# Optimal control of amplifier flows using system identification

**Aurélien. Hervé[a], Denis. Sipp[a], Peter.J. Schmid[b], Manuel Samuelides[c]**

a. *ONERA DAFE, 8 rue des Vertugardins, 92190 Meudon, France*
b. *Ladhyx, Ecole Polytechnique, 91128 Palaiseau, France*
c. *ONERA-DTIM, 2, av. Edouard Belin, 31055 Toulouse*

## Résumé :

*Le contrôle linéaire quadratique gaussien (LQG) d'un écoulement convectivement instable est réalisé, utilisant une modélisation basée sur l'identification d'un estimateur, et le calcul classique d'un contrôleur LQG. La physique de l'écoulement est utilisée pour interpréter et paramétrer un modèle ARMAX. Le formalisme proposé est entièrement orienté vers une possible réalisation expérimentale, et utilise exclusivement l'observation du système via une série de capteurs physiques pour créer un modèle réduit adapté au contrôle.*

## Abstract :

*This paper aims to suppress unsteadiness in a convectively unstable flow configuration, using a linear-quadratic-gaussian (LQG) compensator. To model the flow dynamics, an data-based system-identification technique is used for the estimator, whereas a classical LQG framework will be used to derive the compensator; Physical insight into the specifics of the flow is used to tailor the various terms of an ARMAX model. Due to its reliance on only time-sequences of observable data, the proposed technique should be attractive in the design of control strategies directly from experimental data.*

**Mots clefs : System identification ; LQG control ; Noise amplifier**

## 1 Introduction

The design of an efficient Linear Quadratic Gaussian (LQG) compensator for very large-scale fluid systems usually requires the prior calculation of a reduced order model (ROM) that closely represents the input-output behavior of the full system. Such models are commonly derived from a Galerkin projection of the Navier-Stokes equations onto a specified basis, such as, e.g., a POD-basis. Using this technique, both the full flow state and the underlying model (Navier-Stokes equations) are required.

The aim of this study is the introduction of a system-identification technique to a flow control problem and its link with a classical model-based control design process for the

effective manipulation of noise-amplifier flows. To illustrate a noise amplifier flow behavior, a 2D backward facing step at $Re = 500$ (based on step height) is used. This flow configuration is linearly stable but displays strong transient growth effects as upstream perturbations are amplified along the shear layer through a Kelvin-Helmoltz convective instability. If the amplitude of the incoming upstream perturbations is sufficiently small, the underlying dynamics can be described by a linear system whereas a stronger excitation of the system will provide a non-linear behavior.

The main issue that appears when designing a ROM for a noise amplifier flow is that the unsteadiness is entirely triggered by the upstream perturbations that are, by definition, unknown and often difficult to detect. When using Galerkin-based methods[1], the perturbations are recovered trough a Kalman filter that derives, among others, from a knowledge -or at least an estimation- of the spatial, and temporal distributions of the upstream forcing although they would be very difficult to evaluate in a real experiment setup. Using system-identification techniques will allow an experimentally oriented LQG design process that solely relies on data that could actually be extracted from a lab experiment. The design process will hence neither require knowledge about the upstream perturbations, nor preliminary calculation such as, e.g., a projection basis computation.

## 2  Configuration

The configuration studied in this article consists of a two-dimensional backward facing step that has previously been used, e.g., in [2]. Dimensions are non dimensionalized using the step height, and the upstream centerline velocity. The Reynolds number is chosen as $Re = 500$. The computational domain is taken as $(x, y) \in [-10, 50] \times [-1, 1]$ and is partially sketched in figure 1. The upstream boundary condition is modelled by an inflow of Poiseuille type; the upper and lower boundaries are set to wall conditions $\mathbf{v} = 0$. For the chosen Reynolds number of $Re = 500$, the above flow configuration is globally stable. Nevertheless, the flow exhibits a convective instability along the shear layer extending from the top of the step to about $x = 25$. The flow unsteadiness is due to this local region of convective instability which is contained between upstream and downstream regions of stability. Transient growth of perturbations along the shear layer may arise due to the non-normality of the linearized Navier-Stokes operator.
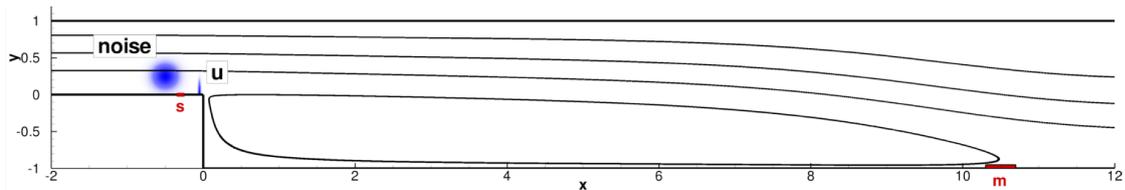


Figure 1: Sketch of different input/outputs. Two skin friction measurements $s$ and $m$ are respectively placed at the top of the step, and at the end of the recirculation bubble. $u$ is the amplitude of actuation, which consists of a space-shaped Gaussian of vertical speed. The upstream forcing is introduced through a similar forcing, of stochastic amplitude $w$. Streamlines show the base flow

The control is to reduce the unsteadiness that grows along the shear layer, which is quantified by the energy of the perturbations. Since the energy is not easily observable in a real system, a downstream skin friction measurement $m$ will be used. It is located near the end of the first recirculation bubble at $x = 10.5$ (see figure 1), and will cease to fluctuate as the bubble is stabilized by our control effort. The measurement $m$ can therefore be taken as our control objective, which shall be minimized; it is then expected that the same control also reduces the global energy of the perturbations.

The external perturbations that will amplify along the shear layer and then impact the objective $m$ originate within the upstream boundary layer. An upstream sensor $s$ is placed at $x = -0.3$ and will be used as an input to the compensator. If the sensor $s$ is sufficiently sensitive to the external perturbations, its measurements will provide important information about the effects of noise on the system.

The control consists of a gaussian shaped forcing momentum, located at the top of the step, as shown in Fig. 1. It will introduce a vertical forcing momentum at the beginning of the convectively unstable zone so that a small control should substantially affect the system and, in particular, the objective $m$.

## 3 LQG design

The linear dynamical model for LQG design is commonly expressed in standard finite-dimensional time-invariant state-space form. We have

$$
\begin{align}
X(t+1) \quad &= AX(t) \quad +Bu(t) + B_w w(t) \tag{1a}\\
s(t) \quad &= C_s X(t) \quad +D_s w(t) + g_s(t) \tag{1b}\\
m(t) \quad &= C_m X(t) \quad +g_m(t) \tag{1c}
\end{align}
$$

where $u, m, s$ describe the control law, and two measurements, respectively, and $\Sigma = (A, B, C, D)$ defines the ROM dynamics. $B_w w(t)$ is referred as the Plant noise and $(g_m, g_s)$ denotes the measurement noises. The former is to represent the unknown external perturbations that enters the system whereas the latter allows to model what would be the imprecision of real sensors. The design of an LQG compensator requires the computation of both a Kalman filter $L$, and a control gain $K$ that are used as sketched in Fig. 2.

In this study we introduce a control design based on system identification that is adoptable for later applications to physical experiments. The system identification consists in recovering the dynamics of a system through the observation of its input-output behavior. However, the flow dynamics is here driven by unknown external perturbations. Input/output data that are required to perform the identification are therefore not straightforwardly available. Nonetheless, the classical linear framework shows the existence of an optimal Kalman filter that allows to estimate the state from partial available informations. Such a filter consists of a linear dynamic system that estimates a state $\hat{X}$, and consequently an output $\hat{m}$, as sketched in 2. The Kalman filter statistically minimizes the error $\|\hat{X} - X\|_2$ where $\hat{X}$ is the estimated state, and $L$ the Kalman filter based on $s$. The estimator can be viewed as a linear dynamic system where both the inputs - namely the control law $u$ and the measurement $s$ - , and the output $m$ are
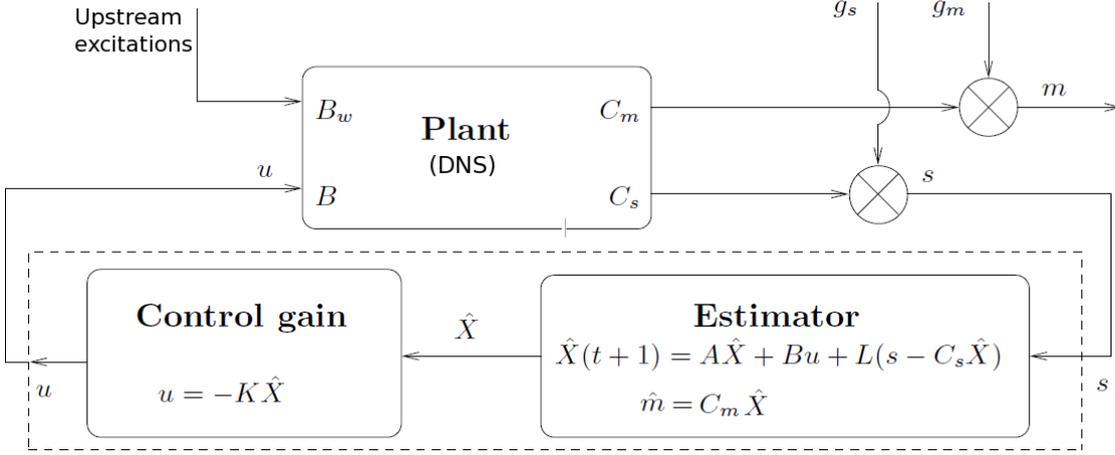
Figure 2: Block diagram of a typical LQG-control configuration. The plant is excited by external noise; the compensator consists of an estimator and a compensator.

known. Hence, instead of directly trying to identify the dynamics of the flow, the first step will consist in identifying an estimator equation (§3.1). Further computations (§3.2) will then allow to identify a full model that reads as (1).

## 3.1   Identifying the estimator

The estimator equation (see figure 2) can be written as a transfer function

$$m(t) = \sum_{i=0}^{t-1} \left[ C_m A_e^{t-i-1} \left( Bu(i) + Ls(i) \right) \right] + C_m A_e^t X_0, \tag{2}$$

where $A_e = A - LC_s$. This transfer function will be identified using an Auto Regressive eXogenous Moving Average (ARMAX) model equation:

$$\underbrace{m(t) + \sum_{k=1}^{n_a} a_k m(t-k)}_{auto\ regressive} = \underbrace{\sum_{k=0}^{n_{bu}-1} b_k^u u(t-k-n_{du}) + \sum_{k=0}^{n_{bs}-1} b_k^s s(t-k-n_{ds})}_{exogenous} + E(t) \tag{3}$$

$$E(t) = \underbrace{\sum_{k=1}^{n_c} c_k e(t-k)}_{moving\ average} + e(t). \tag{4}$$

To perform the identification, some datasets are required. An experiment or simulation that uses an arbitrary control law is run, so that a time-series of $(u(t), s(t), m(t))$ can be recorded, and used for the identification process. The ARMAX regression consists in finding a set of coefficients to obtain a model equation that fits as best as possible to the available datasets $u(t), s(t), m(t)$. The coefficients $(n_a, n_{bu}, n_{bs}, n_{du}, n_{ds})$ have first to be

set. They directly derive from some physical properties of the flow, such as its convective time, or the characteristic length of the perturbations (which is found by evaluating the autocorrelation of the $m(t)$ signal).

## 3.2 Noise identification

The identification process described above allows us to define an efficient estimator for the linear system which can, in turn, be used for designing an LQG-type control. However, the full ROM definition is not yet complete, and the matrix $A = A_e + LC_s$ has still to be defined in order to compute a control gain. It could be tempting to use the estimator equation as a model where $w$ is substituted by $s$. But since $s$ is a sensor, it surely cannot be considered as a source of white noise; rather, it is, most likely, strongly autocorrelated in time.

The $A$ matrix can be recovered by first identifying a $C_s$ matrix that verifies

$$s(t) = C_s X(t) + w(t) \tag{5}$$

where $w(t)$ is white in time, and uncorrelated to the state $X(t)$. Let us emphasis that (5) is used to define a white noise $w$, but not to provide any accurate prediction of $s$.

Equation (5) can be solved by performing a linear regression between a time-series of $s$ from a known dataset, and a time series of $X(t)$ computed through the estimator that has been fed with the same dataset. This step allows to identify a final ROM where the external perturbation is, by construction, *white noise*. Hence, the classical LQG framework can be used to compute an optimal control gain $K$ from the obtained $A, B, C_m$ matrices, leading to a combination of both a full ROM that reads as (1), an estimator, and a control gain. The estimator only takes $(s, u)$ as inputs so that the $m$ measurement is only needed for the design process. The resulting compensator will thus only use the $s$ measurement to run.

## 4 Results: control of the backward facing step

In this section, we present results from direct numerical simulations (DNS) as the control — designed by system identification, and LQG-techniques — is applied. The compensator is driven by measurements from the DNS; the DNS, in turn, uses the control law that the compensator provides. Figure 3 shows the overall control performances of the obtained compensator. The average energy reduction is equal to 98%, and is achieved by using a compensator which consists of a 17x17 state space equation.

Figure 4 shows the average turbulent kinetic energy of both the non-controlled, and the controlled simulations. Although the compensator is designed to minimize the skin-friction fluctuations at the foot of the recirculation bubble, the resulting control efficiently reduces the kinetic energy of the entire flow. The turbulent kinetic energy is reduced by 96% at $x \approx 25$, where the non controlled turbulent kinetic energy is maximum.

## 5 Conclusions

A robust and compelling flow control procedure has been introduced that is data-based in the estimator design and follows classical methods for model-reduction and control
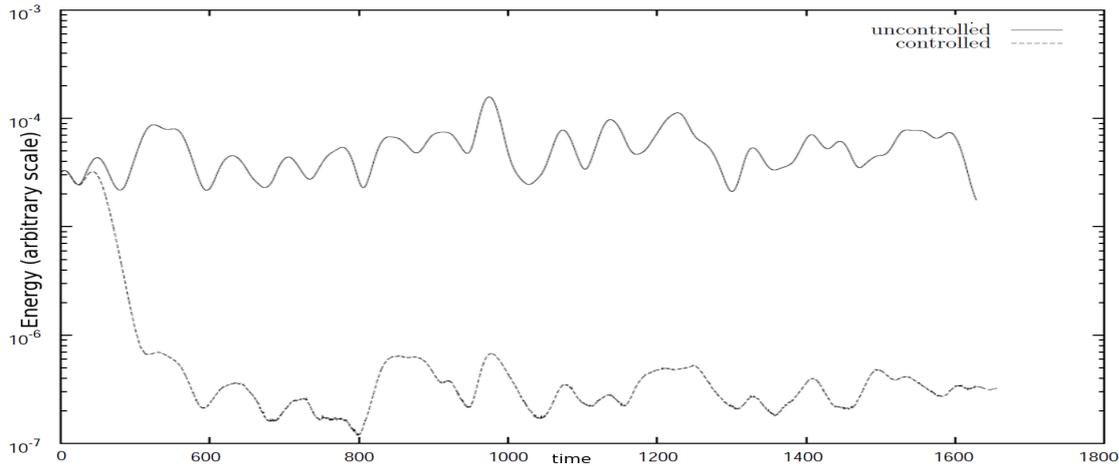
Figure 3: Energy of the perturbations in the dns vs time (semilog scale). Both controlled, and uncontrolled simulations that use the exact same source of random noise are compared. The compensator uses 17 modes. The average energy is reduced by 98%.
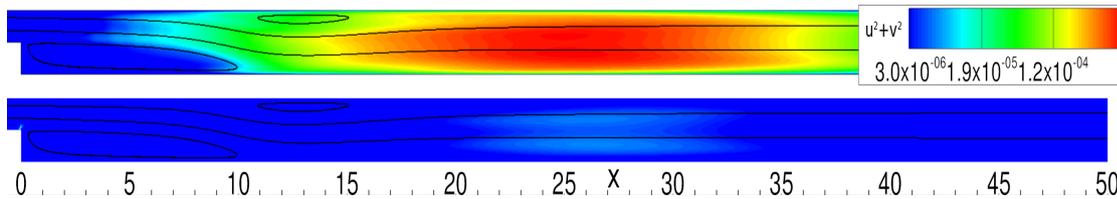


Figure 4: Control results: Contours of the mean turbulent kinetic energy from the numerical simulation (the vertical coordinate is stretched for more clarity). Top: uncontrolled simulation. Bottom: controlled simulation. Streamlines of the base flow are also shown. The maximum peak of turbulent kinetic energy is reduced by 96%.

layout. This technique is Particularly attractive for amplifier flows, where the accurate modeling of noise sources and their influence on system dynamics and sensor output is imperative for a successful compensator performance. It should also appeal to experimental efforts and practical applications of closed-loop flow control.

# References

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