Single Machine Scheduling with Rejection:
Minimizing Total Weighted Completion Time and Rejection Cost

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ABSTRACT

In this paper, the authors consider a single machine scheduling problem with rejection. In traditional research, it is assumed all jobs must be processed. However, in the real-world situation, certain jobs can be rejected. In this study, the jobs can be either accepted and scheduled or be rejected at the cost of a penalty. Two objective functions are considered simultaneously: (1) minimization of the sum of weighted completion times for the accepted jobs, and (2) minimization of the sum of penalties for the rejected jobs. The authors apply two-phase method (TPM), which is a general technique to solve bi-objective combinatorial optimization problems, to find all supported and non-supported solutions for small-sized problems. The authors present a mathematical model for implementing both phases. On the other hand, three different bi-objective simulated annealing algorithms have also been developed to find a good estimation of Pareto-optimal solutions for large-sized problems. Finally the authors discuss the results obtained from each of these algorithms.

Keywords: *Computer Science, Multi-Objective Optimization, Rejection, Scheduling, Simulated Annealing, Two-Phase Method (TPM)*

INTRODUCTION

Most of traditional scheduling theories begin with a set of *n* jobs to be scheduled in a special machine environment so as to optimize a particular optimality criterion. At times, however, a higher-level decision has to be made: given a set of jobs and limited available capacity; choose only a subset of these jobs to be scheduled, while perhaps incurring some penalty for the jobs that are not scheduled. In another word, we should consider the "pre-scheduling negotiation" in which one considers the capacity of the production environment and then agrees to schedule certain jobs at the requested quality of service, while not accepting other jobs together; whether outsourcing or rejecting; in

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any case the rejection/outsourcing costs will be charged. Knapsack is a good example of the problem, where we care only about which subset to "accept". In scheduling, however, the set of accepted jobs must also be scheduled at some cost (Engels et al., 2003).

Here, the main goal is to choose the correct balance between the penalties of the jobs rejected and the increase in completion time for the accepted jobs.

This problem is essentially bi-criteria, so we can model it in three different ways: 1) lexicographical order of criteria, 2) convex combination of criteria, and 3) determination of all Pareto-optimal solutions by considering both criteria simultaneously. Most of the research cited in the section "literature review" aim at model 2. They have given approximation algorithms for on-line (that is, no information is known about the future) and off-line (all information about the instance is available at time zero) environments to solve the problem while aggregating two objective functions, in which one of them is rejection cost. In this paper we address to model 3, consider two objective functions: minimization of total weighted completion times for accepted jobs and minimization of total penalties for the rejected jobs. We are searching to find Pareto-optimal solutions so two strategies are presented: (1) searching for the exact Pareto-optimal solutions by developing an exact method, and (2) searching for estimated Pareto-optimal solutions by adapting three bi-objective meta-heuristic algorithms.

An adaptation of two-phase method (TPM) (Ulungu & Teghem, 1995) for this problem is presented in order to find all supported and non-supported solutions. As TPM is an exact method, it has some limitations for implementation regarding problem size and computational time, so by doing some modifications, we adapt three multi-objective simulated-annealing algorithms (MOSA) proposed by Mansouri (2006), Varadharajan and Rajendran (2005), and Ulungu et al. (1999) to our problem in order to find near Parto-optimal solutions for large-sized problems.

This paper is organized as follows: The next section introduces the problem, followed by the literature review. Then we describe the implementation of TPM in order to find exact Pareto-optimal solutions. Three different MOSA algorithms are presented. Numerical results are reported and discussed, and the last section provides conclusions.

PROBLEM DEFINITION

We are given n jobs j=1,..., n, each with a non-negative processing time p_j, a weight value w_j, and a rejection penalty r_j in an off-line environment. For each job j we must decide whether to accept and schedule it (on a machine that can handle only one job at a time) or to reject it. If we schedule job j, we denote its completion time by C_j. Preemption is not allowed, that is, once processing on a job has been started, we must finish it without interruption. If we reject job j, we pay its rejection penalty r_j.

Our goal is to choose a subset S of the n jobs to schedule on a single machine so as to minimize the sum of weighted completion times of the scheduled jobs and penalties of the rejected jobs. We denote the set of rejected jobs by $\bar{S}$. Engels et al. (2003) have considered the summation of these two objective functions and treated the problem as a single objective one. They have denoted the overall optimality criterion as $1\Big| \Big|\sum_S w_j C_j + \sum_{\bar{S}} r_j$. It has been shown that adding the option of rejection makes the problem NP-hard (Engels et al., 2003).

In this paper, we study these two objective functions simultaneously. First we implement two-phase method to find all exact Pareto-optimal solutions for small-sized problem, and then we adapt three meta-heuristics (MOSA) to find a set of estimated Pareto-optimal solutions for large-sized instances.

We remind that a feasible schedule σ is Pareto-optimal, or non-dominated, with respect to the performance criteria f and g in a minimization problem, if there is no feasible schedule σ' such that $f\left(\sigma'\right) \leq f\left(\sigma\right)$ and