A Sequential Monte Carlo Filter for Joint Linear/Nonlinear State Estimation with Application to DS-CDMA

Ronald A. Iltis
Department of Electrical and Computer Engineering
University of California,
Santa Barbara, CA 93106
Tel: (805) 893-4166, Fax: (805) 893-3262, E-mail: iltis@ece.ucsb.edu

EDICS Classification 2-ESTM

To appear in the IEEE Transactions on Signal Processing.
This work was supported in part by the National Science Foundation under Grant No. CCR-0073214, and by a grant from the International Foundation for Telemetering

Abstract

A sequential Monte Carlo filter is considered which combines previously developed Sequential Importance Sampling (SIS) techniques for conditional linear Gaussian models with measurement linearization for construction of approximate simulation densities. The resulting Sequential Monte Carlo Kalman Filter (SMC-KF) consists of a bank of conventional Kalman filters individually tuned to sampled trajectories of the nonlinear state variables. Sampling is according to a Gaussian distribution, with mean and covariance determined by extended Kalman filter-type equations. The SMC-KF is then applied to joint delay and multipath channel estimation in Direct-Sequence Code-Division Multiple Access (DS-CDMA). A combined analytical/simulation technique is employed to compare performance of the SMC-KF and a previously derived EKF-based DS-CDMA channel estimator.

I. Introduction

The problem of joint estimation of linear and nonlinear state variables remains challenging, especially in multiuser communications applications where the state dimension is large and signal-to-noise ratio (SNR) is low. Classical solutions to nonlinear state estimation include the extended Kalman filter (EKF) [1], Gaussian sum filters [2],[3],[4] and grid-based methods [5]. More recently, sequential Monte Carlo (also known as particle filtering, bootstrap, condensation, and sampling/resampling) techniques have been developed for nonlinear state estimation with a focus on target-tracking problems [6],[7],[8],[9]. Applications of Monte Carlo filtering to digital communications include [10], which introduced the mixture Kalman filter for flat-fading channel estimation, and [11], which addressed the problem of DS-CDMA joint amplitude/data estimation in the case of known delays.

It has been shown in [8],[12] and [10] that joint sampling of a mixed linear/nonlinear state space in a conditional linear Gaussian model [12] is inefficient. For example, in a Jump Markov Linear System (JMLS) it is only necessary to sample from the nonlinear discrete-valued Markov process. The sampling is governed by a Gaussian distribution with mean and variance computed via ordinary Kalman filter equations. However, in applications such as DS-CDMA, both the nonlinear and linear state variables are continuous-valued. Specifically, both the channel bulk delays and channel complex gains are modeled by continuous-valued Gaussian autoregressive processes [13] in DS-CDMA, and hence the JMLS method of [8] is not directly applicable. The

DS-CDMA measurement model can be viewed as conditional linear Gaussian, with conditioning on the code delays.

The Monte Carlo filter proposed here is based on a combination of SIS methods for conditional linear Gaussian models [8] with measurement linearization [12],[14] for construction of a practical simulation density. The resulting algorithm is called the Sequential Monte Carlo Kalman Filter to emphasize its structure. It should be stressed that the SMC-KF algorithm is largely based on existing methods from [8] and [12]. However, the SMC-KF is somewhat different in that measurement linearization is only applied to the nonlinear state variables, with the linear state variables estimated using conventional Kalman filters. Hence, the SMC-KF is best viewed as a synthesis of conditional Gaussian SIS [8] and measurement linearization methods [12],[14] tailored to DS-CDMA channel estimation.

The SMC-KF resembles several other algorithms in the literature, but with key differences. For example, in [15], the problems of tracking and navigation are considered when the observations are nonlinear in the position part of the state vector. The nonlinear (position) state variables are estimated via a Sampling-Importance-Resampling (SIR) technique, whereas the velocity/acceleration (linear) variables are tracked via a conventional Kalman filter. However, the algorithm in [15] assumes that the observations are independent of the linear velocity/acceleration variables. Hence, the partitioning approach in [15] is not directly applicable to the DS-CDMA model, whose measurements depend on both delay (nonlinear) and channel (linear) state variables. Similarly, the idea of conditional Kalman filtering appears in [16], but in the context of unknown time-invariant parameters. The method of [16] could be applied to DS-CDMA if the code delays were time-invariant, but this assumption is not realistic. The SMC-KF can also be viewed as a Rao-Blackwellized filter following [17], in which Kalman filters are updated conditioned on samples generated by SIR or SIS. However, the method of [17] is still not directly applicable to the DS-CDMA channel estimation problem, since the measurement function is assumed independent of the conditionally linear process. Finally, the sampling method in the SMC-KF has similarities to the Gaussian Sum Particle Filter (GSPF) of [18], in which the sampling densities depend on EKF-type likelihoods. However, the GSPF does not directly exploit the conditional linear Gaussian model.

The state estimation problem in DS-CDMA is especially challenging due to the highly nonlinear nature of the channel bulk delay, and the presence of multiuser interference. Previous nonlinear channel estimators for DS-CDMA include the EKF in [19],[20] and [13]. However, at higher Doppler spreads ($> 1 \times 10^{-3}/T$, where $1/T$ is the symbol rate) the EKF was found to be subject to divergence [13]. The unscented filter (UF) [21] was proposed for DS-CDMA channel/delay tracking in [22] to reduce the incidence of divergence. The UF is related to the sequential Monte Carlo filter approach, in that multiple one-step predictions of each state variable are generated using a sampling strategy. However, the UF computes a single measurement update for each state variable based on an "averaged" innovations, and does not have a direct importance sampling interpretation.

In the sequel, the problem of sequential Monte Carlo filtering for continuous-valued Markov processes is discussed in Section II. The linearization approximation is reviewed and resulting SMC-KF algorithm is derived in Section III. The SMC-KF is modified and applied to DS-CDMA channel estimation in Section IV, with results and conclusions following in Sections V and VI respectively.

II. Sequential Monte Carlo Filter for Continuous State-Space Markov Processes

The following nonlinear additive Gaussian measurement model is assumed in the sequel, where $\mathbf{r}(n) \in \mathcal{C}^M$, $\mathbf{x}(n) \in \mathcal{C}^{N_x}$ and $\mathbf{c}(n) \in \mathcal{C}^{N_c}$.

$$\begin{aligned} \mathbf{r}(n) &= \mathbf{H}(\mathbf{c}(n))\mathbf{x}(n) + \mathbf{n}(n) \\ \mathbf{x}(n+1) &= \mathbf{F}_x\mathbf{x}(n) + \mathbf{w}_x(n+1) \\ \mathbf{c}(n+1) &= \mathbf{F}_c\mathbf{c}(n) + \mathbf{w}_c(n+1), \end{aligned} \quad (1)$$

where $\mathbf{x}(n)$ and $\mathbf{c}(n)$ are independent circular Gaussian autoregressive (AR) processes [1] The driving terms $\mathbf{w}_x(n) \in \mathcal{C}^{N_x}$, $\mathbf{w}_c(n) \in \mathcal{C}^{N_c}$ and measurement noise $\mathbf{n}(n) \in \mathcal{C}^M$ are mutually independent white Gaussian processes, with covariance matrices $\mathbf{Q}_x, \mathbf{Q}_c$ and $\mathbf{R}$, respectively. The nonlinear measurement function is given by $\mathbf{H}() \in \mathcal{C}^{M \times N_x}$.

If $\mathbf{c}(n)$ was a discrete-valued Markov process, then (1) would correspond to the JMLS in [8].

[1] Circular Gaussian AR means that the processes are complex-valued, with independent real and imaginary parts.

When $\mathbf{c}(n)$ is continuous-valued, (1) then corresponds to the conditional linear Gaussian model of [12]. The SMC-KF uses the following SIS estimates as in [8], [12].

$$\left(\hat{\mathbf{x}}^P(n|n), \hat{\mathbf{c}}^P(n|n)\right) = \sum_{i=1}^{P} \left(\hat{\mathbf{x}}_i(n|n), \mathbf{c}_i(n)\right) \frac{\frac{p(\mathbf{c}_i^n|\mathbf{r}^n)}{\pi(\mathbf{c}_i^n|\mathbf{r}^n)}}{\sum_{j=1}^{P} \frac{p(\mathbf{c}_j^n|\mathbf{r}^n)}{\pi(\mathbf{c}_j^n|\mathbf{r}^n)}}, \tag{2}$$

where $\mathbf{r}^n = \{\mathbf{r}(0), \ldots, \mathbf{r}(n)\}$ is the cumulative measurement history, and similarly $\mathbf{c}_i^n$ is the i-th cumulative sampled sequence of nonlinear state vectors. The conditional state estimate $\hat{\mathbf{x}}_i(n|n)$ is computed via the Kalman filter equations, and is defined by

$$\hat{\mathbf{x}}_i(n|n) = E\left\{\mathbf{x}(n)|\mathbf{c}_i^n, \mathbf{r}^n\right\}. \tag{3}$$

The density $p(\mathbf{c}_i^n|\mathbf{r}^n)$ is the true probability density function of $\mathbf{c}_i^n$ given the cumulative measurements. The "simulation density" $\pi(\mathbf{c}_i^n|\mathbf{r}^n)$ [8] is selected so as to approximately minimize the variance of the estimation error. As shown in [8], the SIS estimates in (2) converge a.s. to the true minimum variance estimates $E\{\mathbf{x}(n)|\mathbf{r}^n\}$. However, the SIS estimates are not unbiased for finite sample sizes due to division by the sum of the weights in (2).

The simulation density is restricted to the form $\pi(\mathbf{c}^n|\mathbf{r}^n) = \pi(\mathbf{c}(n)|\mathbf{r}^n, \mathbf{c}^{n-1})\pi(\mathbf{c}^{n-1}|\mathbf{r}^{n-1})$ [8] in order to obtain a purely recursive sequential Monte Carlo filter. Consider the choice of density $\pi(\mathbf{c}(n)|\mathbf{r}^n, \mathbf{c}^{n-1}) = p(\mathbf{c}(n)|\mathbf{r}^n, \mathbf{c}^{n-1})$. In this case, the weighting factor defined below is a deterministic function of $\mathbf{c}^{n-1}, \mathbf{r}^n$, and hence its variance w.r.t. the distribution of $\mathbf{c}(n)$ is zero [8].

$$w^i(n) = \frac{p(\mathbf{c}_i^n|\mathbf{r}^n)}{\pi(\mathbf{c}_i^n|\mathbf{r}^n)}. \tag{4}$$

The weight factor update is then

$$w^i(n) = \frac{1}{p(\mathbf{r}(n)|\mathbf{r}^{n-1})} p(\mathbf{r}(n)|\mathbf{c}_i^{n-1}, \mathbf{r}^{n-1}) w^i(n-1). \tag{5}$$

The normalization factor $1/p(\mathbf{r}(n)|\mathbf{r}^{n-1})$ is difficult to calculate analytically, but is not required when the SIS estimator in (2) is used.

The optimal sampling density (in the sense of minimizing the variance of the weights) is given by the following Proposition.

Proposition 1: The sampling density $p(\mathbf{c}(n)|\mathbf{r}^n, \mathbf{c}_i^{n-1})$ for the measurement and process model

(1) is given by

$$p(\mathbf{c}(n)|\mathbf{r}^n, \mathbf{c}_i^{n-1}) = \frac{1}{c_i}\mathcal{N}\left(\mathbf{r}(n); \mathbf{H}(\mathbf{c}(n))\hat{\mathbf{x}}_i(n|n-1), \Sigma_i(n|n-1)\right)\mathcal{N}\left(\mathbf{c}(n); \mathbf{F}_c\mathbf{c}_i(n-1), \mathbf{Q}_c\right), \quad (6)$$

where

$$\begin{aligned} \hat{\mathbf{x}}_i(n|n-1) &= E\{\mathbf{x}(n)|\mathbf{r}^{n-1}, \mathbf{c}_i^{n-1}\} \quad (7) \\ \Sigma_i(n|n-1) &= \mathbf{H}(\mathbf{c}(n))\mathbf{P}_i(n|n-1)\mathbf{H}(\mathbf{c}(n))^H + \mathbf{R} \\ \mathbf{P}_i(n|n-1) &= E\left\{[\mathbf{x}(n) - \hat{\mathbf{x}}_i(n|n-1)][\quad]^H|\mathbf{r}^{n-1}, \mathbf{c}_i^{n-1}\right\}. \end{aligned}$$

Proof: Eq. (6) holds when $\mathbf{c}(n)$ is either continuous-valued or finite-state Markov, and hence the proof follows directly from [8] and [12].

The nonlinearity of $\mathbf{H}(\mathbf{c}(n))$ in $\mathbf{c}(n)$ precludes combining the product of Gaussian densities in (6) into a single Gaussian, and hence obtaining a tractable sampling distribution. The EKF-type approximation from [12] needed to circumvent this problem is now reviewed.

III. Development of the SMC-KF

The following linearization of the measurement function and approximation of the innovations covariance leads to a practical sampling distribution based on the exact form (6).

$$\begin{aligned} &\mathbf{H}(\mathbf{c}(n))\hat{\mathbf{x}}_i(n|n-1) \approx \quad (8) \\ &\quad \mathbf{H}(\mathbf{F}_c\mathbf{c}_i(n-1))\hat{\mathbf{x}}_i(n|n-1) + \left.\frac{\partial \mathbf{H}(\mathbf{c})\hat{\mathbf{x}}_i(n|n-1)}{\partial \mathbf{c}}\right|_{\mathbf{c}=\mathbf{F}_c\mathbf{c}_i(n-1)} [\mathbf{c}(n) - \mathbf{F}_c\mathbf{c}_i(n-1)], \\ &\quad \Sigma_i(n|n-1) \approx \tilde{\Sigma}_i(n|n-1) = \mathbf{H}(\mathbf{F}_c\mathbf{c}_i(n-1))\mathbf{P}_i(n|n-1)\mathbf{H}(\mathbf{F}_c\mathbf{c}_i(n-1))^H + \mathbf{R}, \end{aligned}$$

where $\partial/\partial\mathbf{c}$ is the row derivative operator.

Substitution of (8) into (6) yields the following proposition describing the approximate sampling distribution.

Proposition 2: The true sampling distribution $\pi(\mathbf{c}(n)|\mathbf{r}^n, \mathbf{c}_i^{n-1})$ can be approximated as follows using the linearization (8).

$$\pi(\mathbf{c}(n)|\mathbf{r}^n, \mathbf{c}_i^{n-1}) = \mathcal{N}(\mathbf{c}(n); \overline{\mathbf{c}}_i(n), \mathbf{P}_{c,i}(n)), \quad (9)$$

where

$$\overline{\mathbf{c}}_i(n) = \quad (10)$$

$$\mathbf{F}_c\mathbf{c}_i(n-1) + \mathbf{P}_{c,i}(n)\mathbf{J}_i(n)^H\tilde{\Sigma}_i(n|n-1)^{-1}[\mathbf{r}(n) - \mathbf{H}(\mathbf{F}_c\mathbf{c}_i(n-1))\hat{\mathbf{x}}_i(n|n-1)]$$

$$\mathbf{P}_{c,i}(n)^{-1} = \mathbf{J}_i(n)^H\tilde{\Sigma}_i(n|n-1)^{-1}\mathbf{J}_i(n) + \mathbf{Q}_c^{-1}.$$

The Jacobian matrix $\mathbf{J}_i(n) \in \mathcal{C}^{M\times N_c}$ is defined by

$$\mathbf{J}_i(n) = \left.\frac{\partial \mathbf{H}(\mathbf{c})\hat{\mathbf{x}}_i(n|n-1)}{\partial \mathbf{c}}\right|_{\mathbf{c}=\mathbf{F}_c\mathbf{c}_i(n-1)}. \tag{11}$$

Proof: Substitution of the linearization (8) into the exact simulation density (6) yields an overall Gaussian density for $\mathbf{c}(n)$ conditioned on $\mathbf{r}^n, \mathbf{c}_i^{n-1}$, [12] by following the standard EKF derivation (e.g. [[1], Chap. 8]).

The next sample $\mathbf{c}_i(n)$ corresponding to the trajectory $\mathbf{c}_i^{n-1}$ is thus taken from a Gaussian distribution, with mean and covariance matrix given by (10). The updates for the mean vector $\overline{\mathbf{c}}_i(n)$ and covariance matrix $\mathbf{P}_{c,i}(n)$ are strongly reminiscent of, but not identical to the EKF and Gaussian sum filter [2]. In order to obtain the recursion for the weighting factor $w^i(n)$ (5), the following lemma is required.

Lemma 1: Assume that $p(\mathbf{r}(n)|\mathbf{r}^{n-1}, \mathbf{c}_i^n)$ and $p(\mathbf{c}_i(n)|\mathbf{c}_i(n-1))$ are complex Gaussian, and replace $\mathbf{H}(\mathbf{c}(n))$ by its linearized approximation, and $\Sigma_i(n|n-1)$ by $\tilde{\Sigma}_i(n|n-1)$ (8), so that

$$p(\mathbf{r}(n)|\mathbf{r}^{n-1}, \mathbf{c}_i^n) \approx \mathcal{N}(\mathbf{r}(n); \overline{\mathbf{r}}_i(n), \tilde{\Sigma}_i(n|n-1)) \tag{12}$$
$$p(\mathbf{c}_i(n)|\mathbf{c}_i(n-1)) = \mathcal{N}(\mathbf{c}_i(n); \mathbf{F}_c\mathbf{c}_i(n-1), \mathbf{Q}_c),$$

where

$$\overline{\mathbf{r}}_i(n) = \mathbf{H}(\mathbf{F}_c\mathbf{c}_i(n-1))\hat{\mathbf{x}}_i(n|n-1) + \mathbf{J}_i(n)\left[\mathbf{c}(n) - \mathbf{F}_c\mathbf{c}_i(n-1)\right]. \tag{13}$$

Then the marginal density $p(\mathbf{r}(n)|\mathbf{r}^{n-1}, \mathbf{c}_i^{n-1})$ under the approximations (8) is

$$p(\mathbf{r}(n)|\mathbf{r}^{n-1}, \mathbf{c}_i^{n-1}) = \mathcal{N}\left(\mathbf{r}(n); \mathbf{H}(\mathbf{F}_c\mathbf{c}_i(n-1))\hat{\mathbf{x}}_i(n|n-1), \mathbf{M}_i(n)\right), \tag{14}$$

where

$$\mathbf{M}_i(n) = \tilde{\Sigma}_i(n|n-1) + \mathbf{J}_i(n)\mathbf{Q}_c\mathbf{J}_i(n)^H. \tag{15}$$

Proof: The product of the Gaussian densities $p(\mathbf{r}(n)|r^{n-1}, c_i^n)$ and $p(\mathbf{c}_i(n)|\mathbf{c}_i^{n-1}, \mathbf{r}^{n-1})$ is integrated w.r.t. $\mathbf{c}_i(n)$. This is a standard analysis, similar to the GSF derivation which can be found in [2] and [[1], Chap. 8].

TABLE I

SMC-KF ALGORITHM

For $i = 1, 2, \ldots P$

Given: $\mathbf{P}_i(n|n-1), \hat{\mathbf{x}}_i(n|n-1), \mathbf{c}_i(n-1)$

Compute Jacobian matrix

$$\mathbf{J}_i(n) = \left.\frac{\partial \mathbf{H}(c)\hat{\mathbf{x}}_i(n|n-1)}{\partial \mathbf{c}}\right|_{\mathbf{c}=\mathbf{F}_c\mathbf{c}_i(n-1)}$$

Compute approximate innovations covariance matrix

$$\tilde{\Sigma}_i(n|n-1) =$$

$$\mathbf{H}(\mathbf{F}_c\mathbf{c}_i(n-1))\mathbf{P}_i(n|n-1)\mathbf{H}(\mathbf{F}_c\mathbf{c}_i(n-1))^H + \mathbf{R}$$

Compute sampling distribution mean/covariance

$$\mathbf{P}_{c,i}(n)^{-1} = \mathbf{Q}_c^{-1} + \mathbf{J}_i(n)^H\tilde{\Sigma}_i(n|n-1)^{-1}\mathbf{J}_i(n)$$

$$\overline{\mathbf{c}}_i(n) = \mathbf{F}_c\mathbf{c}_i(n-1) + \mathbf{P}_{c,i}(n)\mathbf{J}_i(n)^H\tilde{\Sigma}_i(n|n-1)^{-1}\left[\mathbf{r}(n) - \mathbf{H}(\mathbf{F}_c\mathbf{c}_i(n-1))\hat{\mathbf{x}}_i(n|n-1)\right]$$

Sample $\mathbf{c}_i(n) \sim \mathcal{N}\left(\mathbf{c}_i(n); \overline{\mathbf{c}}_i(n), \mathbf{P}_{c,i}(n)\right)$

Compute Kalman updates

$$\mathbf{P}_i(n|n)^{-1} = \mathbf{P}_i(n|n-1)^{-1} + \mathbf{H}(\mathbf{c}_i(n))^H\mathbf{R}^{-1}\mathbf{H}(\mathbf{c}_i(n))$$

$$\hat{\mathbf{x}}_i(n|n) = \hat{\mathbf{x}}_i(n|n-1) + \mathbf{P}_i(n|n)\mathbf{H}(\mathbf{c}_i(n))^H\mathbf{R}^{-1}\left[\mathbf{r} - \mathbf{H}(\mathbf{c}_i(n))\hat{\mathbf{x}}_i(n|n-1)\right]$$

$$\hat{\mathbf{x}}_i(n+1|n) = \mathbf{F}_x\hat{\mathbf{x}}_i(n|n)$$

$$\mathbf{P}_i(n+1|n) = \mathbf{F}_x\mathbf{P}_i(n|n)\mathbf{F}_x^H + \mathbf{Q}_x$$

Next i

Using Lemma 1 in the recursion for the weights (5) yields the following approximate update for $w_i(n)$.

$$w_i(n) = \frac{1}{c}\mathcal{N}\left(\mathbf{r}(n); \mathbf{H}(\mathbf{F}_c\mathbf{c}_i(n-1))\hat{\mathbf{x}}_i(n|n-1), \mathbf{M}_i(n)\right) w_i(n-1), \tag{16}$$

where c is a normalization constant.

The SMC-KF algorithm is summarized in Table I.

IV. Application of the SMC-KF to DS-CDMA Channel/Delay Estimation

The SMC-KF is now applied to the problem of joint delay/channel estimation in DS-CDMA. The signal model and approximations employed to obtain a tractable estimator follow the results of [13]. The conventional uplink DS-CDMA signal model with K users in low-pass equivalent form is

$$r_{lp}(t) = \sum_{m=-\infty}^{\infty} \sum_{k=1}^{K} \sum_{l=0}^{N_f-1} b_k(m) f_{k,l}(t) s_k(t - mT - lT_s - Re\{T_k(t)\}) + n(t), \tag{17}$$

where $b_k(n) = \pm 1$ is the data symbol for user k, $s_k(t)$ is the k-th signature waveform with support $[0, T)$ s., and $f_{k,l}(t) \in \mathcal{C}$ is the channel complex gain for path l and user k. The thermal noise $n(t)$ is circular white Gaussian with power spectral density $2N_0$. Details of the signature waveform $s_k(t)$ and its derivative are given in [23] and [13]. The Nyquist sampling interval T_s is set to $T_s = T_c/2$, or half the chip duration. Note that the delay $T_k(t) \in \mathcal{C}$ will be treated as a complex-valued process for consistency with the SMC-KF derivation. However, only the real part of $T_k(t)$ is observable, hence the signature waveform $s_k()$ only depends on $Re\{T_k(t)\}$.

Following [13], a delayed-sampling technique per user eliminates intersymbol interference (ISI) under the key assumption $N_f T_s << T$. That is, the multipath spread is assumed much shorter than the symbol duration. Define the delayed-sampled vector as

$$\mathbf{r}_k(n) = [r(nN_s + d_k(n)), r(nN_s + 1 + d_k(n)), \ldots, r((n+1)N_s - 1 + d_k(n))], \tag{18}$$

where $r(n) = r_{lp}(nT_s)$, and $d_k(n) = \lfloor Re\{\hat{T}_k(n-1)\}/T_s \rfloor$ is the integer part of a prior delay estimate in units of Nyquist samples.

The equivalent received vector signal for each user is given by

$$\mathbf{r}_k(n) = \mathbf{S}_k(Re\{T_k(n)\} - d_k(n)T_s)\mathbf{f}_k(n) b_k(n) + \mathbf{v}_k(n) + \mathbf{n}_k(n), \tag{19}$$

where $\mathbf{v}_k(n)$ is the effective multiaccess interference (MAI) seen by user k, and $\mathbf{n}_k(n)$ is an i.i.d. Gaussian sequence, with covariance matrix $\frac{2N_0}{T_s}\mathbf{I}$. The matrix $\mathbf{S}_k(\tau) \in \mathcal{C}^{N_s \times N_f}$ is determined by the signature sequence $s_k(t)$, with

$$(\mathbf{S}_k(\tau))_{i,j} = s_k((i-j)T_s - \tau), \tag{20}$$

for $i = 1, \ldots, N_s$, $j = 1, \ldots, N_f$. Channel vectors are defined by $\mathbf{f}_k(n) = [f_{k,0}(n), \ldots, f_{k,N_f-1}(n)]^T \in \mathcal{C}^{N_f}$. The MAI is expressed in terms of the signal matrices $\mathbf{S}_{k'}(\tau)$ and channel vectors as

$$\mathbf{v}_k(n) = \sum_{k' \neq k} \sum_{l=-1,1} \mathbf{S}_{k'}(Re\{T_{k'}(n)\} - d_k(n)T_s - lT)\mathbf{f}_{k'}(n)b_{k'}(n-l). \tag{21}$$

Following [13], the MAI $\mathbf{v}_k(n)$ is approximated as circular Gaussian with zero mean and covariance matrix $\mathbf{R}_k(n)$. The channel vectors and delays propagate according to AR models with transition matrices $\mathbf{F}$ and f_τ, respectively, (assumed identical for all users) with driving noise covariances $\mathbf{Q}_f, q_\tau$.

It is now seen that the delayed-sampled DS-CDMA received vector in (19) corresponds to the mixed linear/nonlinear state-variable model in (1). Specifically, $\mathbf{x}(n) = \mathbf{x}_k(n) = \mathbf{f}_k(n)$ is the linear part of the state, and $\mathbf{c}(n) = c_k(n) = T_k(n)$ is the nonlinear state variable. At this point, the SMC-KF could be applied directly if the data sequence $b_k(n)$ and covariance matrix $\mathbf{R}_k(n)$ were known. Unfortunately, an optimal estimator that fully accounted for the data $b_k(n)$ would require an exponentially growing number of filters in time, to account for the 2^n data hypotheses per user in the binary case [24],[20]. An alternative strategy would be to jointly sample from the data and delay spaces, following the mixture Kalman filter approach of [10]. However, we have found that a systematic unimodal approximation for the filtering densities, combined with minimum mean-squared error (MMSE) detection of the $b_k(n)$ as in [13] yields excellent performance without the need to impute data sequences.

The modifications to the SMC-KF unimodal density approximations applicable to DS-CDMA are now presented. The weighting factors, conditional channel/delay estimates and unconditional estimates are defined by

$$\begin{aligned} w_k^i(n) &= \frac{1}{c}\frac{p(T_k^{i,n}|\mathbf{r}_k^n)}{\pi(T_k^{i,n}|\mathbf{r}_k^n)} \\ \hat{\mathbf{f}}_k^i(n|n-1) &= E\{\mathbf{f}_k(n)|\mathbf{r}_k^{n-1}, T_k^{i,n-1}\} \\ \hat{\mathbf{f}}_k^i(n|n) &= E\{\mathbf{f}_k(n)|\mathbf{r}_k^{n}, T_k^{i,n}\} \\ \hat{\mathbf{f}}_k(n|n) &= \sum_{i=1}^{P} \hat{\mathbf{f}}_k^i(n|n)w_k^i(n) \\ \hat{T}_k(n) &= \sum_{i=1}^{P} T_k^i(n)w_k^i(n). \end{aligned} \tag{22}$$

Note that the weighting factors $w_k^i(n)$ are normalized by the constant c to form a probability vector.

The covariance matrix $\mathbf{R}_k(n)$ of the effective MAI plus noise for user k can be estimated using IS methods, however, the resulting form is unnecessarily complicated. In our simulations, we have found that a simpler estimate of $\hat{\mathbf{R}}_k(n)$ based on unconditional channel and delay predictions can be employed as follows:

$$\hat{\mathbf{R}}_k(n) = \sum_{k' \neq k} \sum_{l=-1}^{1} \left[\mathbf{S}_{k'}(f_\tau Re\{\hat{T}_k(n-1)\} - d_k(n)T_s - lT)\mathbf{F}\hat{\mathbf{f}}_{k'}(n-1)\right]\left[\quad\right]^H + \frac{2N_0}{T_s}\mathbf{I}. \tag{23}$$

In order to develop the sampling distribution and avoid exponential growth in complexity due to the data, the following unimodal approximation is introduced for the channel estimates as in [13].

$$p\left(\mathbf{f}_k(n)|\mathbf{r}_k^{n-1}, T_k^{i,n-1}\right) = \mathcal{N}\left(\mathbf{f}_k(n); \hat{\mathbf{f}}_k^i(n|n-1), \mathbf{P}_k^i(n|n-1)\right). \tag{24}$$

Given (24), and the estimated MAI plus noise covariance (23), the innovations likelihood of $\mathbf{r}_k(n)$ averaged over the unknown data $b_k(n)$ is

$$p\left(\mathbf{r}_k(n)|T_k^{i,n}, \mathbf{r}_k^{n-1}\right) = \frac{1}{2}\sum_{b=\pm 1} \mathcal{N}\left(\mathbf{r}_k(n); \mathbf{S}_k(Re\{T_k^i(n)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1)b, \Sigma_k^i(n|n-1)\right), \tag{25}$$

where the innovations covariance is

$$\Sigma_k^i(n|n-1) = \mathbf{S}_k(Re\{T_k^i(n)\} - d_k(n)T_s)\mathbf{P}_k^i(n|n-1)\mathbf{S}_k(Re\{T_k^i(n)\} - d_k(n)T_s)^H + \hat{\mathbf{R}}_k(n). \tag{26}$$

A practical sampling density for the DS-CDMA delays is obtained using the linearization and innovations covariance approximations in (8). However, care must be taken since the delay $T_k(n)$ is treated as a complex-valued process, with only the real part being observable. Identifying $\mathbf{H}()$ with $\mathbf{S}_k()$ yields

$$\begin{aligned} &\mathbf{S}_k(Re\{T_k(n)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1) \approx \\ &\quad \mathbf{S}_k(f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1) + \mathbf{J}_k^i(n)\left[T_k(n) - f_\tau(T_k^i(n-1) - Im\{T_k^i(n-1)\})\right] \\ &\quad \tilde{\Sigma}_k^i(n|n-1) = \\ &\quad \mathbf{S}_k(f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s)\mathbf{P}_k^i(n|n-1)\mathbf{S}_k(f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s)^H + \hat{\mathbf{R}}_k(n), \end{aligned} \tag{27}$$

where the Jacobian is

$$\mathbf{J}_k^i(n) = \left. \frac{\partial \mathbf{S}_k(\tau)\hat{\mathbf{f}}_k^i(n|n-1)}{\partial \tau} \right|_{\tau = f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s} . \tag{28}$$

Comment: The linearization in (27) is an exact Taylor's series expansion about $f_\tau Re\{T_k^i(n-1)\}$ when the imaginary part of $T_k(n)$ is zero. For $T_k(n) \in \mathcal{C}$, the linearization is approximate, but required for consistency with the previous SMC-KF derivation. The quantity $Im\{T_k^i(n-1)\}$ acts as a correction term in the innovations, and simulations show has little effect on estimator performance [25]. Thus, following [25], this correction term is dropped in the sequel.

Under the linearization (27) and unimodal Gaussian approximation (24), the sampling density for $T_k^i(n)$ can be derived and is given by the following proposition.

Proposition 3: The sampling density for the DS-CDMA delay estimator, under the linearized approximation for the signal matrix, is

$$\begin{aligned} &\pi(T_k(n)|T_k^{i,n-1}, \mathbf{r}_k^n) \\ &\quad = \frac{q_k^{i,+1}(n)\mathcal{N}(T_k(n); \overline{T}_k^{i,+1}(n), P_{T,k}^i(n)) + q_k^{i,-1}(n)\mathcal{N}(T_k(n); \overline{T}_k^{i,-1}(n), P_{T,k}^i(n))}{q_k^{i,+1}(n) + q_k^{i,-1}(n)}, \end{aligned} \tag{29}$$

where

$$q_k^{i,b}(n) = \mathcal{N}\left(\mathbf{r}_k(n); b\mathbf{S}_k(f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1); \mathbf{M}_{T,k}^i(n)\right). \tag{30}$$

The means of the sampling density are given by the EKF-type equations

$$\begin{aligned} &\overline{T}_k^{i,b}(n) = f_\tau T_k^i(n-1) + \\ &\quad P_{T,k}^i(n)\mathbf{J}_k^i(n)^H \tilde{\Sigma}_k^i(n|n-1)^{-1}\left[\mathbf{r}_k(n) - b\mathbf{S}_k(f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1)\right], \end{aligned} \tag{31}$$

where the covariance matrices for $q_k^{\pm}(n)$ and $\mathbf{M}_{T,k}^i(n)$ are

$$\begin{aligned} P_{T,k}^i(n)^{-1} &= \frac{1}{q_\tau} + \mathbf{J}_k^i(n)^H \tilde{\Sigma}_k^i(n|n-1)^{-1}\mathbf{J}_k^i(n) \\ \mathbf{M}_k^i(n) &= \tilde{\Sigma}_k^i(n|n-1) + \mathbf{J}_k^i(n) q_\tau \mathbf{J}_k^i(n)^H. \end{aligned} \tag{32}$$

Proof: See Appendix A.

The implication of Proposition 3 is that given the sample history $T_k^{i,n-1}$, $T_k^i(n)$ is selected by choosing a Gaussian random variable with probability $q_k^{\pm 1}(n)$, whose mean and covariance is derived from an EKF-type update conditioned on the data $\pm b_k(n)$. As shown in Appendix A, the resulting weight update using (29) is

$$w_k^i(n) = \frac{1}{c}\left(q_k^{i,+1}(n) + q_k^{i,-1}(n)\right) w_k^i(n-1) \tag{33}$$

The last step required to obtain a practical SMC-KF after sampling $T_k^i(n)$ is the recursion for $\hat{\mathbf{f}}_k^i(n|n)$. Given the unimodal approximation (24), the measurement update for $\mathbf{f}_k(n)$ is now obtained via an ordinary Kalman filter equation, when conditioned on $b_k(n)$, i.e.

$$\begin{aligned} E\left\{\mathbf{f}_k(n)|\mathbf{r}_k^n, T_k^{i,n}, b_k(n)\right\} &= \hat{\mathbf{f}}_k^i(n|n-1) + \mathbf{P}_k^i(n|n)\times \\ &\mathbf{S}_k(Re\{T_k^i(n)\} - d_k(n)T_s)^H \hat{\mathbf{R}}_k(n)^{-1}\left[\mathbf{r}_k(n)b_k(n) - \mathbf{S}_k(Re\{T_k^i(n)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1)\right]. \end{aligned} \tag{34}$$

As in [13], an unconditional estimate of $\mathbf{f}_k(n)$ is computed by replacing $b_k(n)$ in (34) with its MMSE estimate.

$$\begin{aligned} \hat{b}_k^i(n) &= \\ E\{b_k(n)|\mathbf{r}_k^n, T_k^{i,n}\} &= \tanh\left(2Re\left\{\mathbf{r}_k(n)^H \Sigma_k^i(n|n-1)^{-1}\mathbf{S}_k(Re\{T_k^i(n)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1)\right\}\right). \end{aligned} \tag{35}$$

When the MAI predominates, so that $\Sigma_k^i(n|n-1) \approx \hat{\mathbf{R}}_k(n)$, the MMSE detector (35) can be approximated by the version for known channels [26], [27]

$$\hat{b}_k^i(n) = \mathrm{sgn}\left(Re\{\mathbf{r}_k(n)^H \hat{\mathbf{R}}_k(n)^{-1}\mathbf{S}_k(Re\{T_k^i(n)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1)\right). \tag{36}$$

The MMSE detector in (36) is still computationally expensive – a separate symbol decision must be made for every sample trajectory i. The following averaged MMSE detector, which is independent of i, has been found to perform well in simulations, and is employed here.

$$\begin{aligned} \mathbf{h}_{k,SMC-KF}(n) &= \hat{\mathbf{R}}_k^{-1}(n)\sum_{i=1}^{P}\mathbf{S}_k(f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1)w_k^i(n-1) \\ \hat{b}_k(n) &= \mathrm{sgn}\left(Re\{\mathbf{r}_k(n)^H \mathbf{h}_{k,SMC-KF}\}(n)\right). \end{aligned} \tag{37}$$

The overall SMC-KF delay/channel estimator for DS-CDMA is summarized in Table II.

V. Results

The analytic bit-error rate (BER) averaged over the thermal noise and multiuser interference amplitudes was computed using the method of [13] as a function of the SMC-KF detector $\mathbf{h}_{k,SMC-KF}(n)$. The analytic BER was also computed for the matched filter and ideal MMSE

TABLE II

SMC-KF ESTIMATOR FOR DS-CDMA

For $k = 1, 2, \ldots, K$

Update estimated noise plus MAI covariance matrix $\hat{\mathbf{R}}_k(n)$ using eq. (23)

Compute MMSE decision $\hat{b}_k(n) = \text{sgn}(Re\{\mathbf{r}_k(n)^H \mathbf{h}_{k,SMC-KF}(n)\})$

For $i = 1, 2, \ldots P$

Given: $\mathbf{P}_k^i(n|n-1), \hat{\mathbf{f}}_k^i(n|n-1), T_k^i(n-1)$

Compute Jacobian vector

$$\mathbf{J}_k^i(n) = \left.\frac{\partial \mathbf{S}_k(\tau)\hat{\mathbf{f}}_k^i(n|n-1)}{\partial \tau}\right|_{\tau = f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s}$$

Compute approximate innovations covariance and sampling covariance

$$\tilde{\Sigma}_k^i(n|n-1) = \hat{\mathbf{R}}_k(n) +$$

$$\mathbf{S}_k(f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s)\mathbf{P}_k^i(n|n-1)\mathbf{S}_k(f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s)^H$$

$$P_{T,k}^i(n)^{-1} = q_\tau^{-1} + \mathbf{J}_k^i(n)^H \tilde{\Sigma}_k^i(n|n-1)^{-1}\mathbf{J}_k^i(n)$$

Compute means for the sampling density

$$\overline{T}_k^{i,b}(n) = f_\tau T_k^i(n-1) + bP_{T,k}^i(n)\mathbf{J}_k^i(n)^H \tilde{\Sigma}_k^i(n|n-1)^{-1} \times$$

$$[\mathbf{r}_k(n) - b\mathbf{S}_k(f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1)]$$

Compute $q_k^{i,b}$ terms for sampling density with $b = \pm 1$

$$q_k^{i,b}(n) = \mathcal{N}\left(\mathbf{r}_k(n); b\mathbf{S}_k(f_\tau Re\{T_k^i(n-1)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1), \mathbf{M}_k^i(n)\right)$$

Sample $T_k^i(n) \sim \sum_{b=\pm 1} q_k^{i,b}(n) \frac{\mathcal{N}(T_k^i(n); \overline{T}_k^{i,b}(n), P_{T,k}^i(n))}{q_k^{i,+1} + q_k^{i,-1}}$

Update the weight for sample i and user k

$$w_k^i(n) = \frac{1}{c}(q_k^{i,+1}(n) + q_k^{i,-1}(n))w_k^i(n-1)$$

Compute Kalman channel coefficient estimates

$$\mathbf{P}_k^i(n|n)^{-1} = \mathbf{P}_k^i(n|n-1)^{-1} +$$

$$\mathbf{S}_k(Re\{T_k^i(n)\} - d_k(n))^H \hat{\mathbf{R}}_k(n)^{-1}\mathbf{S}_k(Re\{T_k^i(n)\} - d_k(n)T_s)$$

$$\hat{\mathbf{f}}_k^i(n+1|n) = \mathbf{F}\left[\hat{\mathbf{f}}_k^i(n|n-1) + \mathbf{P}_k^i(n|n)\mathbf{S}_k(Re\{T_k^i(n)\} - d_k(n)T_s)^H \times\right.$$

$$\left.\hat{\mathbf{R}}_k(n)^{-1}[\mathbf{r}_k(n)\hat{b}_k(n) - \mathbf{S}_k(Re\{T_k^i(n)\} - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1)]\right]$$

Next sample trajectory i

Next user k

detectors defined in [13] as a function of symbol number n and resulting channels/delays $\{\mathbf{f}_k(n)\}$ and $\{T_k(n)\}$. The ideal MMSE and MF detectors, which assume exact knowledge of the channels/delays are

$$\begin{aligned} \mathbf{h}_{k,MMSE} &= \mathbf{R}_k(n)^{-1}\mathbf{S}_k(Re\{T_k(n)\} - d_k(n)T_s)\mathbf{f}_k(n) \\ \mathbf{h}_{k,MF} &= \mathbf{S}_k(Re\{T_k(n)\} - d_k(n)T_s)\mathbf{f}_k(n). \end{aligned} \tag{38}$$

The SMC-KF algorithm was further compared with the MMSE-EKF method of [13]. Note that the MMSE-EKF corresponds to a single extended Kalman filter, with covariance matrix $\hat{\mathbf{R}}_k(n)$ estimated in a manner similar to (23). The MMSE-EKF detector is then

$$\mathbf{h}_{k,MMSE-EKF} = \hat{\mathbf{R}}_k(n)^{-1}\mathbf{S}_k(Re\{\hat{T}_k(n|n-1)\} - d_k(n)T_s)\hat{\mathbf{f}}_k(n|n-1), \tag{39}$$

where $\hat{T}_k(n|n-1)$ and $\hat{\mathbf{f}}_k(n|n-1)$ are EKF-derived one-step predictions.

For an arbitrary linear detector $\mathbf{h}_k(n)$, and given that $\mathbf{R}_k(n)$ is the true MAI plus thermal noise covariance matrix, the BER for differential detection at iteration n is [13]

$$P_b(n) = \frac{1}{2}\exp\left(-\frac{||\mathbf{h}_k(n)^H\mathbf{S}_k(Re\{T_k(n)\} - d_k(n)T_s)\mathbf{f}_k(n)||^2}{\mathbf{h}_k(n)^H\mathbf{R}_k(n)^{-1}\mathbf{h}_k(n)}\right). \tag{40}$$

In order to avoid convergence of any single weight to unity, the sampling/resampling method of [8], [7] was employed. That is, the delay samples $T_k^i(n)$ are periodically resampled with probability equal to the normalized weights, and all weights then reset to a constant.

Figures 1,2 and 3 correspond to a single-user $K = 1$ system. The AR coefficients $\mathbf{F}, f_\tau$ were chosen to yield a Doppler spread of $B_D = 1 \times 10^{-3}/T$ Hz., where $1/T$ is the symbol rate. The channel length in this and all subsequence simulations was set to $N_f = 3$. The AR transition matrix equaled $\mathbf{F} = \rho\mathbf{I}$, with $\rho = f_\tau$ so that the channel coefficients and delays were i.i.d. with identical bandwidths B_D. In addition, a linear drift term of $.01T_c$ sec. every T seconds was added to $T_k(n)$. The delay samples $T_k^i(n)$ and true delay are plotted in Fig. 1 for this case. The sampling/resampling method is seen to be effective in eliminating divergent delay samples. Furthermore, Figures 2 and 3 show that the SMC-KF delay error and bit-error rate $P_b(n)$ are both significantly smaller than the respective MMSE-EKF quantities.

A single-user system is also considered in Figure 4 and 5, with $P = 128$ samples. Here, the drift term is zero, and ensemble averaged analytic BER and absolute timing error are computed, with averaging over $N_r = 64$ simulation runs. The delay estimation error is clearly smaller for the SMC-KF than for the MMSE-EKF in Fig. 5. The ensemble averaged analytic BER in Fig. 4 was also averaged over symbol number. The resulting overall BER was .010 for the SMC-KF, and .0130 for the MMSE-EKF.

A $K = 5$ user DS-CDMA system was considered in Fig. 6, with $E_b/N_0 = 14$ dB for user 1. The Doppler spread was equal to $B_D = 1 \times 10^{-5}/T$ Hz. The number of samples in the SMC-KF was fixed at $P = 32$. Users 2 through 5 were set at a power level of $J/S = 10$ dB above user 1. In this case, the time and ensemble averaged analytic BER for the SMC-KF was slightly higher than for the MMSE-EKF estimator, equaling .00695 (SMC-KF) and .00588 (MMSE-EKF). At this small Doppler spread, the channel evidently varies slowly enough that the MMSE-EKF is nearly optimal, while the SMC-KF suffers from an occasional divergence phenomenon.

In contrast to Fig. 6, a $K = 5$ user system with larger Doppler spread of $1 \times 10^{-3}/T$ Hz. is considered in Figures 7 and 8. In this case, both the BER and delay error performance is better for the SMC-KF than the MMSE-EKF. The time and ensemble-averaged BER was .02567 for the SMC-KF, and .03682 for the MMSE-EKF.

The weight update in (16) is approximate due to the EKF-type linearization. An exact weight update is feasible if SIR is used with the state vector per user redefined as $\mathbf{x}_k(n) = \left[T_k(n), \mathbf{f}_k(n)^T\right]^T$. Note that this SIR method requires inefficient sampling from both the linear channel and nonlinear delay variables. Following [6], SIS becomes SIR if the simulation density $\pi(\mathbf{x}_k(n)|\mathbf{x}_k^{i,n-1}, \mathbf{r}_k^n)$ is set to $p(\mathbf{x}_k(n)|\mathbf{x}_k^i(n-1))$. The latter transition density is Gaussian with mean $\mathbf{F}\mathbf{x}_k^i(n-1)$ and covariance $\mathbf{Q}$. The SIR weights are then updated according to

$$w_k^i(n) = \frac{1}{c} \sum_{b=\pm 1} \mathcal{N}\left(\mathbf{r}_k(n); b\mathbf{S}_k(T_k^i(n) - d_k(n)T_s)\mathbf{f}_k^i(n), \hat{\mathbf{R}}_k(n)\right) w_k^i(n-1). \quad (41)$$

The SIR algorithm was simulated for the $K = 1$ user case, and a Doppler spread of $1 \times 10^{-3}/T$ Hz., with the resulting delay estimation error plotted in Figure 9. Ensemble performance over $N_r = 32$ runs is shown, with comparison to the SMC-KF with $P = 32$ samples and MMSE-EKF. The SIR performance is much poorer than the SMC-KF or MMSE-EKF. Increasing the number of SIR samples from $P = 32$ to $P = 128$ leads to a slight improvement at the end of $n = 256$ symbols, but clearly sampling from both the linear and nonlinear state variables is highly inefficient, as shown analytically in [8].

VI. Conclusions

A sequential Monte Carlo filter (SMC-KF) was presented which combines the conditional linear Gaussian SIS and measurement linearization methods of [8] and [12]. The resulting algorithm consists of a bank of P Kalman filters for the linear state variables, with the nonlinear state components updated via a sampling algorithm. The means and covariances of the sampling densities are given by EKF-type equations. The SMC-KF was applied to DS-CDMA delay/channel estimation. In the single user case, the SMC-KF clearly outperformed a previous MMSE-EKF algorithm. The SMC-KF significantly outperformed the MMSE-EKF when an unmodeled linear drift term was added to the delay process.

In the multiuser case, SMC-KF performance was superior to that of the MMSE-EKF for the case of a large Doppler spread, with both smaller delay estimation error and BER. For small Doppler spreads, the SMC-KF in a set of $N_r = 64$ simulation runs performed slightly worse than the MMSE-EKF. The combination of estimation noise in the covariance matrix $\hat{\mathbf{R}}_k(n)$ and the fact that the MMSE-EKF is operating in the linear regime at small Doppler spread accounts for the lack of superiority of the SMC-KF.

The results here for DS-CDMA channel estimation suggest that the SMC-KF provides better performance than an EKF when modeling errors and/or wideband processes are present. Again, for more benign scenarios, when the state variable processes are narrowband and modeling is exact, the EKF algorithm may perform as well or even outperform the SMC-KF, especially when the number of samples P is small. This is to be expected, since the EKF performance approaches that of the minimum-variance estimator when the linearization error is negligible.

To conclude, the SMC-KF applies previously-developed Monte Carlo filtering methods to conditional linear Gaussian models occurring in CDMA communications systems. The SMC-KF may be preferable to more conventional algorithms when modeling errors are present, or when the EKF linearization approximation is inaccurate.

Appendix

I. Appendix – Derivation of the Delay Sampling Distribution for DS-CDMA

In order to prove Proposition 3, first write the sampling density as

$$p(T_k(n)|T_k^{i,n-1}, \mathbf{r}_k^n) = \frac{1}{c} p(\mathbf{r}_k(n)|T_k^{i,n}, \mathbf{r}_k^{n-1}) p(T_k^i(n)|T_k^i(n-1)), \tag{I.1}$$

where $c = p(\mathbf{r}_k(n)|T_k^{i,n-1}, \mathbf{r}_k^{n-1})$ also determines the weight update in (33). Due to the data uncertainty, the likelihood of $\mathbf{r}_k(n)$ when conditioned on $T_k^{i,n}, \mathbf{r}_k^{n-1}$ is a sum of Gaussians. Recall that $T_k(n)$ is a first order Gaussian AR process, with parameters f_τ, q_τ. Then under the linearization approximation (27), the sampling density in (I.1) becomes

$$p(T_k(n)|T_k^{i,n-1}, \mathbf{r}_k^n) \approx \frac{1}{c} \left(\sum_{b=\pm 1} \mathcal{N}\left(\mathbf{r}_k'^{\,b}(n); \mathbf{J}_k^i(n) T_k(n), \tilde{\Sigma}_k^i(n|n-1)\right)\right) \mathcal{N}\left(T_k(n); f_\tau T_k^i(n-1), q_\tau\right), \tag{I.2}$$

where

$$\mathbf{r}_k'^{\,b}(n) = b\mathbf{r}_k(n) - \mathbf{S}_k(f_\tau T_k^i(n-1) - d_k(n)T_s)\hat{\mathbf{f}}_k^i(n|n-1) + \mathbf{J}_k^i(n) f_\tau T_k^i(n-1). \tag{I.3}$$

Standard analysis, such as in the Gaussian Sum Filter derivation [[1], Chap. 8] allows the product of Gaussian densities in (I.2) to be re-expressed as

$$\begin{aligned} &\mathcal{N}\left(\mathbf{r}_k'^{\,b}(n); \mathbf{J}_k^i(n) T_k(n), \tilde{\Sigma}_k^i(n|n-1)\right) \mathcal{N}\left(T_k(n); f_\tau T_k^i(n-1), q_\tau\right) \\ &= \frac{1}{\pi^{N_s+1} |\tilde{\Sigma}_k^i(n|n-1)| |q_\tau|} e^{\left(-[\mathbf{r}_k(n) - b\mathbf{S}_k(f_\tau T_k^i(n-1) - d_k(n)T_s)]^H [\Sigma_k^i(n|n-1) + \mathbf{J}_k^i(n) q_\tau \mathbf{J}_k^i(n)^H]^{-1} [\quad]\right)} \times \\ &e^{\left([T_k(n) - \overline{T}_k^i(n)]^H P_{T,k}^i(n)^{-1} [\quad]\right)}. \end{aligned} \tag{I.4}$$

Combining the definitions of $q_k^{i,b}(n)$ in (30), $\mathbf{M}_k^i(n)$ in (32) with the exponential product result in (I.4), and finally inserting in (I.2) yields the sampling density in (29).

References

[1] B. Anderson and J. Moore, *Optimal Filtering.* Englewood Cliffs, NJ: Prentice-Hall, 1979.

[2] H. Sorenson and D. Alspach, "Recursive Bayesian estimation using Gaussian sums," *Automatica*, vol. 7, pp. 465–479, 1971.

[3] D. Alspach and H. Sorenson, "Nonlinear Bayesian estimation using Gaussian sum approximations," *IEEE Transactions on Automatic Control*, vol. AC-17, pp. 439–448, Aug. 1972.

[4] K. Ito and K. Xiong, "Gaussian filters for nonlinear filtering problems," *IEEE Transactions on Automatic Control*, vol. 45, pp. 910–927, May 2000.

[5] R. Bucy and K. Senne, "Digital synthesis of non-linear filters," *Automatica*, vol. 7, pp. 287–298, 1971.

[6] M. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking," *IEEE Transactions on Signal Processing*, vol. 50, pp. 174–188, Feb. 2002.

[7] N. Gordon, D. Salmond, and A. Smith, "Novel approach to nonlinear/non-Gaussian Bayesian state estimation," *IEEE Proceedings-F*, vol. 140, pp. 107–113, April 1993.

[8] A. Doucet, N. Gordon, and V. Krishnamurthy, "Particle filters for state estimation of jump Markov linear systems," *IEEE Transactions on Signal Processing*, vol. 49, pp. 613–624, March 2001.

[9] P. Djurić, "The use of bootstrap in computer-intensive Bayesian methods," in *Asilomar Conference on Signals, Systems, and Computers*, (Pacific Grove, CA), pp. 24–27, Oct. 1999.

[10] R. Chen, X. Wang, and J. Liu, "Adaptive joint detection and decoding in flat-fading channels via mixture Kalman filtering," *IEEE Transactions on Information Theory*, vol. 46, pp. 2079–2094, Sept. 2000.

[11] X. Wang and R. Chen, "Adaptive Bayesian multiuser detection for synchronous CDMA with Gaussian and impulsive noise," *IEEE Transactions on Signal Processing*, vol. 47, pp. 2013–2028, July 2000.

[12] A. Doucet, S. Godsill, and C. Andrieu, "On sequential Monte Carlo sampling methods for Bayesian filtering," *Statistics and Computing*, vol. 10, pp. 197–208, 2000.

[13] R. Iltis, "A DS-CDMA tracking mode receiver with joint channel/delay estimation and MMSE detection," *IEEE Transactions on Communications*, vol. 49, pp. 1770–1779, Oct. 2001.

[14] M. Pitt and N. Shepard, "Filtering via simulation: Auxiliary particle filters," *Journal of the American Statistical Association*, vol. 94, pp. 590–599, June 1999.

[15] F. Gustafsson, F. Gunnarsson, N. Bergman, U. Forssell, J. Jansson, R. Karlsson, and P. Nordlund, "Particle filters for positioning, navigation, and tracking," *IEEE Transactions on Signal Processing*, vol. 50, pp. 425–435, Feb. 2002.

[16] G. Storvik, "Particle filters for state-space models with the presence of unknown static parameters," *IEEE Transactions on Signal Processing*, vol. 50, pp. 281–289, Feb. 2002.

[17] C. Andrieu and A. Doucet, "Particle filtering for partially observed Gaussian state space models." To appear in the Journal of the Royal Statistical Society, Series B, 2002.

[18] J. H. Kotecha and P. M. Djurić, "Gaussian sum particle filtering for dynamic state space models," in *Proceed-*

ings of the IEEE International Conference on Acoustics, Speech and Signal Processing, vol. 6, pp. 3465–3468, May 2001.

[19] T. J. Lim and L. Rasmussen, "Adaptive symbol and parameter estimation in asynchronous multiuser CDMA detectors," *IEEE Transactions on Communications*, vol. 45, pp. 213–220, Feb. 1997.

[20] R. Iltis and L. Mailaender, "An adaptive multiuser detector with joint amplitude and delay estimation," *IEEE Journal on Selected Areas in Communications*, vol. 12, pp. 774–785, June 1994.

[21] S. Julier, J. Uhlmann, and H. Durrant-Whyte, "A new approach for filtering nonlinear systems," in *Proceedings of the 1995 American Control Conference*, (Seattle, WA), pp. 1628–1632, June 1995.

[22] J. Caffery and G. Stüber, "Nonlinear multiuser parameter estimation and tracking in CDMA systems," *IEEE Transactions on Communications*, vol. 48, pp. 2053–2063, Dec. 2000.

[23] R. Iltis, "Joint estimation of PN code delay and multipath using the extended Kalman filter," *IEEE Transactions on Communications*, vol. 38, pp. 1677–1685, October 1990.

[24] X. Wang and H. Poor, "Adaptive joint multiuser detection and channel estimation in multipath fading CDMA channels," *Wireless Networks*, vol. 4, pp. 453–470, 1998.

[25] R. Iltis and A. Fuxjaeger, "A digital DS spread-spectrum receiver with joint channel and doppler shift estimation," *IEEE Transactions on Communications*, vol. 39, pp. 1255–1267, Aug. 1991.

[26] U. Madhow and M. Honig, "MMSE interference suppression for direct-sequence spread- spectrum CDMA," *IEEE Transactions on Communications*, vol. 42, pp. 3178–3188, Dec. 1994.

[27] S. L. Miller, "An adaptive direct-sequence code-division multiple-access receiver for multiuser interference rejection," *IEEE Transactions on Communications*, vol. 43, pp. 1746–1755, 1995.

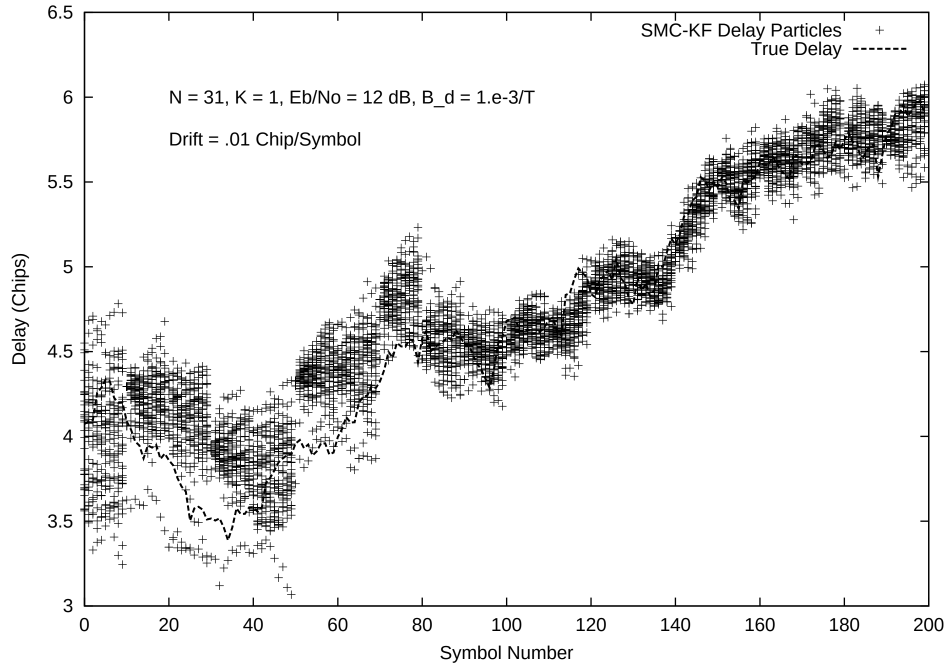

Fig. 1. Delay samples for single-user, $.01T_c$ s. per symbol drift.

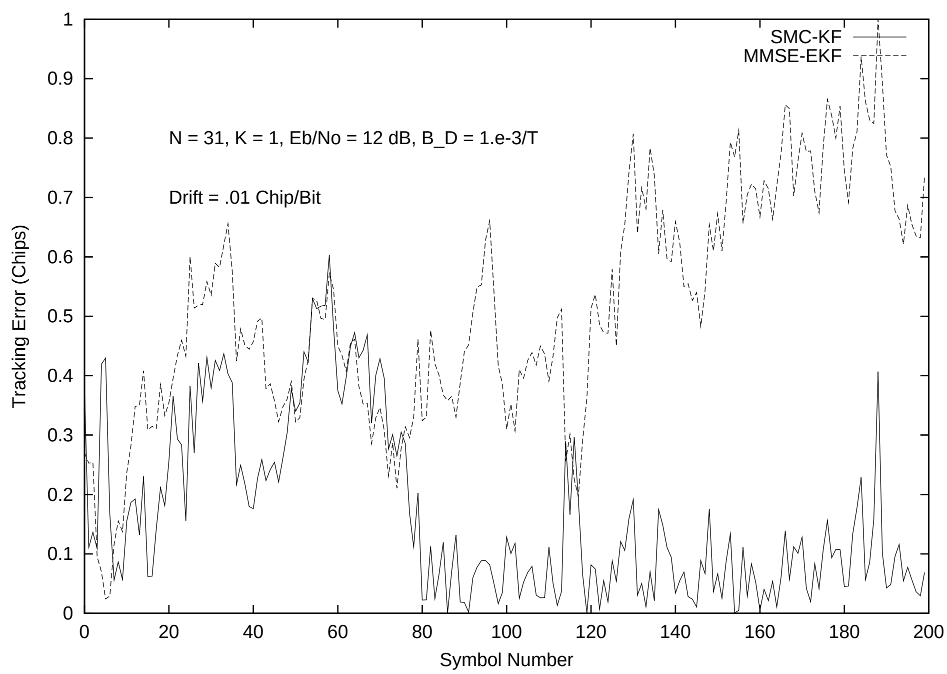

Fig. 2. Delay estimation error for single-user, $.01T_c$ s. per symbol drift.

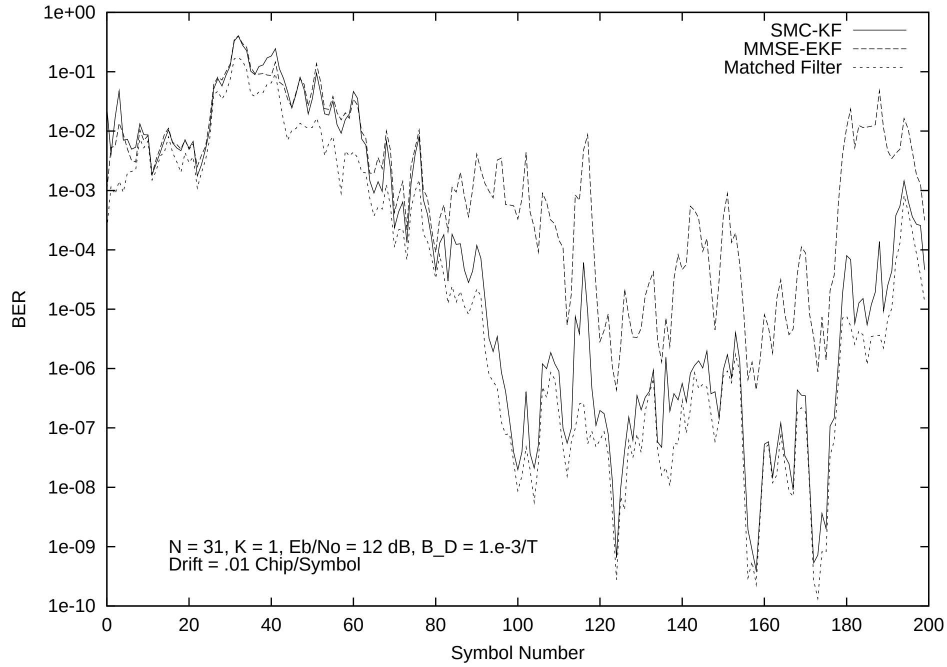

Fig. 3. Bit-error rate for single-user, $.01T_c$ s. per symbol drift.

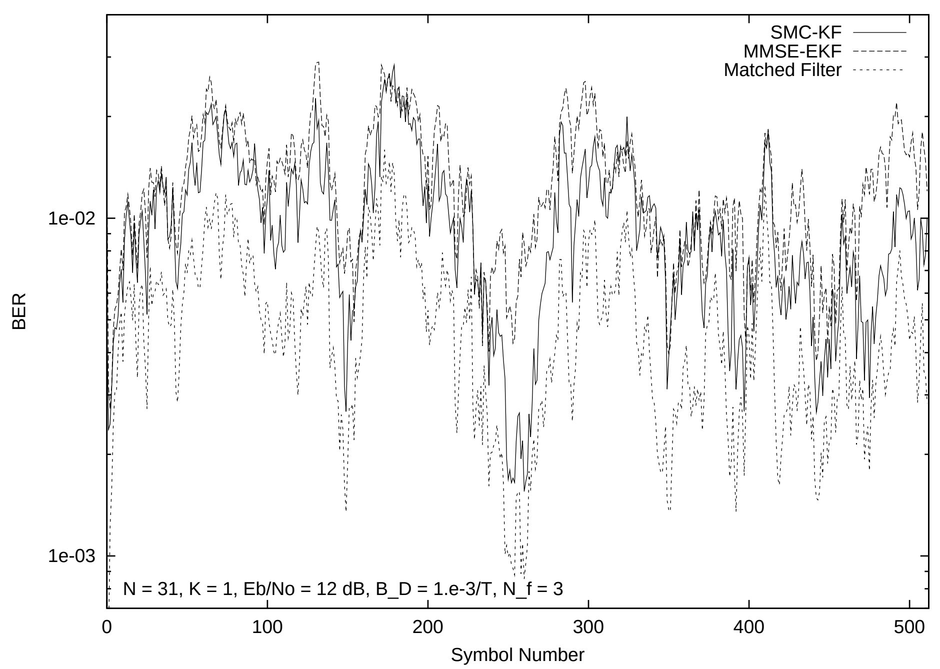

Fig. 4. Bit-error rate for single-user case, averaged over $N_r = 64$ runs

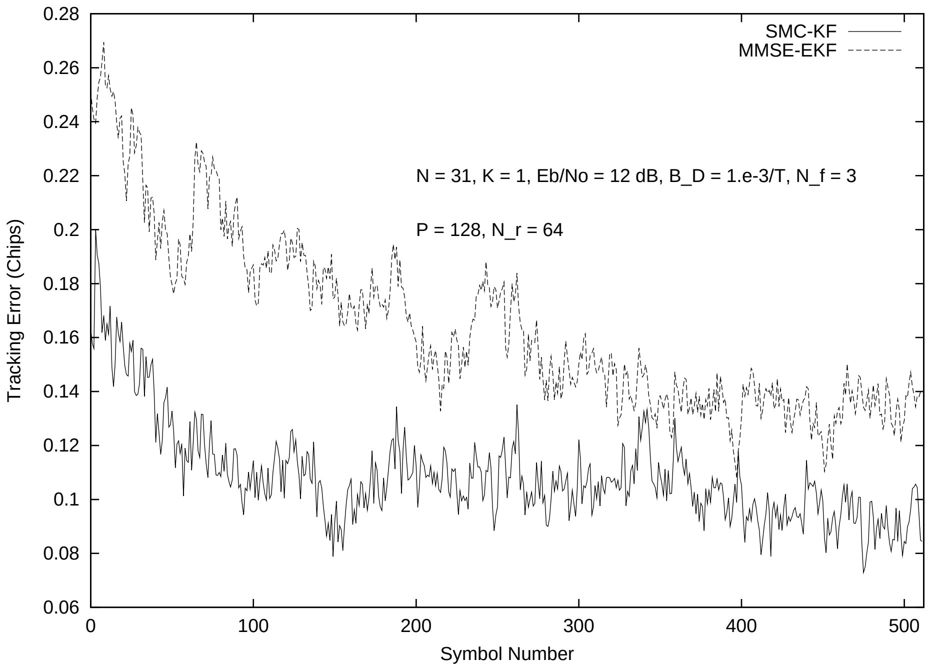

Fig. 5. Delay estimation error for single-user case, averaged over $N_r = 64$ runs

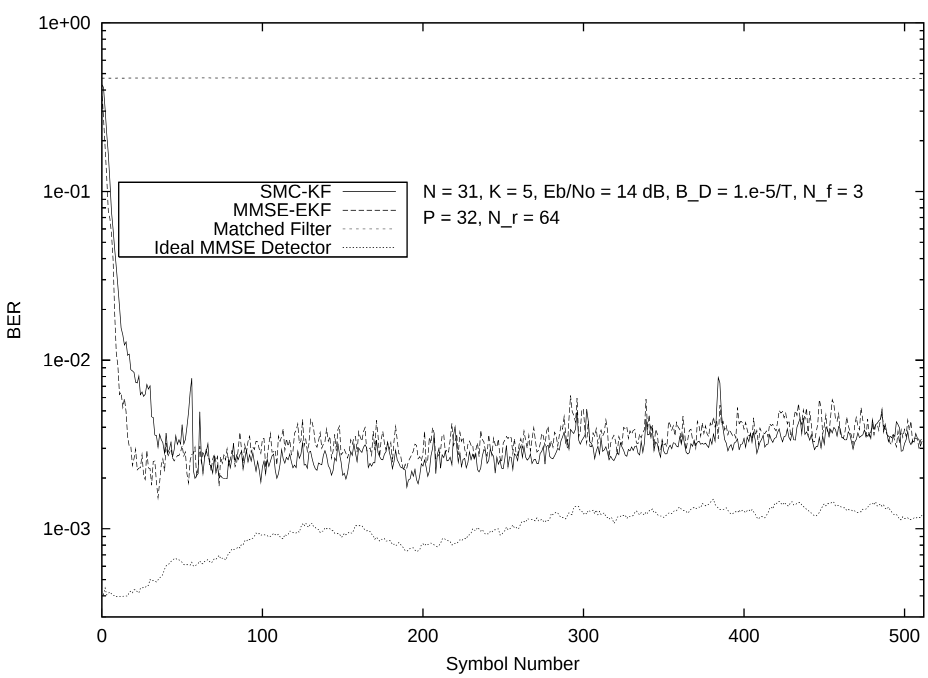

Fig. 6. Weak-user bit-error rate for $K = 5$ user case, averaged over $N_r = 64$ runs, $J/S = 10$ dB. Doppler spread $B_D = 1 \times 10^{-5}/T$.

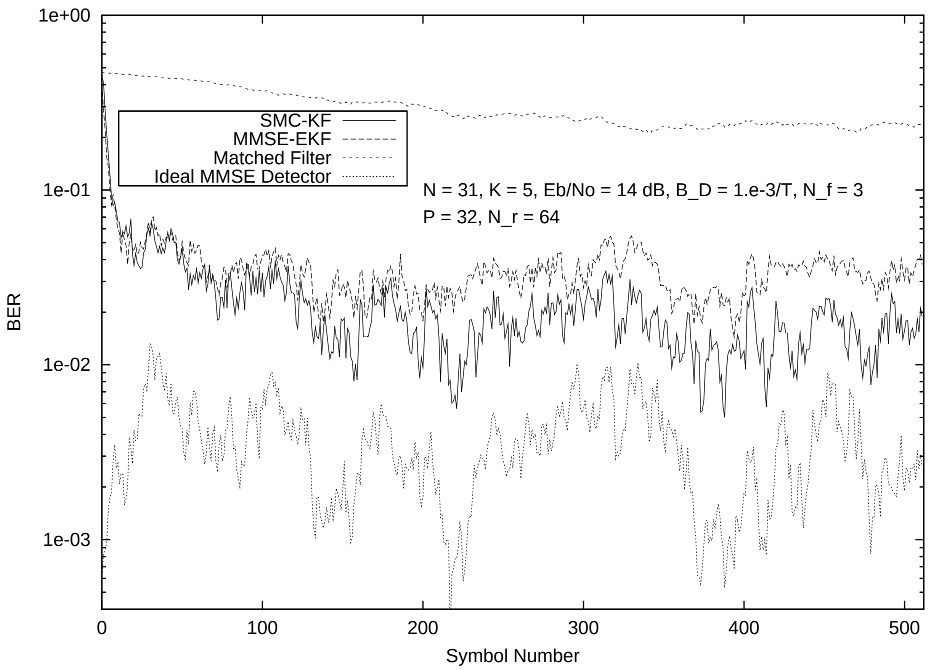

Fig. 7. Weak-user bit-error rate for $K = 5$ user case, averaged over $N_r = 64$ runs, $J/S = 10$ dB. Doppler spread $B_D = 1 \times 10^{-3}/T$.

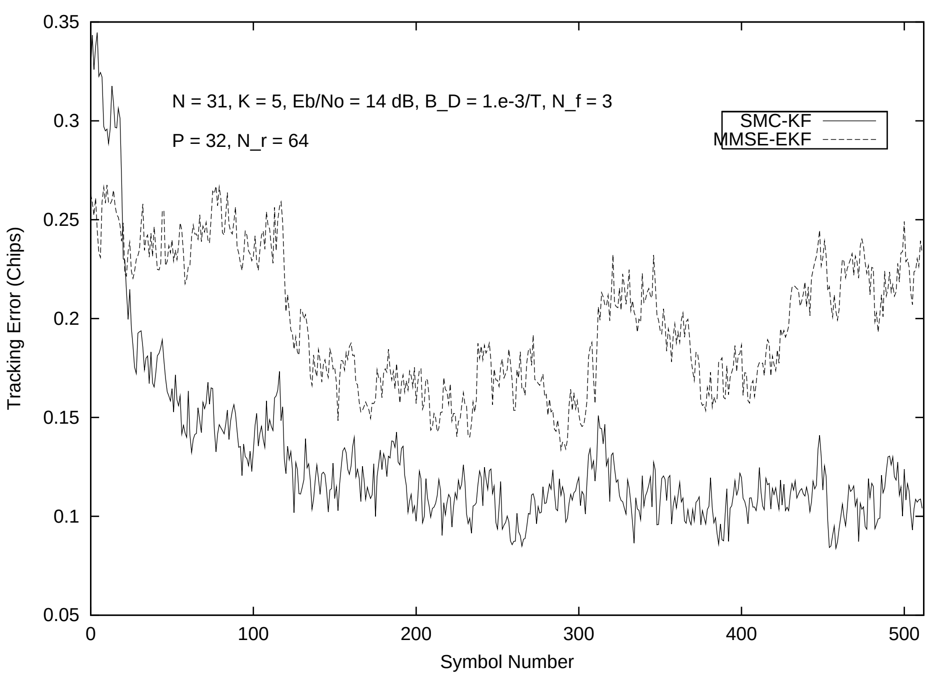

Fig. 8. Weak-user delay estimation error for $K = 5$ user case, averaged over $N_r = 64$ runs, $J/S = 10$ dB. Doppler spread $B_D = 1 \times 10^{-3}/T$

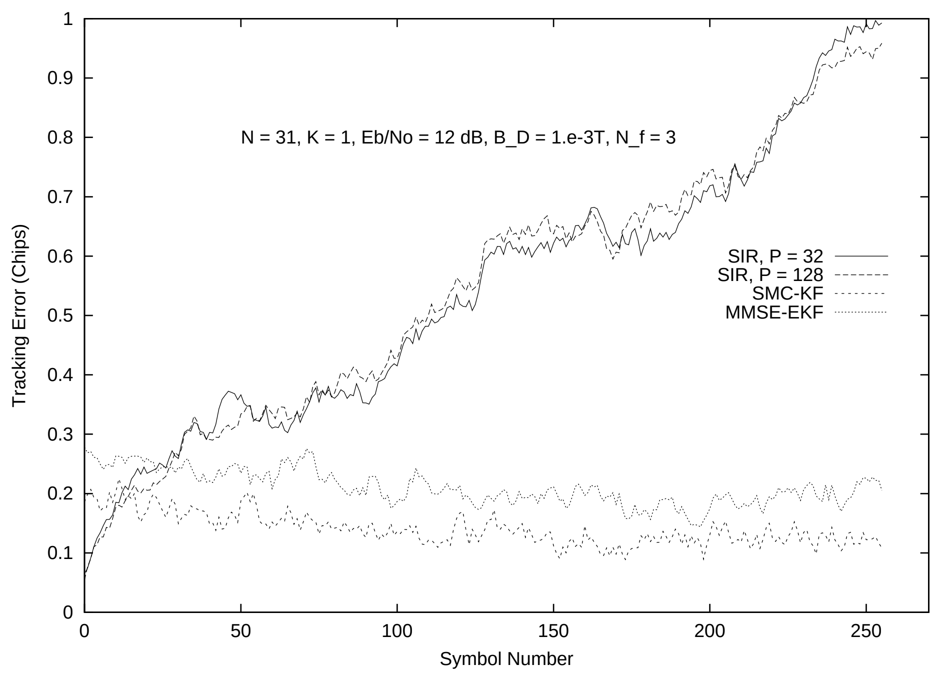

Fig. 9. Comparison of SIR with exact weight update, versus SMC-KF and MMSE-EKF for $K = 1$ user, averaged over $N_r = 32$ runs. Doppler spread $B_D = 1 \times 10^{-3}/T$.