A New Alternative Method for Checking Total Stations

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Abstract: Today, just by activating a button on the display panel of a total station, one can measure horizontal and vertical angles or calculate distances and coordinates of points. For this reason, it has become necessary to improve the check of a total station in order to respond to its contemporary way of use. The aim of this research is to create a new method for checking total stations using only one check—field, in a different way of the today's standards. The basic principle, of the comparison of an instrument's measurements to those of another instrument with higher accuracy, is implemented into a mathematical model such as the adjustment of an indoor geodetic network. Thus, the check of the proper function of a total station is applied. An a-priori calculation of σ_o of the network's adjustment can be calculated by modeling the uncertainties using statistical methods as the random number generators by means of the Monte Carlo method. The prospective σ_o can then be compared to the a-posteriori σ_o provided by the adjustment of the measurements which are carried out by using the total station under check.

Key words: Single control network, geodetic network, Monte Carlo techniques.

Nomenclature

TS_R	A first order, reference total station	
TS_{ch}	The total station under check	
$X_{t_R}, Y_{t_R}, Z_{t_R}$	The reference coordinates of each point t of the network provided by the total networks' adjustment using the TS_R measurements ($t = 1$ to 6)	$\sigma_{O(simH)}, \sigma_{O(simV)}, \sigma_{O(simD)}$
$\sigma_{X_{t_R}}, \sigma_{Y_{t_R}}, \sigma_{Z_{t_R}}$	The standard error of the reference coordinates $X_{t_R}, Y_{t_R}, Z_{t_R}$	The a-posteriori σ_o of each trial Adj_H, Adj_V, Adj_D correspondently of the network provided by $n = 1$ to N Monte Carlo simulations
H_{l_1}	The horizontal angle between three points of the network	The mean a-posteriori σ_o of the N -trials Adj_H, Adj_V, Adj_D calculated by Eq. (8), correspondently
V_{l_2}	The vertical angle between two points of the network	The uncertainty of $\sigma_{O(simH)}, \sigma_{O(simV)}, \sigma_{O(simD)}$ correspondently
D_{l_3}	The distance between two points of the network	The a-posteriori σ_o of the Adj_H, Adj_V, Adj_D by using correspondently the measurements of the TS_{ch}
Adj_R	The total reference adjustment of the network using the measurements of the TS_R	σ_{Dir} The nominal uncertainty of the TS_{ch} for a single direction
Adj_H, Adj_V, Adj_D	The adjustment of the network using only the horizontal angles, the vertical angles, the distances correspondently	σ_H The nominal uncertainty of the TS_{ch} for a horizontal angle measurement
δl_H	The matrix ($1 \times l_1$). The elements of the matrix are formed as the difference between the calculated horizontal angles by using the reference coordinates and the l_1 measured horizontal angles by the TS_{ch}	σ_{comp} The nominal uncertainty of the compensator of the TS_{ch}
δl_V	The matrix ($1 \times l_2$). The elements of the matrix are formed as the difference between the calculated vertical angles by using the reference coordinates and the l_2 measured vertical angles by the TS_{ch}	σ_V The nominal uncertainty of the TS_{ch} for a vertical angle measurement
δl_D	The matrix ($1 \times l_3$). The elements of the matrix are formed as the difference between the calculated distances by using the reference coordinates and the l_3 measured distances by the TS_{ch}	σ_D The nominal uncertainty of the TS_{ch} for the distance measurement
		$c_{H_{l_1}}, c_{V_{l_2}}, c_{D_{l_3}}$ The calculated uncertainty of each $H_{l_1}, V_{l_2}, D_{l_3}$ of the network correspondently by using the coordinates X_{t_R}, Y_{t_R} and Z_{t_R} taking into consideration their standard errors and their covariance's
		n The number of the trials (1 to $N = 10^7$)
		r The freedom degree of the model
		U The residual matrix
		v The residual matrix

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1. Introduction

The International Standards 3 and 4 of ISO (International Organization for Standardizations) 17123 series describe the procedures for checking the proper function of the theodolites (for horizontal directions and zenith angles) and EDMs (electro-optical distance meters), respectively. These standards are also used for checking total stations. Three different check-field procedures are used and separate calculations for each type of measurement are required [1, 2].

This special management of the total station's measuring values does not correspond to its contemporary way of function. In addition, this way of checking is extremely time consuming.

The Standard ISO 17123-5 [3] which refers exclusively to total stations does not provide information on the proper function of the total station but only the uncertainty of the generated coordinates which were calculated by using the specific auxiliary equipment each time (tripods, prisms, etc.) [4].

The proposed methodology uses one only check field, as well as a standardized calculation procedure philosophy.

The check is being performed by using the simultaneous measurements (angles and distances). Three separate adjustments Adj_H , Adj_V and Adj_D of a random indoor geodetic network are carried out by using the least square method considering one point and one azimuth fixed. The measurements used for each adjustment are considered of equal weight.

Thus the independent evaluation of the standard error σ_o for horizontal angles, vertical angles and distances provided by the TS_{ch} is determined separately, avoiding the interaction between them in a total adjustment.

By comparing the results of each adjustment to those theoretically calculated for this network according to the specifications of the TS_{ch} and the network's sensitivity, a reliable answer can be obtained for the total station's proper function.

The method for checking total stations, which is presented in this paper, provides: (1) a total check by using only an indoor 3D geodetic network and the simultaneous measurement of horizontal angles, vertical angles and distances; (2) easy calculation of the network's sensitivity by using a TS_R or a laser tracker; (3) automation, convenience and fast execution of the procedure [5].

2. Method Description

A random 3D/control network of six points is established indoors in a laboratory hall. All the network's elements angles (horizontal and vertical) and distances are measured by using a first order total station TS_R . The reference coordinates of the network's points $X_{t_R}, Y_{t_R}, Z_{t_R}$ ($t = 1$ to 6), their standard errors $\sigma_{X_{t_R}}, \sigma_{Y_{t_R}}, \sigma_{Z_{t_R}}$ and their covariances are calculated by a total network's adjustment Adj_R . It is worth noting that alternatively a laser tracker could be used for the $X_{t_R}, Y_{t_R}, Z_{t_R}$ acquisition.

Continuously, three separate simulation adjustments, Adj_H , Adj_V and Adj_D of the network, are carried out by using the same mathematical model and the Monte Carlo tool. As input data to the Monte Carlo simulations, the calculated expected uncertainties $C_{H_{t_1}}, C_{V_{t_2}}$ and $C_{D_{t_3}}$ are used. These uncertainties, which present the sensitivity of the network, are calculated by using the $X_{t_R}, Y_{t_R}, Z_{t_R}$, the $\sigma_{X_{t_R}}, \sigma_{Y_{t_R}}$ and $\sigma_{Z_{t_R}}$ and their covariance by using the generalized variance-covariance propagation error law. In addition, the nominal uncertainties σ_H, σ_V and σ_D of the TS_{ch} provided by the manufacture are taken into consideration.

The trials should be $N = 10^7$ according to the BIMP (Bureau International des Poids et Mesures) [6]. The $n = 1$ to 10^7 simulations of each one of Adj_H , Adj_V and Adj_D give n values of $\sigma_{o_n(simH)}, \sigma_{o_n(simV)}$ and $\sigma_{o_n(simD)}$, which follow the normal distribution. Their mean values $\sigma_{o(simH)}, \sigma_{o(simV)}$ and $\sigma_{o(simD)}$ as well as their standard errors $\sigma_{\sigma_{o(simH)}}, \sigma_{\sigma_{o(simV)}}$ and $\sigma_{\sigma_{o(simD)}}$ can be calculated for confidence level 95%. Namely

$\sigma_{o(simH)}$, $\sigma_{o(simV)}$ and $\sigma_{o(simD)}$ are the theoretical permissible or acceptable errors for the concrete network if measured by the TS_{ch} .

Then by using the TS_{ch} , the specific network is measured. Three separate adjustments Adj_H , Adj_V and Adj_D are carried out by using the TS_{ch} measurements and following closely the same mathematical models as in the previous step. Each adjustment provides the a-posteriori $\sigma_{o(chH)}$, $\sigma_{o(chV)}$ and $\sigma_{o(chD)}$. The comparison between the theoretical $\sigma_{o(simH)} \pm \sigma_{\sigma_{o(simH)}}$, $\sigma_{o(simV)} \pm \sigma_{\sigma_{o(simV)}}$, $\sigma_{o(simD)} \pm \sigma_{\sigma_{o(simD)}}$ and the succeeded $\sigma_{o(chH)}$, $\sigma_{o(chV)}$, $\sigma_{o(chD)}$ answer the question whether the TS_{ch} has a proper function or not. Fig. 1 presents the main stages of the methodology.

3. Mathematical Model

The mathematical models of the geodetic networks adjustment are based on the least square method.

The Adj_H , Adj_V and Adj_D performed separately by using the well known Eqs. (1-3) for horizontal angles, vertical angles and slope distances, respectively. These equations come out according to Taylor's derivation for no linear relations between the parameters:

$$\left(\frac{y_j^o - y_i^o}{(S_{ij}^o)^2} - \frac{y_k^o - y_i^o}{(S_{ik}^o)^2} \right) \cdot \delta x_i + \left(\frac{x_k^o - x_i^o}{(S_{ik}^o)^2} - \frac{x_j^o - x_i^o}{(S_{ij}^o)^2} \right) \cdot \delta y_i - \frac{y_j^o - y_i^o}{(S_{ij}^o)^2} \cdot \delta x_j + \frac{x_j^o - x_i^o}{(S_{ij}^o)^2} \cdot \delta y_j + \frac{y_k^o - y_i^o}{(S_{ik}^o)^2} \cdot \delta x_k - \frac{x_k^o - x_i^o}{(S_{ik}^o)^2} \cdot \delta y_k = \delta l + v \quad (1)$$

$$-\frac{(x_j^o - x_i^o) \cdot (z_j^o - z_i^o)}{(D_{ij}^o)^2 S_{ij}^o} \cdot \delta x_i - \frac{(y_j^o - y_i^o) \cdot (z_j^o - z_i^o)}{(D_{ij}^o)^2 S_{ij}^o} \cdot \delta y_i + \frac{S_{ij}^o}{(D_{ij}^o)^2} \cdot \delta z_i + \frac{(x_j^o - x_i^o) \cdot (z_j^o - z_i^o)}{(D_{ij}^o)^2 S_{ij}^o} \cdot \delta x_j + \frac{(y_j^o - y_i^o) \cdot (z_j^o - z_i^o)}{(D_{ij}^o)^2 S_{ij}^o} \cdot \delta y_j - \frac{S_{ij}^o}{(D_{ij}^o)^2} \cdot \delta z_j = \delta l + v \quad (2)$$

$$-\frac{x_j^o - x_i^o}{D_{ij}^o} \cdot \delta x_i - \frac{y_j^o - y_i^o}{D_{ij}^o} \cdot \delta y_i - \frac{z_j^o - z_i^o}{D_{ij}^o} \cdot \delta z_i + \frac{x_j^o - x_i^o}{D_{ij}^o} \cdot \delta x_j + \frac{y_j^o - y_i^o}{D_{ij}^o} \cdot \delta y_j + \frac{z_j^o - z_i^o}{D_{ij}^o} \cdot \delta z_j = \delta l + v \quad (3)$$

where,

D is the slope distance between two points i, j ;

S is the horizontal distance between two points i, j ;

i, j and k are points of the network;

x^o, y^o and z^o are the a-priori estimated coordinates of the network's points.

The procedure has the advantage of the use of non-permanent points. Every user can choose different positions for the network's points, but each formed network should be measured by both TS_R and TS_{ch} and should be used for the Monte Carlo estimation procedure.

Fig. 1 The basic steps of the method.

Every alternative network provides different sensitivity due to the different observation equations (different matrix A) and geometry. The δl matrix is formed according to the used mathematical model due to the geometry of the network. Moreover, with the on-the-fly realization of the network's points, the errors of centering, levelling and marking are avoided as permanent tribraches are placed and levelled once the measurements commence.

4. Monte Carlo Simulations

The Monte Carlo method takes advantage of the pseudo—random numbers from generators of any used distribution.

The application of the Monte Carlo method requires input quantities as well as their respective distributions in order to give the output quantities according to the mathematical model which is being simulated [7].

For each trial of the procedure, one output quantity is produced. After N repetitions, it is feasible to extract the average value of the output quantity as well as its uncertainty.

The number of the trials which should be performed is significant and affects the final results. For this reason, the proposals of the guideline “Propagation of the Distributions Using a Monte Carlo Method” [6], for the suitable number of repetitions as described in Section 7.2 of the JCGM (Joint Committee for Guides in Metrology), must be followed. This guide was created to supplement the guide of the expression of the uncertainty in measurement [6, 7].

The general guideline proposes a number of repetitions of about 10^7 which is often expected to give a 95% confidence level for the output quantity in a way that this interval has to be right for one or two significant decimal digits of the output quantity.

Consequently, the Monte Carlo method, by using random values for the input quantities in accordance with their distributions (i.e., their mean value and their standard error), can calculate the output quantity and its standard error objectively. As long as the calculation is

derived from N repetitions, where N is equal to 10^7 , practically infinite, the true value of the output quantity can be attained.

The execution of these experiments needs the huge processing power that computers have today, as the simulations of the specific mathematical models and the calculations of the expected uncertainty of the results are provided by thousands or even millions of trials [8].

For the implementation of the Monte Carlo method, in order to determine the $\sigma_{o(simH)}$, $\sigma_{o(simV)}$ and $\sigma_{o(simD)}$ through the Adj_H , Adj_V and Adj_D , respectively, three δl matrices are used as input quantities.

The definition of each element H_{l_1} , V_{l_2} , D_{l_3} in the matrices δl_H , δl_V and δl_D is the difference between the measured value by the TS_{ch} and the calculated value which comes out by the Adj_R (namely by using the $X_{t_R}, Y_{t_R}, Z_{t_R}$) for each elements.

If no error occurred in the measurements by the TS_{ch} , each element of the matrix δl should be 0. As the above hypothesis could not be true, the maximum permissible value for each element of the matrix δl is the total expected error, due to the nominal uncertainty of the TS_R and the uncertainty of each H_{l_1} , V_{l_2} and D_{l_3} given by the Adj_R ($c_{H_{l_1}}$, $c_{V_{l_2}}$, $c_{D_{l_3}}$). Thus, the elements of matrix δl , for each one of the three models Adj_H , Adj_V and Adj_D are given by the following equations respectively:

As the process is repeated N times, thus N matrices of δl_H , δl_V and δl_D , are created respectively.

$$Adj_H : \delta l_H = \begin{bmatrix} \sigma_{H_1} - c_{H_1} \\ \sigma_{H_2} - c_{H_2} \\ \dots \\ \sigma_{H_{l_1}} - c_{H_{l_1}} \end{bmatrix}$$

$$Adj_V : \delta l_V = \begin{bmatrix} \sigma_{V_1} - c_{V_1} \\ \sigma_{V_2} - c_{V_2} \\ \dots \\ \sigma_{V_{l_2}} - c_{V_{l_2}} \end{bmatrix} \quad (4)$$

$$Adj_D : \delta \mathbf{l}_D = \begin{bmatrix} \sigma_{D_1} - c_{D_1} \\ \sigma_{D_2} - c_{D_2} \\ \dots \\ \sigma_{D_{l_3}} - c_{D_{l_3}} \end{bmatrix}$$

Moreover for each input quantity, the distribution must be defined. In this case, the normal distribution is the proper one.

The elements $c_{H_{l_1}}$, $c_{V_{l_2}}$ and $c_{D_{l_3}}$ of the matrices $\delta \mathbf{l}_H$, $\delta \mathbf{l}_V$ and $\delta \mathbf{l}_D$ have arisen from the reference coordinates X_{t_R} , Y_{t_R} and Z_{t_R} , their standard errors $\sigma_{X_{t_R}}$, $\sigma_{Y_{t_R}}$ and $\sigma_{Z_{t_R}}$ and their covariance by following the generalized variance-covariance propagation error law.

The elements $(\sigma_{H_{l_1}}, \sigma_{V_{l_2}}, \sigma_{D_{l_3}})$ are calculated by the nominal uncertainty of the TS_{ch} , which are provided by the manufacturer's specifications.

As the manufacturers provide the uncertainty of the horizontal directions σ_{Dir} as "angular uncertainty" [1], the nominal uncertainty of a measured horizontal angle is calculated according to Eq. (5) as an horizontal angle comes out by the difference between two directions:

$$\sigma_H = \pm \sqrt{\sigma_{Dir}^2 + \sigma_{Dir}^2} \Rightarrow \sigma_H = \pm \sigma_{Dir} \cdot \sqrt{2} \quad (5)$$

Also, the vertical angular uncertainty is provided by the following equation:

$$\sigma_V = \pm \sqrt{\sigma_{Dir}^2 + \sigma_{comp}^2} \quad (6)$$

The output quantity which is derived from each trial of the Monte Carlo method, by each one of the Adj_H , Adj_V and Adj_D , is the mean square errors which are given by the equation:

$$\begin{aligned} \sigma_{o_n(simH)} &= \sqrt{\frac{\mathbf{U}^T \mathbf{U}}{r}} \\ \sigma_{o_n(simV)} &= \sqrt{\frac{\mathbf{U}^T \mathbf{U}}{r}} \\ \sigma_{o_n(simD)} &= \sqrt{\frac{\mathbf{U}^T \mathbf{U}}{r}} \end{aligned} \quad (7)$$

After N repetitions, (or "pseudo-adjustments" of the network), the mean value of the $\sigma_{o_n(simH)}$,

$\sigma_{o_n(simV)}$ and $\sigma_{o_n(simD)}$ is attained due to the theory of large numbers as follows [9]:

$$\begin{aligned} \sigma_{o(simH)} &= \frac{\sum_{n=1}^N \sigma_{o_n(simH)}}{N} \\ \sigma_{o(simV)} &= \frac{\sum_{n=1}^N \sigma_{o_n(simV)}}{N} \\ \sigma_{o(simD)} &= \frac{\sum_{n=1}^N \sigma_{o_n(simD)}}{N} \end{aligned} \quad (8)$$

Due to the fact that the number of the repetitions is very large (10^7), practically infinite, for each one of the $\sigma_{o(simH)}$, $\sigma_{o(simV)}$ and $\sigma_{o(simD)}$, it is considered to be the true values of the instrument permissible error for each measuring quantity H , V and D in these conditions.

Also $\sigma_{\sigma_{o(simH)}}$, $\sigma_{\sigma_{o(simV)}}$ and $\sigma_{\sigma_{o(simD)}}$ are calculated as follows:

$$\begin{aligned} \sigma_{\sigma_{o(simH)}} &= \sqrt{\frac{vv}{N-1}} \\ \sigma_{\sigma_{o(simV)}} &= \sqrt{\frac{vv}{N-1}} \\ \sigma_{\sigma_{o(simD)}} &= \sqrt{\frac{vv}{N-1}} \end{aligned} \quad (9)$$

Fig. 2 shows the sequence of the application of the Monte Carlo method used in the theoretical approach.

For each population of $\sigma_{o_n(simH)}$, $\sigma_{o_n(simV)}$ and $\sigma_{o_n(simD)}$, one histogram is arising. In these histograms (Fig. 3), according to Eqs. (8) and (9), the means $\sigma_{o(simH)}$, $\sigma_{o(simV)}$, $\sigma_{o(simD)}$ and their standard errors $\sigma_{\sigma_{o(simH)}}$, $\sigma_{\sigma_{o(simV)}}$, $\sigma_{\sigma_{o(simD)}}$ are depicted.

The Monte Carlo method's output quantities are following the normal distribution. This can be easily confirmed by observing the histograms, elsewhere a fit, such as of the criterion of KS (Kolmogorov-Smirnov) [10], for a sample must be applied. Most modern software comes with such specific algorithms.

Fig. 2 The Monte Carlo approach.

Fig. 3 The produced histograms from Monte Carlo.

5. Evaluation of the TS_{ch}

By using the TS_{ch} network's measurements and the corresponding mathematical models Adj_H , Adj_V and Adj_D , the $\sigma_{o(chH)}$, $\sigma_{o(chV)}$ and $\sigma_{o(chD)}$ were calculated. Eq. (10) should be satisfied if the TS_{ch} have a proper function:

$$\begin{aligned} \sigma_{o(simH)} - \sigma_{\sigma_{simH}} &\leq \sigma_{o(chH)} \leq \sigma_{o(simH)} + \sigma_{\sigma_{simH}} \\ \sigma_{o(simV)} - \sigma_{\sigma_{simV}} &\leq \sigma_{o(chV)} \leq \sigma_{o(simV)} + \sigma_{\sigma_{simV}} \\ \sigma_{o(simD)} - \sigma_{\sigma_{simD}} &\leq \sigma_{o(chD)} \leq \sigma_{o(simD)} + \sigma_{\sigma_{simD}} \end{aligned} \quad (10)$$

6. Remarks and Conclusions

The goal of this paper is to propose an alternative

methodology for checking total stations. This methodology focuses mainly on the calculation of the a-posteriori σ_o of a network's adjustment in relation to the nominal accuracy of the total station and the specific network sensitivity.

This proposal can be encoded as follows:

- (1) implementation of a control network in laboratory conditions;
- (2) obtain the reference coordinates of the network's points their standard errors and their covariance's by total 3D adjustment by using horizontal angles, zenith angles and distances which were measured by a first order total station (TS_R). Alternative, the reference

coordinates could be provided by a laser tracker if it is available;

(3) use the Monte Carlo method with the following input quantities:

- the previous reference coordinates and their uncertainties;
- the nominal uncertainties of the TS_{ch} ;
- the distributions of the above;

(4) thus N independent simulations of the a-posteriori σ_o for the specific control network are carried out. Bear in mind that separate simulations of adjustment occur each time involving only the horizontal angles, the zenith angles and the distances in order to avoid interactions;

(5) calculation of the a-posteriori σ_o for the three different mathematical models by using the TS_{ch} measurements (H_{l_1} , V_{l_2} , D_{l_3});

(6) evaluation of the TS_{ch} with separated statistical tests between the permissible a-posteriori σ_o given by the Monte Carlo method and the result a-posteriori σ_o respectively for each one of H , V and D .

The application of the proposed method requires that TS_{ch} should be one order lower than the TS_R . In case that a laser tracker is used for X_{t_R} , Y_{t_R} and Z_{t_R} acquisition, the $\sigma_{X_{t_R}}$, $\sigma_{Y_{t_R}}$ and $\sigma_{Z_{t_R}}$ are eliminated and give the checking opportunity for larger instrument's gamut. If the uncertainties of the reference coordinates are known, the max order of the TS_{ch} can be determined according to the network's geometry, the measured element and the mathematical model.

The advantages of the proposed methodology for checking the total stations are that they require only one check field for the measurements. Also, the required time for measuring the network and the procedure is reduced significantly comparing to the ISO 17123 field procedures. Moreover, the processing power of the high-end computers makes the use of the Monte Carlo method applicable and time efficient. Monte Carlo method can be used effortlessly on difficult simulation models, such as the adjustment of a geodetic network. It is clear that satisfies the modern

function of total stations, since horizontal angles, vertical angles and distances are measured and registered simultaneously. Moreover, the user can choose any arbitrary network for the application.

The outcomes of the procedure are both the achievable accuracy of the measured H , V and D by using the specific TS_{ch} and also the proper function of the TS_{ch} in the concrete measuring conditions separately for horizontal angles, vertical angles and distances.

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