
A study of cooperative advertising in a one-manufacturer two-retailer supply chain based on the multi-stage dynamic game theory

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Abstract: In this paper, the coordination of cooperative advertising decisions is analysed in a supply chain with one manufacturer and two retailers. Suppose the manufacturer invests in national advertising and one retailer invests in local advertising, the manufacturer agrees to share part of the local advertising cost with the retailer. Meanwhile, the other retailer refuses to take part in cooperative advertising. The manufacturer and retailer who put investment in cooperative advertising could choose cooperative or non-cooperative attitude, but the other retailer always chooses non-cooperative attitude. We select four decision variables including local advertising effort, two retailers' marginal profits, and price of product to discuss seven three-stage dynamic game models according to the parties' attitudes being cooperative or not. Seven game models, including one non-cooperative model, five partial cooperative models and one cooperative model, are investigated in detail based on a whole mathematical analysis. By comparing the proposed seven models, several interesting propositions are obtained and the corresponding interesting results are also acquired via these propositions.

Keywords: cooperative advertising; multi-stage dynamic model; supply chain; game theory.

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1 Introduction

As an important aspect in supply chain coordination, cooperative advertising plays an important role. Cooperative advertising is an interactive relationship between the manufacturer and the retailer in which the manufacturer and retailer invest in national advertising and local advertising, respectively. Meanwhile, the manufacturer pays a part of the retailer's local advertising expenditure. The manufacturer's national advertising is intended to influence potential consumers to consider its brand and help developing brand knowledge and preference. The retailer's local advertising is to stimulate consumers' buying

behaviour (Huang and Li, 2001). Therefore, the local one sometimes has greater effects on the buying decision of consumers, especially for some durable consumer items, which also explains manufacturer's encouragement for retailers to increase their local advertising effort.

Cooperative advertising has been employed in the USA since the early 1900s. Warner Brother, a maker of corsets, issued the first cooperative advertising agreement in 1903 (Yue et al., 2006). The use of cooperative advertising spread to grocery stores, hard good stores, the automobile industry and then to fashion (Somers et al., 1990; Green, 2000). In 1987, cooperative advertising cost expended by US

companies amounted to ten billion. About 20 billion was used in 1993 for cooperative advertising (Davis, 1994). About 25%–40% of the local advertising was based on cooperation (Sen, 1994). In 2002, approximately 60 to 65 billion was given to retailers by manufacturers to promote their products (Arnold, 2003). Nager (2006) reported that, in 2000, companies in the USA spent 15 billion, which shows four-fold increase compared with 900 million in 1970. From the data above mentioned, the application of cooperative advertising is very wide and important for companies. So, cooperative advertising is worth studying.

In recent years, cooperative advertising has been paid substantial attention. According to the investigation of Gerhard et al. (2014), there are 110 scientific articles, conference papers, and working papers of scientific institutions in English dealing with cooperative advertising, of the major part published in academic journals. Beger (1972) was the first researcher to use mathematical methodology to analyse cooperative advertising between a manufacturer and a retailer. Berger's research indicated that quantitative analysis may yield significantly better solutions to cooperative advertising decisions than those solutions previously used.

Several researchers have focused a supply chain of one manufacturer and one retailer (Huang and Li, 2001; Yue et al., 2006; Hutchins, 1953; Berger, 1972; Young and Greyser, 1983; Shantanu et al., 1995; Jorgensen et al., 2000, 2001; Li et al., 2002; Nager, 2006; Amrouche et al., 2008; Buratto and Zaccour, 2009; Yan, 2010; Javid and Hoseinpour, 2001; Ahmadi and Hoseinpour, 2012). They adopted different mathematical models to study the impact of cooperative advertising on all parties' profits. Shantanu et al. (1995) made an empirical investigation to see how they vary around the consumers and industrial products, as well as between convenience and non-convenience consumer products. They also studied how participation and accrual rates vary through the level of manufacturer concentration across product categories. Jorgensen et al. (2000) investigated a two-member channel in which a manufacturer and an exclusive retailer could make advertising costs that have both short and long term impacts on the retailer's sales. The manufacturer can also support both retailers' advertising efforts through a cooperative advertising program. They discussed four different attitudes that the manufacturer adopted. Huang et al. (2001) observed that manufacturer not only paid brand name investments, but also paid part of local advertisement costs incurred by retailers. Three cooperative advertising game models including two non-cooperative game models and one cooperative game model were used to discuss a vertical coop advertising within one manufacturer and one retailer. Susan et al. (2002) developed three strategic models for equilibrium marketing and investment effort levels for a manufacturer and a retailer in a two-member supply chain. They addressed that the impact of brand name investments, local advertising and sharing policy on cooperative advertising programs in these models. Yue et al. (2006) studied the coordination of cooperative advertising in a

manufacturer-retailer supply chain when the manufacturer offered price deductions to customers. Mir et al. (2011) studied vertical cooperative advertising along with pricing decisions in a supply chain under one manufacturer and one retailer where demand is influenced by both price and advertisement. Four game-theoretic models were established in order to study the effect of supply chain power balance on the optimal decisions of supply chain members. Zhang et al. (2013) proposed a dynamic cooperative advertising model for a manufacturer-retailer supply chain and analysed how the reference price effect would influence the decisions of all the channel members. Utilising differential game models, this paper formulated the optimal decisions of the manufacturer and the retailer in two different game scenarios: Stackelberg game and cooperative game.

Some other researchers have focused on a one manufacturer and two retailers' supply chain. Zhang and Zhong (2011) expanded the research to one manufacturer and two retailers. They introduced the concepts of product life cycle and spillover effect. Spillover effect means the local advertising that one retailer invests will affect the other retailer's sales. At different stages in the product life cycle, the influence of spillover effect on sales is different. This paper mainly discussed how the spillover effect influences the local advertising, sales and profit. Wang et al. (2011) considered cooperative advertising issues of a monopolistic manufacturer with competing duopolistic retailers. Four possible game structures, Stackelberg-Cournot, Stackelberg-Collusion, Nash-Cournot and Nash-Collusion, were discussed under each of four game structures, a decision model was developed for the three partners to design the optimal cooperative advertising policies. Ghadimi et al. (2013) considered the coordination of cooperative advertising when the relationships among the manufacturer and two retailers are symmetric and cooperate to increase their profits. Meanwhile, they used concepts of cooperative games to find fair allocation schemes for dividing the total profit of the grand coalition among the three members. Giri and Sharma (2014) studied the manufacturer's pricing strategy in a supply chain with a single manufacturer and two retailers. The manufacturer, as a Stackelberg leader, specifies wholesale prices to two retailers who face advertising dependent demand. Based on this game structure, two mathematical models are developed. At last, a numerical example is taken to illustrate the theoretical results derived.

In addition to study one kind mentioned above separately, there is one situation that two kinds are both researched. Amin and Farid (2014) considered pricing and cooperative advertising decisions in two-stage supply chain and developed a monopolistic retailer and duopolistic retailer's model. In these models, the manufacturer and the retailers play the Nash, Manufacturer-Stackelberg and cooperative games to make optimal pricing and cooperative advertising decisions.

Based on previous interviews, we can obtain some results. Firstly, most articles discussed cooperative

advertising in one-manufacturer one-retailer supply chain, few studies focused on one-manufacturer two-retailer supply chain. In real world, a manufacturer would deal with multiple retailers at the same time. Secondly, almost no papers research multi-stage game model on cooperative advertising. Thirdly, the existing papers researching duopolistic retailers seldom consider the influence of one retailer's local advertising effort on the other retailer's sales, especially in different stages of the product life cycle.

Therefore, in this paper, we intend to fill this gap and consider cooperative advertising in a one-manufacturer two-retailers supply chain. Suppose that the manufacturer invests in national advertising and one retailer invests in local advertising. Meanwhile, the manufacturer agrees to share part of the local advertising cost with the retailer. But the other retailer refuses to take part in cooperative advertising. The manufacturer and the retailer who put investment in cooperative advertising could choose cooperative or non-cooperative attitude, but the other retailer always chooses non-cooperative attitude. We select four decision variables including local advertising effort, two retailers' marginal profits and prices, and then develop seven three-stage dynamic game models according to the parties' attitude whether cooperative or not. Furthermore, we solve each model and obtain a equilibrium result. By comparing with these results, we can get some propositions.

The rest of the paper is organised as follows. In Section 2, the assumptions and model description are presented. Section 3 depicts decision order and each player's attitude. In this part, we suppose that decision-making order has three stages, and the attitude who participates in the cooperative advertising adopt could be cooperative or non-cooperative. But the player who does not invest on cooperative advertising always chooses non-cooperative attitude. Section 4 is the key part of this paper, here, we introduce seven dynamic game models, one non-cooperative model, five partial cooperative models and one cooperative model. Then, the solution procedure of each model is given. Section 5 gives the results of the whole paper by comparing the price, retailer's marginal profit, and local advertising level among the seven models, several interesting results are presented as Propositions 1 to 5. The last part is Section 6, which summarises the conclusions of this paper and gives some problems to be investigated further.

2 Assumptions and model description

To proceed, in the whole paper, we assume that

- a Suppose that in a two-echelon supply chain system there are a manufacturer M and two retailers R_1 and R_2 , who are independent decision makers. R_1 is willing to cooperate with manufacturer M and invest in local advertising, while R_2 refuses local advertising directly.
- b Let k be the manufacturer's investment level in national advertising, f_1 is the retailer R_1 's investment level in local advertising. As advertising cost is monotonic

increasing function with the advertising efforts level, we may consider the cost of local advertising as

$$c_1 = \frac{n}{2} f_1^2, \text{ the cost of national advertising as}$$

$$c_0 = \frac{r_0}{2} k^2, \text{ here, } r_1 > 1, r_0 > 1 \text{ are positive constants, named advertising difficulty coefficient.}$$

- c The fraction of the local advertising expenditure that the manufacturer agrees to share is t , $0 < t < 1$, this presents the manufacturer's cooperative advertising reimbursement policy.
- d The retailer R_1 's sales response volume function of the product is assumed to be affected mainly by the selling price, the retailer's local advertising level and the national advertising level. The main purpose of cooperate advertising between them is to increase short-term sales, therefore we assume that the retailer R_1 's sales response volume function is $q_1 = a(1+k) - bp + f_1 + \bar{\xi}$.

Except for factors like the selling price and the manufacturer's national advertising level, the retailer R_2 's sales response volume function of the product is also affected by R_1 's advertising level. So we assume $\beta \in (-1, 1)$ is spillover effect coefficient, meaning the influence of R_1 's advertising level. The retailer R_2 's sales response volume function is

$$q_2 = a(1+k) - bp + \beta f_1 + \bar{\xi}. \text{ } a > 0 \text{ is the market basis, } b > 1 \text{ is correlation index of price elasticity, } k, f_1 \text{ stands for the national advertising level and the retailer } R_1 \text{'s local advertising level. } \bar{\xi} \text{ is the environment uncertainly with mean zero.}$$

β is spillover effect coefficient. It means that if one retailer changes the investment of local advertising, that will influence the others. $\beta \in (0, 1)$ means the product is in rise period, the product's demand is very large, thus, whoever invests the local advertising must cause all retailers' sales increase. $\beta \in (-1, 0)$ means the product is in mature period, the product's demand is saturated, one retailer's local advertising must reduce other retailer's sales. The higher the value of $|\beta|$, the more change of others.

- e This study focuses on how manufacturer's pricing, retailer's local advertising investment and marginal profit influence manufacturer, retailers and the whole system efficiency. We assume k and t are constant.
- f The manufacturer M , retailer R_1 , and retailer R_2 's expectation profit functions, the sum profits of retailer R_1 and manufacturer M expectation profit function are the following:

$$\begin{aligned} \pi_m = & (p - n_1)[a(1+k) - bp + f_1] \\ & + (p - n_2)[a(1+k) - bp + \beta f_1] \\ & - \frac{r_0}{2} k^2 - t \frac{n}{2} f_1^2 \end{aligned} \quad (1)$$

$$\pi_1 = n_1[a(1+k) - bp + f_1] - (1-t)\frac{n_1}{2}f_1^2 \quad (2)$$

$$\pi_2 = n_2[a(1+k) - bp + \beta f_1] \quad (3)$$

$$\begin{aligned} \pi = \pi_1 + \pi_m = & p[a(1+k) - bp + f_1] \\ & + (p - n_2)[a(1+k) - bp + \beta f_1] \\ & - \frac{n_0}{2}k^2 - \frac{n_1}{2}f_1^2 \end{aligned} \quad (4)$$

3 Decision order and attitude

3.1 Decision order

In a large number of game models in real life, besides a static game in which all players make decisions at the same time, there exist a lot of dynamic games. It means all players make decisions one by one, while sometimes some players may make multi-stage decisions. In this paper, we study a three stage dynamic game model, its decision-making order has three stages as follows.

- stage 1 retailer R_1 decides the local advertising effort
- stage 2 retailer R_1 and retailer R_2 decide their marginal profit separately
- stage 3 the manufacturer M decides the price of product.

3.2 Attitude

In the past, studies mainly focused on non-cooperative model, it means all players maximise their own profit and each decision-making behaviour is based on the purpose of benefit maximisation. With the change of external competitive environment, the relationship of players in the market is becoming more and more complex. Based on the change of external environment, this paper mainly researches manufacturer and retailers' competition and cooperation relationship when they adopt different attitude, non-cooperation or cooperation. Competition and cooperation mean what attitude the player adopts. If the player adopts non-cooperative strategy, he will optimise the decision variable only considering his own profit. Otherwise, if the player chooses cooperative attitude, he will consider other's benefit. It is worth mentioning that the player whether chooses cooperation or not just happening on the cooperative advertising participants. If one player does not take part in the cooperative advertising, he must always choose non-cooperative attitude. In this paper, retailer R_2 does not invest in local advertising, so in all models, retailer R_2 always insists on optimising his own profit. When we research the competition and cooperation relationship between manufacturer and retailers, the different attitudes are follows.

3.2.1 Non-cooperation

Under the non-cooperation condition, manufacturer and retailers make decisions completely based on the principle

of maximising their own profits, and they do not take other players' profit into consideration.

3.2.2 Partial cooperation

In the competition and cooperation relationship between manufacturer and retailers, if one player makes decision only considering his own profit, it could make the total profits unable to achieve maximum. This is a well known phenomenon, called double marginalisation. The objective of cooperation is to build a mechanism to lead all players to make right decisions to maximise the total profits. Mostly, cooperation will make the profit increase. The partial cooperation refers to holistic optimisation only on part of the decision variables, not all. We divide the partial cooperation into three types, manufacturer coordination, retailer coordination, manufacturer and retailer both coordination.

- Type 1 is manufacturer coordination. It means only manufacturer adopts cooperative attitude, but retailers do not. In order to stimulate retailer to make promotion, manufacturer is willing to maximise the sum profits when he makes price decision. To explain what is sum profits, sum profits means the sum of manufacturer M 's profit and retailer R_1 's profit. Because retailer R_2 always chooses non-cooperative attitude, it will maximise the sum profits when manufacturer or retailer R_1 chooses cooperative attitude. The sum profit mentioned below refers to this meaning.
- Type 2 is retailer coordination. It refers to the retailer who is willing to invest local advertising make decision on marginal profit, local advertising effort, or both, he will maximise the sum profits. However, manufacturer M and retailer R_2 choose non-cooperative attitude.
- Type 3 is manufacturer and retailer R_1 both choose coordination. This concept is relative to total cooperation. In this condition, manufacturer optimises sum profits on price, but retailer R_1 optimises sum profits on marginal profit or local advertising.

3.2.3 Cooperation

Under the cooperation condition, manufacturer and retailer R_1 make decision completely based on the principle of maximise the sum profits, while retailer R_2 is keeping non-cooperative attitude.

4 Cooperative advertising game models and solutions

Considering all players' decision attitude, this paper introduces seven dynamic game models, one non-cooperative model, five partial cooperative models and one cooperative model. All models and solution procedure are follows.

4.1 Non-cooperative model

In this section, we consider a non-cooperative dynamic game model as follows.

Model A supply chain dynamic game model in non-cooperative condition.

Stage 1 retailer R_1 chooses the local advertising level f_1 to maximise his own profit, as $\max_{f_1} \pi_1$.

Stage 2 retailer R_1 chooses the marginal profit n_1 to maximise his own profit, as $\max_{n_1} \pi_1$. Retailer R_2 chooses the marginal profit n_2 to maximise his own profit, as $\max_{n_2} \pi_2$.

Stage 3 manufacturer decides the price p to maximise his own profit, as $\max_p \pi_m$.

In order to determine Stackelberg equilibrium, we first solve for the reaction function in the third stage of the game. The manufacturer M 's reaction function is $\max_p \pi_m$, where

$$\begin{aligned} \pi_m = & (p - n_1)[a(1 + k) - bp + f_1] \\ & + (p - n_2)[a(1 + k) - bp + \beta f_1] \\ & - \frac{r_0}{2} k^2 - t \frac{n_1}{2} f_1^2 \end{aligned}$$

So, the optimal value of the price is determined by setting the first derivative of p , with respect to p to be zero

$$\frac{\partial \pi_m}{\partial p} = 2a(1 + k) + f_1(1 + \beta) + b(n_1 + n_2) - 4bp = 0 \quad (5)$$

Then, we have

$$p = \frac{1}{4b} [2a(1 + k) + f_1(1 + \beta)] + \frac{n_1 + n_2}{4} \quad (6)$$

Next, the value of n_1 is determined by maximising retailer R_1 's profit and the value of n_2 is determined by maximising retailer R_2 's profit, above formula both subject to the constraint imposed by equation (6). So, Substituting equation (6) into equations (2) and (3),

$$\begin{aligned} \pi_1 = & n_1 \left[\frac{2a(1 + k) + f_1(3 - \beta)}{4} - \frac{b(n_1 + n_2)}{4} \right] \\ & - \frac{1}{2}(1 - t)n_1 f_1^2 \end{aligned} \quad (7)$$

$$\pi_2 = n_2 \left[\frac{2a(1 + k) + f_1(3\beta - 1)}{4} - \frac{b(n_1 + n_2)}{4} \right] \quad (8)$$

So, the optimal value of marginal profit n_1 is determined by setting the first derivative of π_1 with respect to n_1 to be zero. The optimal value of marginal profit n_2 is determined by setting the first derivative of π_2 with respect to n_2 to be zero.

$$\begin{aligned} \frac{\partial \pi_1}{\partial n_1} = & \frac{1}{4} [2a(1 + k) + f_1(3 - \beta) - b(2n_1 + n_2)] \\ = & 0 \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\partial \pi_2}{\partial n_2} = & \frac{1}{4} [2a(1 + k) + f_1(3\beta - 1) - b(n_1 + 2n_2)] \\ = & 0 \end{aligned} \quad (10)$$

Then, we have

$$n_1 = \frac{1}{3b} [2a(1 + k) + (7 - 5\beta)f_1] \quad (11)$$

$$n_2 = \frac{1}{3b} [2a(1 + k) + (7\beta - 5)f_1] \quad (12)$$

Finally, the value of f_1 is determined by maximising retailer R_1 's profit subject to the constraint imposed by equations (6), (11) and (12),

$$\pi_1 = \frac{1}{36b} [2a(1 + k) + (7 - 5\beta)f_1]^2 - \frac{1}{2}(1 - t)n_1 f_1^2 \quad (13)$$

So, the optimal value of local advertising level f_1 is determined by setting the first derivative of π_1 with respect to f_1 to be zero.

$$\frac{\partial \pi_1}{\partial f_1} = \frac{[2a(1 + k) + (7 - 5\beta)]}{18b} - (1 - t)n_1 f_1 = 0 \quad (14)$$

Solving equation (14) for f_1 and substituting it into equations (11) and (12)

$$f_1^{NC} = \frac{2a(1 + k)(7 - 5\beta)}{18b(1 - t)n_1 - (7 - 5\beta)^2} \quad (15)$$

$$n_1^{NC} = \frac{1}{3b} 2a(1 + k) \frac{18b(1 - t)n_1}{18b(1 - t)n_1 - (7 - 5\beta)^2} \quad (16)$$

$$n_2^{NC} = \frac{1}{3b} \left[2a(1 + k) + \frac{2a(1 + k)(7 - 5\beta)(7\beta - 5)}{18b(1 - t)n_1 - (7 - 5\beta)^2} \right] \quad (17)$$

The superscript *NC represents the game equilibrium of model A. Substituting equations (15) to (17) into equation (6), we have

$$p^{NC} = \frac{5}{12b} \left[2a(1 + k) + \frac{2a(1 + k)(1 + \beta)(7 - 5\beta)}{18b(1 - t)n_1 - (7 - 5\beta)^2} \right] \quad (18)$$

Substituting equations (15) to (18) into equations (1) to (4), we have

$$\pi_1^{NC} = \frac{[2a(1 + k)]^2(1 - t)n_1}{2[18b(1 - t)n_1 - (7 - 5\beta)^2]} \quad (19)$$

$$\pi_2^{NC} = \frac{[2a(1 + k)]^2[3b(1 - t)n_1 - 2(7 - 5\beta)(1 - \beta)]^2}{b[18b(1 - t)n_1 - (7 - 5\beta)^2]^2} \quad (20)$$

$$\pi_m^{NC} = \frac{[2a(1+k)]^2}{4b[18b(1-t)r_1 - (7-5\beta)^2]^2} \{3b(1-t)r_1[3b(1-t)r_1 - 5(1-\beta)(7-5\beta)] + [3b(1-t)r_1 - 3(1-\beta)(7-5\beta)][3b(1-t)r_1 - 2(1-\beta)(7-5\beta)]\} - \frac{r_0}{2}k^2 - \frac{t_1[2a(1+k)]^2(7-5\beta)^2}{2[18b(1-t)r_1 - (7-5\beta)^2]^2} \quad (21)$$

$$\pi_m^{NC} = \frac{[2a(1+k)]^2}{4b[18b(1-t)r_1 - (7-5\beta)^2]^2} \{15b(1-t)r_1[3b(1-t)r_1 - (1-\beta)(7-5\beta)] + [3b(1-t)r_1 - 3(1-\beta)(7-5\beta)][3b(1-t)r_1 - 2(1-\beta)(7-5\beta)]\} - \frac{r_0}{2}k^2 - \frac{n_1[2a(1+k)]^2(7-5\beta)^2}{2[18b(1-t)r_1 - (7-5\beta)^2]^2} \quad (22)$$

4.2 Partial cooperative model

In this section, we consider five partial cooperative dynamic game models as follows.

4.2.1 Manufacturer coordination

Model B Supply chain dynamic game model with manufacturer coordination

Stage 1 retailer R_1 chooses the local advertising level f_1 to maximise his own profit, as $\max_{f_1} \pi_1$.

Stage 2 retailer R_1 chooses the marginal profit n_1 to maximise his own profit, as $\max_{n_1} \pi_1$. Retailer R_2 chooses the marginal profit n_2 to maximise his own profit, as $\max_{n_2} \pi_2$.

Stage 3 manufacturer decides the price p to maximise the sum profits, as $\max_p \pi$.

Use backward induction, we first solve for the reaction function in the third stage of the game. Since π is a concave function of p , the optimal value of price is determined by setting the first derivative of π with respect to p to be zero.

$$\frac{\partial \pi}{\partial p} = [a(1+k) - bp + f_1] - bp + [a(1+k) - bp + \beta f_1] - b(p - n_2) = 0 \quad (23)$$

Then, we have

$$p = \frac{1}{4b}[2a(1+k) + f_1(1+\beta)] + \frac{n_2}{4} \quad (24)$$

Next, in second stage of the game, the value of n_1 is determined by maximising retailer R_1 's profit and the value of n_2 is determined by maximising retailer R_2 's profit, above formula both subject to the constraint imposed by

equation (24). So, Substituting equation (24) into equations (2) and (3),

$$\pi_1 = n_1 \left[\frac{2a(1+k) + f_1(3-\beta) - bn_2}{4} - \frac{1}{2}(1-t)\frac{r_1}{f_1^2} \right] \quad (25)$$

$$\pi_2 = n_2 \left[\frac{2a(1+k) + f_1(3\beta-1) - bn_2}{4} \right] \quad (26)$$

So, the optimal value of marginal profit n_1 is determined by setting the first derivative of π_1 with respect to n_1 to be zero. The optimal value of marginal profit n_2 is determined by setting the first derivative of π_2 with respect to n_2 to be zero.

$$\frac{\partial \pi_1}{\partial n_1} = \frac{1}{4}[2a(1+k) + f_1(3-\beta) - bn_2] \quad (27)$$

$$\frac{\partial \pi_2}{\partial n_2} = 14[2a(1+k) + f_1(3\beta-1) - 2bn_2] = 0 \quad (28)$$

Solving equation (28) for n_2 ,

$$n_2 = \frac{2a(1+k) + f_1(3\beta-1)}{2b} \quad (29)$$

Substituting equation (29) into equation (27),

$$\frac{\partial \pi_1}{\partial n_1} = \frac{2a(1+k) + f_1(7-5\beta)}{8} > 0 \quad (30)$$

Equation (30) illustrates that retailer R_1 hopes his marginal profit is bigger, because the more the marginal profit is, the more profits he has. Analysing the reason, due to the attitude that the manufacturer adopts is cooperative, there is not constraint to retailer R_1 , so R_1 wants more marginal profits to obtain money. It is obviously that in this model just the manufacturer coordinate is unreasonable.

4.2.2 Retailer coordination

Model C supply chain dynamic game model with retailer R_1 coordinate on local advertising level

Stage 1 retailer R_1 chooses the local advertising level f_1 to maximise the sum profits, as $\max_{f_1} \pi$.

Stage 2 retailer R_1 chooses the marginal profit n_1 to maximise his own profit, as $\max_{n_1} \pi_1$. Retailer R_2 chooses the marginal profit n_2 to maximise his own profit, as $\max_{n_2} \pi_2$.

Stage 3 manufacturer decides the price p to maximise his own profit, as $\max_p \pi_m$.

It is obvious that the third and second stages of backward induction in model C are the same as model A, so substituting equations (6), (11) and (12) into equation (4), we have

$$\pi = \pi_1 + \pi_m = \frac{5}{144b} [2a(1+k) + f_1(7-5\beta)] + \frac{1}{144b} [2a(1+k) + f_1(1+\beta)] \quad (31)$$

So, the optimal value of local advertising level f_1 is determined by setting the first derivative of π with respect to f_1 to be zero.

$$\frac{\partial \pi}{\partial f_1} = \frac{1}{12b} [2a(1+k)(5-3\beta) + f_1(-31\beta^2 + 50\beta - 15)] - \eta_1 f_1 = 0 \quad (32)$$

Solving equation (32) for f_1 and substituting it into equations (11) and (12)

$$f_1^{PC_1} = \frac{2a(1+k)(5-3\beta)}{31\beta^2 - 50\beta + 15 + 12b\eta_1} \quad (33)$$

$$n_1^{PC_1} = \frac{4a(1+k)[23\beta^2 - 48\beta + 25 + 6b\eta_1]}{3b[31\beta^2 - 50\beta + 15 + 12b\eta_1]} \quad (34)$$

$$n_2^{PC_1} = \frac{4a(1+k)[6b\eta_1 - 5(1-\beta^2)]}{3b[31\beta^2 - 50\beta + 15 + 12b\eta_1]} \quad (35)$$

The superscript *PC_1 represents the game equilibrium of model C. Substituting equations (33) to (35) into equation (6), we have

$$p^{PC_1} = \frac{10a(1+k)[3b\eta_1 - (7\beta-5)(1-\beta)]}{3b[31\beta^2 - 50\beta + 15 + 12b\eta_1]} \quad (36)$$

Substituting equations (33) to (36) into equations (1) to (4), we have

$$\pi_1^{PC_1} = [2a(1+k)]^2 \frac{2[23\beta^2 - 48\beta + 25 + 6b\eta_1]^2 - 9b(1-\beta)\eta_1(5-3\beta)^2}{18b[31\beta^2 - 50\beta + 15 + 12b\eta_1]^2} \quad (37)$$

$$\pi_2^{PC_1} = [2a(1+k)]^2 \frac{[6b\eta_1 - 5(1-\beta^2)]^2}{9b[31\beta^2 - 50\beta + 15 + 12b\eta_1]^2} \quad (38)$$

$$\begin{aligned} \pi_m^{PC_1} = & \{ [3b\eta_1 + 5(7-5\beta)(1-\beta)] \\ & [5\beta^2 - 5 + 6b\eta_1(6-5t)] \\ & + 5[3b\eta_1 - (7\beta-5)(1-\beta)] \\ & [23\beta^2 - 48\beta + 25 + 6b\eta_1(6-5t)] \\ & - 2[23\beta^2 - 48\beta + 25 + 6b\eta_1]^2 \\ & - 9b\eta_1(5-3\beta)^2 \} \\ & \times \frac{2[a(1+k)]^2}{9b(31\beta^2 - 50\beta + 15 + 12b\eta_1)^2} \\ & - \frac{r_0}{2} k^2 \end{aligned} \quad (39)$$

$$\begin{aligned} \pi^{PC_1} = & \pi_1 + \pi_m \\ = & \{ [3b\eta_1 + 5(7-5\beta)(1-\beta)] \\ & [5\beta^2 - 5 + 6b\eta_1(6-5t)] \\ & \times \frac{2[a(1+k)]^2}{9b(31\beta^2 - 50\beta + 15 + 12b\eta_1)^2} \\ & + 5[3b\eta_1 - (7\beta-5)(1-\beta)] \\ & [23\beta^2 - 48\beta + 25 + 6b\eta_1(6-5t)] \\ & \times \frac{2[a(1+k)]^2}{9b(31\beta^2 - 50\beta + 15 + 12b\eta_1)^2} \\ & - \frac{r_0}{2} k^2 - \frac{\eta_1 [2a(1+k)]^2 (5-3\beta)^2}{2(31\beta^2 - 50\beta + 15 + 12b\eta_1)^2} \end{aligned} \quad (40)$$

Model D supply chain dynamic game model with retailer R_1 coordination on marginal profit

Stage 1 retailer R_1 chooses the local advertising level f_1 to maximise his own profit, as $\max_{f_1} \pi_1$.

Stage 2 retailer R_1 chooses the marginal profit n_1 to maximise the sum profits, as $\max_{n_1} \pi$. Retailer R_2 chooses the marginal profit n_2 to maximise his own profit, as $\max_{n_2} \pi_2$.

Stage 3 manufacturer decides the price p to maximise his own profit, as $\max_p \pi_m$.

It is obviously that the third stage of backward induction in model D is the same as model A. Substituting equation (6) into equations (3), (4), we have

$$\begin{aligned} \pi = & \frac{1}{16b} [2a(1+k) + f_1(3-\beta) - b(n_1 + n_2)] \\ & [2a(1+k) + f_1(1+\beta) + b(n_1 + n_2)] + \frac{1}{16b} \\ & [2a(1+k) + f_1(3\beta-1) - b(n_1 + n_2)] \\ & [2a(1+k) + f_1(1+\beta) + b(n_1 - 3n_2)] \\ & - \frac{r_0}{2} k^2 - \frac{\eta_1}{2} f_1^2 \\ \pi_2 = & n_2 \left[\frac{2a(1+k) + f_1(3\beta-1) - b(n_1 + n_2)}{4} - \frac{b(n_1 + n_2)}{4} \right] \end{aligned} \quad (41) \quad (42)$$

So, the optimal value of marginal profit n_1 is determined by setting the first derivative of π with respect to n_1 to be zero. The optimal value of marginal profit n_2 is determined by setting the first derivative of π_2 with respect to n_2 to be zero.

$$\frac{\partial \pi}{\partial n_1} = 4bn_1 = 0 \quad (43)$$

$$\begin{aligned} \frac{\partial \pi_2}{\partial n_2} = & \frac{1}{4} [2a(1+k) + f_1(3\beta-1) - b(n_1 + 2n_2)] \\ = & 0 \end{aligned} \quad (44)$$

Then, we have

$$n_1 = 0 \quad (45)$$

$$n_2 = \frac{2a(1+k) + f_1(3\beta - 1)}{2b} \quad (46)$$

Finally, the value of f_1 is determined by maximising retailer R_1 's profit subject to the constraint imposed by equations (6), (45) and (46),

$$\pi_1 = -\frac{1}{2}(1-t)r_1f_1^2 \quad (47)$$

So, the optimal value of local advertising level f_1 is determined by setting the first derivative of π_1 with respect to f_1 to be zero.

$$\frac{\partial \pi_1}{\partial f_1} = -(1-t)r_1f_1 = 0 \quad (48)$$

Solving equation (48) for f_1 and substituting it into equations (45), (46)

$$f_1^{PC_2} = 0 \quad (49)$$

$$n_1^{PC_2} = 0 \quad (50)$$

$$n_2^{PC_2} = \frac{a(1+k)}{b} \quad (51)$$

The superscript *PC_2 represents the game equilibrium of model D. Substituting equations (49) to (51) into equation (6), we have

$$p^{PC_2} = \frac{3a(1+k)}{4b} \quad (52)$$

Substituting equations (49) to (52) into equations (1) to (4), we have

$$\pi_1^{PC_2} = 0 \quad (53)$$

$$\pi_2^{PC_2} = \frac{[2a(1+k)]^2}{16b} \quad (54)$$

$$\pi_m^{PC_2} = \frac{[2a(1+k)]^2}{16b} - \frac{r_0}{2}k^2 \quad (55)$$

$$\pi^{PC_2} = \pi_1 + \pi_m = \frac{[2a(1+k)]^2}{16b} - \frac{r_0}{2}k^2 \quad (56)$$

Model E supply chain dynamic game model with retailer R_1 coordination on local advertising and marginal profit

Stage 1 retailer R_1 chooses the local advertising level f_1 to maximise the sum profits, as $\max_{f_1} \pi$.

Stage 2 retailer R_1 chooses the marginal profit n_1 to maximise the sum profits, as $\max_{n_1} \pi$. Retailer R_2 chooses the marginal profit n_2 to maximise his own profit, as $\max_{n_2} \pi_2$.

Stage 3 manufacturer decides the price p to maximise his own profit, as $\max_p \pi_m$.

It is obviously that the third and second stages of backward induction in model E is the same as model D, substituting equations (6), (45), (46) into equation (4), we have

$$\begin{aligned} \pi &= \pi_1 + \pi_m \\ &= \frac{1}{64b} [2a(1+k) + f_1(7-5\beta)] \\ &\quad [6a(1+k) + f_1(1+5\beta)] \\ &\quad + \frac{1}{64b} [2a(1+k) + f_1(3\beta-1)] \\ &\quad [2a(1+k) + f_1(7\beta-5)] \\ &\quad - \frac{r_0}{2}k^2 - \frac{r_1}{2}f_1^2 \end{aligned} \quad (57)$$

Finally, the optimal value of f_1 is determined by maximising the sum profits, so setting the first derivative of π with respect to f_1 to be zero.

$$\frac{\partial \pi}{\partial f_1} = \frac{1}{8b} [4a(1+k) + f_1(3-\beta)(1+\beta)] - r_1f_1 = 0 \quad (58)$$

Solving equation (58) for f_1 and substituting it into equations (45) and (46),

$$f_1^{PC_3} = \frac{4a(1+k)}{8br_1 - (3-\beta)(1+\beta)} \quad (59)$$

$$n_1^{PC_3} = 0 \quad (60)$$

$$n_2^{PC_3} = \frac{2a(1+k)[8br_1 - (5+\beta)(1-\beta)]}{2b[8br_1 - (3-\beta)(1+\beta)]} \quad (61)$$

The superscript *PC_3 represents the game equilibrium of model E. Substituting equations (59) to (61) into equation (6), we have

$$p^{PC_3} = \frac{2a(1+k)[24br_1 - (3\beta+7)(1-\beta)]}{8b[8br_1 - (3-\beta)(1+\beta)]} \quad (62)$$

Substituting equations (59) to (62) into equations (1) to (4), we have

$$\pi_1^{PC_3} = -\frac{2(1-t)r_1[2a(1+k)]^2}{[8br_1 - (3-\beta)(1+\beta)]^2} \quad (63)$$

$$\pi_2^{PC_3} = \frac{[2a(1+k)]^2[8br_1 - (5+\beta)(1-\beta)]^2}{16b[8br_1 - (3-\beta)(1+\beta)]^2} \quad (64)$$

$$\begin{aligned} \pi_m^{PC_3} &= \frac{1}{16b} [2a(1+k)]^2 \\ &\quad \left\{ \frac{4[16br_1 - (3-\beta)(1+\beta)]}{[8br_1 - (3-\beta)(1+\beta)]^2} + 1 \right\} \\ &\quad - \frac{r_0}{2}k^2 - \frac{2tr_1[2a(1+k)]^2}{[8br_1 - (3-\beta)(1+\beta)]^2} \end{aligned} \quad (65)$$

$$\begin{aligned} \pi^{PC_3} &= \pi_1 + \pi_m \\ &= \frac{1}{16b} [2a(1+k)]^2 \\ &\quad \left\{ \frac{4[16bn_1 - (3-\beta)(1+\beta)]}{[8bn_1 - (3-\beta)(1+\beta)]^2} + 1 \right\} \\ &\quad - \frac{r_0}{2} k^2 - \frac{2n_1[2a(1+k)]^2}{[8bn_1 - (3-\beta)(1+\beta)]^2} \end{aligned} \quad (66)$$

4.2.3 Manufacturer and retailer R_1 both coordinate with partial decision variable

Model F supply chain dynamic game model with retailer R_1 coordinate with marginal profit and manufacturer coordinate with price of product

Stage 1 retailer R_1 chooses the local advertising level f_1 to maximise his own profit, as $\max_{f_1} \pi_1$.

Stage 2: retailer R_1 chooses the marginal profit n_1 to maximise the sum profits, as $\max_{n_1} \pi$. Retailer R_2 chooses the marginal profit n_2 to maximise his own profit, as $\max_{n_2} \pi_2$.

Stage 3 manufacturer decides the price p to maximise the sum profits, as $\max_p \pi$.

The third stage of backward induction in model F is the same as model B, substituting equation (24) into equations (4) and (3), we have

$$\begin{aligned} \pi &= \frac{1}{16b} [2a(1+k) + f_1(1+\beta) + bn_2] \\ &\quad [2a(1+k) + f_1(3-\beta) - bn_2] \\ &\quad + \frac{1}{16b} [2a(1+k) + f_1(1+\beta) - 3bn_2] \\ &\quad [2a(1+k) + f_1(3\beta-1) - bn_2] \\ &\quad - \frac{r_0}{2} k^2 - \frac{n_1}{2} f_1^2 \\ \pi_1 &= n_2 \left[\frac{2a(1+k) + f_1(3\beta-1) - bn_2}{4} - \frac{bn_2}{4} \right] \end{aligned} \quad (67)$$

$$\pi_1 = n_2 \left[\frac{2a(1+k) + f_1(3\beta-1) - bn_2}{4} - \frac{bn_2}{4} \right] \quad (68)$$

Next, in second stage, the optimal value of n_1 is determined by maximise the sum profits. The optimal value of n_2 is determined by maximising the retailer R_2 's own profit. So,

$$\frac{\partial \pi}{\partial n_1} = 0 \quad (69)$$

$$\frac{\partial \pi_2}{\partial n_2} = \frac{1}{4} [2a(1+k) + f_1(3\beta-1) - 2bn_2] = 0 \quad (70)$$

Then, we have n_1 is uncertainly value.

$$n_2 = \frac{2a(1+k) + f_1(3\beta-1)}{2b} \quad (71)$$

Substituting equation (71) into equation (2),

$$\pi_1 = n_1 \frac{2a(1+k) + f_1(7-5\beta)}{8} - \frac{1}{2} (1-t)n_1 f_1^2 \quad (72)$$

Finally, the optimal value of f_1 is determined by maximising his own profit, so setting the first derivative of π with respect to f_1 to be zero.

$$\frac{\partial \pi_1}{\partial f_1} = \frac{(7-5\beta)n_1}{8} - (1-t)n_1 f_1 = 0 \quad (73)$$

Then, we have

$$f_1 = \frac{(7-5\beta)n_1}{8(1-t)n_1} \quad (74)$$

According to equation (74), retailer R_1 's local advertising level is related to the marginal profit n_1 , because n_1 is an uncertainly value, the level of local advertising is not sure. We do not discuss this situation further more.

4.3 Cooperation model

In this section, we consider a cooperative dynamic game model as follows.

Model G supply chain dynamic game model in cooperative condition

Stage 1 retailer R_1 chooses the local advertising level f_1 to maximise the sum profits, as $\max_{f_1} \pi$.

Stage 2 retailer R_1 chooses the marginal profit n_1 to maximise the sum profits, as $\max_{n_1} \pi$. Retailer R_2 chooses the marginal profit n_2 to maximise his own profit, as $\max_{n_2} \pi_2$.

Stage 3 manufacturer decides the price p to maximise the sum profits, as $\max_p \pi$.

It is obviously that the third and second stages in model G is the same as model F, substituting equations (24), (71) into equation (4), we have

$$\begin{aligned} \pi &= \frac{1}{64b} [6a(1+k) + f_1(1+5\beta)] \\ &\quad [2a(1+k) + f_1(7-5\beta)] \\ &\quad - \frac{1}{64b} [2a(1+k) + f_1(7\beta-5)] \\ &\quad [2a(1+k) + f_1(3\beta-1)] \\ &\quad - \frac{r_0}{2} k^2 - \frac{n_1}{2} f_1^2 \end{aligned} \quad (75)$$

Finally, the optimal value of f_1 is determined by maximising the sum profits, so setting the first derivative of π_1 with respect to f_1 to be zero.

$$\begin{aligned} \frac{\partial \pi}{\partial f_1} &= \frac{1}{16b} [2a(1+k)(7-5\beta) - (23\beta_2 - 26\beta - 1)f_1] \\ &\quad - n_1 f_1 = 0 \end{aligned} \quad (76)$$

Solving equation (76) for f_1 and substituting it into equations (71), we have

$$f_1^{TC} = \frac{2a(1+k)(7-5\beta)}{16bn_1 + 23\beta_2 - 26\beta - 1} \quad (77)$$

n_1^{TC} is uncertainly value.

$$n_2^{TC} = \frac{16a(1+k)[2bn_1 + \beta_2 - 1]}{2b[16bn_1 + 23\beta_2 - 26\beta - 1]} \quad (78)$$

The superscript $*^{TC}$ represents the game equilibrium of model G.

Substituting equations (77), (78) into equation (24), we have

$$p^{TC} = \frac{2a(1+k)[12bn_1 + 11\beta_2 - 12\beta + 1]}{2b[16bn_1 + 23\beta_2 - 26\beta - 1]} \quad (79)$$

Substituting equations (77) to (79) into equations (1) to (4), we have

$$\pi_1^{TC} = \frac{2a(1+k)m_1[16bn_1 + 3(1-\beta)^2]}{8[16bn_1 + 23\beta^2 - 26\beta - 1]} - \frac{(1-t)\eta_1[2a(1+k)]^2(7-5\beta)^2}{2[16bn_1 + 23\beta^2 - 26\beta - 1]^2} \quad (80)$$

$$\pi_2^{TC} = \frac{8[2a(1+k)]^2(2bn_1 + \beta^2 - 1)^2}{2b[16bn_1 + 23\beta^2 - 26\beta - 1]^2} \quad (81)$$

$$\pi_m^{TC} = \left[\frac{2a(1+k)(12bn_1 + 11\beta^2 - 12\beta + 1)}{2b(16bn_1 + 23\beta^2 - 26\beta - 1)} - n_1 \right] \frac{2a(1+k)(2bn_1 + \beta^2 - 12\beta + 6)}{16bn_1 + 23\beta^2 - 26\beta - 1} + \frac{[2a(1+k)]^2(2bn_1 + \beta^2 - 1)}{2b[16bn_1 + 23\beta^2 - 26\beta - 1]^2} + \frac{[-4bn_1 + 3(1-\beta)(3-\beta)]}{2b[16bn_1 + 23\beta^2 - 26\beta - 1]^2} - \frac{r_0}{2}k^2 - \frac{t\eta_1(7-5\beta)^2[2a(1+k)]^2}{2(16bn_1 + 23\beta^2 - 26\beta - 1)^2} \quad (82)$$

$$\pi^{TC} = \frac{[2a(1+k)]^2(12bn_1 + 11\beta^2 - 12\beta + 1)(2bn_1 + \beta^2 - 12\beta + 6)}{2b[16bn_1 + 23\beta^2 - 26\beta - 1]^2} + \frac{[2a(1+k)]^2(2bn_1 + \beta^2 - 1)}{2b(16bn_1 + 23\beta^2 - 26\beta - 1)^2} + \frac{[-4bn_1 + 3(1-\beta)(3-\beta)]}{2b(16bn_1 + 23\beta^2 - 26\beta - 1)^2} - \frac{r_0}{2}k^2 - \frac{\eta_1(7-5\beta)^2[2a(1+k)]^2}{2[16bn_1 + 23\beta^2 - 26\beta - 1]^2} \quad (83)$$

5 Results and analysis

Comprehending above calculation and analysis results, we will compare with price, retailer's marginal profit, local advertising level among the seven cooperative advertising game models.

In model B and model F, manufacturer who is at the end of decision-making order makes decision about price of product both considering the sum profits maximisation.

Proposition 1: The final decision-maker always cannot achieve system optimisation either by adopting coordination by himself or combining with other player do partial coordination. The reasons are:

- 1 in model B, retailer R_1 hopes his marginal profit will be bigger in manufacturer coordination model
- 2 in model F, retailer R_1 's marginal profit may be any value.

Proposition 1 illustrates that if the player at the end of decision-making order makes single or partial coordination without coordinating with players at the front of decision-making order, as the result, the front decision-makers will lack constraint. Thus, retailer R_1 hopes to get more marginal profit, infinitely great is better. It is obvious that the two models in which manufacturer adopts cooperative single or partial cooperation are unreasonable.

Proof of Proposition 1: see in equations (30) and (69).

In models C, D and E, we analyse the change of retailer R_1 's marginal profit, local advertising level and profit when just retailer R_1 adopts cooperative attitude.

Proposition 2: Suppose retailer R_1 just adopts cooperative attitude,

- 1 in model C, retailer R_1 's marginal profit and local advertising level are greater than zero
- 2 in model D, retailer R_1 's marginal profit, local advertising level and profit are equal to zero
- 3 in model E, retailer R_1 's marginal profit is equal to zero, local advertising level is greater than zero, profit is less than zero.

Proposition 2 shows that when retailer focuses on coordinate local advertising transfer to his marginal profit, it will cause marginal profit and local advertising level to decrease. That means when retailer focuses on shifting, he will reduce local advertising level until it drops to zero. Manufacturer finds that retailer R_1 does not invest in local advertising, so he is not willing to coordinate with retailer, which will cause retailer R_1 's marginal profit to drop to zero. Comparing these with these three models, we see that, as long as retailer R_1 is willing to coordinate with local advertising, whether he coordinates marginal profit or not, the local advertising is always greater than zero, and this is good for manufacturer. Furthermore, when retailer who is in the front of decision-making order coordinates singly, the game equilibrium is bad for retailer.

Proof of the Proposition 2 see in Appendix.

Analysis of price in different model.

Proposition 3: Suppose the product is in rise period, the price in model D which assumes that only retailer

coordinates on marginal profit is greater than the price in model E and model G. When the product is in mature period, there are two cases. One is that if the spillover effect coefficient is less than 1/5, the result is the same as that in rise period. However, the other is that if the spillover effect coefficient is more than 1/5, the result is opposite.

$$1 \quad p^{PC_2} > p^{PC_3}, \text{ as } -1 < \beta < \frac{1}{5};$$

$$p^{PC_2} = p^{PC_3}, \text{ as } \beta = \frac{1}{5};$$

$$p^{PC_2} < p^{PC_3}, \text{ as } \frac{1}{5} < \beta < 1$$

$$2 \quad p^{PC_2} > p^{TC}, \text{ as } -1 < \beta < \frac{1}{5};$$

$$p^{PC_2} = p^{TC}, \text{ as } \beta = \frac{1}{5};$$

$$p^{PC_2} < p^{TC}, \text{ as } \frac{1}{5} < \beta < 1.$$

Proposition 3 reveals that when just retailer R_1 chooses cooperative attitude on marginal profit, it cannot achieve the goal, then this does not optimise the supply chain. Besides, when the product is in rise period, the behaviour that retailer who coordinate on marginal profit singly also do not make the system optimisation. However, the result is opposite in mature period after the initial.

Proof of Proposition 3 is given in Appendix.

Analysis of retailer R_1 's local advertising level.

Proposition 4: If retailer R_1 coordinates on his marginal profit, the local advertising level be zero, that is

$$1 \quad f_1^{PC_2} = 0.$$

Proposition 4 illustrates that, in order to make the local advertising greater than zero of the supply chain coordination, it is necessary to avoid retailer choosing model D's behaviour, that is retailer coordinates on marginal profit. Analysis of retailer R_1 and R_2 's marginal profit n_1, n_2 is located in the following Proposition 5.

Proposition 5:

1 Retailer R_1 's marginal profit is equal to zero in model $D^{(PC_2)}$ and model $E^{(PC_3)}$ and any value in model $G^{(TC)}$

2 Retailer R_2 's marginal profit is a constant in model $D^{(PC_2)}$ which is not affected in the product life.

Proposition 5 is from the following formulas.

$$n_1^{PC_2} = n_1^{PC_3} = 0.$$

n_1^{TC} is any value.

$$\frac{\partial n_2^{PC_2}}{\partial \beta} = 0$$

Proposition 5 illustrates that when retailer R_1 chooses coordinate on marginal profit or both marginal profit and local advertising, his marginal profit decreases to zero. It shows that manufacturer do not want to retailer coordinate as his marginal profit, such situation lead to retailer's negotiation ability decreasing. Besides, in cooperation model, retailer R_1 's marginal profit can be any value, a mechanism is required to constraint manufacturer and retailer so as to achieve the whole supply chain optimisation.

Proof of Proposition 5 is given in Appendix.

6 Summary and discussion

Through seven dynamic game models (one non-cooperative model, five partial cooperative models and one cooperative model), the changes of the price, retailer's marginal profit and local advertising level are explored when game parties adopting different attitudes.

Contributions of this paper include:

- 1 building a three-stage dynamic game model
- 2 three kinds of attitude, non-cooperation, partial cooperation and cooperation, are analysed in details
- 3 seven models are discussed and compared.

There are two implications for future research. First, this paper discusses changes of the price, retailer's marginal profit and local advertising level but does not refer to the profits of retailers and manufacturer or the sum profits, it will be discuss in the future. Second, we assume that there is only one manufacturer in the market and it is certain we can expand one manufacturer to two manufacturers or more.

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Appendix

Proof of Proposition 2

- 1 From equation (34), since $\beta \in (-1, 1)$, $b > 1$, $r_1 > 1$, $0 < t < 1$ so, we have

$$23\beta^2 - 48\beta + 25 + 6bt_1 > 0 \quad (84)$$

$$31\beta^2 - 50\beta + 15 + 12bt_1 > 0 \quad (85)$$

And because $a > 0$, $k > 0$, so, we have

$$n_1^{PC_1} = \frac{4a(1+k)[23\beta^2 - 48\beta + 25 + 6bt_1]}{3b[31\beta^2 - 50\beta + 15 + 12bt_1]} > 0$$

From equation (33), since $a > 0$, $k > 0$, $\beta \in (-1, 1)$ and equation (86), we have

$$f_1^{PC_1} = \frac{2a(1+k)(5 - 3\beta)}{31\beta^2 - 50\beta + 15 + 12bt_1} > 0$$

- 2 From equations (49), (50) and (53), we can prove that.
- 3 From equation (60), we know that $n_1^{PC_3} = 0$.

From equation (59), since $b > 1$, $r_1 > 1$, so, we have

$$8bt_1 - (3 - \beta)(1 + \beta) > 0$$

And because $a > 0$, $k > 0$, we have

$$f_1^{PC_3} = \frac{4a(1+k)}{8bt_1 - (3 - \beta)(1 + \beta)} > 0$$

From equation (63), since $0 < t < 1$, $r_1 > 1$, so, we have

$$\pi_1^{PC_3} = -\frac{2(1-t)t_1[2a(1+k)]^2}{[8bt_1 - (3 - \beta)(1 + \beta)]^2} < 0$$

Proof of Proposition 3.

From equations (52), (62),

$$\begin{aligned}
 p^{PC_2} - p^{PC_3} &= \frac{3a(1+k)}{4b} \\
 &\quad - \frac{2a(1+k)[24br_1 - (3\beta+7)(1-\beta)]}{8b[8br_1 - (3-\beta)(1+\beta)]} \\
 &= \frac{a(1+k)(1-5\beta)}{2b[8br_1 - (3-\beta)(1+\beta)]}
 \end{aligned}$$

since $b > 1$, $r_1 > 1$, so, we have

$$8br_1 - (3-\beta)(1+\beta) > 0$$

And because $a > 0$, $k > 0$, we have

$$\begin{aligned}
 p^{PC_2} - p^{PC_3} &= \frac{a(1+k)(1-5\beta)}{2b[8br_1 - (3-\beta)(1+\beta)]} > 0, \\
 \text{as } -1 < \beta < \frac{1}{5} &= 0, \\
 \text{as } \beta = \frac{1}{5} < 0, \text{ as } \frac{1}{5} < \beta < 1
 \end{aligned}$$

From equations (52), (79),

$$\begin{aligned}
 p^{PC_2} - p^{TC} &= \frac{3a(1+k)}{4b} \\
 &\quad - \frac{2a(1+k)[12br_1 + 11\beta^2 - 12\beta + 1]}{2b[16br_1 + 23\beta^2 - 26\beta - 1]} \\
 &= \frac{a(1+k)(5\beta+1)(5\beta-7)}{4b(16br_1 + 23\beta^2 - 26\beta - 1)}
 \end{aligned}$$

since $b > 1$, $r_1 > 1$, so, we have

$$16br_1 + 23\beta^2 - 26\beta - 1 > 0$$

And because $a > 0$, $k > 0$, we have

$$\begin{aligned}
 p^{PC_2} - p^{TC} &= \frac{a(1+k)(5\beta+1)(5\beta-7)}{4b(16br_1 + 23\beta^2 - 26\beta - 1)} > 0, \\
 \text{as } -1 < \beta < 1 &= 0, \\
 \text{as } \beta = \frac{1}{5} < 0, \text{ as } \frac{1}{5} < \beta < 1
 \end{aligned}$$

Proof of Proposition 5.

From equations (50) and (60),

$$n_1^{PC_2} = n_1^{PC_3} = 0.$$

n_1^{TC} is any value.

From equations (51),

$$\frac{\partial n_2^{PC_2}}{\partial \beta} = 0$$