Technical Notes and Correspondence

A Frequency-Domain Approach to Identification of Mechanical Systems with Friction

Seung-Jean Kim and In-Joong Ha

Abstract—**This note describes a novel frequency-domain approach to identification of mechanical systems with friction on the basis of their periodic steady-state motions. Most importantly, the proposed method does not require acceleration information for its implementation. An interesting feature is that any periodic excitation input with sufficiently large amplitude and/or frequency can ensure the feasibility of the proposed method. We also present some experimental results to illuminate further its practical use.**

Index Terms—**Friction, identification, mechanical system.**

I. Introduction

Motion control systems such as machine tools, microelectronics manufacturing equipments, and automatic inspection machines usually exhibit steady-state tracking errors and/or oscillations, if the controllers are designed without considering friction [9]. In this context, friction identification has been one of the important issues in the design of high-performance motion control systems [5]–[8]. On the other hand, the controllers used for precise tracking of desired trajectories in advanced motion control systems also require the mass information. Hence, identification of the mass is equally important for the full performance of those controllers.

In general, identification methods for mechanical systems with friction can be classified into two categories, online identification methods and offline identification methods [3]. The most well-known online identification methods, which have been applied to mechanical systems with friction, are the least mean squares (LMS) and recursive least squares (RLS) methods. Good convergence properties can be established for the LMS and RLS methods, provided that the persistence of excitation (PE) is met [10]. However, the direct application of these methods to mechanical systems with friction usually requires the acceleration information [3]. Furthermore, it is usually impossible to check before identification process whether or not the PE property can be met. On the other hand, most offline identification methods, which have been applied to mechanical systems with friction, involve acquiring the acceleration information numerically, which can cause a large parameter identification error.

In this note, we present a new offline identification method for mechanical systems with friction. The proposed method is based on a kind of frequency-domain linear regression model, which is derived from the Fourier analysis of the periodic steady-state motions of mechanical systems with friction. Most importantly, the proposed method does not require the acceleration information for its implementation. An interesting feature is that any periodic excitation input with sufficiently large amplitude and/or frequency can ensure the feasibility of the proposed method. For the theoretical completeness of our work, we show that the simple proportional-derivative (PD) controller can ensure the existence of periodic steady-state motions in closed-loop systems under periodic excitation inputs. Finally, some experimental results are presented to illuminate further the practical use of the proposed method.

Manuscript received January 21, 2000; revised December 8, 1999 and October 6, 2000. Recommended by Associate Editor O. Egeland. This work was supported by SNU BK21-IT program.

The authors are with the School of Electrical Engineering, Seoul National University, Kwanak-Ku, Seoul 151-742, Korea (e-mail: ijha@asri.snu.ac.kr).

Publisher Item Identifier S 0018-9286(01)03606-6.

II. Preliminary Results

The one-dimensional (1-D) motion of a mass, which moves on a surface with friction, is governed by

$$m\dot{v}(t) + F(v(t), u(t)) = u(t) \tag{1}$$

where

- m mass;
- v velocity of the mass;
- F friction force;
- u control force applied to the mass [1], [2].

Specifically speaking, the function F in (1) is modeled as

$$F(v,u) \triangleq F_s(v) + F_0(v,u). \tag{2}$$

Here, the function F_s presents the slip friction force at nonzero velocity and is continuous except at zero. On the other hand, the stick friction F_0 presents the friction force at zero velocity, and its mathematical model is given by

$$F_0(v,u) \triangleq \begin{cases} 0, & \text{if } v \neq 0 \\ \lambda(u), & \text{if } v = 0 \end{cases} \tag{3}$$

where

$$\lambda(u) \triangleq \begin{cases} F_s(0+), & \text{if } u \geq F_s(0+) > 0 \\ u, & \text{if } F_s(0-) < u < F_s(0+) \\ F_s(0-), & \text{if } u \leq F_s(0-) < 0. \end{cases}$$

Our main result requires that the motion of the system in (1) to a periodic excitation input exhibits steady-state oscillation. To ensure the existence of steady-state oscillation, the control force u is governed by the following PD controller:

$$u(t) = K_p[r(t) - x(t)] + K_d[\dot{r}(t) - v(t)]. \tag{4}$$

Here, x is the position of the mass, i.e., $\dot{x} = v$. Moreover, the excitation input r is assumed to be continuously differentiable and T-periodic, i.e.,

$$r(t+T) = r(t), \quad \forall t \in R. \tag{5}$$

In our analysis, we make the following assumptions on the slip friction F_s, which can provide mathematical tractability without loss of the practicality of the friction model.

A1) There exist a function F_n, a scalar K_v, and a positive constant F_M such that

$$F_s(v) = K_v v + F_n(v), \quad |F_n(v)| \leq F_M \quad \forall v \in R.$$

A2) There exists a constant K_F such that

$$\frac{F_s(v_1) - F_s(v_2)}{v_1 - v_2} \geq K_F, \quad \text{if } v_1 \neq v_2.$$

As a matter of fact, all the previously known static models of friction including the Gaussian model and the Lorentzian model [1], [3] can be easily shown to satisfy the assumptions A1) and A2).

Then, the closed-loop system consisting of the system in (1) and the PD controller in (4) can be written as

$$\dot{z} = f(t, z) \triangleq Az + \alpha(t, z) + b\hat{r}(t) \tag{6}$$

where

$$z \triangleq \begin{bmatrix} x \\ v \end{bmatrix}, \quad A \triangleq \begin{bmatrix} 0 & 1 \\ -\frac{K_p}{m} & -\frac{K_d + K_v}{m} \end{bmatrix}, \quad b \triangleq \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$
$$\alpha(t, z) \triangleq \begin{bmatrix} 0 \\ -\frac{1}{m}F_n(v) - \frac{1}{m}F_0(v, -K_d v - K_p x + \hat{r}(t)) \end{bmatrix}$$
$$\hat{r}(t) \triangleq K_d \dot{r}(t) + K_p r(t).$$

Here, it should be pointed out that the system in (6) admits a solution in the Carathèodory sense, although its right-hand side is discontinuous with respect to z due to the nature of friction at zero velocity. The proof can be found in [12].

Now, we are ready to state the following theorem.

Theorem 1 (Existence of Periodic Steady-State Solution): Suppose that the gains of the PD controller in (4) are chosen to satisfy

$$K_p > 0, \quad K_d > \max\{-K_F, -K_v\}. \tag{7}$$

Then, any solution z of the system in (6) converges to an absolutely continuous T-periodic function $\bar{z} \triangleq [\bar{x} \bar{v}]^{\mathrm{T}} : R \to R^2$. ■

To prove Theorem 1, we need the following two lemmas.

Lemma 1: Suppose that the assumption A2) is satisfied. Then

$$(v_1 - v_2)[F(v_1, u_1) - F(v_2, u_2)] \geq K_F |v_1 - v_2|^2 \quad \forall v_1, v_2, u_1, u_2 \in R. \tag{8}$$

Proof: We need to consider four cases: i) $v_1 = v_2 = 0$; ii) $v_1 \neq 0, v_2 = 0$; iii) $v_1 = 0, v_2 \neq 0$; and iv) $v_1, v_2 \neq 0$. However, we only show that (8) holds for the case ii) since the proof for the other three cases is similar or trivial. When $v_1 > 0$ and $v_2 = 0$, we can see from assumption A2) and the definition of the function λ in (3) that

$$\begin{aligned}(v_1 - v_2)[F(v_1, u_1) - F(v_2, u_2)] &= v_1[F_s(v_1) - \lambda(u_2)] \\ &= v_1[F_s(v_1) - F_s(0+)] \\ &\quad + v_1[F_s(0+) - \lambda(u_2)] \\ &\geq v_1[F_s(v_1) - F_s(0+)] \\ &\geq K_F |v_1|^2.\end{aligned} \tag{9}$$

Similarly, it can be shown that (8) also holds for the other case of $v_1 < 0$ and $v_2 = 0$. ■

Lemma 2: Suppose that all the hypotheses of Theorem 1 are satisfied. Let $z_1 \triangleq (x_1, v_1)$ and $z_2 \triangleq (x_2, v_2)$ denote, respectively, any two solutions of the system in (6). Then, it holds for any $t_0 \geq 0$ that

$$\sup_{t \geq t_0} \|z_1(t) - z_2(t)\|_V \leq \|z_1(t_0) - z_2(t_0)\|_V \tag{10}$$
$$\lim_{t \to \infty} |v_1(t) - v_2(t)| = 0 \tag{11}$$

where $\|z_1 - z_2\|_V \triangleq \sqrt{K_p/2(x_1 - x_2)^2 + m/2(v_1 - v_2)^2}$ denotes a weighted distance between two points $z_1 \triangleq (x_1, v_1), z_2 \triangleq (x_2, v_2) \in R^2$.

Proof: By Lemma 1 along with (7), the time derivative of the square of the weighted distance between z_1 and z_2 satisfies that for almost all $t \geq t_0$

$$\begin{aligned}\frac{d}{dt}\|z_1(t) - z_2(t)\|_V^2 =& K_p[x_1(t) - x_2(t)][v_1(t) - v_2(t)] \\ & - [v_1(t) - v_2(t)] \\ & \times [F(v_1(t), u_1(t)) \\ & \quad + K_d v_1(t) + K_p x_1(t) \\ & \quad - F(v_2(t), u_2(t)) \\ & \quad - K_d v_2(t) - K_p x_2(t)] \\ =& - [v_1(t) - v_2(t)] \\ & \times [F(v_1(t), u_1(t)) - F(v_2(t), u_2(t))] \\ & + K_d |v_1(t) - v_2(t)|^2 \\ \leq& - (K_F + K_d)|v_1(t) - v_2(t)|^2\end{aligned} \tag{12}$$

where $u_1 \triangleq K_p x_1 - K_d v_1 + \hat{r}$ and $u_2 \triangleq -K_p x_2 - K_d v_2 + \hat{r}$. Then, this inequality implies (10). Furthermore, $\|z_1(\infty) - z_2(\infty)\|_V$ is well defined and

$$\begin{aligned}\int_{t_0}^{t} |v_1(\tau) - v_2(\tau)|^2 d\tau \leq& - \frac{1}{K_F + K_d} \\ & \times [\|z_1(\infty) - z_2(\infty)\|_V^2 \\ & \quad - \|z_1(t_0) - z_2(t_0)\|_V^2], \\ & \forall t \geq t_0.\end{aligned}$$

Thus, it is clear that $(v_1 - v_2) \in \mathcal{L}_2[0, \infty)$. Note from the assumption (A.1) that the function α in (6) is bounded. Also, note from (7) that A is a Hurwitz matrix. Hence, z_1 and z_2 are also bounded. It is therefore obvious that $(\dot{v}_1 - \dot{v}_2) \in \mathcal{L}_\infty$. Finally, this along with the Babalat's Lemma [14] impies (11). ■

Now, we are ready to prove Theorem 1.

Proof of Theorem 1: Define the solution map $z(\cdot; t_0, z_0) = [x(\cdot; t_0, z_0), v(\cdot; t_0, z_0)]^{\mathrm{T}}$ by $z(t; t_0, z_0) \triangleq z(t + t_0)$, where z denotes the solution of the system in (6) with the initial condition $z(t_0) = z_0$. Then, (10) implies that the above solution map $z(\cdot; t_0, z_0)$ is well defined and is bounded. Furthermore, the following properties easily follow from Lemma 2 along with the T-periodicity of the right-hand side of the system in (6).

$$z(t; t_0 + T, z_0) = z(t; t_0, z_0), \quad \forall t \geq 0. \tag{13}$$
$$z(t; s + t_0, z(s; t_0, z_0)) = z(t + s; t_0, z_0), \quad \forall t, s \geq 0. \tag{14}$$
$$\lim_{n \to \infty} \|z(t; t_0, z_n) - z(t; t_0, z^*)\|_V = 0, \quad \forall t \geq 0 \tag{15}$$

where $\{z_n\}$ is any sequence converging to z^*.

Define Z^* as the set of all the cluster points of the bounded sequence $\{z(nT, t_0, z_0) : n \in N\}$. Then, the well-known Bolzano–Weirstrass theorem [11] implies that Z^* is nonempty. Now, suppose that $z^* \in Z^*$. Then, there exists a subsequence $\{z(n_k T, t_0, z_0) : k \in N\}$ which converges to z^*, i.e.,

$$\lim_{k \to \infty} z(n_k T, t_0, z_0) = z^*. \tag{16}$$

On the other hand, (13) and (14) imply that

$$\begin{aligned} z(t;t_0,z(n_kT,t_0,z_0)) &= z(t;t_0+n_kT,z(n_kT;t_0,z_0)) \\ &= z(t+n_kT;t_0,z_0) \\ &\forall t \geq 0 \text{ and } \forall k=1,2,\ldots. \end{aligned} \tag{17}$$

Similarly, we can show that

$$z(t+T;t_0,z(n_kT,t_0,z_0)) = z(t+n_kT;t_0,z(T;t_0,z_0)) \quad \forall t \geq 0 \text{ and } \forall k=1,2,\ldots. \tag{18}$$

From (15) – (18) along with (11), it then follows that:

$$\begin{aligned} &v(t;t_0,z^*) - v(t+T;t_0,z^*) \\ &= \lim_{k\to\infty}[v(t;t_0,z(n_kT,t_0,z_0)) \\ &\qquad - v(t+T;t_0,z(n_kT,t_0,z_0))] \\ &= \lim_{k\to\infty}[v(t+n_kT;t_0,z_0) \\ &\qquad - v(t+n_kT;t_0,z(T,t_0,z_0))] \\ &= 0, \qquad \forall t \geq 0. \end{aligned} \tag{19}$$

By this along with the boundedness of $z(\cdot;t_0,z_0)$, we can easily show that the solution of the system in (6), whose initial condition is a cluster point z^* of $\{z(nT,t_0,z_0)\}$, must be T-periodic. In particular, it holds that

$$z(nT;t_0,z^*) = z^*, \quad \forall z^* \in Z^* \text{ and } \forall n = 0,1,2,\ldots. \tag{20}$$

Furthermore, it is not difficult to see from Lemma 2 that Z^* must be a singleton. Thus, the sequence $\{z(nT,t_0,z_0)\}$ is convergent to the unique cluster point z^*. For any $\epsilon > 0$, there then exists an integer N such that

$$\|z(nT,t_0,z_0) - z^*\|_V \leq \epsilon, \ \forall n \geq N.$$

By this, along with (10) and (20)

$$\begin{aligned} &\sup_{t\geq NT} \|z(t;t_0,z_0) - z(t;t_0,z^*)\|_V \\ &\leq \|z(NT,t_0,z_0) - z(NT,t_0,z^*)\|_V \\ &= \|z(NT,t_0,z_0) - z^*\|_V \leq \epsilon. \end{aligned}$$

Hence

$$\lim_{t\to\infty} \|z(t;t_0,z_0) - z(t;t_0,z^*)\|_V = 0.$$

Finally, denoting $\bar{z}(t) = z(t,t_0,z^*)$ completes the proof. ■

The identification method proposed in the next section is based on the steady-state solution $\bar{z}$ whose existence is ensured by Theorem 1.

III. Main Results

For mathematical tractability, we assume that the slip friction F_s is described by the following linear-in-parameters model with p unknown friction parameters $\lambda_1, \cdots, \lambda_p$:

$$F_s(v) = \psi^{\mathrm{T}}(v)\lambda + \Delta f(v) \tag{21}$$

where

$$\psi(v) \triangleq [\psi_1(v),\ldots,\psi_p(v)]^{\mathrm{T}} \quad \lambda \triangleq [\lambda_1,\ldots,\lambda_p]^{\mathrm{T}}.$$

Here, $\psi_i, i=1,\ldots,p$ are the model basis functions and Δf represents the model parameterization error of the slip friction. The system in (1) can then be written as the following linear regression model:

$$u(t) = \phi^{\mathrm{T}}(t)\theta + e(t) \tag{22}$$

where

$$\begin{aligned} \theta \triangleq \begin{bmatrix} m \\ \lambda \end{bmatrix} \quad \phi(t) \triangleq \begin{bmatrix} \dot{v}(t) \\ \psi(v(t)) \end{bmatrix} \\ e(t) \triangleq \Delta f(v(t)) + F_0(v(t),u(t)). \end{aligned}$$

Here, the function e represents the systematic error due to the effect of the stick friction as well as the model parameterization error of the slip friction.

In our development, we make the following assumptions on the model basis function $\psi_i, i=1,\ldots,p$:

A3) $\psi_i, \ i=1,\ldots,p$ are piecewise continuous;

A4) $\mu^*(\psi_1,\ldots,\psi_p) \triangleq \inf\{\mu > 0 : \psi_i, i=1,\ldots,p$ are linearly independent on$[-\mu,\mu]\}$ is finite.

The assumption A4) simply requires that the model basis functions $\psi_i, i=1,\ldots,p$ are linearly independent over a sufficiently large domain $[-\mu,\mu]$. In fact, it can be shown that all the model basis functions $\psi_i, i=1,\ldots,p$ of the previously known models of the slip friction satisfy $\mu^*(\psi_1,\cdots,\psi_p) = 0$.

In addition to the assumptions A3) and A4), we make the following assumption on the excitation input r:

A5) the periodic excitation input r is chosen so that

$$[-\mu^*(\psi_1,\ldots,\psi_p), \mu^*(\psi_1,\ldots,\psi_p)] \subset (v_{\min}, v_{\max}) \tag{23}$$

where

$$v_{\min} \triangleq \min_{t\in[0,\mathrm{T}]} \bar{v}(t), \quad v_{\max} \triangleq \max_{t\in[0,\mathrm{T}]} \bar{v}(t). \tag{24}$$

In particular, assumption A5) is reduced to $v_{\min} \neq v_{\max}$ in the case of $\psi_i, i=1,\ldots,p$ with $\mu^*(\psi_1,\ldots,\psi_p) = 0$. In other words, it simply requires that the mass should not move at a constant velocity. Note that the only constant-velocity motion of the system in (6) with the periodic input r is no motion. Hence, it is equivalent to the condition that the mass is not always in the stick condition at steady state, that is, $\bar{v} \not\equiv 0$.

At steady state, the linear regression model in (22) becomes

$$\bar{u}(t) = \bar{\phi}^{\mathrm{T}}(t)\theta + \bar{e}(t) \tag{25}$$

where

$$\begin{aligned} \bar{u}(t) &\triangleq K_p[r(t) - \bar{x}(t)] + K_d[\dot{r}(t) - \bar{v}(t)] \\ \bar{\phi}(t) &\triangleq \begin{bmatrix} \dot{\bar{v}}(t) \\ \psi(\bar{v}(t)) \end{bmatrix} \\ \bar{e}(t) &\triangleq \Delta f(\bar{v}(t)) + F_0(\bar{v}(t),\bar{u}(t)). \end{aligned}$$

Taking the Fourier series of $\bar{u}$, $\bar{\phi}$, and $\bar{e}$ in (25) yields the following linear regression model described in terms of their Fourier series coefficients.

$$u_k = \phi_k^{\mathrm{T}}\theta + e_k, \qquad \forall k = 0,\pm1,\pm2,\ldots \tag{26}$$

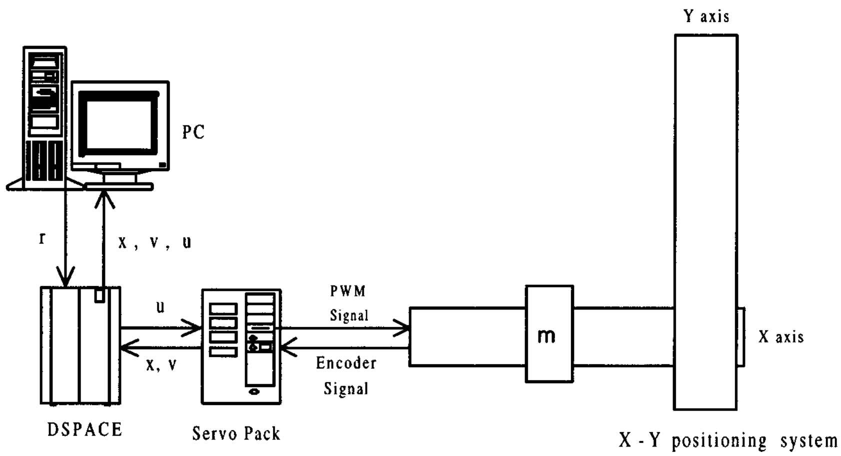

Fig. 1. Schematic diagram of experimental setup.

where

$$u_k \triangleq \frac{1}{T}\int_0^T \bar{u}(t)e^{-jk\omega t}dt \quad \phi_k \triangleq \frac{1}{T}\int_0^T \bar{\phi}(t)e^{-jk\omega t}dt$$
$$e_k \triangleq \frac{1}{T}\int_0^T \bar{e}(t)e^{-jk\omega t}dt. \ \omega \triangleq \frac{2\pi}{T}.$$

In contrast with the regression vector ϕ in (22), the regression vector ϕ_k in (26) can be calculated using only the velocity information without the acceleration information.

$$\phi_k = \begin{bmatrix} jk\omega v_k \\ \frac{1}{T}\int_0^T \psi(\bar{v}(t))e^{-jk\omega t}dt \end{bmatrix}$$
$$v_k \triangleq \frac{1}{T}\int_0^T \bar{v}(t)e^{-jk\omega t}dt. \quad (27)$$

However, since v_k is multiplied by $jk\omega$, the noise present in the computation of v_k can be amplified to appear in ϕ_k for $|k| \gg 1$. In this context, we instead use the following frequency-weighted linear regression model for the parameter identification:

$$\tilde{u}_k = \tilde{\phi}_k^{\mathrm{T}}\theta + \tilde{e}_k, \qquad \forall k = 0, \pm 1, \pm 2, \ldots \quad (28)$$

where

$$\tilde{\phi}_k \triangleq H(j\omega k)\phi_k = \begin{bmatrix} jk\omega H(jk\omega)v_k \\ H(jk\omega)\frac{1}{T}\int_0^T \psi(\bar{v}(t))e^{-jk\omega t}dt \end{bmatrix}$$
$$\tilde{u}_k \triangleq H(j\omega k)u_k \quad \tilde{e}_k \triangleq H(j\omega k)e_k.$$

Here, $H(s)$, which is not necessarily causal, is any transfer function satisfying the following condition:

$$H(jk\omega) \neq 0, \quad \forall k = 0, 1, 2, \ldots. \quad (29)$$

Observe that $sH(s)$ is proper, if $H(s)$ is strictly proper. Accordingly, the noise effect on computation of $\tilde{\phi}_k$ and $\tilde{u}_k$ can be reduced significantly through a proper choice of $H(s)$.

Finally, we can design the optimal estimate θ^* by solving the following least-squares optimization problem:

$$(rmP): \hat{\theta}(M) = \arg\min_{\theta\in R^{p+1}} \sum_{k=-M}^{M} |\tilde{u}_k - \tilde{\phi}_k^{\mathrm{T}}\theta|^2 dt. \quad (30)$$

If the matrix $\sum_{-M}^{M} \tilde{\phi}_k\tilde{\phi}_k^{\mathrm{T}}$ is nonsingular, the optimal estimate $\hat{\theta}(M)$ can then be computed as

$$\hat{\theta}(M) = \left[\sum_{k=-M}^{M} \tilde{\phi}_k\tilde{\phi}_k^H\right]^{-1} \sum_{k=-M}^{M} \tilde{\phi}_k\tilde{u}_k^H. \quad (31)$$

Here and elsewhere, A^H denotes the Hermitian of A. Obviously, $\hat{\theta}(M)$ is a real vector.

The following theorem concerns the feasibility of the estimator in (30).

Theorem 2: There exists an integer K such that

$$\text{the matrix } \sum_{k=-M}^{M} \tilde{\phi}_k\tilde{\phi}_k^H \text{ is nonsingular for all } M \geq K. \quad (32)$$

Proof: To begin with, observe that

$$P \triangleq \frac{1}{T}\int_0^T \phi(\bar{v}(t))\phi^{\mathrm{T}}(\bar{v}(t))dt = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad (33)$$

where

$$P_{11} \triangleq \frac{1}{T}\int_0^T |\dot{\bar{v}}(t)|^2 dt \in R$$
$$P_{12} \triangleq \frac{1}{T}\int_0^T \dot{\bar{v}}(t)\psi^{\mathrm{T}}(\bar{v}(t))dt \in R^{1\times p}$$
$$P_{21} \triangleq P_{12}^{\mathrm{T}} \in R^{p\times 1}$$
$$P_{22} \triangleq \frac{1}{T}\int_0^T \psi(\bar{v}(t))\psi^{\mathrm{T}}(\bar{v}(t))dt \in R^{p\times p}.$$

As will be shown soon

$$P_{21} = P_{12}^{\mathrm{T}} = 0_{p\times 1} \quad (34)$$
$$P_{11} > 0 \quad (35)$$
$$P_{22} \in R^{p\times p} \text{is nonsingular.} \quad (36)$$

Hence, P is nonsingular. By this fact, along with the well-known Parseval's theorem [13], there exists an integer $K > 0$ such that $\sum_{k=-M}^{M} \phi_k\phi_k^H$ is nonsingular for all $M \geq K$. Consequently, the $(p+1)\times(2M+1)$ matrix $[\phi_{-M}, \ldots, \phi_0, \ldots, \phi_M]$ is of full rank. Then, it is easy to see from (29) that the $(p+1)\times(2M+1)$ matrix

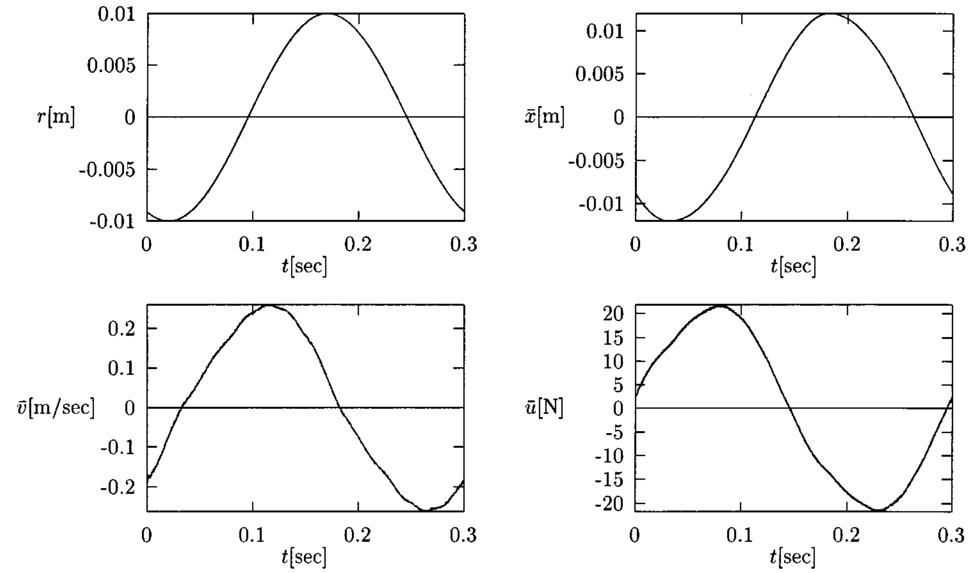

Fig. 2. Steady-state responses of the closed-loop system.

$[\tilde{\phi}_{-M}, \cdots, \tilde{\phi}_0, \cdots, \tilde{\phi}_M]$ is also of full rank. Finally, this fact means that (32) holds.

Now, we show that (34)–(36) are true. We first show that (34) is true. By the piecewise continuity of the function ψ, there exists a sequence of continuous functions $\{g_n : R \to R^p\}$ such that

$$\lim_{n\to\infty} g_n(v) = \psi(v), \quad \forall v \in R.$$

Then, the well-known Lebesque Convergence theorem [11] along with the T-periodicity of $\bar{v}$ implies that

$$\begin{aligned} P_{21} = P_{12}^{\mathrm{T}} &= \int_0^T \dot{\bar{v}}(t)\psi(\bar{v}(t))dt \\ &= \lim_{n\to\infty}\int_0^T \dot{\bar{v}}(t)g_n(\bar{v}(t))dt \\ &= \lim_{n\to\infty}[G_n(\bar{v}(T)) - G_n(\bar{v}(0))] = 0 \end{aligned}$$

where $G_n(v) \triangleq \int_0^v g_n(s)ds$. On the other hand, it is easy to see from the assumption A5) that the function $\bar{v}$ is not identically constant on $[0, T]$ Hence, (35) holds clearly.

Finally, we show by way of contradiction that (36) is true. Suppose that P_{22} is singular. Then, there exists a nonzero vector $c \in R^p$ such that

$$\begin{aligned} c^{\mathrm{T}} P_{22} c &= \frac{1}{T}\int_0^T c^{\mathrm{T}}\psi(\bar{v}(t))\psi^{\mathrm{T}}(\bar{v}(t))c dt \\ &= \frac{1}{T}\int_0^T |c^{\mathrm{T}}\psi(\bar{v}(t))|^2 dt = 0. \end{aligned} \tag{37}$$

This implies that $c^{\mathrm{T}}\psi(\bar{v}(t)) =$ a.e. on $[0, T]$. This fact along with the piecewise continuity of the function ψ implies that $c^{\mathrm{T}}\psi(v^*) = 0,$ a.e. on $[v_{\min}, v_{\max}]$, which is contradictory to the assumptions A4) and A5). Thus, (36) is true. ■

To sum up, the above theorem shows that any periodic excitation input r with sufficiently large amplitude and/or frequency (so that assumption A5) is met) can ensure the feasibility of the estimator in (30).

IV. A Practical Example

In this section, some experimental results are presented to illuminate further the practical use of the proposed method. Before presenting our experimental results, we briefly explain our experimental setup. The schematic diagram of our experimental setup is shown in Fig. 1. It is made up of a ball-screw type X–Y positioning mechanism, a servo pack, and a DSPACE system equipped with a custom DSP board and an encoder signal processing board. Here, the ball-screw type X–Ypositioning mechanism is drived by two brushless DC motors (BLDCMs), but it suffices to activate only its$X$ axis for our experimental results. An optical encoder with the resolution of 8192 pulses/rev is mounted on each BLDCM as a position sensor. On the other hand, the pitch of each ball-screw type axis is 2 cm. This implies that the linear resolution is approximately 2.44×10^{-4} cm. The velocity information is obtained using the well-known M/T method based on the encoder signal.

The control gains of the PD controller used to guarantee the existence of steady-state oscillation are chosen as

$$K_d = 50\left[\frac{\mathrm{Ns}}{\mathrm{m}}\right] \quad K_p = 5000\left[\frac{\mathrm{N}}{\mathrm{m}}\right]. \tag{38}$$

We have also chosen the following periodic function as an excitation input to the closed-loop system:

$$r(t) = 0.01 \sin\frac{2\pi}{0.3}t \text{ [m]}. \tag{39}$$

The steady-state responses of the position x, the velocity v, and the control force u of the closed-loop system, which are denoted, respectively, by $\bar{x}$, $\bar{v}$, and $\bar{u}$, are given in Fig. 2, together with the excitation input r.

In our experimental work, we use, as the parametric model of the slip friction, the asymmetric Coulomb + viscous friction model, which takes the form in (21) with

$$\begin{aligned} \psi(v) &\triangleq [|v|\mu_+(v), \mu_+(v), |v|\mu_-(v), \mu_-(v)]^{\mathrm{T}} \\ \lambda &\triangleq [F_{v+}, F_{c+}, F_{v-}, F_{c-}]^{\mathrm{T}} \end{aligned} \tag{40}$$

TABLE I
IDENTIFIED VALUES OF MODEL PARAMETERS USINGTHE ASYMMETRIC COULOMB + VISCOUS FRICTION MODEL

model parameter	value
$\hat{m}$	2.89 [kg]
$\hat{F}_{c+}$	3.93 [N]
$\hat{F}_{v+}$	38.5 [Ns/m]
$\hat{F}_{c-}$	3.75 [N]
$\hat{F}_{v-}$	38.6 [Ns/m]

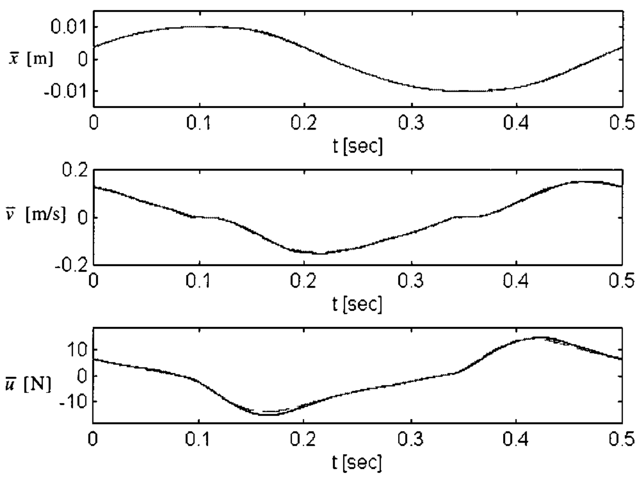

Fig. 3. Indirect evaluation of identification accuracy ($r(t) = 0.01 \sin 2\pi/0.5t$ [m]).

where

$$\mu_+(v) \triangleq \begin{cases} 1, & \text{if } v > 0 \\ 0, & \text{if } v \leq 0 \end{cases} \quad \mu_-(v) \triangleq -\mu_+(-v).$$

Here, $m, F_{v+}, F_{c+}, F_{v-},$ and F_{c-} are the model parameters to be identified. Then, it is easy to see that $\mu^*(|v|\mu_+(v), \mu_+(v), |v|\mu_-(v), \mu_-(v)) = 0$. Thus, in principle, any periodic excitation input r ensuring $\bar{v} \not\equiv 0$ works for the proposed method. Finally, the constant M and the weighting factors $\{\alpha_k\}$ in (30) are chosen, respectively, as

$$M = 10, \quad \alpha_k = |H(jk\omega)|^2, \quad H(s) \triangleq \frac{\alpha}{s+\alpha},$$
$$\alpha = 2\pi * 10 * \frac{1}{0.3}.$$

The identification results using the asymmetric Coulomb + viscous friction model are summarized in Table I.

Finally, we evaluate the accuracy of the experimental identification results in an indirect way. Using the identification results in Table I, we first determine the following simulation model for the closed-loop system consisting of the mechanical system in (1) and the PD controller in (4).

$$\hat{m}\ddot{x} + \hat{F}_s(\dot{x}) + K_d\dot{x} + K_p x = K_d\dot{r} + K_p r. \tag{41}$$

Here, the constant $\hat{m}$ and the function $\hat{F}_s$ represent, respectively, the identified values of the mass and the asymmetric Coulomb + viscous friction model with the identified values of model parameters in Table I The controller gains K_d and K_p are chosen to be the same as before. Then, the steady-state responses of the closed-loop system obtained from the experiment are compared with those obtained from the simulation using (41) in Fig. 3. In both simulation and experiment, the periodic excitation input r is chosen as $r(t) = 0.01 \sin 2\pi/0.5t$, which differs from that used for the identification of the model parameters. According to Fig. 3, the proposed method has identified the model parameters of the X axis positioning system with fairly good accuracy.

V. CONCLUSION

In this note, we have proposed a new method for the identification of mechanical systems with friction, and have demonstrated its practical use through some experimental results. The proposed method does not require the acceleration information for its implementation. Moreover, we have shown that the the feasibility of the proposed method can be ensured under natural conditions. Nonetheless, from a practical point of view, further research toward finding those excitation inputs, which can reduce the effect of the model parameterization error, should be carried out.

REFERENCES

[1] B. Armstrong-Helouvry, *Control of Machines with Friction*. Boston, MA: Kluwer, 1991.
[2] H Olsson, *Control Systems with Friction*. Lund, Sweden: Lund Inst. Technol., 1996.
[3] B. Helouvry, P. Dupont, and C. de Wit, “A survey of models, analysis tools and compensation methods for the control of machines with friction,” *Automatica*, vol. 30, no. 7, pp. 1083–1138, 1994.
[4] C. de Wit, K.J. Aström, and K. Braun, “Adaptive friction compensation in DC motor drives,” *IEEE Trans. Robot. Automat.*, vol. RA-3, Dec. 1987.
[5] J. W. Gilbart and G. C. Winston, “Adaptive compensation for an optical tracking telescope,” *Automatica*, vol. 10, no. 2, pp. 125–31, 1974.
[6] B. Friedland and Y. J. Park, “On adaptive friction compensation,” *IEEE Trans. Automat. Contr.*, vol. 37, pp. 1609–1612, Oct. 1992.
[7] W. Li and X. Cheng, “Adaptive high-precision control of positioning tables,” *IEEE Trans. Control Syst. Technol.*, vol. 2, pp. 265–270, Sept. 1994.
[8] S. C. Southward, C. J. Radcliffe, and C. R. MacCluer, “Robust nonlinear stick-slip friction compensation,” *ASME J. Dyn. Syst., Meas., Control*, vol. 113, pp. 639–645.
[9] H. S. Lee and M. Tomizuka, “Robust motion controller design for high-accuracy positioning systems,” *IEEE Trans. Ind. Electron.*, vol. 43, pp. 48–55, Oct. 1996.
[10] L. Ljung, *System Identification : Theory for the User*. Englewood Cliffs, NJ: Prentice-Hall, 1987.
[11] E. Hewitt and K. Stromberg, *Real and Abstract Analysis*. New York: Springer-Verlag, 1967.
[12] S. J. Kim and I. J. Ha, “On the existence of Carathèodory Solutions in mechanical systems with friction,” *IEEE Trans. Automat. Contr.*, vol. 44, pp. 2086–2089, Nov. 1999.
[13] R. F. Edwards, *Fourier Series : A Modern Introduction*. New York: Holt, Rinehart and Winston, 1967.
[14] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*. Englewood Cliffs, NJ: Prentice-Hall, 1989.