

Strategic Inventories in Vertical Contracts^a

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Abstract

The literature on contract theory has mostly focused on models - static or dynamic - without inventories. On the other hand, inventory theory has been developed primarily within a single decision-maker optimization framework. This paper bridges these two streams of research by studying the role of inventories in a dynamic vertical contract between a supplier and a buyer. In the model, the buyer may carry inventory across periods. Under zero fixed costs, zero lead times, no uncertainty, and stationary demand, all the classical reasons for inventories are eliminated. Yet (as we prove), the buyer's optimal strategy in equilibrium is to carry inventories, and the supplier is unable to prevent this. These inventories arise out of purely *strategic* considerations not identified before in the literature, and have a significant impact on the equilibrium solution, as well as supplier, buyer and channel profits. We find that strategic inventories are eliminated if the supplier can preempt the buyer's gaming behavior by committing early to wholesale prices. We compare the performance of such commitment contracts with (uncommitted) dynamic contracts.

We extend the case of linear wholesale prices to two-part tariff vertical contracts, and demonstrate that strategic inventories continue to play a pivotal role. When the buyer can carry inventories strategically, two-part tariffs do not lead to optimal channel performance, nor can the supplier extract away all of the channel profits.

Our results are robust to assumptions of general (arbitrary) demand functions, arbitrary contractual structures and general (finite or infinite) horizon lengths - the threat of carrying inventories plays a significant strategic role under all these conditions. Our results imply that firms can and must carry inventories strategically, and that the ability to carry inventories is an important factor in optimal contract design.

[*JEL classification*: L12, L42. *Keywords*: Vertical contracts; Inventories; Industrial Organization.]

1 Introduction

Much of the literature on contract theory has focused on models that don't take inventories into account.¹ The primary focus of this body of literature has been on incentives for motivating the right action under moral hazard or asymmetric information. Likewise, the extensive literature in Industrial Organization on incentives for vertical controls also ignores the effect of inventories: This body of work studies various mechanisms for vertical control under a deterministic environment with sales originating from current production in each period (Tirole, 1990). Classical inventory theory focuses on optimization under a single decision-maker, ignoring the role of incentives. This paper is a contribution towards bridging these two research streams - one that focusses on incentives for motivation and coordination ignoring inventories and the other that focuses on inventories ignoring incentive issues. We demonstrate in our dynamic model that *strategic inventories* are an artifact of contractual structure, and significantly alter the contractual outcome (the equilibrium solution, as well as supplier, buyer and channel profits, consumer surplus and welfare). Hence, firms can and must carry inventories strategically, and optimal contract design must take the possibility of inventories into account.

Inventory may be carried for a number of well-documented reasons (cf. Anupindi *et al.* (1999)). First, due to economies of scale in procurement or production, it may be cheaper to procure or produce in quantities larger than what is immediately needed, resulting in *cycle inventory*. Second, *pipeline inventory* may be carried to ensure availability of goods in the face of delivery or production delays. Third, a firm may carry *safety inventory* to cope with uncertainty in demand or supply. Fourth, firms may carry *speculative inventory* to hedge against price fluctuations. Finally, while economies of scale lead to cycle inventory, a firm with production diseconomies (e.g., increasing marginal production cost) and fluctuating demand may carry inventory to smooth production and thus lower production costs; see Holt *et al.* (1960).

The classical reasons for holding inventories, discussed above, arise even in a non-competitive context, due to the economics of matching supply and demand. Vertical control issues in such environments have been studied, for example, by Deneckere *et al.* (1996). More recently, there is growing interest in incentives for coordination in supply chains in environments with demand uncertainty; see Cachon (2001) for a review of the work on vertical coordination. However, there is no strategic value for carrying inventories in such settings. In contrast, some researchers have demonstrated that inventories may play a strategic role under horizontal competition. Specifically, Saloner (1986) considers a two-period duopoly model in which firms

¹For instance, the index of the monumental Laffont and Tirole (1993), devoted to procurement contracts and regulation, does not even mention inventories.

produce and sell in the first period but may salvage inventory carried over to the next period. He shows that the firm with the first-mover advantage at the production stage is unable to achieve the Stackelberg equilibrium, because it cannot credibly commit to selling its entire production in the first period - this is true even though no inventories are carried in equilibrium. Rotemberg and Saloner (1989) consider a duopoly model in which firms keep inventories even though *none* of the classical reasons for holding inventories exist. These inventories are used to sustain collusive profits by the threat of reversion to competitive behavior. Yet another stream of work considers the strategic value of inventories in an oligopolistic setting where a firm may invest in inventories to pre-empt its competitor's future production; see, for example, Mollgaard et. al. (2000) and references therein.

Like the papers cited above, none of the classical reasons for holding inventories exist in our model as well. However, in contrast to these models of horizontal duopolistic / oligopolistic competition, our focus is on a *vertical* value chain with a supplier and a buyer.

The Static Problem

The classical static (single-period) version of our model, with linear costs and prices, is analyzed in many economics textbooks (cf. Tirole (1990)). Consider a vertical value chain with a single supplier and a solitary buyer. The supplier has constant economies of scale in procurement (or, production), and sells to the buyer at a linear wholesale price. The buyer in turn sells her purchased quantity in a market with a known (linear) demand curve. When supplier and buyer act as a *team*², the total channel profits are maximized. However, when they maximize their individual profits, the resulting incentive misalignment leads to the well-known *double marginalization*,³ i.e., the supplier and buyer add their margins to the price quoted downstream, and the final (retail) price is higher than what is optimal for the channel. The quantity sold is correspondingly lower than the *team* case, and total channel profits fall. Clearly, in the static problem, the buyer sells the entire quantity that she buys, and carries no inventory.

The Dynamic Problem

In this paper we focus on the dynamic (two-period) version of the classical model.⁴ The buyer may carry inventories across periods, while incurring holding costs. None of the classical reasons for holding inventories exist in our model. Furthermore, there is no lateral competition (between buyers) as in Rotemberg and Saloner (1989) and Mollgaard et. al. (2000). Therefore, we would expect that inventories are not carried, and that the equilibrium solution

²A *team* consists of multiple entities who coordinate their actions to fulfill a shared objective such as maximizing their joint profits.

³See Spengler (1950) for an early reference.

⁴We later extend the analysis to arbitrary horizon lengths.

for the dynamic problem simply mimics the static solution in each period. We prove that this “intuition” does not hold. In fact, in the two-period model, the buyer *does* carry inventories in equilibrium. Further, the equilibrium wholesale price is higher for the first period than for the second; hence, *ceteris paribus*, the buyer actually pays more for the inventory she carries than she would have if she had simply purchased the same quantity in the second period. The presence of inventory holding costs compounds this puzzle. Hence we postulate that the inventories carried in our model are motivated purely from strategic considerations that override the obvious costs of paying higher unit prices and holding costs; we use the term *strategic inventories* for these inventories.

In our model, strategic inventories arise from misalignment of contractual incentives in the channel. Since inventories are a costly drain to the channel (through holding costs), one would expect that eliminating them would lead to higher channel and supplier (and perhaps even buyer) profits. Since the buyer’s use of strategic inventories forces the supplier to lower his second-period wholesale price, the supplier might be willing to directly offer the lower second-period wholesale price in exchange for reduced inventories. The problem is that the supplier cannot credibly commit in advance to a lower second-period price.

On the other hand, if the supplier could credibly commit to a future lower price, under an alternative contracting mechanism, then the buyer’s need for strategic inventories would perhaps be eliminated. We explore such a *commitment contract*, where the supplier quotes wholesale prices for both periods at the beginning of the first period, and the buyer then decides on her purchase quantities. As expected we find that such commitment contracts do eliminate inventories from the system, and the equilibrium for the dynamic game mimics the solution for the one-period game in each period. Surprisingly, however, we find that the profits do not move in the direction we would expect. For a wide range of holding costs, buyer, supplier and channel profits are *greater* under the short-term (dynamic) contracting mechanism than under the commitment contract; thus, the commitment contract is in fact pareto-dominated by the dynamic contract. Further, consumer surplus and welfare are both greater under the dynamic contract than under the commitment contract, even though inventory holding costs are incurred in the former.

These results imply that strategic inventories, far from being just a drain on channel profits through the holding costs, have actually improved channel coordination. We postulate that the option of carrying inventories indirectly expands the feasible “contract space” between the two parties, and reduces the effect of double marginalization. Further, for the most part, the “contract-expansion” effect of strategic inventories dominates the “inventory-drain” effect, leading to a pareto-improving equilibrium.

Several questions arise at this point. (a) Are strategic inventories, and the pareto-improvement induced by them, a consequence only of the restrictive contract space of linear wholesale prices, which the possibility of carrying inventories expands? (b) Do the results for linear demand curves extend to more general demand functions? (c) Do inventories continue to play a strategic role in longer-horizon problems? To address the first issue, we relax the contract space by extending the vertical contract to two-part tariffs which we discuss next. Subsequently, we address the other two questions by relaxing the contract and demand space for both two-period and longer horizons.

Two part tariffs

Two-part tariffs are a form of quantity discounting. In the static problem, the supplier quotes a fixed fee K in addition to a linear wholesale price w . As is well-known, two-part tariffs lead to the first-best solution, and the supplier extracts all of the channel profits (Tirole, 1990). The solution is for the supplier to set w equal to his marginal cost, and extract all of the buyer's profits through K . For the two-period model, we analyze both dynamic contracts (without *ex ante* commitment to the second period wholesale price) and commitment contracts (with *ex ante* supplier commitment to the second period wholesale price).

We find that inventories continue to play a pivotal strategic role in the equilibrium outcomes as well as individual and channel profits. Under dynamic contracting, and in contrast to the single-period case, two-part tariffs do *not* lead to the first-best solution. This outcome is driven by two factors, both connected to strategic inventories: (i) double marginalization, as revealed by positive wholesale prices, and induced by the threat of inventories, and (ii) the inventory drain effect due to holding costs. Further, the buyer carries inventories even though the equilibrium first-period wholesale price is greater than that of the second period, and even when holding costs are positive. However, as in the static model, the supplier manages to extract all of the buyer's profits.

The equilibrium under commitment contracts is even more subtle. Here, the buyer does not carry inventories in equilibrium, and yet the channel does not achieve first-best. This is because, while there is no direct drain through inventory holding costs, the *threat* of carrying inventories induces double marginalization (to forestall it), and lowers channel profits. Further, by using the threat of carrying inventories, the buyer makes positive residual profits that the supplier is unable to extract. These results confirm that strategic inventories do not arise merely because of the restrictiveness of linear contract spaces, but are also observed under non-linear contracts.

With two-part tariffs, unlike in the case of linear pricing, the channel always does better under commitment contracting than under dynamic contracting. The results show that the

buyer is always better off with commitment contracts, a reversal of the general rule that it is a disadvantage to move later in the game (particularly with no uncertainty-resolution, as in our model). In fact, the supplier, who moves earlier under commitment contracting, does better under dynamic pricing in almost all cases.

Generalizations

To investigate the robustness of our results, we relax the demand structure to allow for very general demand functions and consider the space of the most general (arbitrary) *dynamic* contracts for general (finite or infinite) horizon lengths. We show that the ability to carry inventories significantly affects the equilibrium outcome, regardless of whether inventories are actually carried by the buyer in equilibrium. Specifically, we show that the supplier cannot enforce the first-best solution and simultaneously extract all of the buyer's residual profits. This phenomenon persists under linear and quantity-discount commitment contracts for general demand functions and longer horizon problems. However, under arbitrary commitment contracts, the supplier can obtain first-best profits.

The rest of the paper is organized as follows. In the next section, we present the analysis and implications of the main model (with linear wholesale prices). In section 3 we analyze the impact of two-part tariff wholesale contracts. Then, in section 4, we extend our results to general contract spaces, general demand functions, and longer horizon problems. Finally, we summarize and conclude with a discussion of future research extensions in section 5. Proofs of all results appear in the appendix.

2 Model and Analysis with Linear Wholesale Prices

We formulate a dynamic (two-period) model of vertical contracting under full information and no uncertainty.⁵ The buyer may carry inventories from the first period to the second. Following Rotemberg and Saloner (1989) and many other precedents in the academic literature (see Tirole (1990)), we (initially) focus on linear demand curves. Further, we assume that the slope and intercept of the demand curve are known to both parties, and identical for both periods. While the static (single period) model, with linear wholesale prices, is well known, we present this here to facilitate comparisons with the dynamic model. Then we analyze the dynamic model with inventories.

Linear wholesale prices are widely observed in practice on account of their simplicity. They

⁵As will become clear in the analysis, these assumptions are necessary to isolate the *strategic* interactions between buyer and supplier via inventories, without muddying the waters through other effects that are not the focus of this paper.

have been extensively used in economic models as well, and their use helps the supplier forestall buyer arbitrage (Tirole, 1990). So we initially focus on linear prices. Later, we will study the performance of two-part tariff wholesale contracts, and eventually extend the analysis to arbitrarily general vertical contracts.

2.1 The Static Model

The sequence of events in the static model is as follows. The supplier first quotes a linear wholesale price w , and the buyer responds with her purchase quantity Q . The buyer then sells the quantity q in the market, at a price given by the linear demand curve $p(q) = a - b \cdot q$; a and b are common knowledge. The unit production cost for the supplier is normalized to zero. To eliminate arbitrage, we assume here and throughout this paper that the salvage value of the good is zero. Hence the buyer will sell all of the purchased goods in the market (i.e., $q = Q$), at the price $p(q)$. Both buyer and supplier maximize their individual profits.

The solution to this problem is given below.⁶ (Throughout this paper, we use the subscripts B, S and C to denote the Buyer, the Supplier and the Channel respectively.)

Proposition 2.1 *For the static game described above, the following is the unique equilibrium outcome:*

1. *Wholesale Price:* $w = \frac{a}{2}$.
2. *Purchase (and Sales) Quantity:* $Q = q = \frac{a-w}{2b} = \frac{a}{4b}$.
3. *Retail Price:* $p = \frac{3a}{4}$.
4. *Profits:* $\Pi_B = \frac{a^2}{16b}$; $\Pi_S = \frac{a^2}{8b}$; $\Pi_C = \Pi_B + \Pi_S = \frac{3a^2}{16b}$.

Observe that had the channel been owned by a single player, or functioned as a *team*, the optimal outcome would be $q^{fb} = \frac{a}{2b}$; $p^{fb} = \frac{a}{2}$; and $\Pi_C^{fb} = \frac{a^2}{4b}$, where the superscript fb denotes the *first-best* solution: one that maximizes channel profits. When supplier and buyer maximize their individual profits, double marginalization leads to a retail price higher than optimal for the channel, and a correspondingly lower sales quantity. The loss for the channel from double marginalization is $\Pi_C^{fb} - \Pi_C = \frac{a^2}{16b}$, which is a full *quarter* of the first-best profits.

⁶Proofs of all results appear in the appendix.

2.2 The Dynamic Model

We extend the static model to two periods. We assume that the demand curve is identical in each period, and given by $p(q) = a - b \cdot q$. Further, the buyer can hold part or all of his first-period purchases as inventories, to sell in the second period.⁷ We assume that the buyer's holding cost is linear, at h per unit.⁸ The sequence of events is now as follows. The supplier quotes a wholesale price w_1 in the first period, and the buyer responds with her purchase quantity Q_1 . The buyer sells the quantity q_1 , at the price $p(q_1)$, and carries inventory $I = Q_1 - q_1$. In the second period, the supplier quotes the wholesale price w_2 . Then the buyer purchases Q_2 and sells $q_2 = (Q_2 + I)$. This game has the unique subgame-perfect solution given below.

Theorem 2.1 *For the dynamic game described above, the unique subgame-perfect equilibrium is as follows:*

1. *Wholesale Prices:* $w_1 = \frac{9a-2h}{17}$; $w_2 = \frac{6a+10h}{17}$.
2. *Purchase Quantities:* $Q_1 = \frac{13a-18h}{34b}$; $Q_2 = \frac{3a+5h}{17b}$.
3. *Inventory:* $I = \frac{5(a-4h)}{34b}$.
4. *Sales Quantities:* $q_1 = \frac{4a+h}{17b}$; $q_2 = \frac{11a-10h}{34b}$.
5. *Retail Prices:* $p_1 = \frac{13a-h}{17}$; $p_2 = \frac{23a+10h}{34}$.
6. *Profits:* $\Pi_B = \frac{155a^2-118ah+304h^2}{1156b}$; $\Pi_S = \frac{9a^2-4ah+8h^2}{34b}$.

Observe that the equilibrium under the dynamic contract does *not* simply mimic the static solution of Proposition 2.1 in each period. The buyer's ability to carry inventories drives this asymmetry. To see this, imagine that h is prohibitively high (e.g. $h \geq \frac{a}{4}$). If the buyer cannot carry inventory, the two periods will be effectively decoupled, and the static solution of Proposition 2.1 is replicated in each period. Also observe that:

1. The model assumes no fixed (or non-linear) costs. In the absence of economies of scale, *no cycle inventories need be carried.*

⁷Since the unit production cost is identical in the two periods and production is immediate, the supplier does not ever carry inventories.

⁸Throughout the paper, we assume that $h < \frac{a}{4}$. Since a is the demand intercept as well as the maximum possible price, this assumption is consistent with most settings. Further, relaxing this assumption leads to the trivial case where the holding cost is too high for inventories to be feasible and, in effect, the dynamic problem decouples into two separate single-period problems.

2. Zero lead times preclude any *pipeline inventory* in the model.
3. The absence of demand or supply uncertainty precludes *safety inventory*.
4. There are no exogenous price-effects. In fact, it is easily checked that the endogenously determined wholesale prices are related as $w_1 > w_2$. So the inventory carried was procured at the more costly first-period wholesale price, and cannot be attributed to *forward buying*.⁹
5. In addition, the buyer incurs positive holding costs of h per unit for her inventories.

Thus none of the classical reasons for carrying inventories (identified in the Introduction) exist and yet inventories are carried in equilibrium. Why is this so?

In equilibrium, the buyer *anticipates* the supplier's second-period wholesale price as a *strategic* response to her first-period actions (purchased quantities). Hence, the *strategic* value of carrying inventories to the buyer must have overridden the drawbacks of higher unit prices and holding costs. We use the term *strategic inventories* for inventories that arise purely from such strategic considerations.

When the buyer carries strategic inventories, she forces the supplier to price only for the buyer's *residual* demand, leading to a lower second-period wholesale price w_2 . Thus, the buyer curtails the supplier's monopoly power in the second period, by inducing (Cournot-like) supply-side competition between the supplier and the buyer's inventories. Since the supplier anticipates this, he wants to discourage the buyer from carrying inventories. Hence he raises the first-period wholesale price. The problem with raising w_1 is that this reduces profits from sales in the first period; thus, the supplier sets w_1 to balance these two considerations. In equilibrium, this leads to $w_1 > w_2$. As a result, retail prices are higher in the first period (i.e., $p_1 > p_2$), and the sales quantities are related inversely (i.e., $q_1 < q_2$).

Strategic inventories arise from gaming considerations. Since inventories are costly to the channel, one would expect that eliminating them would lead to higher channel and supplier (and perhaps even buyer) profits. For example, in the preceding analysis, inventories arose because the buyer wanted to reduce w_2 . Even if the supplier were willing to offer a lower w_2 in return for a reduction in buyer inventories, he cannot credibly commit to it in the previous model. On the other hand, if the supplier could credibly commit to a second-period wholesale price (under an alternative contracting mechanism), the buyer's need for strategic inventories would be eliminated. We explore such a *commitment contract* in the next section.

⁹*Forward buy* is a term used in the retail industry to denote inventory carried forward in response to an announced or anticipated future price increase.

2.3 Commitment Contract

The commitment contract is identical to the dynamic contract of the previous section, with one exception: Now the supplier can commit to both periods' wholesale prices in advance. Thus, the supplier announces w_1 and w_2 at the start of the horizon, and the buyer subsequently decides on order and sales quantities, as well as inventories. The theorem below gives the unique subgame-perfect equilibrium outcome.

Theorem 2.2 *The unique subgame-perfect equilibrium for the commitment contract is as follows:*

1. *Wholesale Prices:* $w_1 = w_2 = \frac{a}{2}$.
2. *Purchase Quantities:* $Q_1 = Q_2 = \frac{a}{4b}$.
3. *Inventory:* $I = 0$.
4. *Sales Quantities:* $q_1 = q_2 = \frac{a}{4b}$.
5. *Retail Prices:* $p_1 = p_2 = \frac{3a}{4}$.
6. *Profits:* $\Pi_B = \frac{a^2}{8b}$; $\Pi_S = \frac{a^2}{4b}$.

Note that (i) the equilibrium outcome mimics the solution of the static model in each period, and (ii) no inventories are carried. So the outcome here is very different from that of the dynamic contract analyzed previously. The next section compares the prices and profits (for supplier, buyer and channel) under the two contracts.

2.4 Comparisons

From Theorems 2.1 and 2.2, only one variable - the ratio $\frac{h}{a}$ - matters for qualitative comparisons of prices and profits under dynamic and commitment contracts. This is easily checked by taking the ratio of price (wholesale/retail) and profit (supplier/buyer/channel) expressions under dynamic and commitment contracts: these are functions of $\frac{h}{a}$. Since we vary this ratio all the way from 0 to 25% (the maximum value, beyond which inventories are simply not feasible), the qualitative insights from our numerical comparisons are entirely general. In the examples, we fix $a = \$100$ and $b = 1$; h then varies from 0 to \$25.

We first compare the (average) wholesale and retail prices under dynamic and commitment contracts. For the dynamic contract, since the quantities purchased and sold are different in each period, we compute the weighted average wholesale and retail prices. Using

Theorem 2.1, these are $w_{avg}^d = \frac{153a^2 - 68ah + 136h^2}{17(19a - 8h)}$ and $p_{avg}^d = \frac{461a^2 - 84ah - 104h^2}{34(19a - 8h)}$.¹⁰ For the commitment contract, the average wholesale price is the same as the price in either period, and given by Theorem 2.2. Figure 1 shows that w_{avg}^d is bounded from above by w^c . In fact, w_{avg}^d is increasing in h (for a fixed a), while w^c is constant; the two are equal only when $h = \frac{a}{4}$, at which point, inventories are too costly to be feasible, and dynamic and commitment equilibria are identical. Thus we conclude that strategic inventories force the supplier to reduce his wholesale prices.

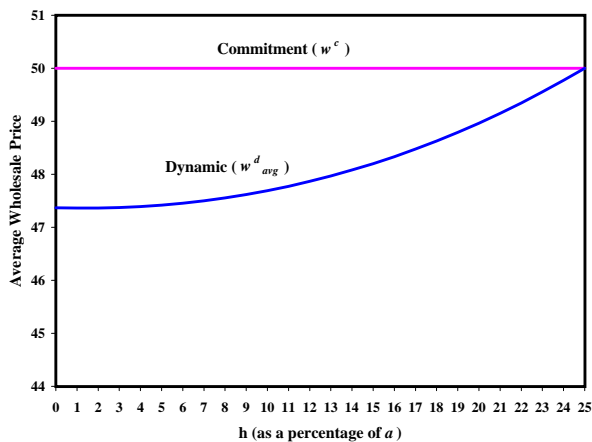


Figure 1: Comparison of Average Wholesale prices.

The lower wholesale price is reflected in lower average retail price, as seen in Figure 2. In fact, the behavior of the retail prices as h varies mimics that of the wholesale prices: p_{avg}^d is increasing in h , and reaches its upper bound (p^c) only at $h = \frac{a}{4}$ – the maximum value of the unit holding cost in our model. While the double marginalization effect persists under both contracts (retail prices are greater than p^{fb}), it is diminished under dynamic contracts, especially for smaller values of h . Hence we conclude that *strategic inventories reduce the double marginalization effect*.

We illustrate these effects graphically (Figure 3), using averages over the two periods (The averages provide insight because demand is certain in the model; hence demand curves may be horizontally summed or averaged). MR is the marginal revenue line. We know from our previous analysis that the retail prices are related as $p^c = p^s > p_{avg}^d > p^{fb}$. Correspond-

¹⁰We will use the following superscripts throughout: 'd' for dynamic contract, 'c' for commitment contract, 'fb' for first-best and 's' for the static equilibrium.

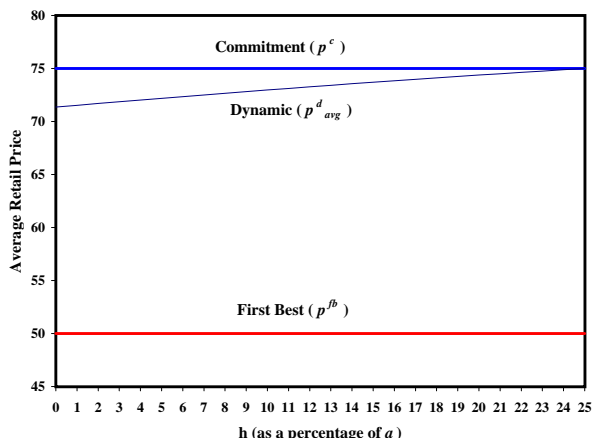


Figure 2: Comparison of Average Retail prices.

ingly, sales quantities are ordered as $q^c = q^s < q^d_{avg} < q^{fb}$, and wholesale prices are related as $w^c = w^s (= p^{fb}) > w^d_{avg}$. The graph of Figure 3 captures these relationships. The channel profits under first-best, the dynamic contract and the commitment contract are given by the areas of rectangles GHIL, DFJL, and BCKL respectively. By definition of first-best, and since $p^{fb} < p^d_{avg} < p^c$, $area(GHIL) > area(DFJL) > area(BCKL)$; i.e., the channel benefits more from the dynamic contract (revenues given by $area(DFJL)$) than from the commitment contract, except for the loss due to inventories (represented by the rectangle with area hI). Obviously, for $h = 0$ (and by continuity, small values of h), channel profits are greater under dynamic contracts than under commitment contracts. This is illustrated in Figure 4. Under the dynamic contract, both revenues and profits for the channel decrease with h . In fact the channel profits are higher under dynamic contracts for all $\frac{h}{a} < 19\%$, i.e., for all holding costs up to 38% of the static wholesale price, given by $\frac{a}{2} = \$50$. Thus we conclude that for most reasonable values of holding costs, the channel is better off with dynamic contracts. However, in situations with very high values of h , the channel cost of carrying inventories outweighs the benefit from lower dynamic prices, and the channel does better under commitment rather than dynamic contracts.

We now study how the two parties fare *individually* under each type of contract. Intuitively, we would expect that the buyer is better off under the dynamic contract because she can (and does) use strategic inventories to reduce the supplier's second period monopoly power and obtain lower second period wholesale prices (The buyer induces competition by forcing the

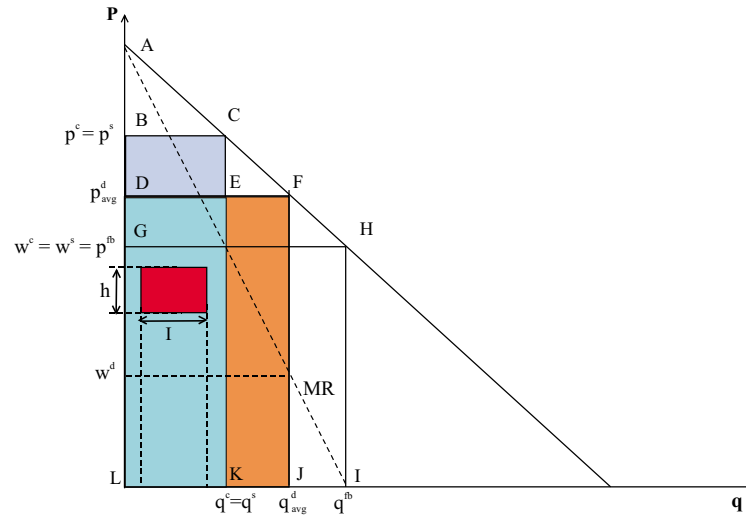


Figure 3: Double Marginalization Effects.

supplier to compete with her inventories). On the other hand, for precisely these reasons, the supplier might be better off under the commitment contract, where he preempts the buyer by committing early to the second period wholesale price. The following Proposition, which summarizes the profit comparisons under the two kinds of contracts, demonstrates that our ‘intuition’ does not hold.

Proposition 2.2 *The channel profits are higher under the dynamic contract for $\frac{h}{a} < \frac{55}{288} \approx 19\%$; otherwise the channel does better under the commitment contract. The buyer prefers the dynamic contract for $\frac{h}{a} < \frac{21}{152} \approx 14\%$. Finally, the supplier always prefers the dynamic contract.*

As expected, we find that for a wide range of holding costs, the buyer’s profits are greater under the dynamic contract than under the commitment contract (Figure 5). However, we find that the supplier is also (*always*) better off under dynamic pricing (Figure 6); i.e., his profits fall when he as the early-mover can commit credibly and early to *both* periods’ wholesale prices (before the buyer chooses purchase quantities). Thus, for a wide range of reasonable values of h (up to 28% of the static wholesale price), the dynamic equilibrium is a *pareto-improvement* over the commitment equilibrium. Hence, inventories, far from merely being a drain on the channel, have actually improved channel coordination.

Finally, we compare the consumer surplus and welfare under the two contracts. Since the average retail price is lower under the dynamic contract, the consumer surplus under the dynamic contract (given by the $area(ADF)$ in Figure 3) is greater than that under the

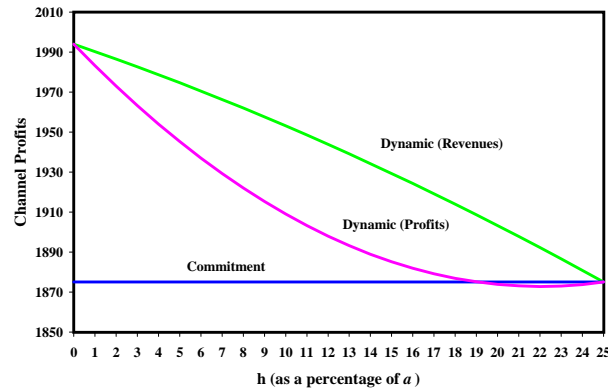


Figure 4: Comparison of Channel Profits.

commitment contract (given by the $area(ABC)$). As Figure 7 shows, welfare (the sum of consumer surplus and channel profits) is always greater under the dynamic contract than under the commitment contract. Proposition 2.3 formally establishes these results.

Proposition 2.3 *The total consumer surplus and welfare are always higher under the dynamic contract.*

We summarize and provide more intuition into the main results of our analysis thus far. We extended the classical static model of a vertical value chain to a dynamic context with inventories, and found that the equilibrium optimal policy was non-stationary even under a stationary demand and cost environment: wholesale and retail prices, as well as quantities, were non-identical across periods. By eliminating scale economies, lead times and demand uncertainty in our model, we precluded the possibility of any cycle, pipeline or safety inventories. Further, positive holding costs and endogenously *decreasing* equilibrium wholesale prices, as well as the absence of exogenous demand shocks, eliminated any incentive for forward buying by the buyer. Nevertheless, inventories were carried under our model, for purely *strategic* reasons arising from incentive misalignment between supplier and buyer.

By carrying inventories from the first period into the second, the buyer forces the supplier to compete with her inventories and price for residual demand in the second period. Thus, inventories reduce the supplier's second period monopoly power, and lower the second period wholesale price. The supplier anticipates this, and increases his first period wholesale price

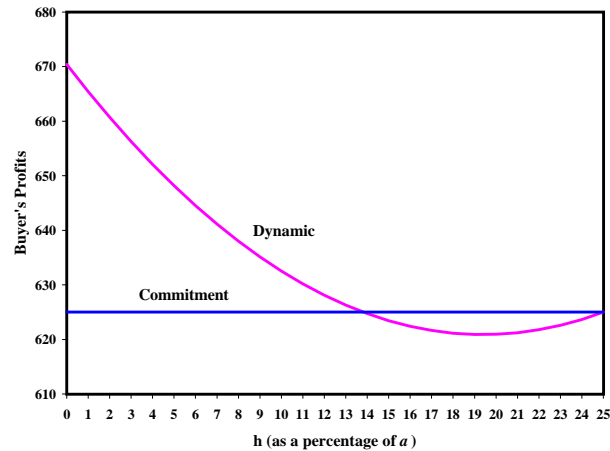


Figure 5: Comparison of Buyer's profits.

to discourage the buyer from carrying inventories. However, he cannot indefinitely raise his wholesale price, since this cuts into his first-period profits through reduced sales. The equilibrium wholesale prices reflect this tradeoff.

To eliminate channel losses due to strategic inventories, we analyzed the case of commitment contracts. We showed that the commitment contract equilibrium mimics the static equilibrium in each period, thus eliminating strategic inventories. However, we found to our puzzlement that, compared to the commitment contract, the dynamic contract usually leads to higher buyer and channel profits, and *always* results in higher supplier profits, greater consumer surplus and improved welfare. The reason for these effects is a surprising dynamic inadvertently induced by strategic inventories.

We know that linear prices lead to channel and welfare losses, due to both unextracted consumer surplus and dead-weight loss (customers who value the good above its marginal cost are not served). Double marginalization exacerbates the dead-weight loss and the welfare loss. However, under the dynamic contract, by carrying inventories in the first period, the buyer can in effect source for the second period at two different prices: (i) from the supplier, at his second period wholesale price, or (ii) from her own inventories at a different price. Although the supplier anticipates this and prices accordingly, he is unable to entirely eliminate inventories from the channel. *Thus, strategic inventories relax (expand) the effective space of vertical contracts, and induce non-linear wholesale pricing in the second period— lowering average wholesale and retail prices, and reducing the double-marginalization effect.* However,

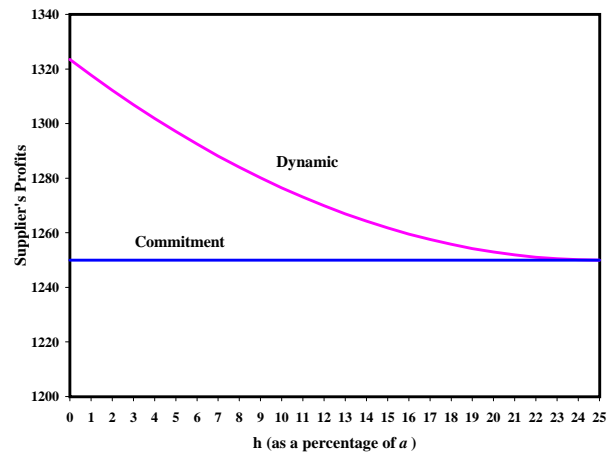


Figure 6: Comparison of Supplier's profits.

a second, countervailing effect is the drain on buyer (and channel) profits due to inventory holding costs. In most cases (small to medium values of h), the *contract-space-expansion* effect dominates the *inventory-drain* effect, and supplier, buyer and channel profits are greater under dynamic pricing. However, when h is high enough, the inventory-drain effect outweighs the contract-space-expansion effect of the equilibrium, and the buyer and channel begin to do better under commitment contracts (The supplier in our model is always better off under the dynamic contract, reversing the conventional wisdom on the value of commitment to the early-mover.).

Given our findings, two questions arise regarding strategic inventories: (i) Are strategic inventories a consequence merely of linear wholesale pricing; i.e., will they arise in other contexts? and (ii) Will the contract-space-expansion effect, which leads to counter-intuitive results on individual and channel profits, vanish or be dominated by the inventory-drain effect when the vertical contract is not restricted to linear pricing? In other words, are strategic inventories significant in our model only because of the overly restrictive nature of the linear vertical contract space (which they indirectly expand)? What would happen if we relaxed the contract space? To address these questions, we study the case of two-part tariff vertical contracts in the following section.

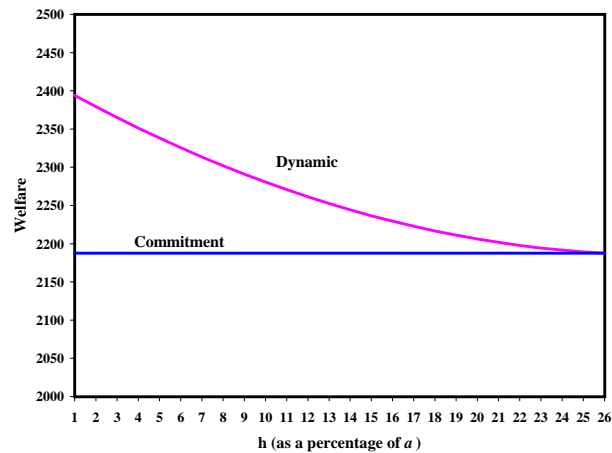


Figure 7: Comparison of Welfare.

3 Two-Part Tariffs

Under linear wholesale prices, inventories improve channel profits by inducing an expansion in the vertical contract space, but the first-best solution is never achieved. In contrast, under two-part tariffs, the first-best is attained even in the static problem, with the supplier extracting all of the residual channel profits. We examine whether inventories can play any (strategic) role under two-part tariff vertical contracts, in a dynamic setting.

The solution to the static problem (section 2.1) under two-part tariffs entails setting the marginal wholesale price w equal to the supplier's marginal production cost (zero here), which maximizes channel profits. The supplier charges a fixed fee of $K = \frac{a^2}{4b}$ to extract away all of the retailer's surplus. The two-part tariff scheme has two possible interpretations under the static setting: (i) The supplier 'sells the firm' to the buyer at the price K (which allows the marginal production cost to pass through); and (ii) The supplier offers quantity (or volume) discounts to the buyer via this instrument— the average purchase price is decreasing in the purchased quantity. These two interpretations of the optimal solution are indistinguishable under the static setting.

In the dynamic context, however, the two approaches ('selling the firm' and 'volume discounting') are not equivalent. The supplier can sell his production/procurement unit to the buyer at a price equal to the expected channel profits over the horizon. Then the erstwhile buyer, who now owns the entire channel, implements the first-best solution. For many rea-

sons (well-documented in the literature), selling the firm is often not a viable strategy; after all, multiple firms do exist in practice.¹¹ Hence, in this section, we focus on the more interesting case of using two-part tariffs as a volume discount instrument. We address the following issues: (i) Are inventories carried under dynamic two-part tariff vertical contracts? (ii) If so, what is their effect on supplier, buyer, and channel profits? (iii) What would be the impact of the supplier's early price commitment on supplier, buyer, and channel profits? (iv) Finally, do inventories play any role under commitment two-part tariff contracts?

We assume that the demand and cost structures are the same as before. As we did for linear wholesale prices, we first analyze the *dynamic* two-part tariff contract. Then we study the two-part tariff contract with *commitment*, in which the supplier commits to the wholesale price schedules (fixed fees and linear prices) for both periods in advance.

3.1 Dynamic Two-part Tariff

In this contract, the supplier first quotes a price schedule (K_1, w_1) for the first period, where K_1 is the fixed fee payable upon purchase of non-zero quantities, and w_1 is the linear, marginal wholesale price. The buyer can purchase $Q_1 > 0$ units at the price $K_1 + w_1 \cdot Q_1$, or elect not to purchase any quantity at all and pay nothing. She can then sell $q_1 \leq Q_1$ in the market, and carry inventory $I = Q_1 - q_1$ into the second period. In the second period, the supplier quotes the price schedule (K_2, w_2) . The buyer purchases $Q_2 \geq 0$ units as per this price schedule, and sells $Q_2 + I$ in the second period.

This game has a unique subgame-perfect equilibrium, which is derived in Theorem 3.1 below.

Theorem 3.1 *The unique subgame-perfect equilibrium outcome is as follows.*

1. *Marginal Wholesale Prices:* $w_1 = \frac{2}{3}a; w_2 = 0$.
2. *Fixed Fees:* $K_1 = \frac{1}{b} \left[\left(\frac{a}{6}\right)^2 + \left(\frac{a-3h}{6}\right)^2 \right]; K_2 = \frac{1}{b} \left[\frac{a^2}{4b} - \left(\frac{5a}{6} + \frac{h}{2}\right) \left(\frac{a}{6} - \frac{h}{2}\right) \right]$.
3. *Purchase Quantities:* $Q_1 = \frac{a}{3b} - \frac{h}{2b}; Q_2 = \frac{a}{3b} + \frac{h}{2b}$.
4. *Inventory:* $I = \frac{a}{6b} - \frac{h}{2b}$.
5. *Sales Quantities:* $q_1 = \frac{a}{6b}; q_2 = \frac{a}{2b}$.

¹¹Furthermore, 'selling the firm' will fail to garner the first-best profits to the supplier under a host of reasonable relaxations of the problem - the obvious ones being any kind of demand uncertainty coupled with asymmetric information and/or risk aversion. As discussed previously, we prefer to isolate the phenomenon of strategic inventories by not introducing these confounding issues.

6. *Retail Prices:* $p_1 = \frac{5a}{6}; p_2 = \frac{a}{2}$.

7. *Profits:* $\Pi_B = 0; \Pi_S = \frac{1}{18b} [7a^2 - 3ah + 9h^2]$.

Theorem 3.1 shows that strategic inventories once again induce a non-stationary equilibrium solution in the otherwise stationary model. The buyer carries strategic inventories even though $w_1 > w_2 = 0$ in equilibrium, and even when $h > 0$. These inventories drain channel profits, ensuring that, unlike in the static setting, two-part tariffs do not lead to the first-best solution. To discourage inventories (that cut into his second period profits), the supplier sets w_1 to be very high ($= \frac{2}{3}a$); this induces a significant level of double-marginalization under two-part tariffs, and further erodes channel profits. However, the supplier manages to extract all of the buyer's profits.

3.2 Two-part Tariff with Commitments

We assume that the supplier still employs two-part tariff wholesale prices, but commits early to his second period price schedule. In a manner analogous to the linear commitment contract, the supplier announces his comprehensive price schedule $[(K_1, w_1), (K_2, w_2)]$ at the outset. Then the buyer decides on purchase quantities Q_1 and Q_2 , and inventory I . Thus the sales quantities in each period are $Q_1 - I$ and $Q_2 + I$, for which revenues are realized. This game has a unique subgame-perfect equilibrium, which is derived in Theorem 3.2 below.

Theorem 3.2 *The unique subgame-perfect equilibrium is as follows:*

1. *Wholesale Prices:* $w_1 = \frac{a-h}{2}; w_2 = 0$.

2. *Fixed Fees:* $K_1 = \frac{(a+h)^2}{16b}; K_2 = \frac{a^2}{4b} - \frac{(a-h)^2}{16b}$.

3. *Purchase Quantities:* $Q_1 = \frac{a+h}{4b}; Q_2 = \frac{a}{2b}$.

4. *Inventory:* $I = 0$.

5. *Sales Quantities:* $q_1 = \frac{a+h}{4b}; q_2 = \frac{a}{2b}$.

6. *Retail Prices:* $p_1 = \frac{3a-h}{4}; p_2 = \frac{a}{2}$.

7. *Profits:* $\Pi_B = \frac{(a-h)^2}{16b}; \Pi_S = \frac{3a^2 + 2ah - h^2}{8b}$.

Inventories play a subtle strategic role in this equilibrium. Observe that prices, quantities and profits are all functions of the holding cost h . Even though the buyer does not carry

inventories in equilibrium, her *threat* to do so forces the supplier to hike up his first-period wholesale price w_1 above marginal cost, leading to double marginalization and channel losses. Nevertheless, the buyer makes positive residual profits that the supplier is unable to extract. In fact, since the threat of carrying inventories diminishes as h increases, w_1 is a decreasing function of h , and equals marginal cost (zero) only when $h = a$. The *channel loss*, the difference between first-best channel profits and those under two-part tariffs with commitment, is $\frac{(a-h)^2}{16b}$; this is also decreasing in h , and converges to zero as h approaches a . These results confirm that inventories can be strategically important even for non-linear contracts; they are not just artifacts of the restrictive space of linear contracts.

3.3 Comparisons

Proposition 3.1 compares the channel, buyer and supplier profits for the two kinds of two-part tariff contracts analyzed above.

Proposition 3.1 *Consider two-part tariffs. Both channel and buyer profits are always greater under the commitment contract than under the dynamic contract. Supplier profits are greater under the dynamic contract whenever $\frac{h}{a} < \frac{1}{3(5+2\sqrt{5})} \approx 3.5\%$; otherwise they are greater under the commitment contract.*

We now compare the channel performance under all four vertical contracts analyzed previously. Building on Proposition 2.2 (that compares the two linear contracts) and Proposition 3.1 (that compares the two contracts under two-part tariffs), we have the following result:

Proposition 3.2 *Comparing the four kinds of vertical contracts – dynamic linear pricing, commitment linear pricing, dynamic two-part tariffs and commitment two-part tariffs – channel profits are highest under commitment two-part tariffs.*

To summarize, we find that first-best is never achieved in the dynamic (two-period) setting, even with two-part tariff vertical contracts. Strategic inventories (or the threat of carrying them) are a powerful weapon in the hands of the buyer. Responding to this threat, the supplier raises his first-period wholesale price. This induces some level of double marginalization in the channel, and erodes channel profits. Under dynamic two-part tariffs, the supplier is unable to prevent the buyer from carrying inventories, but extracts the entire channel profits. The reverse is true under commitment two-part tariffs: Here, the supplier prevents the buyer from carrying inventories, but the buyer makes positive residual profits.

4 Generalizations

In this Section we prove that inventories play a key strategic role in the space of the most general (arbitrary) dynamic contracts, even under a general (not necessarily linear) demand structure, and for general (finite or infinite) horizon lengths. The ability to carry inventories affects the equilibrium outcome, regardless of whether inventories are actually carried by the buyer in equilibrium or not. Theorem 4.1 below states the key result for a two-period dynamic contract: When the buyer can carry inventories, the supplier cannot enforce the first-best solution and simultaneously extract all of the buyer's residual profits, irrespective of the contracting mechanism he uses. This result is an artifact solely of the buyer's *ability* to carry inventories strategically (in the equilibrium, she may or may not carry inventory). Theorem 4.2 shows that the result extends to general horizon lengths.

The results from our analysis of commitment contracts are mixed. On the one hand, when the form of contracting is restricted to the widely used linear or quantity-discount commitment contracts, the strategic effect of inventories persists (Theorem 4.3). However, as Theorem 4.4 shows, when the space of commitment contracts is expanded to allow for the most general (arbitrary) contracts, the supplier manages to garner first-best profits: the contracts that achieve this are variants of the idea of selling the firm to the buyer, to eliminate incentive misalignment. (It is worth emphasizing that such 'selling the firm' solutions will fail under many plausible model relaxations; e.g., demand uncertainty coupled with asymmetric information and/or risk aversion. These relaxations will induce *safety* and/or *speculative* inventories in addition to strategic inventories.)

Thus, our previous results on the role of strategic inventories are robust to horizon lengths under both dynamic and commitment contracts: the results from our two-period model were by no means driven by any 'end-of-horizon' effect.

Consider the case of a general demand function, stationary across periods, given by $P(q)$, where $P(\cdot)$ is the price as a function of quantity. The revenue function, $R(q)$, is given by $R(q) = P(q) \cdot q$. In our analysis, we work with the revenue function. We assume that future periods are discounted, and model this via a per-period multiplicative discount factor of δ ; without loss of generality, $0 < \delta \leq 1$. Thus, M dollars received in period k are worth $\delta \cdot M$ dollars in period $k - 1$. The undiscounted case corresponds to $\delta = 1$. Our results also extend to the infinite horizon, provided $\delta < 1$ strictly: this condition ensures that total discounted profits over the infinite horizon are bounded. To minimize technical distractions, we assume that the revenue function $R(q)$ satisfies the following properties:

1. $R(\cdot)$ is twice-differentiable everywhere, and has a well-defined inverse $R^{-1}(\cdot)$ which is

also differentiable.

2. $R(\cdot)$ is increasing and concave in q . Thus, selling more quantity in the market will yield greater revenues, but at a decreasing marginal rate. Mathematically, $R'(q) > 0$ and $R''(q) < 0, \forall q \geq 0$. This implies further that the function $R^{-1}(\cdot)$ is increasing and convex (Thus, the function $R^{-1'}(\cdot) > 0$).
3. $R(0) = 0$ (selling zero quantities will yield zero revenues) and $\delta \cdot R'(0) > h$. The latter condition states that the *maximum* marginal revenue from carrying inventory to the next period, discounted to the current period, is greater than the marginal holding cost, a very reasonable assumption.

Most non-pathological demand functions can be characterized or at least approximated by revenue functions that satisfy the preceding properties. We use the single-period, first-best solution as a benchmark. Assuming as before that the marginal production cost is zero¹² the *unique* optimal solution is $q_{fb} : R'(q_{fb}) = 0$, i.e., $q_{fb} = R'^{-1}(0)$, and the corresponding first-best profits are $R(q_{fb}) = R(R'^{-1}(0))$.¹³ In the rest of this Section, we work with the general revenue function specified above.

4.1 Dynamic Contracts

4.1.1 Two-Period Problem under General Dynamic Contracts

Theorem 4.1 below proves that inventories play a strategic role in the dynamic (two-period) setting under a general, stationary demand function and in the space of the most general (arbitrary) dynamic vertical contract. The Theorem shows that the supplier *cannot* implement a contract with the buyer that will give him the first-best profits of $(1 + \delta) \cdot R(q_{fb}) = (1 + \delta) \cdot R(R'^{-1}(0))$:¹⁴ Either the buyer will make residual profits (using strategic inventories) or (even if the supplier is able to prevent this) the equilibrium outcome will be different from the first-best outcome and generate lower profits.

¹²Relaxing this assumption is straight-forward but omitted in the interests of clarity of the presentation. The condition we really need is that the net profit function, given by $R(\cdot) - C(\cdot)$ satisfies the properties specified above for $R(\cdot)$, where $C(\cdot)$ is the production cost function. Thus, all we need is that $C(\cdot)$ is "less concave" than $R(\cdot)$, which is guaranteed for all linear and convex production cost functions.

¹³In the special case of the linear demand curve given by $P(q) = a - bq$, the marginal revenue curve is given by $R'(q) = a - 2b \cdot q$. Hence, $R'^{-1}(y) = \frac{a-y}{2b}$, and $q_{fb} = R'^{-1}(0) = \frac{a}{2b}$. The per-period sales revenue from the first-best solution is $R(q_{fb}) = R(\frac{a}{2b}) = \frac{a^2}{4b}$.

¹⁴Recall that in the undiscounted case, $\delta = 1$.

Theorem 4.1 *Consider the two-period model under dynamic contracting, with a general demand (and revenue) function as specified above. When the buyer can carry inventories, there exists no dynamic vertical contract using which the supplier can simultaneously implement the first-best solution and extract away all of the buyer's residual profits.*

Corollary 4.1 *Strategic inventories ensure that the supplier can never make first-best profits, even if he manages to extract away all of the buyer's residual profits.*

To establish that it is the buyer's ability to carry inventories that drives the result of Theorem 4.1, consider the case that the buyer (for whatever reason) *cannot* carry inventories¹⁵. Here, the periods are effectively decoupled, and so the supplier can easily structure a dynamic contract that implements the first-best solution and extracts all of the buyer's residual profits. For example, the two-part tariff contract in each period, with a fixed fee of $R(R'^{-1}(0))$ and marginal-cost unit pricing (0 in this case), ensures first-best profits for the supplier. Thus, under dynamic contracts, the strategic role of inventories is *not* an artifact of restrictions such as linear or two-part tariff contracts or the linear demand function.

4.1.2 General Horizon Lengths under General Dynamic Contracts

We now show that in the general space of dynamic contracts (not confined to linear pricing or two-part tariffs), for a general class of demand functions and for general horizon lengths, inventories play a strategic role.

Theorem 4.2 *Consider the finite horizon model (n -period repeated game) under dynamic contracting, with a general demand (and revenue) function as specified above. When the buyer can carry inventories, there exists no dynamic vertical contract using which the supplier can simultaneously implement the first-best solution and extract away all of the buyer's residual profits.*

Corollary 4.2 *Strategic inventories ensure that the supplier can never make first-best profits, even if he manages to extract away all of the buyer's residual profits.*

Theorem 4.2 applies to both undiscounted and discounted finite horizons (i.e., $\delta \leq 1$), as well as the discounted infinite horizon ($\delta < 1$): The latter restriction for the infinite horizon is to ensure that total profits are not unbounded.

To complete the loop, we need to establish that it is the buyer's strategic option to carry inventories that drives the result of Theorem 4.2. We can show this by proving that if the

¹⁵The assumption that $\delta \cdot R'(0) > h$ is not applicable in this case.

buyer cannot carry inventories, the supplier can structure an appropriate contract to garner first-best profits. The buyer's inability to carry inventories effectively decouples the successive periods. In this case, a simple two-part tariff contract in each period with a fixed fee of $R(R'^{-1}(0))$ and marginal price of 0 accomplishes the supplier's objectives.

4.2 Commitment Contracts

In Theorem 3.2, we showed that inventories play a strategic role under a quantity discount commitment contract for the two-period problem. Theorem 4.3 below extends this result to the general n -period problem.

Theorem 4.3 *Consider the finite horizon model (n -period repeated game) under commitment contracting, where the vertical contract in each period is a quantity discount scheme implemented using two-part tariffs. We assume a general demand (and revenue) function as specified above. When the buyer can carry inventories, there exists no two-part tariff commitment contract using which the supplier can simultaneously implement the first-best solution and extract away all of the buyer's residual profits.*

Corollary 4.3 *Strategic inventories ensure that the supplier can never make first-best profits, even if he manages to extract away all of the buyer's residual profits.*

As was true with Theorem 4.2, Theorem 4.3 applies to both undiscounted and discounted finite horizons (i.e., $\delta \leq 1$), as well as the discounted infinite horizon ($\delta < 1$).

Once again, it is easily established that the result of Theorem 4.3 is driven by the buyer's ability to carry inventories: When the buyer cannot carry inventories, the supplier can obtain first-best profits by proposing a simple two-part tariff contract in each period with a fixed fee of $R(R'^{-1}(0))$ and marginal price of 0.

Since linear commitment contracts are a special case of two-part tariffs, with the fixed fee set to zero, Theorem 4.3 also applies to this case. The following Theorem studies the case of *general* commitment contracts (not confined to linear pricing or two-part tariffs).

Theorem 4.4 *In the space of the most general of commitment contracts, the supplier can always achieve first-best profits, i.e., simultaneously implement the first-best solution and extract away all of the buyer's residual profits, even when the buyer can carry inventories strategically.*

Theorem 4.4 holds for any arbitrary (finite or infinite) horizon length, and for any arbitrary demand (and revenue) function. The proof of Theorem 4.4 is by construction of a contract that

actually implements first-best profits for the supplier. Contracts that ensure first-best profits to the supplier are variants of the idea of 'selling the firm' to the buyer, at a fixed fee equal to the value of the combined firm.

To summarize the results of this Section, inventories play an unassailable strategic role for the buyer under dynamic vertical contracts. The intuition is that, however rich the dynamic vertical contract constructed by the supplier is, the contract's effectiveness is restricted to *within* the period of its enforcement. The buyer maneuvers around the string of intra-period dynamic contracts by exploiting the inter-period link enabled by her inventories. This inter-period dynamic of inventories (wherein inventories acquired in earlier periods serve as a deterrent to the supplier's monopoly power in later periods) is less effective when the supplier can design a commitment contract that anticipates this inter-period dynamic. The result of such commitment contracts is in fact equivalent to "selling the firm" to the buyer at an appropriate fee that (barely) satisfies the buyer's participation constraint. We see that inventories are strategically effective for the buyer under linear or quantity-discount commitment contracts, since these are not equivalent to the supplier's "selling his firm". Finally, we established that the strategic role of inventories is robust to general demand functions and arbitrarily long, discounted or undiscounted horizon lengths.

5 Postscript: Taking Stock

In an increasingly competitive marketplace, firms are recognizing that their supply chain structures, including their inventory management policies and their relationships with suppliers and vendors, play a critical role in determining their profits. The grocery industry's popular Efficient Consumer Response movement (Salmon & Associates, 1993) and the apparel industry's Quick Response initiative (Abernathy, Dunlop, Hammond & Weil, 1999; Lowson, King & Hunter, 1999), which focus on optimizing firms' supply chains, are cases in point. In this spirit, our paper studies the relationship between inventories and contracting.

In traditional inventory management, the buyer told the supplier when and how much of the good she needed, and the supplier attempted to meet these demands. This "arms-length" relationship, characterized by incentive misalignment and upstream variability, led to excess inventories and costs. Firms are now experimenting with new and creative methods of inventory management, while appropriately tailoring their relationships with their suppliers and buyers. More cooperative vertical relationships such as Vendor Managed Inventories (VMI) and consignment are gaining in popularity. Under VMI, the supplier uses the available consumer demand data and his own product expertise to actively manage the retailer's inventory.

The consignment system takes this a step further whereby the supplier even *owns* the inventory at the retailer's. In effect, the supplier absorbs the entire financial holding costs for his products, and charges the retailer only after sales are made.

Under consignment, by bearing the entire financial holding costs, the supplier provides a strong incentive to the buyer to stock his products. A more modest incentive scheme is to subsidize the buyer's holding costs. When shelf-space is limited (as it often is), holding cost subsidies are a way of sharing risk. In fact, generous payment terms commonly offered by suppliers are a form of such subsidies. If the product does not move quickly enough off the display aisles, the supplier bears some of the costs. Thus, such schemes enable the supplier to credibly signal his confidence in the demand for his products.

The contractual implications of such relationships are not well understood. Our paper is a first step. Once one recognizes that inventories play a strategic role in the relationship, it is clear that holding cost subsidies must also do so - in ways that are different from simple wholesale price discounts. Future work, currently under way, studies the effect of holding-cost subsidies on contractual structure.

In complex supply chains, with extended networks of suppliers and buyers, each firm purchases its production inputs from multiple suppliers, and in turn supplies its output(s) to many buyers. Inventories play an important role in both types of relationships. On the *demand* side, safety inventories can mitigate the effect of demand spikes. While there is a well-developed literature studying the relationship between inventories and demand-side phenomena, our paper focuses on the relationship between inventories and the supply side. In our model, inventories provide a strategic *option*, with their option value related to the mitigation of the supplier's monopoly. A recent paper by Erhun *et.al.* (2000) builds on this idea in studying the role of capacity options. In their model, the supplier 'sells' manufacturing capacity, and the buyer can purchase capacity options at different times.

In this paper, we assumed no uncertainty in demand, in order to focus on upstream interactions at the wholesale level. (Under demand uncertainty, the presence of safety inventory would have made it difficult to isolate, and hence analyze, the strategic role of inventory.) Previous academic research has shown that under demand uncertainty, information interacts with inventories in subtle but important ways (Anand, 1999). Under asymmetric information, with the buyer having more accurate demand information than the supplier, inventories may play an even more important role. In essence, the supplier tries to infer the demand structure for future periods from the buyer's orders. Inventories enable the buyer to decouple her demand from her orders to the supplier, and thus to conceal her actual demand. On the other hand, by building up adequate levels of inventories, the consequences to the buyer of

revealing her expected future demand may be less important. We speculate that if the latter effect dominates, the entire channel may be better off with inventories, and in fact the supplier may want to offer holding cost subsidies to induce revelation. We are currently exploring these issues, in an effort to fully characterize the interaction of Inventories, Incentives, and Information (I^3) in a value chain.

Our model focused on a bilateral monopoly to throw the spotlight on the strategic role of inventory in a vertical contract. An interesting extension would be to the case of retailer (and supplier) competition; we speculate that inventory would play other kinds of strategic roles in these settings.

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A Appendix: Proofs of Theorems and Lemmas

Proof of Theorem 2.1:

In the analysis of this sequential game, we start with the second period, assuming that the retailer carries inventories I and that the supplier has quoted a wholesale price w_2 . The retailer's problem is now the quadratic:

$$\max_{Q_2} (a - b(Q_2 + I))(Q_2 + I) - w_2 Q_2.$$

The supplier's problem in the second period is:

$$\max_{w_2} w_2 Q_2 = w_2 \left(\frac{a - w_2}{2 \cdot b} - I \right)^+.$$

Solving for Q_2 for the buyer, we get that $Q_2 = \left(\frac{a - w_2}{2 \cdot b} - I \right)^+$. Assuming $I < \frac{a - w_2}{2 \cdot b}$, (without which the supplier's wholesale price is irrelevant), substituting for Q_2 into the supplier's profit function, and optimizing we get: $w_2 = \frac{a}{2} - b \cdot I$. This implies that $Q_2 = \frac{a}{4 \cdot b} - \frac{I}{2}$. The second-period profit functions of the supplier and the buyer as a function of buyer's inventories are given by

$$\begin{aligned} \Pi_{B,2}(I) &= \frac{1}{4b} [a - w_2]^2 + w_2 I \\ &= \frac{1}{4b} \left[\frac{a}{2} + bI \right]^2 + \left[\frac{a}{2} - bI \right] I, \text{ and} \end{aligned} \quad (1)$$

$$\Pi_{S,2}(I) = \frac{1}{2b} \left[\frac{a}{2} - bI \right]^2. \quad (2)$$

Now, moving to the first period, when the supplier quotes a wholesale price w_1 , the buyer's problem becomes

$$\max_{q_1, I} (a - bq_1)q_1 - w_1(q_1 + I) - hI + \frac{1}{4b} \left[\frac{a}{2} + bI \right]^2 + \left[\frac{a}{2} - bI \right] I, \quad (3)$$

where the buyer buys $Q_1 = q_1 + I$ in the first period, sells q_1 and carries inventory I to sell in the second period. Imposing non-negativity constraints on q_1 and I , the Lagrangian is

$$\mathcal{L}(q_1, I, \lambda, \mu) = (a - bq_1)q_1 - w_1(q_1 + I) - hI + \frac{1}{4b} \left[\frac{a}{2} + bI \right]^2 + \left[\frac{a}{2} - bI \right] I + \lambda q_1 + \mu I,$$

with λ and μ non-negative. The first-order and complementary slackness conditions give us two possible cases, depending on the parameter values:

(i) $\underline{w_1 + h \geq \frac{3a}{4}}$: $I = 0$; $Q_1 = q_1 = \frac{a - w_1}{2b}$; $\lambda = 0$; $\mu = w_1 + h - \frac{3a}{4}$.

(ii) $\underline{w_1 + h < \frac{3a}{4}}$: $I = \frac{2}{3b} \left[\frac{3a}{4} - w_1 - h \right]$; $q_1 = \frac{a - w_1}{2b}$; $Q_1 = \frac{1}{b} \left[a - \frac{7}{6}w_1 - \frac{2}{3}h \right]$; $\lambda = 0$; $\mu = 0$.

The first period profits of the supplier, as a function of w_1 , are:

$$\Pi_{S,1} = \begin{cases} w_1 q_1 = w_1 \left(\frac{a - w_1}{2b} \right), & \text{if } w_1 + h > \frac{3a}{4}; \\ w_1 (q_1 + I) = \frac{w_1}{b} \left[a - \frac{7}{6}w_1 - \frac{2}{3}h \right], & \text{otherwise.} \end{cases}$$

From equation (2), we see that the second period supplier profits are:

$$\Pi_{S,2} = \begin{cases} \frac{a^2}{8b}, & \text{if } w_1 + h > \frac{3a}{4}; \\ \frac{2}{9b} (w_1 + h)^2, & \text{otherwise.} \end{cases}$$

Hence, total supplier profits (over both periods) are

$$\Pi_S = \begin{cases} w_1 \left(\frac{a-w_1}{2b} \right) + \frac{a^2}{8b}, & \text{if } w_1 + h > \frac{3a}{4}; \\ \frac{w_1}{b} \left[a - \frac{7}{6}w_1 - \frac{2}{3}h \right] + \frac{2}{9b} (w_1 + h)^2, & \text{otherwise.} \end{cases}$$

The unconstrained maximum of the profit expression $w_1 \left(\frac{a-w_1}{2b} \right) + \frac{a^2}{8b}$ is at $w_1 = \frac{a}{2}$, at which point his profits are $\frac{a^2}{4b}$. Thus, if the supplier picks a w_1 such that $w_1 + h > \frac{3a}{4}$, his profits are bounded from above by $\frac{a^2}{4b}$. Further, the unconstrained maximum of the profit expression $\frac{w_1}{b} \left[a - \frac{7}{6}w_1 - \frac{2}{3}h \right] + \frac{2}{9b} (w_1 + h)^2$ is at $w_1 = \frac{9a-2h}{17}$, at which point his profits are $\frac{9a^2-4ah+8h^2}{34b}$. Further algebra reveals that $\frac{9a^2-4ah+8h^2}{34b} \geq \frac{a^2}{4b}$. Hence, the supplier's profit maximizing first-period wholesale price is given by $w_1 = \frac{9a-2h}{17}$, and his profits over both periods are $\frac{9a^2-4ah+8h^2}{34b}$.

The buyer's total profits can be calculated using equations (1) and (3), and the known values of w_1 and I . Similarly, it is now a simple matter to calculate the purchase and sales quantities, inventories, and wholesale and retail prices by plugging in these results into the previous expressions. ■

Proof of Theorem 2.2:

We first solve for the buyer's optimal response to a given w_1 and w_2 . Since his decision variables are Q_1, Q_2 and I , the buyer's decision problem is

$$\max_{Q_1, Q_2, I} (a - b \cdot (Q_1 - I)) \cdot (Q_1 - I) + (a - b \cdot (Q_2 + I)) \cdot (Q_2 + I) - h \cdot I - w_1 \cdot Q_1 - w_2 \cdot Q_2,$$

subject to non-negativity constraints $Q_2, I \geq 0$, and the additional constraint that $Q_1 \geq I$. Subsequently, we solve for the supplier's optimal pricing decision.

We ignore the last constraint in formulating the Lagrangian; we will check that that it holds in the solution to the relaxed problem. The Lagrangian (using the remaining constraints) is then

$$\begin{aligned} \mathcal{L}(Q_1, Q_2, I, \lambda, \mu) = & (a - b(Q_1 - I))(Q_1 - I) + (a - b(Q_2 + I))(Q_2 + I) \\ & - hI - w_1Q_1 - w_2Q_2 + \lambda I + \mu Q_2, \end{aligned}$$

with λ and μ non-negative. The first-order and complementary slackness conditions are:

$$\begin{aligned} I &= \frac{Q_1 - Q_2}{2} + \frac{\lambda}{4b} - \frac{h}{4b}; \\ Q_1 - I &= \frac{a - w_1}{2b}; \\ Q_2 + I &= \frac{\mu + a - w_2}{2b}; \\ \lambda I &= 0; \quad \text{and} \\ \mu Q_2 &= 0. \end{aligned}$$

Examining the various possible values of the parameters, and simplifying, the solution reduces to two possible cases, which are:

- (i) $w_1 + h \geq w_2$: $Q_1 = q_1 = \frac{a-w_1}{2b}$; $Q_2 = q_2 = \frac{a-w_2}{2b}$; $I = 0$; $\lambda = w_1 + h - w_2$; $\mu = 0$.
- (ii) $w_1 + h < w_2$: $Q_1 = \frac{a-w_1}{b} - \frac{h}{2b}$; $q_1 = \frac{a-w_1}{2b}$; $Q_2 = 0$; $q_2 = I = \frac{a-w_1}{2b} - \frac{h}{2b}$; $\lambda = 0$; $\mu = \frac{w_2 - (w_1 + h)}{2b}$.

Under both these cases, the constraint $Q_1 \geq I$ is satisfied; so the solution is both feasible and optimal.

The supplier profits (over *both* periods) are $\pi_S = w_1 Q_1 + w_2 Q_2$, which reduce to

$$\Pi_S = \begin{cases} w_1 \left(\frac{a-w_1}{2b} \right) + w_2 \left(\frac{a-w_2}{2b} \right), & \text{if } w_1 + h \geq w_2; \\ w_1 \left(\frac{a-w_1}{b} - \frac{h}{2b} \right), & \text{otherwise.} \end{cases}$$

Now the supplier optimizes her profit function over w_1 and w_2 . For the first case (assuming $w_1 + h \geq w_2$), we get that the optimal prices are $w_1 = w_2 = \frac{a}{2}$ giving the supplier optimal profits of $\frac{a^2}{4b}$. Clearly $w_1 + h \geq w_2$. For the second case (assuming $w_1 + h < w_2$), we derive $w_1 = \frac{a}{2} - \frac{h}{4}$ with optimal profits of $\frac{(a-h/2)^2}{4b}$. It is straightforward to see that the supplier makes a higher profit by implementing the prices given by the first case. The remaining quantities - sales, inventories, etc. - follow. ■

Proof of Proposition 2.2:

Follows by straightforward comparison of the appropriate profit expressions given in Theorems 2.1 and 2.2. ■

Proof of Proposition 2.3:

Formally, the consumer surplus in any given period is given by $\frac{1}{2} b q_i^2$ where q_i is the sales quantity in period i . Thus the total consumer surplus (S) across both periods for the dynamic contract is:

$$S^d = \frac{185a^2 - 188ah + 104h^2}{2312b}$$

and for the commitment contract is $S^c = \frac{a^2}{16b}$. The difference between the two is:

$$S^d - S^c = \frac{81a^2 - 376ah + 208h^2}{4624b}$$

which is always greater than zero for $h < a/4$. The difference in total welfare between the dynamic and commitment contract is computed as:

$$\frac{191a^2 - 1392ah + 2512h^2}{4624b},$$

which can be shown to be always non-negative for $h < a/4$. ■

Proof of Theorem 3.1:

To prove this result, we proceed in a backward fashion beginning from the last period. We will first derive the optimal policy of the buyer for the last period.

Given K_2, w_2 , and I from the supplier, the buyer solves the following optimization problem for period 2.

$$\max_{Q_2 \geq 0} (a - b(Q_2 + I))(Q_2 + I) - w_2 Q_2 - K_2 \mathbf{1}_{Q_2 > 0}. \quad (4)$$

Given buyer's response to K_2 and w_2 , the supplier solves

$$\max_{w_2, K_2} K_2 + w_2 Q_2(w_2, I) \quad (5)$$

The following Lemma describes the optimal policy structure for the second period:

Lemma A.1 *The optimal second period two-part tariff is $(K_2^*(I), w_2^*)$ where*

$$\begin{aligned} K_2^*(I) &= \frac{a^2}{4b} - (a - bI)I \\ w_2^* &= 0 \end{aligned}$$

The optimal purchase quantity for the buyer is $Q_2^ = \max\{\frac{a}{2b} - I, 0\}$. The corresponding optimal profits of the buyer and supplier are as follows:*

$$\begin{aligned} \Pi_{B,2}(I) &= (a - bI)I \\ \Pi_{S,2}(I) &= \frac{a^2}{4b} - (a - bI)I. \end{aligned}$$

Proof: : Solving for the optimal purchase quantity for the buyer, we get:

$$Q_2 = \max \left\{ \frac{a - w_2}{2b} - I, 0 \right\} \quad (6)$$

If $Q_2 > 0$, then the optimal second period profits $\Pi_{B,2}(I)$ for the buyer is:

$$\Pi_{B,2}(I) = \left(a - b \left(\frac{a - w_2}{2b} \right) \right) \left(\frac{a - w_2}{2b} \right) - w_2 \left(\frac{a - w_2}{2b} - I \right) - K_2.$$

Clearly, $Q_2 > 0$ if and only if, $\Pi_{B,2}$ above is at least as large as the profits that the buyer would earn with $Q_2 = 0$ and selling from inventory I . That is, we require that

$$\Pi_{B,2}(I) \geq (a - bI)I.$$

Substituting for $\Pi_{B,2}(I)$ and simplifying, we get that

$$K_2 \leq \frac{(a - w_2)^2}{4b} - (a - w_2 - bI)I.$$

Define

$$K_2(I) \equiv \frac{(a - w_2)^2}{4b} - (a - w_2 - bI)I.$$

Then we have that if $K_2 \leq K_2(I)$, the buyer will purchase a non-negative quantity in period 2.

The supplier, in the second period, solves the following problem:

$$\max_{w_2} K_2(I) + w_2 Q_2$$

where Q_2 is given by (6). Notice that if supplier sets w_2 such that $\frac{a-w_2}{2b} \leq I$ then $Q_2 = 0$ and the supplier makes zero profits in the second period. So let us assume that $w_2 < a - 2bI$ such that $Q_2 > 0$. Substituting for $K_2(I)$ and Q_2 in the supplier's profit function and optimizing for w_2 , we get that $w_2 = 0$. Of course, we need $I \leq \frac{a}{2b}$ for $Q_2 > 0$. Since $w_2 = 0$, it is straightforward to see that the buyer will obey this constraint else loses $(w_1 + h)$ per unit at the margin. Thus,

$$K_2(I) = \frac{a^2}{4b} - (a - bI)I,$$

and

$$Q_2 = \frac{a}{2b} - I.$$

Substituting into the expressions for the profits of the buyer and the supplier, we get the buyer's profits to be:

$$\Pi_{B,2}(I) = (a - bI)I \quad (7)$$

and the supplier profits as:

$$\Pi_{S,2}(I) = \frac{a^2}{4b} - (a - bI)I \quad (8)$$

We now solve for the first period prices and quantities. In the first period, the supplier announces K_1 and w_1 . The buyer then decides to purchase Q_1 , sells $q_1 \leq Q_1$, and perhaps carry inventory of I into the next period. He solves the following optimization problem:

$$\max_{Q_1, I} (a - b(Q_1 - I))(Q_1 - I) - hI - w_1 Q_1 - K_1 + \Pi_{B,2}(I) \quad (9a)$$

$$\text{s.t.} \quad I \geq 0 \quad (9b)$$

$$I \leq Q_1 \quad (9c)$$

Then the buyer's first period strategy is given by the following Lemma.

Lemma A.2 *The optimal purchase quantity Q_1 and optimal inventory I are given as follows:*

$$(Q_1, I) = \begin{cases} \left(\frac{a-w_1}{b} - \frac{h}{2b}, \frac{a-w_1}{2b} - \frac{h}{2b} \right) & \text{if } w_1 < a - h \\ \left(\frac{a-w_1}{2b}, 0 \right) & \text{if } w_1 \geq a - h \end{cases}$$

Proof: : Let λ and μ be the Lagrange multipliers associated with the two constraints in (9). Folding the constraints into the objective function and writing the first order conditions we get

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial Q_1} &= -w_1 + a - b(Q_1 - I) - b(Q_1 - I) + \mu \\ \frac{\partial \mathcal{L}}{\partial I} &= -[a - b(Q_1 - I)] + b(Q_1 - I) - h + a - 2bI + \lambda - \mu \end{aligned}$$

We need to consider two cases:

• **Case (i):** $\mu = 0$ and $\lambda = 0$. From the two first order conditions, we then get

$$Q_1 = I + \frac{a - w_1}{2b}, \quad \text{and} \quad Q_1 - 2I = \frac{h}{2b}.$$

Simplifying, we get

$$I = \frac{a - w_1}{2b} - \frac{h}{2b},$$

and,

$$Q_1 = \frac{a - w_1}{b} - \frac{h}{2b}.$$

Since we require that $I > 0$, we need that $w_1 < a - h$.

• **Case (ii):** $\mu = 0$ and $\lambda > 0$. This implies that $I = 0$. Substituting into the first order conditions, we get that $w_1 \geq a - h$ and

$$Q_1 = \frac{a - w_1}{2b}.$$

Given the buyer's response to K_1 and w_1 , the supplier needs to set these prices. She will do so to extract all of the buyer's current and future profits, where the latter is given by $\Pi_{B,2}(I)$. Thus, she solves the following problem:

$$\max_{K_1, w_1} K_1 + w_1 Q_1(w_1) + \Pi_{S,2}(I(w)) \quad (10)$$

where $Q_1(w_1)$ and $I(w_1)$ are as given by Lemma A.2.

The supplier's optimal response is derived as follows. Suppose $w_1 < a - h$. Then substituting for $Q_1(w_1)$ and $I(w_1)$, simplifying and writing the first order condition with respect to w_1 , we get $w_1 = \frac{2}{3}a$. Since we require that $w_1 < a - h$, this price is feasible whenever $h < a/3$. Substituting this price into expressions for $Q_1(w_1)$ and $I(w_1)$, we get

$$I = \frac{a}{6b} - \frac{h}{2b}; \quad \text{and} \quad Q_1 = \frac{a}{3b} - \frac{h}{2b}.$$

Setting the buyer's profit to zero, we get

$$K_1 = (a - b(Q_1 - I))(Q_1 - I) + (a - bI)I - hI - w_1 Q_1.$$

Substituting for w_1 and the corresponding (Q_1, I_1) and simplifying we get

$$K_1 = \frac{1}{b} \left[\left(\frac{a}{6} \right)^2 + \left(\frac{a - 3h}{6} \right)^2 \right].$$

Substituting for the optimal I into $K_2(I)$ and $Q_2(I)$, we get

$$K_2 = \frac{a^2}{4b} - \left(\frac{5a}{6} + \frac{h}{2} \right) \left(\frac{a}{6b} - \frac{h}{2b} \right),$$

and

$$Q_2 = \frac{a}{3b} + \frac{h}{2b}.$$

Therefore, the supplier's overall profits given by

$$K_1 + K_2 + w_1 Q_1 + w_2 Q_2 = \frac{1}{18b} [7a^2 - 3ah + 9h^2].$$

Now if $h \geq a/3$, then $w_1 \geq a - h$ and from Lemma A.2 we get that $I = 0$ and $Q_1 = \frac{a-w_1}{2b} = \frac{a}{6b}$. We then compute K_1 to extract all profits as:

$$K_1 = (a - bQ_1)Q_1 + -w_1 Q_1.$$

Substituting and simplifying, we get $K_1 = \frac{a^2}{36b}$. Similarly, we compute $K_2 = \frac{a^2}{4b}$ and $Q_2 = \frac{a}{2b}$. The total profits of the supplier is then

$$K_1 + K_2 + w_1 Q_1 + w_2 Q_2 = \frac{7a^2}{18b}.$$

Observe that the supplier will set K_1 to extract all current and future (residual)¹⁶ profits. Once we compute w_1 and K_1 , we can derive K_2 and the total profits of the supplier which equals the channel profits. ■

Proof of Theorem 3.2:

We derive the optimal policy structure working backwards starting with the buyer's response. The buyer solves the following problem:

$$\max_{Q_1, Q_2, I} (a - b(Q - I))(Q - I) - w_1 Q_1 - K_1 \mathbf{1}_{Q_1 > 0} + (a - b(Q_2 + I))(Q_2 + I) - hI - w_2 Q_2 - K_2 \mathbf{1}_{Q_2 > 0} \quad (11a)$$

$$\text{s.t. } I \leq Q_1 \quad (11b)$$

$$I \geq 0 \quad (11c)$$

$$Q_2 \geq 0 \quad (11d)$$

The buyer's optimal response is given by the following Lemma.

Lemma A.3 Given (K_1, w_1) and (K_2, w_2) , the buyer chooses one of the following actions:

No.	Actions			Buyer's Profit	Conditions Needed
	Q_1	I	Q_2		
1	0	0	$\frac{a-w_2}{2b}$	$\Pi_B^{(1)} = -K_2 + \frac{(a-w_2)^2}{4b}$	$w_2 < a$
2	$\frac{a-w_1}{2b}$	0	0	$\Pi_B^{(2)} = -K_1 + \frac{(a-w_1)^2}{4b}$	$a - h \leq w_1 < a$
3	$\frac{2(a-w_1)-h}{2b}$	$\frac{a-w_1-h}{2b}$	0	$\Pi_B^{(3)} = -K_1 + \frac{(a-w_1)^2}{4b} + \frac{(a-w_1-h)^2}{4b}$	$w_1 < a - h$
4	$\frac{a-w_1}{2b}$	0	$\frac{a-w_2}{2b}$	$\Pi_B^{(4)} = -K_1 - K_2 + \frac{(a-w_1)^2}{4b} + \frac{(a-w_2)^2}{4b}$	$\{w_1, w_2\} < a$, and $w_1 + h \geq w_2$

Proof : There are three possible strategies for the buyer. Either (i) $Q_1 = 0$, $Q_2 > 0$; or (ii) $Q_1 > 0$, $Q_2 = 0$; or (iii) $Q_1 > 0$, $Q_2 > 0$. We now consider each of these cases.

¹⁶Whatever is left in period 2 after taking away K_2 .

Case (i) $Q_1 = 0, Q_2 > 0$: In this case the optimization problem for the buyer is

$$\max_{Q_2} -K_2 - w_2 Q_2 + (a - bQ_2)Q_2.$$

Solving for Q_2 gives $Q_2 = \frac{a-w_2}{2b}$ which is non-zero only when $w_2 < a$. Substituting back into the profit function gives us that

$$\Pi_B = -K_2 + \frac{(a - w_2)^2}{4b}.$$

Case (i) $Q_1 > 0, Q_2 = 0$: In this case the buyer does not purchase any quantity in the second period. This implies that he may choose to carry over inventory from the first period and sell it in the second. Thus the buyer's objective function is:

$$\max_{Q_1, I} -K_1 - w_1 Q_1 + (a - b(Q_1 - I))(Q_1 - I) - hI + (a - bI)I.$$

To determine if the buyer would carry inventory for any given Q_1 , we derive the partial of the above profit function w.r.t. I to write:

$$\begin{aligned} \frac{\partial(\cdot)}{\partial I} &= -a + b(Q_1 - I) + (Q_1 - I)b - h + a - 2bI \\ &= 2b(Q_1 - I) - 2bI - h \\ &= 2b(Q_1 - 2I) - h. \end{aligned}$$

Equating this to zero and simplifying, we see that the optimal level of inventory to carry is $I^* = \frac{Q_1}{2} - \frac{h}{4b}$. Thus if $Q_1 \leq \frac{h}{2b}$ then $I = 0$ and no inventory is carried; otherwise $I > 0$.

Now suppose $I = 0$; substituting back into the profit function and solving for optimal Q_1 , we see that $Q_1 = \frac{a-w_1}{2b}$. First we require that $w_1 < a$ for $Q_1 > 0$. Furthermore, we need that $I = 0$ which implies that $Q_1 \leq \frac{h}{2b}$. This implies that $w_1 \geq a - h$.

Next consider the situation when $I > 0$. Substituting the expression for I in the buyer's objective function and solving for the optimal Q_1 by equating the first order condition w.r.t. Q_1 to zero, we get that,

$$Q_1 = \frac{2(a - w_1) - h}{2b}; \quad I = \frac{a - w_1 - h}{2b}.$$

Notice that for $Q_1 > \frac{h}{2b}$ we would need that $w_1 < a - h$.

Thus case(ii) has two solutions: if $w_1 \geq a - h$ then $Q_1 = \frac{a-w_1}{2b}$ and $I = 0$; else if $w_1 < a - h$ then $Q_1 = \frac{2(a-w_1)-h}{2b}$ and $I = \frac{a-w_1-h}{2b}$. Substituting back into the profit function, we derive that the buyer's profits in the former case is $\Pi_B = -K_1 + \frac{(a-w_1)^2}{4b}$ and in the latter it is $\Pi_B = -K_1 + \frac{(a-w_1)^2}{4b} + \frac{(a-w_1-h)^2}{4b}$.

Case (i) $Q_1 > 0, Q_2 > 0$: In this case the buyer's optimization problem is:

$$\begin{aligned} \max_{Q_1, Q_2, I} & -K_1 - K_2 - w_1 Q_1 - w_2 Q_2 \\ & + [a - b(Q_1 - I)](Q_1 - I) + [(a - b(Q_2 + I))(Q_2 + I) - hI \end{aligned}$$

Given that $Q_1, Q_2 > 0$, the analysis of this case is similar to the linear contract under commitments. Recall that for that case we derived that if $w_1 + h \geq w_2$, then $Q_1 = \frac{a-w_1}{2b}$, $Q_2 = \frac{a-w_2}{2b}$, and $I = 0$ giving

$$\Pi_B = -K_1 - K_2 + \frac{(a-w_1)^2}{4b} + \frac{(a-w_2)^2}{4b}.$$

If $w_1 + h < w_2$ then the buyer's optimal action would be $Q_1 > 0$ and $Q_2 = 0$ and we are back to case (ii).

To summarize, the buyer acts to choose one of the four actions described by one in case(i), two in case (ii), and one in case (iii). These correspond to the four cases in the Proposition. ■

The supplier then optimizes the following objective function

$$\max_{K_1, K_2, w_1, w_2} K_1 \mathbf{1}_{Q_1 > 0} + w_1 Q_1 + K_2 \mathbf{1}_{Q_2 > 0} + w_2 Q_2$$

subject to the buyer maximizing her own profit function as per the choices listed in Lemma A.3. Now observe that the buyer's action (1) is a special case of her action (4) and pareto-dominated by $K_1 = K_2 + \epsilon$ and $w_1 = w_2$. That is, the supplier will always choose to force the buyer to pick (4) instead of (1). Similarly, buyer's action (2) is also pareto-dominated by her action (4) with (for example) $K_2 = 0$ and $w_1 = w_2$. Thus the supplier need to only consider buyer's actions (3) and (4) in deciding his pricing scheme.

Let $\Pi_B^{(k)}$ denote buyer's profits under action (k) outlined in Lemma A.3. Now consider the situation that the supplier wishes to enforce buyer's action (3). Then he solves the following problem:

$$\max_{K_1, w_1} K_1 + w_1 \frac{2(a-w_1) - h}{2b} \quad (12a)$$

$$\text{s.t. } w_1 < a - h \quad (12b)$$

$$\Pi_B^{(3)} > \Pi_B^{(4)} \quad (12c)$$

$$\Pi_B^{(3)} \geq 0 \quad (12d)$$

Now consider the situation when the supplier wishes to enforce buyer's action (4). Then he solves the following problem:

$$\max_{K_1, K_2, w_1, w_2} K_1 + K_2 + w_1 \frac{a-w_1}{2b} + w_2 \frac{a-w_2}{2b} \quad (13a)$$

$$\text{s.t. } \Pi_B^{(4)} \geq 0 \quad (13b)$$

$$\Pi_B^{(4)} \geq \Pi_B^{(1)} \quad (13c)$$

$$\Pi_B^{(4)} \geq \Pi_B^{(2)} \quad (13d)$$

$$\Pi_B^{(4)} \geq \Pi_B^{(3)} \quad \text{if } w_1 + h \leq a \quad (13e)$$

$$w_1 + h \geq w_2 \quad (13f)$$

$$w_1 \leq a, \quad w_2 \leq a \quad (13g)$$

Once we derive the solutions of (12) and (13), we need to compare these to determine which of the two actions of the buyer - (3) or (4) - does the supplier wish to induce.

We first derive the solution when the supplier wishes to implement action (3). The unconstrained optimum of (12) is given by

$$w_1 = \frac{a}{2} - \frac{h}{4}.$$

The first constraint of $w_1 < a - h$ is then satisfied as long as $h < \frac{2}{3}a$. The second constraint implies that

$$K_2 > \frac{(a - w_2)^2}{4b} - \frac{(a - w_1 - h)^2}{4b}$$

which is easily satisfied by setting a large enough K_2 . Finally, the supplier will pick K_1 such that $\Pi_B^{(3)} = 0$; that is, set

$$\begin{aligned} K_1 &= \frac{(a - w_1)^2}{4b} + \frac{(a - w_1 - h)^2}{4b} \\ &= \frac{1}{4b} \left[\left(\frac{a}{2} + \frac{h}{4} \right)^2 + \left(\frac{a}{2} - \frac{3h}{4} \right)^2 \right] \end{aligned}$$

This results in the following profits for the buyer and the supplier:

$$\begin{aligned} \Pi_B &= 0 \\ \Pi_S &= \frac{1}{4b} \left[\frac{3}{2}a^2 - \frac{5}{2}ah + \frac{7}{8}h^2 \right]. \end{aligned}$$

Now consider the situation when the supplier wishes to implement action (4); that is he solves (13). Notice that,

$$\text{Constraint (13c)} \Rightarrow K_1 \leq \frac{(a - w_1)^2}{4b}$$

$$\text{Constraint (13d)} \Rightarrow K_2 \leq \frac{(a - w_2)^2}{4b}$$

Constraints (13c) and (13d) \Rightarrow Constraints (13b)

$$\text{Constraint (13e)} \Rightarrow K_2 \leq \frac{(a - w_2)^2}{4b} - \frac{(a - w_1 - h)^2}{4b} \quad \text{if } w_1 + h \leq a$$

This implies that the supplier will choose the following fixed fees:

$$K_1 = \frac{(a - w_1)^2}{4b} \tag{14}$$

$$K_2 = \begin{cases} \frac{(a - w_2)^2}{4b} - \frac{(a - w_1 - h)^2}{4b} & \text{if } w_1 + h \leq a \\ \frac{(a - w_2)^2}{4b} & \text{otherwise} \end{cases} \tag{15}$$

The unconstrained solution for w_1 and w_2 are easily derived as:

$$w_1 = \frac{a - h}{2}; \quad w_2 = 0.$$

Since certainly $h < a$, we have that $w_1 + h < a$. It is also straightforward to see that Constraint (13f) is satisfied. Substituting for w_1 and w_2 to determine the fixed fees, we get

$$K_1 = \frac{(a + h)^2}{16b}; \quad K_2 = \frac{a^2}{4b} - \frac{(a - h)^2}{16b}.$$

Now consider the supplier's solution when she induces buyer action set (3). We have $Q_1, I > 0$ and $Q_2 = 0$, where I is affectively purchased at $w_1 + h$. Now if we let $w_2 = w_1 + h$ and $K_2 = 0$, then

this solution is also feasible for (13). Hence the solution that induces buyer action (4) dominates the solution that induces buyer action (3).

The optimal profit of the supplier is then derived as:

$$\begin{aligned}\Pi_S &= K_1 + K_2 + w_1 Q_1 + w_2 Q_2 \\ &= \frac{3a^2 + 2ah - h^2}{8b}.\end{aligned}$$

Similarly, the optimal profit of the buyer is:

$$\begin{aligned}\Pi_B &= -K_1 - K_2 + \frac{(a - w_1)^2}{4b} + \frac{(a - w_2)^2}{4b} \\ &= \frac{(a - h)^2}{16b}\end{aligned}$$

Proof of Proposition 3.1:

The buyer clearly prefers the commitment contract as she makes zero profits under dynamic contract. The channel profits under dynamic contract is given by:

$$\Pi_C = \frac{7a^2 - 3ah + 9h^2}{18b} \quad (16)$$

Similarly, the channel profits under the commitment contract is

$$\Pi_C = \Pi_B + \Pi_S = \frac{7a^2 + 2ah - h^2}{16b}. \quad (17)$$

It is easy to see that the latter always dominates. Similarly, comparing the supplier's profits under dynamic contract with that under commitment contract, we get the desired result for the supplier's profits. ■

Proof of Proposition 3.2: Recall that the channel preferred dynamic contracts whenever $h < \frac{55a}{288}$. In contrast, with two-part tariffs, the channel always prefers the commitment contract (Proposition 3.1). Hence from a channel perspective, we only need to compare the profits under commitment two part-tariff contracts, with (a) with that under the dynamic linear contract for $h < \frac{55a}{288}$, and (b) with commitment linear contract for $h \geq \frac{55a}{288}$. The channel profits under dynamic linear contract obtained by summing the profits of the buyer and the supplier given in Theorem 2.1 is:

$$\Pi_C = \Pi_B + \Pi_S = \frac{461a^2 - 254ah + 576h^2}{1156b};$$

The channel profits under the commitment linear contract, again obtained by summing the profits of the buyer and the supplier given in Theorem 2.2 is:

$$\Pi_C = \Pi_B + \Pi_S = \frac{3a^2}{8b}.$$

Simple algebraic manipulation leads us to the final result. ■

Proof of Theorem 4.1

We prove this result by contradiction, using an outcomes-based argument. Suppose that such a contract exists, and the supplier can make first-best profits of $(1 + \delta) \cdot R(q_{fb}) = (1 + \delta) \cdot R(R'^{-1}(0))$ over the two periods. Then the *outcome* of the contract will need to satisfy the following conditions:

1. *Sales and Purchase Quantities, and Inventories:* The quantities sold by the buyer in the market in each period *must* be $q_1 = q_2 = q_{fb} = R'^{-1}(0)$. These are the *unique* set of sales quantities that implement the first-best solution in each period, and generate per-period, channel-revenue maximizing sales of $R(q_{fb}) = R(R'^{-1}(0))$, yielding total revenues of $(1 + \delta) \cdot R(R'^{-1}(0))$ over both periods. Further, since inventories will drain the channel via holding costs, the buyer's *purchase* quantities in each period (induced by the dynamic contract) must also be $Q_1 = Q_2 = q_{fb} = R'^{-1}(0)$, ensuring that inventories are not carried.
2. *Transfer Payments from Buyer to Supplier:* The payment from buyer to supplier can be via fixed fees, unit prices, a combination of the two, or any other non-linear device. In making our argument here, we are only concerned with the *total* transfer payments. The buyer can and will choose not to participate in either period if she expects to make a loss by accepting the supplier's terms. This is because, under dynamic contracting, future periods are not contractible in the current period, and past-period contracts have expired. To ensure the buyer's participation, her total per-period transfer payment to the supplier must be bounded from above by her sales revenues of $R(q_{fb}) = R(R'^{-1}(0))$ per period. Thus the supplier's total profits are bounded from above by $(1 + \delta) \cdot R(R'^{-1}(0))$, the maximum total payment he can get from the buyer. By assumption, the supplier's profits must attain this bound under the contract. Thus, the total payment to the supplier by the buyer in each period must be exactly $R(q_{fb}) = R(R'^{-1}(0))$ – the single-period, first-best profit.

To summarize, the *outcome* of the optimal dynamic contract that *both* implements the first-best solution *and* extracts away all of the buyer's residual profits must be as follows: Transfer payments from buyer to supplier¹⁷ are $H_1 = H_2 = R(R'^{-1}(0))$, purchase quantities are $Q_1 = Q_2 = q_{fb} = R'^{-1}(0)$, and Inventory, $I = 0$.

Now we demonstrate that such an outcome cannot arise from any (sub-game perfect) equilibrium, since the buyer can do better by unilateral deviation. Suppose in the first period, the buyer has purchased the quantity $Q_1 = R'^{-1}(0)$, and paid a sum of $H_1 = R(R'^{-1}(0))$ to the supplier. If the buyer sells this entire quantity in the first period, she makes zero residual profits in the first period, and then, in the second period, the supplier can implement the rest of the optimal contract. Knowing that this will be the outcome *if* she sells all her purchased quantities in the first period, the buyer can try to do better by selling some of her purchased quantity in the first period, and the rest in the second period, *by carrying inventory*. Under this strategy, the buyer's optimization is given by:

$$\max_{q_1} \Pi(q_1) = R(q_1) + \delta \cdot R(q_{fb} - q_1) - h(q_{fb} - q_1) - H_1,$$

subject to the constraint $0 \leq q_1 \leq q_{fb}$, where q_1 is the quantity sold in the first period, $(q_{fb} - q_1)$ is the inventory carried and sold in the second period, and $H_1 = R(R'^{-1}(0))$ is the first-period payment to the supplier. Observe that H_1 was paid by the buyer in the first period to procure the quantity q_{fb} , and

¹⁷Recall that we do not specify the form of the payment (which could be via fixed fees, unit prices or any other non-linear device), or the exact structure of the contract that enforces this payment. We are only concerned here with the total emoluments transferred.

is a sunk cost when the buyer decides on how much to sell. Setting aside the constraint on the range of q_1 for now, the First Order Conditions with respect to q_1 are:

$$\Gamma(q_1) = R'(q_1) - \delta \cdot R'(q_{fb} - q_1) + h = 0 \quad (18)$$

Since $R'(\cdot)$ is a decreasing function but positive in the relevant range, the function $\Gamma(q_1)$ is monotonically decreasing in q_1 in the interval $[0, q_{fb}]$. Further, since $R'(q_{fb}) = 0$ (which is the first-best solution in the static case) and $\delta \cdot R'(0) > h$ (by assumption), $\Gamma(0) > 0 > \Gamma(q_{fb})$. Thus, the solution q_1^* to equation (18) satisfies the constraint $0 \leq q_1^* \leq q_{fb}$, and is an interior point of this region. Further, the Second Order Conditions reveal that this solution is a maximum, since $\Gamma'(q_1^*) = R''(q_1^*) + \delta \cdot R''(q_{fb} - q_1^*) < 0$. Hence the buyer will carry inventories (given by $I = q_{fb} - q_1^*$), and sell only a part of her first period purchase q_{fb} in the first period. The buyer's profits at this maximizing solution are $\Pi(q_1^*) > \Pi(q_{fb}) = 0$ (the latter equality holds by construction of H_1). Faced with the threat of residual profits of $\Pi(q_1^*)$ (with the buyer carrying inventories), the supplier can only implement those contracts in the second period that guarantee the buyer at least $\Pi(q_1^*) = R(q_1^*) + \delta \cdot R(q_{fb} - q_1^*) - h(q_{fb} - q_1^*) - H_1$ in profits. Thus, *the contractual outcome specified by conditions (i) and (ii) above, is inconsistent with the requirement of subgame perfection. But we showed that conditions (i) and (ii) must be satisfied by any contract that generates first-best profits of $(1 + \delta) \cdot R(R'^{-1}(0))$ to the supplier. Hence, by contradiction, no such contract that generates first-best profits to the supplier is feasible.* ■

Proof of Theorem 4.2: We focus on the finite (n -period) horizon problem, with the per-period multiplicative discount factor of δ , where $0 < \delta \leq 1$. The proof extends in a straightforward way to the *discounted* infinite horizon ($0 < \delta < 1$).

We know that the first-best profits in each period are $R(q_{fb}) = R(R'^{-1}(0))$. Thus, the total discounted first-best profits over the n period horizon are $\sum_{i=1}^n \delta^{i-1} \cdot R(R'^{-1}(0))$, which simplifies to $\frac{1-\delta^n}{1-\delta} \cdot R(R'^{-1}(0))$ for $0 < \delta < 1$, and $n \cdot R(R'^{-1}(0))$ when $\delta = 1$.

The proof is an extension of the proof for Theorem 4.1, and is also by contradiction, using an outcomes-based argument. Suppose that such a contract exists, and the supplier can both enforce the first-best solution and extract all of the residual profits in each period. Then, arguing as in Theorem 4.1, the *outcome* of the optimal dynamic contract that *both* implements the first-best solution *and* extracts away all of the buyer's residual profits over the n periods must be as follows: Total emoluments transferred from buyer to supplier are $H_1 = H_2 = \dots = H_n = R(R'^{-1}(0))$, purchase quantities are $Q_1 = Q_2 = \dots = Q_n = q_{fb} = R'^{-1}(0)$, and Inventories are $I_1 = I_2 = \dots = I_{n-1} = 0$, where I_j is the inventory carried by the buyer from period j to $j + 1$.

Now we demonstrate that such an outcome cannot arise from any (sub-game perfect) equilibrium, since the buyer can do better by unilateral deviation. Suppose in the first period, the buyer has purchased the quantity $Q_1 = R'^{-1}(0)$, and paid a sum of $H_1 = R(R'^{-1}(0))$ to the supplier. If the buyer sells this entire quantity in the first period, she makes zero residual profits in the first period, and then, from the second period onwards, the supplier can implement the rest of the optimal contract. Knowing that this will be the outcome *if* she sells all her purchased quantities in the first period, the buyer can try to do better by selling some of her purchased quantity in the first period, and carrying the rest as inventory. Observe that, once the buyer has purchased the quantity q_{fb} in the first period, she is

free to sell that quantity or carry it forward to future periods: The supplier has no credible enforcing mechanism to ensure sales of the entire purchased quantity within the buyer's period of purchase. Punishments via future contracts are neither credible (history-dependency in the finite horizon fails to meet the subgame-perfection criterion) nor feasible (since the buyer's residual profits in each period are already driven to her participation constraint). As an example, the buyer could sell whatever she carries forward from the first period in the second period. We analyze the result of such an unilateral deviation.¹⁸ Under this strategy, the buyer's optimization is given by:

$$\max_{q_1} \Pi(q_1) = R(q_1) + \delta \cdot R(q_{fb} - q_1) - h(q_{fb} - q_1) - H_1,$$

subject to the constraint $0 \leq q_1 \leq q_{fb}$, where q_1 is the quantity sold in the first period, $(q_{fb} - q_1)$ is the inventory carried and sold in the second period, and $H_1 = R(R'^{-1}(0))$ is the first-period payment to the supplier.¹⁹ Observe that H_1 was paid by the buyer in the first period to procure the quantity q_{fb} , and is a sunk cost when the buyer decides on how much to sell. Setting aside the constraint on the range of q_1 for now, the First Order Conditions with respect to q_1 are:

$$\Gamma(q_1) = R'(q_1) - \delta \cdot R'(q_{fb} - q_1) + h = 0 \quad (19)$$

Since $R'(\cdot)$ is a decreasing function but positive in the relevant range, the function $\Gamma(q_1)$ is monotonically decreasing in q_1 in the interval $[0, q_{fb}]$. Further, since $R'(q_{fb}) = 0$ (which is the first-best solution in the static case) and $\delta \cdot R'(0) > h$ (by assumption), $\Gamma(0) > 0 > \Gamma(q_{fb})$. Thus, the solution q_1^* to equation (19) satisfies the constraint $0 \leq q_1^* \leq q_{fb}$, and is an interior point of this region. Further, the Second Order Conditions reveal that this solution is a maximum, since $\Gamma'(q_1^*) = R''(q_1^*) + \delta \cdot R''(q_{fb} - q_1^*) < 0$. Hence the buyer will carry inventories (given by $I = q_{fb} - q_1^*$), and sell only a part of her first period purchase q_{fb} in the first period. The buyer's profits at this maximizing solution are $\Pi(q_1^*) > \Pi(q_{fb}) = 0$ (the latter equality holds by construction of H_1). Faced with the threat of residual profits of $\Pi(q_1^*)$ (with the buyer carrying inventories), the supplier can only implement those contracts in the second period that guarantee the buyer at least $\Pi(q_1^*) = R(q_1^*) + \delta \cdot R(q_{fb} - q_1^*) - h(q_{fb} - q_1^*) - H_1$ in profits. Thus, the outcome (derived above) of any contract that generates first-best profits to the supplier is unattainable under subgame perfection. Hence, by contradiction, no such contract is feasible. ■

Proof of Theorem 4.3: We prove the result by contradiction, using an outcomes-based argument.

The exposition below focuses on the finite (n -period) horizon problem, with the per-period multiplicative discount factor of δ , where $0 < \delta \leq 1$. The proof extends in a straightforward way to the *discounted* infinite horizon ($0 < \delta < 1$).

We know that the first-best profits in each period are $R(q_{fb}) = R(R'^{-1}(0))$. Thus, the total discounted first-best profits over the n period horizon are $\sum_{i=1}^n \delta^{i-1} \cdot R(R'^{-1}(0))$, which simplifies to $\frac{1-\delta^n}{1-\delta} \cdot R(R'^{-1}(0))$ for $0 < \delta < 1$, and $n \cdot R(R'^{-1}(0))$ when $\delta = 1$.

¹⁸Other deviations are possible, such as selling the purchased quantity gradually, over multiple periods. Some of these may yield even higher residual profits to the buyer than the deviation we analyze. However, positive residual buyer profits under the simple deviation we consider will be sufficient to demonstrate that the posited first-best dynamic contract is an infeasible equilibrium under subgame-perfection.

¹⁹We fix the strategies and outcomes from periods 3 to n as in the posited contract, so the buyer's profits are zero from period 3 onwards.

Suppose a commitment contract exists such that the supplier can both enforce the first-best solution and extract all of the residual profits in each period. Thus, over the n period horizon, the supplier makes first-best profits of $\frac{1-\delta^n}{1-\delta} \cdot R(R'^{-1}(0))$ for $0 < \delta < 1$ and $n \cdot R(R'^{-1}(0))$ when $\delta = 1$. Then the *outcome* of the contract will need to satisfy the following conditions:

1. *Sales and Purchase Quantities, and Inventories:* The quantities sold by the buyer in the market in each period *must* be $q_1 = q_2 = \dots = q_n = q_{fb} = R'^{-1}(0)$. These are the *unique* set of sales quantities that implement the first-best solution in each period, and generate per-period, channel-revenue maximizing sales of $R(q_{fb}) = R(R'^{-1}(0))$. Further, since inventories will lead to channel losses via holding costs, we must have $I_1 = I_2 = \dots = I_{n-1} = 0$, where I_j is the inventory carried by the buyer from period j to $j + 1$. Thus, the buyer's *purchase* quantities in each period (induced by the dynamic contract) must equal the sales quantities for that period; i.e., $Q_1 = Q_2 = \dots = Q_n = q_{fb} = R'^{-1}(0)$, ensuring that inventories are not carried.
2. *Transfer Payments from Buyer to Supplier:* Let $\{(K_i, w_i)\}_{i=1}^n$ be the schedule of fixed fees (K_i) and unit prices (w_i) under the proposed commitment contract. Thus the total transfer payments induced by the contract in period i must be $H_i = K_i + w_i \cdot q_{fb}$. The buyer can and will choose not to participate in any period i (i.e., set $Q_i = 0$) if she expects to make a loss by accepting the supplier's terms in that period. Thus the supplier must ensure that the $H_i \leq R(q_{fb}) = R(R'^{-1}(0))$. At the same time, the supplier must charge a total fee of $R(q_{fb}) = R(R'^{-1}(0))$ in each period. If he charges less, he cannot make up for this in future periods by charging more, since this would violate the buyer's participation constraint. Thus, the per-period transfer payment from buyer to supplier under the proposed contract must be exactly $H_i = K_i + w_i \cdot q_{fb} = R(R'^{-1}(0))$, for $i = 1, 2, \dots, n$.

To summarize, the *outcome* of the optimal commitment contract that *both* implements the first-best solution *and* extracts away all of the buyer's residual profits over the n periods must be as follows: Total emoluments transferred from buyer to supplier are $H_1 = H_2 = \dots = H_n = R(R'^{-1}(0))$, purchase quantities are $Q_1 = Q_2 = \dots = Q_n = q_{fb} = R'^{-1}(0)$, and Inventories are $I_1 = I_2 = \dots = I_{n-1} = 0$, where I_j is the inventory carried by the buyer from period j to $j + 1$.

Now we demonstrate that such an outcome cannot arise from any (sub-game perfect) equilibrium, since the buyer can do better by unilateral deviation. Suppose in the first period, the buyer has purchased the quantity $Q_1 = R'^{-1}(0)$, and paid a sum of $H_1 = R(R'^{-1}(0))$ to the supplier. If the buyer sells this entire quantity in the first period, she makes zero residual profits in the first period; if she replicates this pattern of buying $R'^{-1}(0)$ in each period and selling all of it within the period of purchase, she makes zero residual profits in every period. Knowing that this will be the outcome *if* she sells all her purchased quantities in the first period, the buyer can try to do better by selling some of her purchased quantity in the first period, and carrying the rest as inventory. Observe that, once the buyer has purchased the quantity q_{fb} in the first period, she is free to sell all or part of that quantity that period and carry the rest forward to future periods: The supplier has no credible enforcing mechanism to ensure sales of the entire purchased quantity within the buyer's period of purchase. As an example, the buyer could sell whatever she carries forward from the first period in the second period, without

making any additional purchases in the second period. We analyze the result of such an unilateral deviation.²⁰ Under this strategy, the buyer's optimization is given by:

$$\max_{q_1} \Pi(q_1) = R(q_1) + \delta \cdot R(q_{fb} - q_1) - h(q_{fb} - q_1) - H_1,$$

subject to the constraint $0 \leq q_1 \leq q_{fb}$, where q_1 is the quantity sold in the first period, $(q_{fb} - q_1)$ is the inventory carried and sold in the second period, and $H_1 = R(R'^{-1}(0))$ is the first-period payment to the supplier.²¹ Observe that H_1 was paid by the buyer in the first period to procure the quantity q_{fb} , and is a sunk cost when the buyer decides on how much to sell. Setting aside the constraint on the range of q_1 for now, the First Order Conditions with respect to q_1 are:

$$\Gamma(q_1) = R'(q_1) - \delta \cdot R'(q_{fb} - q_1) + h = 0 \quad (20)$$

Since $R'(\cdot)$ is a decreasing function but positive in the relevant range, the function $\Gamma(q_1)$ is monotonically decreasing in q_1 in the interval $[0, q_{fb}]$. Further, since $R'(q_{fb}) = 0$ (which is the first-best solution in the static case) and $\delta \cdot R'(0) > h$ (by assumption), $\Gamma(0) > 0 > \Gamma(q_{fb})$. Thus, the solution q_1^* to equation (20) satisfies the constraint $0 \leq q_1^* \leq q_{fb}$, and is an interior point of this region. Further, the Second Order Conditions reveal that this solution is a maximum, since $\Gamma'(q_1^*) = R''(q_1^*) + \delta \cdot R''(q_{fb} - q_1^*) < 0$. Hence the buyer will carry inventories (given by $I = q_{fb} - q_1^*$), and sell only a part of her first period purchase q_{fb} in the first period. The buyer's profits at this maximizing solution are $\Pi(q_1^*) > \Pi(q_{fb}) = 0$ (the latter equality holds by construction of H_1). By carrying inventories, the buyer can make residual profits of at least $\Pi(q_1^*) = R(q_1^*) + \delta \cdot R(q_{fb} - q_1^*) - h(q_{fb} - q_1^*) - H_1$ in the first two periods. Thus the supplier cannot extract away all of the buyer's residual profits under the posited two-part tariff commitment contract. This implies that the outcome (derived above) of any contract that generates first-best profits to the supplier is unattainable under subgame perfection. Hence, by contradiction, no such contract is feasible. ■

Proof of Theorem 4.4: The simplest proof of this result is to construct a commitment contract that (i) ensures buyer-participation, (ii) implements the first-best solution and (iii) extracts away all of the buyer's residual profits. Variants of "selling the firm" to the buyer, at a fee equal to the total discounted first-best profits, will satisfy all three conditions.

We know that the first-best profits in each period are $R(q_{fb}) = R(R'^{-1}(0))$. Thus, the total discounted first-best profits over the n period horizon are $\sum_{i=1}^n \delta^{i-1} \cdot R(R'^{-1}(0))$, which simplifies to $\frac{1-\delta^n}{1-\delta} \cdot R(R'^{-1}(0))$ for $0 < \delta < 1$, and $n \cdot R(R'^{-1}(0))$ when $\delta = 1$. Over the infinite horizon, the total discounted first-best profits are $\sum_{i=1}^{\infty} \delta^{i-1} \cdot R(R'^{-1}(0))$, which simplifies to $\frac{R(R'^{-1}(0))}{1-\delta}$.

A commitment contract with upfront fees equal to the total discounted first-best profits over the horizon (as derived above) and marginal unit-cost pricing (i.e., providing any quantity the buyer desires

²⁰Other deviations are possible, such as selling the purchased quantity gradually, over multiple periods. Some of these may yield even higher residual profits to the buyer than the deviation we analyze. However, positive residual buyer profits under the simple deviation we consider will be sufficient to demonstrate that the posited first-best commitment contract is an infeasible equilibrium under subgame-perfection.

²¹If the buyer does not purchase any quantity in the second period, her transfers H_2 to the supplier are zero. We fix the strategies and outcomes from periods 3 to n as in the posited contract, so the buyer's profits are zero from period 3 onwards.

at zero incremental cost), will accomplish the supplier's objectives. The buyer will optimize and buy the quantity q_{fb} every period, to make profits of $R(q_{fb}) = R(R'^{-1}(0))$ every period. The buyer's total optimal discounted profits over the horizon will be equal to the upfront fee paid to the supplier, and so her residual profits will be driven to zero. The supplier makes first-best channel profits. ■