Omega_{12} \\
\Omega_{12}^* & \Gamma_2
\end{pmatrix}.
\]

Let us detail the different \( 3 \times 3 \) blocks that compose the matrix \( \Sigma \).

1) \( \Gamma_1 \) and \( \Gamma_2 \) correspond to the polarimetric measurements of each of the two antennas and, consequently, contain the polarimetric information from the matrix \( \Sigma \). These matrices have notably been studied in [7] and [29]. The authors defined useful parameters that characterize the randomness of the scattering process and the interaction between the wave and the lightened medium.

2) \( \Omega_{12} \) represents the polarimetric interferometry part of \( \Sigma \). To remove the dependence of \( \Omega_{12} \) on polarimetry, it has been proposed to define the normalized coherence matrix \( M \) with \( M = \Gamma_1^{-1/2} \Omega_{12} \Gamma_2^{-1/2} \). It has been shown in [28] that the larger singular value of \( M \) corresponds to the maximum generalized coherence [28] that can be reached by varying the polarization states of the signals from the two antennas.

Considering the matrices \( \Gamma_1 \) and \( \Gamma_2 \) and the normalized coherence matrix \( M \), we propose to introduce the following notation in order to decompose the matrix \( \Sigma \) and separate polarimetry and interferometry characteristics from this matrix. Let us define the joint polarization matrix \( \Gamma \) that contains the polarimetric information of \( \Sigma \)

\[
\Gamma = \begin{pmatrix}
\Gamma_1 & 0 \\
0 & \Gamma_2
\end{pmatrix}.
\]

If we assume that \( \Gamma \) is invertible (neither \( \Gamma_1 \) nor \( \Gamma_2 \) represents a totally polarized wave), we can also introduce the matrix \( \Phi \)

\[
\Phi = \Gamma^{-1/2} \Sigma \Gamma^{-1/2}
\]

where \( \Gamma^{-1/2} = V D^{-1/2} V^T \) with \( D \) as a diagonal matrix that contains the eigenvalues of \( \Gamma \), and \( V \) contains the eigenvectors of \( \Gamma \). We can develop this matrix in the following way:

\[
\Phi = \Gamma^{-1/2} \Sigma \Gamma^{-1/2} = \begin{pmatrix}
\Gamma_1^{-1/2} & 0 \\
0 & \Gamma_2^{-1/2}
\end{pmatrix} \begin{pmatrix}
\Omega_{12} & 0 \\
0 & \Omega_{12}^*
\end{pmatrix} \begin{pmatrix}
\Gamma_1^{-1/2} & 0 \\
0 & \Gamma_2^{-1/2}
\end{pmatrix}
\]

\[
= \begin{pmatrix}
I_{3 \times 3} & \Omega_{12} \Gamma_1^{-1/2} \\
\Gamma_2^{-1/2} \Omega_{12} \Gamma_1^{-1/2} & I_{3 \times 3}
\end{pmatrix}
\]

where \( I_{3 \times 3} \) represents the \( 3 \times 3 \) identity matrix. Following its definition introduced in [30], one has \( \Phi = \begin{pmatrix}
I_{3 \times 3} & M \\
M^T & I_{3 \times 3}
\end{pmatrix} \) and \( \Sigma = \Gamma_1^{1/2} \Phi^T \Gamma_2^{1/2} \). The polarimetric and interferometric matrix \( \Phi \) can thus be decomposed into an interferometric matrix \( \Phi \) and a polarimetric matrix \( \Gamma \). We can use this decomposition to synthesize PolInSAR images with well-defined polarimetric or interferometric contrast between neighboring regions. Indeed, if two neighboring regions do not have the same matrix \( \Phi \), they
will have an interferometric contrast, and if they do not have the same matrix $G$, they will have a polarimetric contrast.

In Fig. 7, we present the label image that has been used to simulate the data. It is composed of six regions. In the simulated data, the pixels of the image follow multidimensional Gaussian PDF with different covariance matrices in the six regions of the label image. To define these covariance matrices, we consider two polarimetric matrices $G_1$ and $G_2$ and two interferometric matrices $\Phi_1$ and $\Phi_2$, which are defined in the Appendix. We make combinations between these four matrices to obtain PolInSAR covariance matrices. Namely, the covariance matrix in region 1 is described with $\Upsilon_1 = G_2 \Phi_1 G_2$; in region 2, one has $\Upsilon_2 = G_2 \Phi_2 G_2$; in region 3, $\Upsilon_3 = G_1 \Phi_1 G_1$; in region 4, $\Upsilon_4 = G_2 \Phi_1 G_2$; and finally, in region 6, $\Upsilon_6 = G_1 \Phi_2 G_1$. Moreover, we impose that there is no contrast in intensity: The diagonal terms of each matrix $\Upsilon_i$ with $i = 1, 6$ are equal to one. Our purpose is to compare on this image the partitions obtained with the statistical grid algorithm based on PolSAR, InSAR, and PolInSAR models. The PolSAR and the InSAR images are computed as follows from the PolInSAR image.

1) The target vectors of a PolInSAR image for each pixel have the following form: $\vec{K}_{\text{polint}} = (S_{1HH}; S_{1VV}; \sqrt{2S_{1HV}}; S_{2HH}; S_{2VV}; \sqrt{2S_{2HV}})^T$. With this six-component vector, we can construct an InSAR image considering only the signal from the HH polarization leading to the following vector: $\vec{K}_{\text{interf}} = (S_{1HH}; S_{2HH})^T$. In the same way, it is possible to extract a PolSAR image by choosing the full polarimetric response of only one antenna, i.e., $\vec{K}_{\text{pol}} = (S_{1HH}; S_{1VV}; \sqrt{2S_{1HV}})^T$.

2) For each image, we apply the statistical active grid with its corresponding model. Thus, we obtain three different grid partitions in statistically homogeneous regions for PolSAR, InSAR, and PolInSAR image. The results appear in Fig. 7. We can notice that the number of regions and the number and the location of the grid nodes vary from one image to another.

The partition on PolInSAR data enables us to detect all the regions of interest in the image, whereas it is not the case with PolSAR and InSAR. Indeed, in PolSAR image, we only partition the neighboring regions whose polarimetric components (i.e., matrices $G$) are different (for instance, regions 1 and 5) and, in the case of InSAR data, the neighboring regions whose interferometric components (i.e., matrices $\Phi$) are different (regions 1 and 2). This simple example illustrates that the proposed PolInSAR statistical grid method is sensitive to both polarimetric and interferometric contrast. The proposed decomposition of the PolInSAR matrix can also be used to interpret the contrast of regions obtained by partitioning real images.

IV. APPLICATIONS OF THE STATISTICAL ACTIVE GRID

In this section, we focus on the partition of real PolInSAR data with the statistical active grid. Finally, we show that PolInSAR leads to more precise partition in statistically homogeneous regions than PolSAR and InSAR. In this section, we present partitions on real PolInSAR images. For this purpose, we consider two Office National d’Études et Recherches Aéronautiques (ONERA) RAMSES SAR [31] data sets acquired in P- and X-bands. The first one is a RAMSES X-band image acquired in March 2002 over the Institut National de Recherche Agronomique (INRA) site of Avignon. This PolInSAR image has a dimension of 6492 × 1152, the incidence angle is $30^\circ$, and the slant range and azimuth resolution are 0.9 and 0.94 m, respectively. It is a single-pass interferometry image with a 63-cm baseline, and the altitude of ambiguity is about 250 m. Trihedral corners and dihedral reflectors were deployed on the scene for radiometric and polarimetric calibration.

INRA’s site is composed of small fields delimited by cypress hedges and planted with clover, wheat, and orchards like peach trees, pear trees, and apricot trees. The rest of the imaged zone contains some other types of vegetation, like apple trees, pine forest, plane trees, and, notably, poplars (Fig. 8). The ground truth is known for most of INRA’s parcels, which will allow us to evaluate the quality of parameter estimation and classification. We show the partition on this PolInSAR image and the corresponding aerial photography in Fig. 9. We can

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Fig. 8. Extract (1670 × 647) of the PolInSAR X-band Avignon image notably composed of small agricultural parcels, buildings, and roads. (a) Pauli vector image (R: HH + VV; G: HH − VV; B: 2HV). (b) Interferometric phase HH (black: −π to white: π). (c) Coherence HH (black: 0 to white: 1). The red points in image (a) make the correspondence between the SAR image and the aerial image of Fig. 8.

Fig. 9. (a) Final partition obtained with an initial 8 × 8 pixel grid on the PolInSAR data. Since the scene is complex, the final partition contains a large number of regions. (b) Aerial image taken over the corresponding testing area Fig. 8 on the X-band Avignon image.

notice that the final partition contains many regions because of the complexity of the image. The statistical active grid has detected regions which are homogeneous with respect to polarimetry and interferometry. We can remark that very fine details have been partitioned, notably in the building and forest areas, which is important for the analysis of the image. On the other hand, larger areas have been partitioned into several regions by the grid. Indeed, the grid is a partition method that is designed to find the statistically homogeneous regions and not the regions of semantic interest of the image.

The second image has been acquired in P-band by RAMSES over the region of Nezer, France. The campaign was carried out in January 2004. The image size is 3500 × 1550 pixels, and the slant range and azimuth resolution are 2.5 and 3.2 m, respectively. It is a double-pass interferometry image; the altitude of ambiguity is about 45 m, and the incidence angle is 30°. This site is composed of major part of rectangle-shaped parcels (Fig. 10) containing different types of trees. The age and the type of trees are known in most of the parcels. Fig. 11 shows the partition into statistically homogeneous regions of this image on PolSAR and PolInSAR data. We can remark that there are several regions of the grid inside a parcel of forest. This means that a parcel does not constitute a homogeneous region in the sense of polarimetry and interferometry. It is probably due to a change in the canopy density that modifies the polarimetric or interferometric properties of the region. Thus, the grid provides a region mapping that can be used to estimate covariance matrix and polarimetric and interferometric parameters. In the future, it will be interesting to compare the proposed technique of this paper to more refined methods [17], [32] than with a boxcar that take into account the edges between regions and that yields a precise estimation of the covariance matrix. Indeed, these techniques usually involve determination of the suitable size of the window, and it will be interesting to analyze if the proposed technique can be a relevant alternative for PolInSAR data estimation.

V. CONCLUSION

We have proposed a new algorithm for partitioning PolInSAR images into statistically homogeneous regions. It consists of optimizing a polygonal grid and is based on a parameter-free criterion which relies on stochastic complexity. The partition technique contains nevertheless parameters in its optimization such as the size of the initial grid. This algorithm leads to a partition of the image into regions which have homogeneous PolInSAR characteristics, although the 36 parameters of
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The PolInSAR covariance matrix have to be estimated. The performances of this method have been demonstrated on synthetic and real PolInSAR images. It can thus provide an alternative to simple boxcar estimation and to spatially adaptive filtering, notably for agricultural and forest area mapping. Monitoring of crops and planted forests thus constitutes the main application of this technique, whose practical advantages have been illustrated on real PolInSAR images.

This work has many interesting perspectives. The partition obtained with the proposed method of this paper can be a good starting point for a thematic classification task, as shown, for example, in [33], where the authors start from a partition into statistically homogeneous regions in order to obtain an oil slick detection method. We thus intend to study the application of the proposed method to classification and segmentation in the future. Further studies should also be made on interferometry-based partition of terrains with significant slope since, in this case, the phase is not constant in a partition region. Finally, a comparison with refined filters such as Lee’s filter [17] would be useful in assessing the advantages and drawbacks of the different methods for PolInSAR covariance matrix estimation.

APPENDIX

The matrices shown at the bottom of the page have been used in Section III-B to synthesize regions that have a polarimetric and/or an interferometric contrast. The values of these different matrices have been chosen in order to simulate polarimetric
and interferometric contrasts between the six regions in Fig. 7. Indeed, the simulated covariance matrices are all different in the PolInSAR case but not in the simulated PolSAR case since regions 1, 2, and 4, on the one hand, and regions 3, 5, and 6, on the other hand, have the same covariance matrix. For regions 1, 2, and 4, one has

\[ \Upsilon_{\text{pol}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

and for regions 3, 5, and 6

\[ \Upsilon_{\text{pol}} = \begin{pmatrix} 1.33 & 0.2 & 0.8 + 0.8i \\ 0 & 1.0i & 0.04 + 0.04i \\ 0.8 - 0.8i & 0.04 - 0.04i & 1.32 \end{pmatrix}. \]

The regions 1, 2, and 4 and 3 and 6 are thus fused on the PolSAR data with the proposed algorithm. In the same way, regions 1, 3, 4, and 5 and 2 and 6 have the same covariance matrix on the simulated InSAR data in Fig. 7. For regions 1, 3, 4, and 5, one has

\[ \Upsilon_{\text{interf}} = \begin{pmatrix} 1 & 0.7 + 0.1i \\ 0.7 - 0.1i & 1 \end{pmatrix} \]

and for regions 2 and 6

\[ \Upsilon_{\text{interf}} = \begin{pmatrix} 1 & 0.7 + 0.6i \\ 0.7 - 0.6i & 1 \end{pmatrix}. \]

The couple of regions 1 and 5, 3 and 4, and 2 and 6 are thus merged on the InSAR data with the proposed algorithm.

Acknowledgment

The authors would like to thank ONERA/Département Electromagnétisme et Radar (DEMR)/Radar Imageur et Expérimentation (RIM) team for their important effort in providing us calibrated SAR polarimetric and interferometric data, the Physics and Image Processing Group at the Fresnel Institute, particularly G. Delyon, for their support on segmentation methods, the Programme National de Télédétection Spatiale (PNTS), the Radar Image pour les Thématiques Agricoles et de Sol (RITAS) working group, and Direction Générale de l’Armement (DGA)/Service des Techniques et des Technologies Communes (STTC) (French Department of Defense).

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