A New Multi-Objective Genetic Algorithm for Feature Subset Selection in Fatigue Fracture Image Identification

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***Abstract*— Feature subset selection is the most important and difficult task in the field of fatigue fracture image identification. In this paper, a new method which is hybrid of linear prediction, called LP-Based Multi-Objective Genetic Algorithms (LP-MOGA) is proposed for fatigue fracture feature subset selection. In LP-MOGA, predicted new solutions with elite solutions by liner prediction to improve the local search ability. For fatigue fracture identification, texture character and fractal dimension feature are extracted for original features; and then, feature subset selection is performed by LP-MOGA, in which, the objective functions minimize error identification rate, undetected identification rate and selected featured number; at last, the identification is executed by quadratic distance classifier. Compared with other methods, the experiment results of actual data demonstrate the presented algorithm is effective.**

***Index Terms*—Multi-objective Genetic Algorithm, Liner Prediction, Feature Extraction, Feature Subset Selection, Fatigue Fracture Identification**

I. INTRODUNCTION

Fatigue fracture failure analysis is a most import part in failure analysis, and the fatigue fracture identification is a previous work. Feature subset selection is a process of choosing a small subset of features that is necessary and sufficient to describe fatigue fracture. For fatigue fracture identification, the goal of feature subset selection is to find the subset of features that produces the best identification result and requires the least computational effort. The importance of feature subset selection is due to the potential for speeding up the processes of both concept learning and classification, reducing the cost of classification, and improving the quality of classification. Feature selection has long been the focus of researchers of many fields such as pattern recognition, image understanding and machine learning [1, 2, 3, 4, and 5].

Genetic algorithm (GA) is widely used in feature subset selection [6, 7, 8, 9, 10], but in previous literatures, the fitness function of genetic algorithm is considered as a single objective function [11, 12]. In fact, the feature subset selection is a multi-objective optimization which trades off between identification rate, undetected identification rate and cost computation. In the past two decades, several Multi-objectives GA (MOGA) such as Vector Evaluated Genetic Algorithm (VEGA) [13] and Non-dominated Sorted Genetic Algorithm (NSGA) have been proposed [14]. However, it is impractical to find the true Pareto-optimal solutions of combinatorial optimization problems. In this case, a proposed approach is to improve the local search ability of MOGA to try to drive populations to true Pareto-optimal solutions as close as possible for obtaining a variety of near Pareto-optimal solutions. After optimal feature subsets are generated, ensemble classifier can be constituted.

In this paper, we present a feasible approach to fatigue fracture identification based on improved multi-objective genetic algorithm which is hybrid of linear prediction, called LP-MOGA. In LP-MOGA, near Pareto-optimal solutions, *i.e.* feature subsets, through linear prediction which makes them converge to the true Pareto-optimal front better. After that, the most accurate and diverse members, which are trained by corresponding feature subsets, are selected to constitute fatigue fracture identification model.

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TABLE I. LIST OF SYMBOLS

List of symbols	
MO: Multi-objective Optimization	**LP**: Linear Prediction
SO : Signal Objective Optimization	**GLCM**: Gray-Level Co-occurrence Matrix
GA : Genetic Algorithm	**FD**: Fracture Dimension
MOGA: Multi-objective Optimization Genetic Algorithm	**EIR**: Error Identification Rate
NSGA: Non-dominated Sorted Genetic Algorithm	**UIR**: Undetected Identification Rate

The remainder of this paper is organized as follows: Section 2 presents the multi-objective optimization principles. Section 3 describes feature extraction and the mathematical model of feature subset selection consider in this paper. In Section 4, the proposed algorithm namely LPMOGA is offered. The experimental and comparative results are offered in Section 5. Finally, the conclusions are presented in Section 6. The symbols which appear in the paper are listed in Table I.

Ⅱ.MULTI-OBJECTIVE OPTIMIZATION

Considering that we have p objective functions to be optimized (p>1), these objective functions are usually in conflict with each other. Let $f_i(x)$ be the ith objective function, where x is feasible solution. A general multi-objective function optimization problem is defined as follows:

Minimize $f_1(x)$, $f_2(x)$ …$f_i(x)$…$f_p(x)$.

Where x is feasible solution.

In general, there is no solution that minimizes all of the objective functions simultaneously. Non-dominated solution: solution x is said to dominate solution z if only if:

$$(1)\ f_i(x) \le f_i(z) \qquad \forall i \in \{1,2,...,p\}$$

$$(2)\ f_i(x) < f_i(z) \qquad \exists i \in \{1,2,...,p\}$$

Pareto optimality: Let x be a feasible solution. Solution x is called Pareto-optimal if and only there is no feasible solution that dominates solution x. Pareto-optimal solutions are also called non-interior solutions. These Pareto-optimal solutions form the Pareto front [15].

Ⅲ LP-MOGA

There are several well-known MOGAs. NSGA-II [16] is one of the most popular algorithms. NSGA-II was proposed as an improvement of NSGA [14]. Although MOGAs are able to escape from local optima by means of the crossover and mutation operator, they are weak in fine-tuning near local optimum points and disabled to find a perfect solution because of premature convergence [17, 18, and 19]. This makes the obtained solutions be not as close as possible and uniformly spread-out towards the true Pareto-optimal front. To improve the search capability of MOGAs, in this section, a new method hybrid with linear prediction, which is called LP-MOGA, is developed to improve the efficiency of existing multi-objective genetic algorithms. In the LP-MOGA, first, a liner prediction is defined. Then, in each generation of GA, predicted solutions are obtained through elite solutions (Pareto-optimal solutions) performed by liner prediction.

We denote the present elite solutions (chromosomes) and predicted chromosomes as (x_1 $x_{2...}$ x_k) and $\hat{x}$ .The linear predictor inputs the elite solutions (x_{i1} $x_{i2...}$ x_{kL}), and creates the output $\hat{x}$ as a linear combination of the previous k_L inputs. This is formulated as

$$\hat{x} = \sum_{i=1}^{k_L} \omega_i x_i \tag{1}$$

Where ω_i is the weight on the elite solutions and k_L is the linear prediction order. And the weight can be evaluated by Levinson-Durbin method [20].

The statement of LP-MOGA:

Step0: Initialize the NSGA-Ⅱ. Set the crossover and mutation probabilities, the number of generations N_{ol}, and the linear prediction order k_L and max linear prediction number N_{lp}. Set the initial generation i=0 and initial prediction number n=0.

Step1: Run the multi-objective genetic algorithm. Evaluate current population and determine the fist frontier chromosome (elite solution) of current population.

Step 2: Predict the chromosome $\hat{x}$ through the frontier chromosome using Eq. 1, n=n+1.

Step 3: Calculate the fitness values of new chromosome $\hat{x}$. If $\hat{x}$ dominates the frontier chromosome, update the Pareto frontier; else, if $n < N_{lp}$, go to step 2.

Step4: i=i+1. If i< N_{ol}, go to step 1, else go to step 5.

Step5: End.

Figure 1 shows the relationship between the NSGA-Ⅱand LP-MOGA. In this figure, P_t is parent population, Q_t is children population, and F_j is the jth frontier population in NSGA-Ⅱ algorithm.

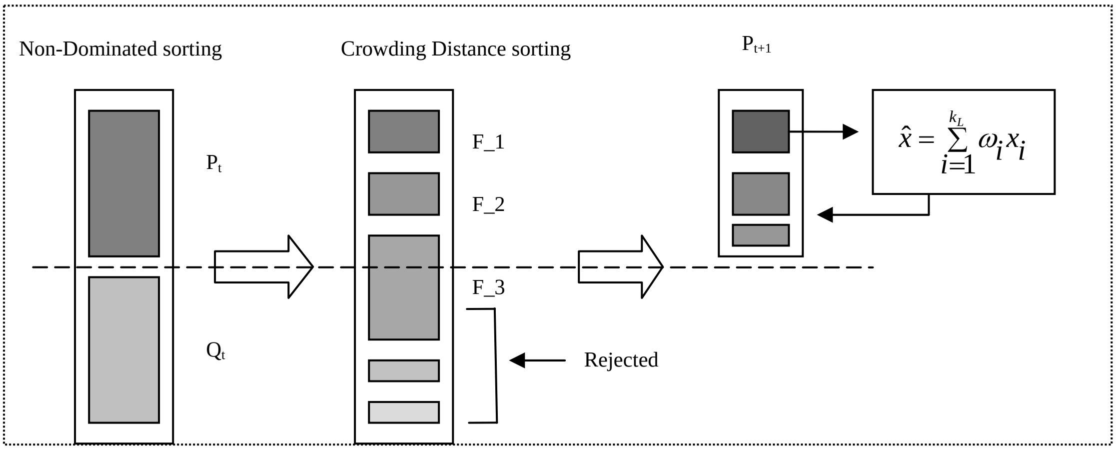

Figure 1. Relationship between the NSGA-Ⅱ and LP-MOGA

Ⅳ. PROBLEM DEFINITION

The feature subset selection in fatigue fracture identification is a multi-objective optimum problem, presented in this paper, the objectives functions should minimize error identification rate, minimize undetected identification rate and cost computation. This section first introduces the features extraction, and then, establishes mathematic mode of feature subset selection, at last feature subset selection is performed by the LP-MOGA which is mentioned in Section 3.

A Feature extraction

The goal of feature extraction is to project the original image data onto a feature space which can reflect the important inherent structure of the original images. Generally, multi-texture is contained in fracture surface image; therefore, statistic property of gray- level co-occurrence matrix (GLCM) which describes texture character can be treated as features. The concept of fractal dimension(FD), used to describe irregular surfaces with a self-similar (or self-affine) nature, provides a basis for the quantitative characterization of the tortuosity of fracture surfaces, and the potential linkage between the fractal dimension and mechanical properties[21, 22], so FD can be as features according to characterize roughness in image.

Feature 1-feature 28

The GLCM is a technique that allows for the extraction of statistical information from the image regarding the distribution of pairs of pixels. It is computed by defining a direction and a distance(d), and pairs of pixels separated by this distance, computed across the defined direction(θ),are analyzed. A count is then made of the number of pairs of pixels that possess a given distribution of gray-level values. There may be many GLCM computed for an image, one for each pair of distances and directions defined. In this paper, a set of 8 GLCM are computed, for two different distances 1 and 3, in the horizontal, vertical, and two diagonal directions. The GLCM $\boldsymbol{G}=[g_{\mathrm{GLCM}}(m,n)]$ is

$$g_{GLCM}(m,n) = P\Big(f(x,y)=m,\ f\big(x+d\cos\theta, y+d\sin\theta\big)=n\Big) \quad (2)$$

In Eq. 2, f (x, y) is the image gray-level at the point (x, y).

The statistic property of GLCM are Angular Second Moment (ASM), Contrast(CON), Correlation (COR),Variance(VAR), Inverse Difference Moment(IDM), Sum Average (SA), Sum Variance(SV), Sum Entropy(SE), Entropy (ENT), Difference Variance (DV), Difference Entropy(DE), Information Measure of CorrelationⅠ(IMCⅠ), Information Measure of CorrelationⅡ(IMCⅡ) and Maximal Correlation Coefficient(MCC). The mathematic expression for each of these can be found in [23]. When d=1, θ=0°,45°,90°,135°, the mean value of each statistic property respectively in four directions is composed of feature1-feature14; similarity, when d=3, θ=0°,45°,90°,135°, the mean value of each statistic property respectively in four directions is composed of feature15-feature28.

Feature 29-feature 33

Mandelbrot et al. [21, 22] first introduced fractal dimension concept to materials science in 1984.FD is a feature proposed to characterize roughness in image. Mandelbrot [21] stated that the FD of a set A in Euclidean n-space can be derived from the relation

$$N_r . r^{FD} = 1 \quad (3)$$

or

$$FD = \lim_{r\to 0} \log(N_r)/\log(1/r) \quad (4)$$

Where N_r, is the union of nonoverlapping copies of A scaled down by a ratio r. However, it is difficult to compute FD by using Eq.3 directly. Sarkar and Chaudhuri[23] described an efficient box-counting approach, named Differential Box-Counting (DBC) that uses differences on computing N_r, and gives satisfactory results in all range of FD. The FD in this method is given by Eq. 4 where N_r is counted in a different manner from the others box-counting methods.

Consider that the image of size $M\times M$ pixels has been scaled down to a size $s\times s$, where $M/2 > s > 1$ and s is an integer. Then we have an estimate of $r = s / M$. Now, consider the image as a 3-D space with (x, y) denoting

2-D position and the third coordinate (z) denoting gray-level. The (x, y) space is partitioned into grids of size $s \times s$. On each grid there is a column of boxes of size $s \times s \times s'$. If the total number of gray levels is G, then [G/s'] = [M/s]. Assign number 1, 2 ... n to the boxes as shown in Figure 2. If the minimum gray-level of the image in the grid (i,j) felled in number k box, and the maximum gray level of the same grid felled in number p box, then

$$n_r(i,j)=p-k+1 \, , \; N_r = \sum n_r(i,j) \tag{5}$$

N_r is counted for different values of r and s. Then applying Eq. 4, FD can be estimated from the least squares linear fit of log (N_r) against log (l/r).

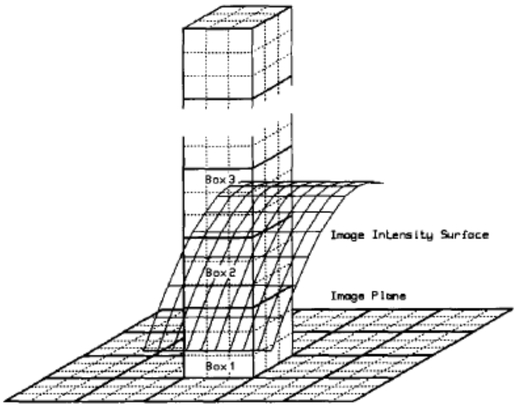

Figure 2 . Determination of Nr by DBC method

We propose to use five features in order to discriminate directional information. They are based on the FD of the original image (feature 29), the horizontally smoothed image (feature 30), the vertically smoothed image (feature 31), diagonally smoothed image (feature 32, feature 33). The smoothed image at different direction (θ) in windows ($2\omega+1\times2\omega+1$) is described as:

$$I_2(i,j)=\frac{1}{2\omega+1}\sum_{k=-\omega}^{\omega} I(i,j+k), \theta=0° \tag{6}$$

$$I_3(i,j)=\frac{1}{2\omega+1}\sum_{k=-\omega}^{\omega} I(i+k,j), \theta=90° \tag{7}$$

$$I_4(i,j)=\frac{1}{2\omega+1}\sum_{k=-\omega}^{\omega} I(i+k,j+k), \theta=45° \tag{8}$$

$$I_5(i,j)=\frac{1}{2\omega+1}\sum_{k=-\omega}^{\omega} I(i+k,j-k), \theta=135° \tag{9}$$

After feature extraction, 33 raw features are obtained, which detailed on Table Ⅱ.

TABLE Ⅱ. RAW FEATURE SET

number	feature	parameter	number	feature	parameter
feature 1	ASM		feature 18	VAR	
feature 2	CON		feature 19	IDM	
feature 3	COR		feature 20	SA	
feature 4	VAR	d=1	feature 21	SV	d=1
feature 5	IDM	θ=0°	feature 22	SE	θ=0°
feature 6	SA	θ=45°	feature 23	ENT	θ=45°
feature 7	SV	θ=90°	feature 24	DV	θ=90°
feature 8	SE	θ=135°	feature 25	DE	θ=135°
feature 9	ENT		feature 26	IMCⅠ	
feature 10	DV		feature 27	IMCⅡ	
feature 11	DE		feature 28	MCC	
feature 12	IMCⅠ		feature 29	FD	
feature 13	IMCⅡ		feature 30	FD	θ=0°
feature 14	MCC		feature 31	FD	θ=90°
feature 15	ASM		feature 32	FD	θ=45°
feature 16	CON		feature 33	FD	θ=135°
feature 17	COR				

B. Mathematic Model

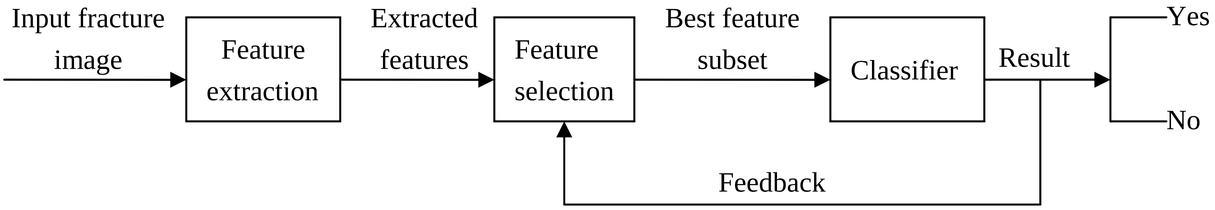

Figure 3. The framework of fatigue fracture image identification

Fatigue fracture image identification is a two-class judgment problem, the inputs are valid feature set, and the output is Yes/No, that is either fatigue fracture image or not. The fatigue fracture image identification framework is show in figure 3. The classifier is fixed in identification, which is a quadratic distance classifier, but the set of features that is input into the classifier is variable. In feature subset selection for classification, the task is discerning out of the several valid features of fracture images to be used for efficient classification. For the fatigue fracture identification, more features means more complex model, hence, lower error identification rate and lower undetected identification rate is expected. We propose the following objective functions for LP-MOGA to minimum, namely minimizing error identification rate (EIR) n_e, undetected identification rate (UIR) n_u and cost computation.

Mathematic model:

chromesome: C_i is a chromosome coding the selected set of features, L is the total number of features extracted from each fracture image, k is the number of features selected (C_i has k bits of 1 and L - k bits of 0).

Multi-objective function: It is easy to see that the fewer the number of features selected, the smaller the number of images misclassified and smaller number of undetected, the classification is better. Thus, the objective functins minimize ($f_1(x)$, $f_2(x)$ $f_3(x)$), and $f_1(x)$=-log(k), $f_2(x)$=log(n_e), $f_3(x)$= log(n_u), and n_e is the number of images misclassified, and n_u is the number of fatigue fracture image judged to be not fatigue.

n_e : Assumed that the fatigue fracture is classed to be Y_1 , non-fatigue fracture class to be Y_2, features subsets are (f_{k1} , f_{k2} , ⋯, f_{kM}, k=1,2, M is the training images number of Y_k), thus, the mean and variance are:

$$\mu_k = \frac{1}{M}\sum_{j=1}^{M} f_{kj}, \Sigma = \frac{1}{M}\sum_{j=1}^{M}\left(f_{kj}-\mu_k\right)\left(f_{kj}-\mu_k\right)^T \quad (10)$$

For a certain images, if the selected feature is f, the quadratic distance between feature vector and class Y_k is:

$$d_k = \left(f-\mu_k\right)^T \Sigma_k^{-1}\left(f-\mu_k\right) \quad (11)$$

For each image, if $d_1 < d_2$, it is judged to be fatigue fracture image, else, to be non-fatigue fracture image. Therefore, n_e can be defined as the total number judging Y_i as Y_j (i,j=1,2; $i \neq j$).

n_u :the total numbers that judging Y_1 to Y_2.

Other parameters:

Population size:	120	Probability of crossover:	0.7
Number of generation:	400	Probability of mutation:	0.1

C. Feature Subset Selection Based on LP-MOGA

The main procedures of feature subset selection by LP-MOGA algorithm are described as follows.

Procedure:

1. Generate randomly an initial population P_0, t=0;
2. Create a children population Q_0 of size n;
3. Combine parent and children population $R_t=P_t \cup Q_t$;
4. Evaluate the populations;
5. Generate all non-dominated fronts F=(F_1,F_2,…) of R_t, let number of F_i front chromosome be h_i ;
6. Predict p_1 new chromosomes V_{t1} through F_1 frontier chromosomes and p_2 new chromosomes V_{t2} through F_2 frontier chromosomes by liner predict which motioned in Section 3, and p_1 =0.2*h_1, p_2 =0.2*h_2, $R_t=P_t \cup Q_t \cup V_{t1} \cup V_{t2}$;
7. Generate all non-dominated fronts F=(F_1,F_2,…) of R_t;
8. Sort the non-dominated fronts. $i \prec j$, if (($i_{rank} < j_{rank}$) or (($i_{rank} = j_{rank}$) and ($i_{distance} > j_{distance}$))) ;
9. Choose the best solutions needed to fill the population;
10. Use selection, crossover and mutation to create a new population Q_{t+1}, t=t+1;
11. If the maximum number of generations is not reached, go to (3), else go to (12);
12. End.

Ⅴ EXPERIMENT

A. Database

The fracture images used in this work were taken from the real failure analysis experiment. The database consists of a set of 242 samples, in which 82 fatigue fracture images, 160 non-fatigue fracture images such as cleavage, intergranular and dimple. Examples for each class fracture surface images are show in Figure 4.

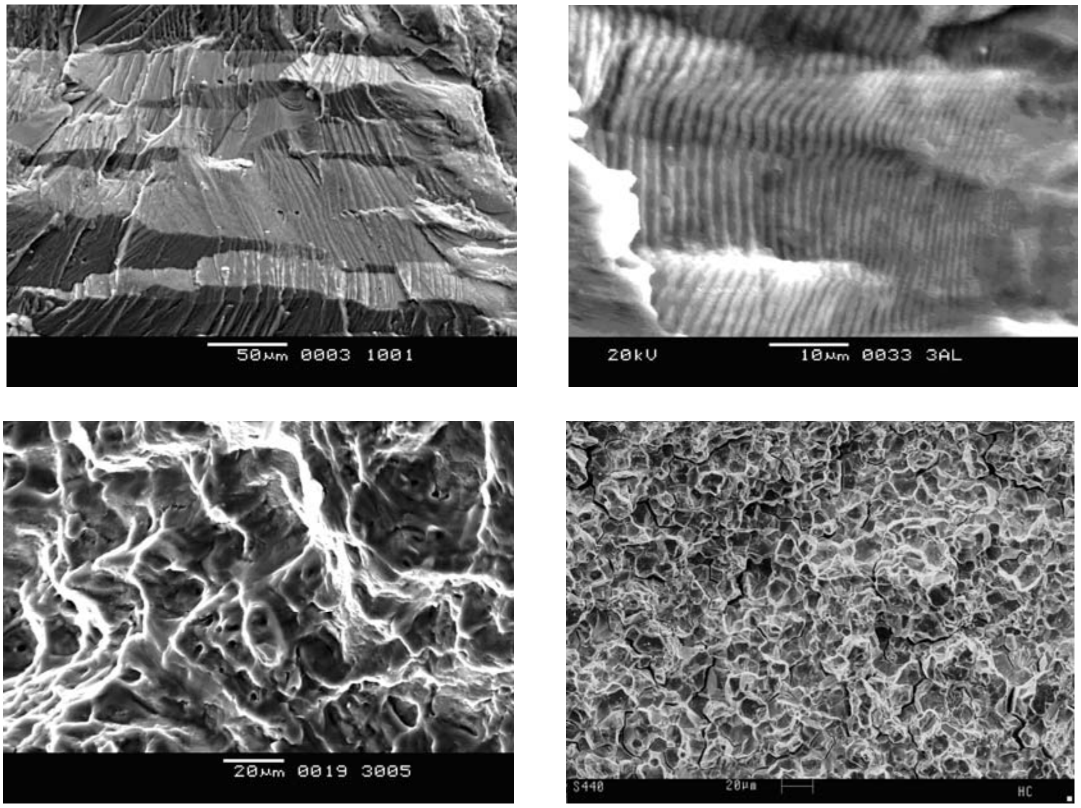

Figure 4. Metal fracture surface images (from left to right, top to bottom, followed by cleavage, fatigue, dimple, intergranular)

B Experiment

In accordance with the established program, the process of the experiment may be summarized by: first, randomly selected 42 fatigue images and 80 non-fatigue images as training database, while the remaining images as test database. Then, extracted features mentioned in section 4 for each training image, and then, the optimal feature subset sequence obtained by performing LP-MOGA to entire training images features set. Feature subset selection using the proposed LP-MOGA based on training database, Experiment execute 10 times independently, the optimal feature subset sequence is [2 3 5 8 11 13 24 26 28 29 30 31 33]. The classification results by using quadratic distance classifier are listed at Table Ⅲ. In order to compare, the classification by all features is also list in Table Ⅲ.

TABLE Ⅲ. THE FATIGUE FRACTURE IMAGE IDENTIFICATION RESULT

	Training database			Test database		
	n_e	n_u	k	n_e	n_u	k
Optimal feature subset	9	1	13	9	0	13
All feature	20	6	33	17	4	33

From the table above, we can see that identification by all features is worse than the feature subset selected by LP-MOGA, because of redundancy between the features.

C. Compared with other GA

In previous literatures [11, 12], the feature selection is viewed as signal objective optimum. Considering the same factors as in MOGA, the fitness function of SO can be modified as Eq. 12

$$F(f) = -\left(q\log_{10}(L) + n_e\log_{10}(n) + n_u\log_{10}(m)\right) \quad (12)$$

Feature selection preformed by SGA, the feature subset and the identification result are shown in Table Ⅳ. The NSGA-Ⅱ is a classical MOGA, so, feature selection by NSGA-Ⅱ is also performed and the result is listed in Table Ⅳ.

TABLE Ⅳ COMPARED THE FATIGUE FRACTURE IMAGE IDENTIFICATION RESULT BASED ON LP-MOGA AND SGA

Methods	Optimal feature subset	Training database			Test database		
		n_e	n_u	k	n_e	n_u	k
LP-MOGA	[2 3 5 8 11 13 24 26 28 29 30 31 33]	9	1	13	9	0	13
NSGA-Ⅱ	[2 4 6 8 11 12 24 26 27 29 30 31 33]	10	4	13	9	2	13
SGA	[1 4 5 6 8 11 24 26 28 29 32]	12	6	11	11	4	11

From table Ⅳ, it can conclude that the feature subset selected by MO is better than SO, that is to say, the feature subset selection problem is a multi-objective optimization. Compared with NSGA-Ⅱ, the selected features number is the same, but the number of the misclassified and undetected images is less, so we can conclude that the LP-MOGA is better than NSGA-Ⅱin feature selection which used in fatigue fracture image identification.

Ⅵ. CONCLUSION

In this paper, we proposed newly LP-MOGA, and introduced LP-MOGA feature subset selection algorithm into a specific application domain to identify the fatigue fracture image from the fracture surface images. Feature extraction, LP-MOGA feature selection and final discrimination are successfully implemented and good results are obtained. Our experimental results show that the LP-MOGA selected a good subset of features. Our experimental results show that it balances the number of features selected and the error rate very well. In the future, we will plan to extend this approach to additional features and more complex background clutter.

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