www.iop.org/journals/physed

Mirages in a bottle

T López-Arias, G Calzà, L M Gratton and S Oss

Department of Physics, University of Trento, 38123 Povo (Trento), Italy

Abstract

A simple experiment is presented to visualize inferior and superior mirages in the laboratory. A quantitative analysis is done using ray tracing with both photographic and computational techniques. The mirage's image, as seen by the eye or the camera lens, can be used to analyse the deflection and inversion of light rays.

M This article features online multimedia enhancements

Introduction

A mirage is a fascinating natural optical phenomenon. Thermal gradients in the atmosphere and the associated refractive index changes make light rays bend. An object can be seen in a higher or lower position than it actually is, creating an optical illusion. One has, for example, the quite common 'wet road' effect, i.e. an inferior mirage one can observe driving on a hot summer day. There is also the fascinating 'Fata Morgana'[1] which nevertheless requires very specific atmospheric conditions and is more difficult to see. Mirages offer an excellent opportunity to address several aspects of optics, such as light propagation, the reversibility of light rays, the refractive index and the physical realization of an optical image, including how it is processed by our brain.

A mirage can be reproduced in the laboratory using different techniques. Those that use air, to simulate the atmosphere, require a heated surface of some kind [1]. However, a mirage effect can also be achieved by using water at different temperatures[2], or mixable liquids of different densities [3, 4]. The key point for these kinds of experiments to be effective is to achieve a stable enough refractive index gradient. As a consequence, a well defined pattern of light rays will be produced.

In the present work we present a very simple way to obtain both inferior and superior mirages using two liquids of different density. Our setup shows the detailed structure of the mirage with the corresponding behaviour of light rays to be observed. We also discuss a ray tracing procedure by means of which the mirage can be qualitatively reconstructed. The observation of the phenomenon and its analysis at a basic level can be done in one single session provided the liquids are prepared well in advance. A more detailed study using ray tracing may be time consuming depending on the degree of competence of the students both with mathematical and computational techniques. The photography did not require any particular setup but requires some expertise with photographic techniques.

The experiment

The inferior mirage

An inferior mirage takes place when an object is seen in a position lower than its actual one

[1] In Italian 'Fata' means fairy. This mirage can be seen in the strait of Messina, where the sharp temperature gradients between the sea surface (cold due to the sea streams) and the air (which can present alternating temperature gradients) together with the winds contribute to the formation of a quickly changing superior mirage. The buildings of the city of Messina appear to the observer as hanging in the sky, continuously moving, providing a beautiful and mysterious effect.

[2] Reference [2] provides a list of several ways to achieve mirages that can be found in the literature

 0031-9120/09/060582+07$30.00

Figure 1. Inferior mirage as seen from the point of view of an observer looking through the container.

(as happens in the 'wet road' effect, where the sky is seen in place of the road). Light rays are bent upwards as the refractive index increases with height.

In order to reproduce this same situation in the laboratory, we used a transparent glass container (31 cm × 20 cm × 6 cm) filled with water[3] (6–7 cm depth) on top of which a thick layer (3–4 cm depth) of ethyl alcohol (95 vol%) was carefully deposited. The boundary region where the two liquids get mixed provides the necessary refractive index gradient which must increase with height to achieve an inferior mirage[4]. The refractive indexes of the alcohol and the water we used were 1.3640 ± 0.0005 and 1.3330 ± 0.0005, respectively, both measured with an Abbe refractometer.

It is better to prepare the water–alcohol layers one day in advance, and to pour the alcohol by making it slide over the container's walls to avoid too much mixing with the water. However, since the gradient changes with time, the solution has to be used within one day of its preparation.

We show in figure 1 the mirage as it would be seen by an observer looking through the container. The mirage appears in the lower part and it is composed of writing on the bottle label which appears twice: once turned upside down and once upright and squeezed. Figure 2 (left) shows a cross view of the container with a photographic sequence of light rays (a green low-power laser, whose path has been enhanced by means of sodium fluorescein in all our experiments) for continuously decreasing incident angles. Multiple exposure was adopted in order to include laser beam paths at different angles. Figure 2 (right) shows the simulation of this setup obtained by

[3] We always used distilled water which had been previously boiled, in order to avoid bubbles on the container's walls.

[4] The teacher may remark that a substance with a lower density can have a higher refractive index in order to distinguish neatly between the density of a substance and its optical density (or refractive index).

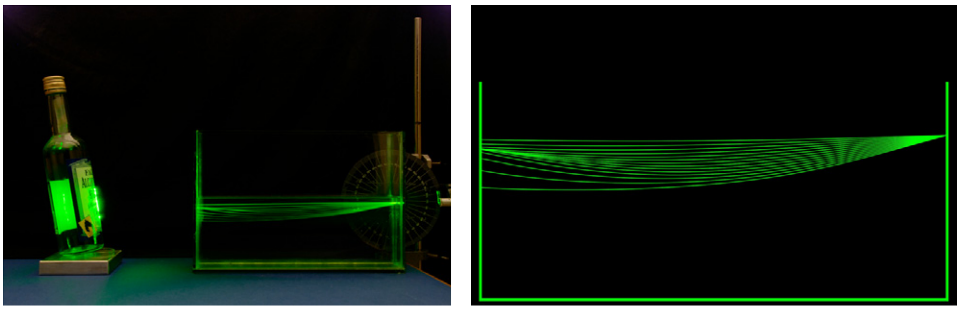

Figure 2. Left: the sequential laser light ray paths which hit the bottle in several points represent the light rays coming from the object. Right: calculated ray paths using a numerical approach, as explained in the text. In order to have equally focused images in figure 1, the bottle was slightly tilted to compensate for the longer optical path length of the curved rays.

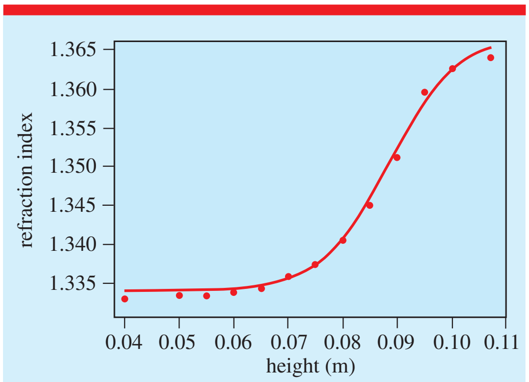

Figure 3. Alcohol–water refractive index gradient as a function of the height. The higher optical density on top leads to an inferior mirage.

numerical solution of a differential equation, as discussed in the following. The excellent agreement enables one to address, from this specific case, the general issue of theoretical models supporting observed data.

The refractive index of the water–alcohol layers was measured with an Abbe refractometer taking a sample of the liquid, from top to bottom, every half centimetre. The samples were collected with a syringe. The results for the refractive index gradient are shown in figure 3. In the boundary region between water and alcohol, the refractive index changes quickly, reaching a constant value at the top and the bottom. The detailed analysis of the ray tracing procedure is discussed in the last section.

The superior mirage

A superior mirage takes place when an object is seen in a position higher than its actual one. The light rays are bent downwards because of the decreasing (with height) refractive index. As a consequence, the object is seen hanging over the real one. (The 'Fata Morgana', mentioned above, is a particular case of this kind of mirage in which the particular vanishing effect of the image comes from an alternating refractive index gradient (increasing and decreasing) due to variations in the air temperature with height.)

To obtain the superior mirage we used sugar syrup and water. The sugar syrup was prepared mixing 25% of sugar with 75% of water. Its refractive index was $n = 1.3725 \pm 0.0005$, measured with an Abbe refractometer. The

Figure 4. Superior mirage as seen from the point of view of an observer looking through the container.

refractive index of the sugar syrup can also be retrieved directly from its concentration using reference tables which relate these two quantities, see for instance [5]. The syrup leads to a higher value of the refractive index at the bottom of the tank thanks to its higher density with respect to water. In order to assure a proper stratification of the syrup and water, we first filled the tank with a layer of water (6 cm depth) and deposited the syrup (7 cm depth) reaching the bottom of the tank with a small funnel and a pipe. The boundary region between water and syrup provides the proper gradient in the refractive index to achieve the superior mirage[5].

The mirage as seen by an observer looking through the container is shown in figure 4. Note that the bottle is now in a lower position with respect to the previous setup. Figure 5 (left) shows a cross view of the container with a photographic

[5] One more layer of alcohol on top of the water would provide an inversion in the gradient to simulate a mirage of the 'Fata Morgana' type.

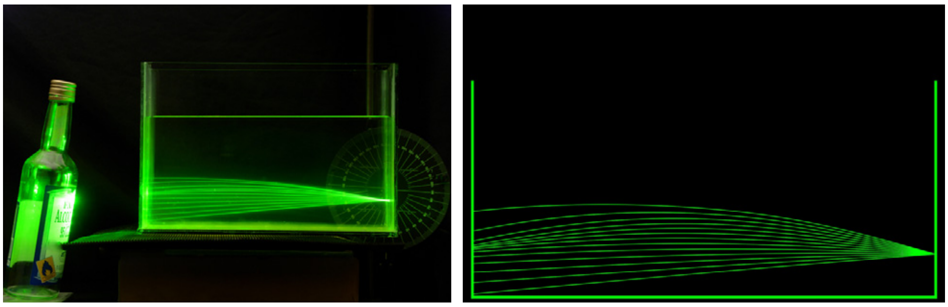

Figure 5. Left: the sequential laser light ray paths which hit the bottle at several points represent the light rays coming from the object. Right: calculated ray paths using a numerical fit of the experimental refractive index gradient, as explained in the text. Note that the bottle stands with a different tilt with respect to the inferior mirage.

sequence of light rays for continuously decreasing incident angles. Once again, in figure 5 (right) the computed ray tracing for this experimental setup is shown.

In this case, to attain a proper refractive index gradient and a good mirage it is necessary to perform the experiment and take the liquid samples within a few hours of its preparation. It is possible to get a refractive index of up to 1.45–1.46 using a sugar–water solution, depending on its temperature [5].

As in the previous case, we performed the sampling of the liquid to obtain the refractive index gradient. The plot is shown in figure 6 as a function of the height in the tank. The procedure and apparatus are the same as those of the inferior mirage. We also mention that there could be slight irregularities in the refractive index variations in both experiments in a horizontal direction through the fluid. These have not been taken into account in this work.

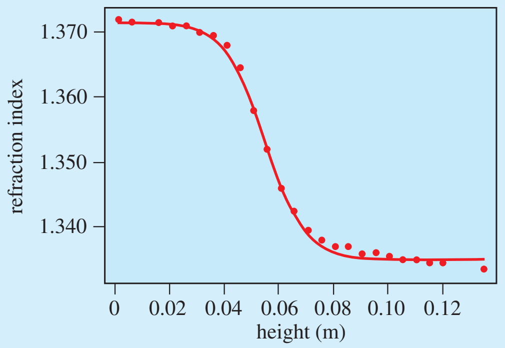

Figure 6. Syrup–water refractive index gradient as a function of the height. The lower optical density on top leads to a superior mirage.

Mirage ray tracing

The use of gradient refractive index media in the laboratory to illustrate various mathematical problems and physical phenomena is well reported in the literature [3]. In particular, ray tracing in a gradient refractive index media is described by Mamola *et al* who solved the differential equation for the light ray paths using Snell's law and used the experimental refractive index gradient to predict the light ray path from knowing the initial position and slope of the light beam.

We propose a similar approach, this time to compare the actual position of the object with the recorded image of the mirage (as directly observed by the eye or recorded by the camera). Our experiment and photographic record of the light rays allows for a quantitative determination of the mirage image, point by point. This gives a good comparison between the predicted (ray-traced) mirage image and the actual one. Moreover, the inversion of the image in the mirage can be studied in detail.

First, we obtained an analytical form for the refractive index gradient by fitting our experimental data with the logistic function

$$n(y) = n_0 + \frac{c}{1 + d\exp(-ey)}$$

in which the parameters n_0, c, d and e are given in table 1 and the resulting curves for alcohol and syrup are shown in figures 3 and 6, respectively. The differential equation for obtaining the light

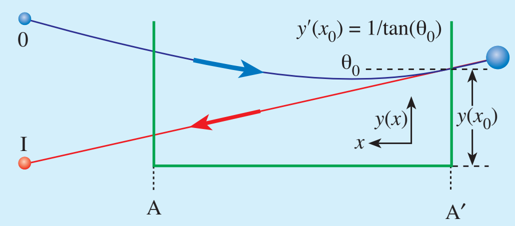

Figure 7. Schematic view of the ray tracing elements. The blue ray is the 'physical' path (refracted). The red ray has the same initial conditions (x_0, y_0) as the blue one (right-hand side) but it follows a straight line path and so constitutes a geometric virtual source.

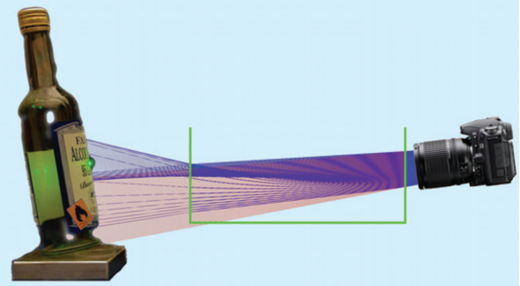

Figure 8. Refracted (blue) and imaginary (red) paths to explain the (inferior) mirage on the bottle.

path $y(x)$ is

$$y''(x) - \frac{n'(y)}{n(y)}[1 + (y'(x))^2] = 0$$

in which y is also seen as the fluid height and x is the horizontal displacement coordinate. Derivatives are denoted as $y'(x) = \mathrm{d}y/\mathrm{d}x$. Boundary conditions are easily obtained in terms of geometrical considerations, as shown in figure 7. More specifically, we consider a single ray starting from the right side of the figure and, following the 'blue' path of figure 7, reaching through the container to the 'screen' on the left side. This corresponds to the same experimental setup shown in figures 2 and 5, in which a laser beam was followed (photographed and ray-traced) from right to left. The actual situation is a bit different, in that light rays start from the 'object' (bottle) from the left side and travel to the right, always along the 'blue' path of figure 7. We can consider both descriptions as valid ones because of the optical reversibility of light rays. The red ray of figure 7 comes instead from the perceived,

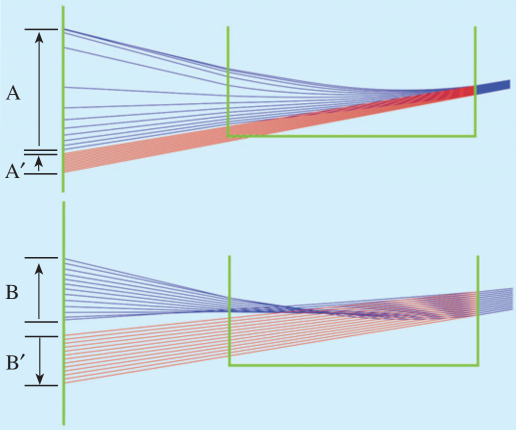

Figure 9. Ray tracings in alcohol (inferior mirage) showing the construction of the non-inverted, squeezed image (top) and of the inverted image (bottom).

Table 1. Coefficients of the analytical expression used to reproduce refraction indices of alcohol and sugar syrup. Parameters are dimensionless.

	Alcohol	Sugar
n_0	1.334	1.335
c	0.032 50	0.03657
d	1.964×10^6	6.778×10^{-4}
e	−164.2	134.6

apparent imagine. It is drawn by superimposing its external segment (at the right side) to the corresponding blue ray (i.e. the blue ray starting from the same point). One can say that the object 'O' is mapped to the imagine 'I' in figure 7. One has also to account for two different light refractions at sections A, A′ of figure 7, in which light passes from air-to-glass-to-liquid and vice versa. The glass interface, though, is not thick enough to allow for a noticeable deviation and has not been included in our computation. The differential equation has been solved numerically with the Mathematica™ 'NDSolve' routine[6].

[6] Mathematica™, as is well known, allows for very effective, compact writing of even complex procedures. As an example, our problem was reduced to the following *single* line of instruction:

```
NDSolve[{y''[x] − (n'[y[x]]/n[y[x]]) ∗ (1 + (y'[x])^2) ==
0, y[0] == y0, y'[0] == 1/Tan[theta0]}, y, {x, 0, xmax};
```

in which `n[x]` is the variable refraction index and `y0` and `theta0` are the initial conditions on the trajectory coordinate y. The solution is provided on the $[0, xmax]$ interval of x values.

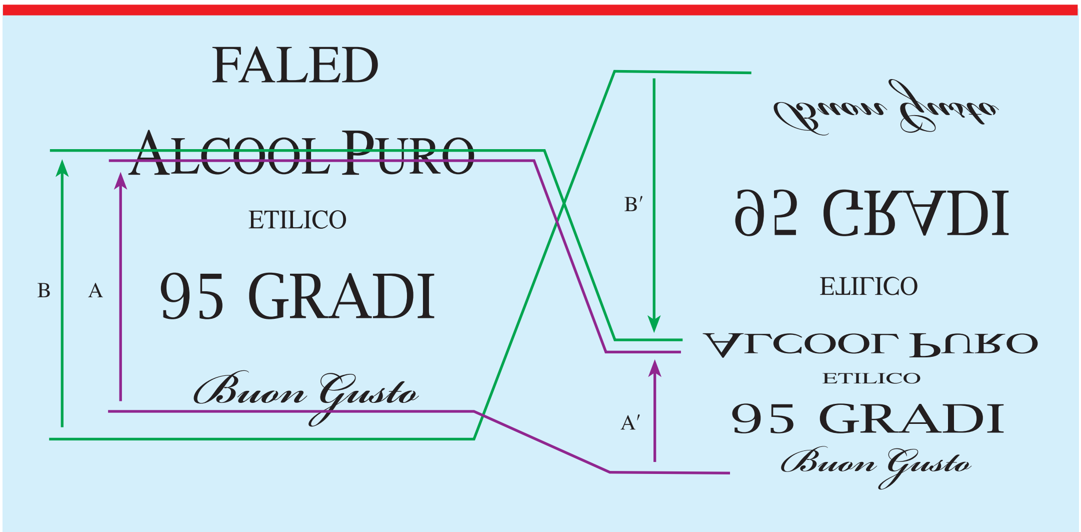

Figure 10. Inferior mirage main features (see figures 1 and 9). 'A' section: squeezed, non-inverted image (figure 9, top); 'B' section: inverted image (figure 9, bottom).

It is now possible to extend the calculation to include rays spanning the whole label on the bottle in order to reproduce the actual perceived mirage. An output of this calculation is shown in figure 8. It is clear that the pattern of ray paths is quite complicated. There are different zones where the image is compressed, expanded and/or inverted. We performed a scan of the observation angle (image point) varying it within fixed intervals and selecting only a few rays to avoid overcrowding the picture. In doing so our intent is to analyse in detail the optical mechanism of the mirage formation. We here present the results for the inferior mirage as the photograph (figure 1) is of better quality and allows for a clear understanding of the phenomenon. Yet, the analysis of the superior mirage is completely analogous.

By selecting two parts of the image, with its corresponding light rays, we can show the main features of the mirage formation, i.e. the squeezing of the image and its inversion upside down. This is shown in figure 9 where two bunches of ray paths, corresponding to two ranges of observation angles, are displayed. A close comparison of our ray tracing, figure 9, and the actual image of the mirage, in figure 1, shows very clearly the accuracy of our prediction. At the same time, we clearly show the critical zones of the mirage, and where and how the image gets distorted due to the refractive index gradient.

We summarize in figure 10 the most relevant effects of light refraction in the inferior mirage and their connections with the ray tracings shown in figure 9.

Conclusions

We have presented a simple experiment to obtain vivid inferior and superior mirages. While direct observation is quite easy and high quality mirages can be obtained with little practice, the photographic recording of the light rays and the image of the mirage require some more practice, particularly to obtain a well focused photograph of the mirage. Determining the refractive index experimentally and recording ray paths accurately makes possible a detailed study of light behaviour, providing a deep understanding of the physical origin of the mirage.

Received 28 May 2009, in final form 16 June 2009
doi:10.1088/0031-9120/44/6/002

References

[1] See, for instance, Fabri E, Fiorio G, Lazzeri F and Violino P 1982 Mirage in the laboratory *Am. J. Phys.* **50** 517
Kosa T and Palffy-Muhoray P 2000 Mirage mirror on the wall *Am. J. Phys.* **68** 1120
Tennakone K 1983 A mirage from a laser *Am. J. Phys.* **51** 270

Richey L, Stewart B and Peatross J 2006 Creating and analyzing a mirage *Phys. Teach.* **44** 460

[2] Barker R, Crofts P R M and Gal M 1989 A superior 'superior' mirage *Am. J. Phys.* **57** 953

[3] Mamola K C, Mueller W F and Regittko B J 1992 Light rays in gradient index media: a laboratory exercise *Am. J. Phys.* **60** 527

[4] Greenler R G 1987 Laboratory simulation of inferior and superior mirages *J. Opt. Soc. Am.* A **4** 589

Tape C H 2000 Aquarium, Computer, and Alaska range mirages *Phys. Teach.* **38** 308

[5] Gratton L M, López-Arias T, Calzà G and Oss S 2009 The whiteness of things and light scattering *Phys. Educ.* **44** 411–4

T López-Arias is a post-doctoral fellow working in the study and experimentation of new teaching activities devoted to the physics of flight. Her original field of research is in nuclear physics.

G Calzà is a high-school maths and physics teacher. He is collaborating with Physics Department staff at the University of Trento, Italy on the construction of new school science projects and curricula, and the development of laboratory materials.

L M Gratton is assistant professor at the Physics Department of the University of Trento, Italy where he is in charge of the coordination of physics laboratories for the high-school teacher training centre. His background is in material science and ionic implantation processes.

S Oss is associate professor of physics and in charge of the Physical Science Communication Laboratory at the Physics Department of the University of Trento, Italy. His background is in atomic physics and theoretical molecular spectroscopy.