

TIME-VARYING FORMATION FINITE-TIME TRACKING CONTROL FOR MULTI-UAV SYSTEMS UNDER JOINTLY CONNECTED TOPOLOGIES*

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Abstract

Purpose- This paper aims to investigate time-varying finite-time formation tracking control problem for multiple unmanned aerial vehicle (multi-UAV) systems under switching topologies, where the states of the UAVs need to form a desired time-varying formations while tracking the trajectory of the virtual leader in finite time under jointly connected topologies.

Design/Methodology/Approach- A consensus-based formation control protocol is constructed to achieve the desired formation. In this paper, the time-varying formation is specified by a piecewise continuously differentiable vector, while the finite-time convergence is guaranteed by utilizing a nonlinear function. Based on graph theory, the finite time stability of the close-loop system with the proposed control protocol under jointly connected topologies is proven by applying LaSalle's invariance principle and the theory of homogeneity with dilation.

Findings- The effectiveness of the proposed protocol is verified by numerical simulations. Consequently, the proposed protocol can successfully achieve the predefined time-varying formation in finite time under jointly connected topologies while tracking the trajectory generated by the leader.

Originality/value- This paper proposes a solution to simultaneously solve the control problems of time-varying formation tracking, finite-time convergence, and switching topologies.

Keywords multi-UAV system; formation tracking control; finite time; jointly connected topologies

Paper type Research paper

1. Introduction

In recent years, formation control for unmanned aerial vehicle (UAV) systems has attracted considerable research interest, and has performed well in a variety of areas, such as target seeking, telecommunication relay, environment monitoring, earthquake rescue, and so on. Compared with single UAV working, multi-UAV coordinated formation working has overwhelming superiority of high efficiency in accomplishing tasks, strong robustness and better flexibility (Xue *et al.*, 2016). The main challenges in formation control for multi-UAV systems are switching communication network, the practical demand of time-varying formation, large time delay, and the rigid requirement of fast convergence speed. Therefore, how to deal with these challenges has become the common concern of researchers from the control engineering and robotics communities.

Three typical approaches, namely, leader-follower based (Wang, 1991; Das *et al.*, 2002), behavior based (Balch and Arkin, 1998) and virtual structure based (Lewis and Tan, 1997) approaches have been applied in the formation control problems. However, according to Beard *et al.*, (2001), all these three methods have their own drawbacks: the leader-following based strategies are lack of robustness with respect to the possible failure of the leader, the behavior based approaches are difficult to analyze mathematically and the stability cannot generally be guaranteed, while the virtual structure based methods are not suitable for distributed implementation as they leads to a heavy burden on computations and communications. With the development of consensus control for multi-agent systems, increasing researchers have extended consensus theory to formation control and numerous results have been obtained (Abdessameud and Tayebi, 2011; Cao *et al.*, 2010; Du *et al.*, 2013; Dong and Hu, 2016; Dong *et al.*, 2016a, b; El-Hawwary, 2015; He *et al.*, 2016; Ge *et al.*, 2016; Guo *et al.*, 2014; Guzey *et al.*, 2015; Liu and Jiang, 2013; Mylvaganam and Astolfi, 2015; Nair and Behera, 2015; Ou *et al.*, 2012; Rahimi *et al.*, 2014; Seo *et al.*, 2012; Tian and Wang, 2013; Wang *et al.*, 2014; Wang *et al.*, 2016; Xiao *et al.*, 2009; Xu *et al.*, 2015). Since the consensus based formation control approaches are fully distributed and only require the local neighbor-to-neighbor information between two agents, they possess strong robustness and high flexibility. Moreover, Ren (2007) showed that many existing leader-follower based, behavior based and virtual structure based formation control approaches can be unified in the framework of consensus based approaches. Hence, the defects of the three traditional formation control schemes can be overcome.

It should be pointed out that the formations in the great majority of the mentioned results are time-invariant. However, in many practical applications, such as target seeking, forming time-varying formations are often required. Several results on time-varying formation have been found in Dong and Hu (2016), Dong *et al.*, (2016a, b), He *et al.*, (2016), Rahimi *et al.*, (2014), and Wang *et al.*, (2016). Noting that the results for time-invariant formation cannot be directly applied to time-varying formations as the

derivative of the time-varying formation information may increase complexity of control law design and system stability analysis in time-varying formation control, it is more meaningful to study the time-varying formation control. Besides, the formation may also need to track the trajectory generated by the virtual/real leader to perform a task, so the time-varying formation tracking problem arises, where a group of UAVs keep the desired time-varying formation while tracking the trajectory of the leader.

It is well known that the interactions among UAVs may switch due to the communication failures or new creations of communication links in practice. Recently, time-varying formation control under switching topologies was studied in Dong *et al.*, (2016a, b). However, Dong *et al.*, (2016a, b) required that each of the possible topologies is connected, which implies that the topologies always maintain connected although they are switching. Since the jointly connected topologies do not require connection all the time, the jointly connected network is actually a more general condition for time-varying formation control.

As for system stability, it can be noted that most of the aforementioned results for the formation control of multi-UAVs or multi-agents are about asymptotical convergence. However, finite-time stability is more desirable compared with asymptotical stability. Besides faster response rates, the closed-loop systems under finite-time control usually demonstrate higher accuracy, better disturbance rejection property, and stronger robustness against uncertainties (Du *et al.*, 2011; Li *et al.*, 2008; Ou *et al.*, 2014; Zhao *et al.*, 2010). Considering such superiorities, some finite-time formation control algorithms were proposed in Du *et al.*, (2013), Cao *et al.*, (2010), Ge *et al.*, (2016), Nair and Behera (2015), Ou *et al.*, (2014), Xiao *et al.*, (2009) and Xu *et al.*, (2015). However, there are no results available to treat the time-varying formation tracking control problem for multi-UAV systems in finite time under jointly connected topologies, which this paper exactly investigates.

Compared with the previous relevant results, the main contributions of this paper can be summarized as follows. Firstly, an integrated protocol to simultaneously solve the problems of time-varying formation tracking, finite-time convergence and switching topologies is proposed in this paper. Secondly, the finite-time stability of the close-loop system is obtained based on LaSalle's invariance principle, the theory of homogeneity with dilation and graph theory as well. Finally, numerical simulations are constructed to verify the effectiveness of the proposed control protocol.

The rest of the paper is organized as follows. Graph theory and the problem formulation are given in Section 2. The control protocol to realize the time-varying formation finite-time tracking control system under jointly connected topologies is proposed and the stability analysis is studied in Section 3. Numerical simulation results are presented in Section 4 to illustrate the effectiveness of the proposed control law. Conclusions are drawn in Section 5.

2. Graph Theory and Problem Description

In this section, the basic concepts on graph theory and the problem description will be introduced.

2.1. Graph theory

In this paper, the multi-UAV system comprising n UAVs can be regarded as a multi-agent system, while the interaction network can be modelled by a graph denoted by $G=(V,E,A)$, where $V=\{v_1,v_2,\dots,v_n\}$, $E\subseteq V\times V$, and $A=[a_{ij}]\in R^{n\times n}$ are the set of nodes, the set of edges, and the associated adjacency matrix, respectively. In addition, $I=\{1,2,\dots,n\}$ represents the number of the nodes. If $(v_j,v_i)\in E$, $j\neq i$, then node v_i can receive the information from node v_j , and vice versa. Moreover, $a_{ij}>0$, $j\neq i$ if and only if $(v_j,v_i)\in E$, $j\neq i$, and $a_{ij}=0$ otherwise. Besides, $a_{ii}=0$. A graph is an undirected digraph if and only if $a_{ij}=a_{ji}$. $N_i=\{v_j\in V:(v_j,v_i)\in E\}$ denotes the neighbor set of node v_i . Let $D=\text{diag}\{d_1,d_2,\dots,d_N\}\in R^{N\times N}$ denote the in-degree matrix, where $d_i=\sum_{v_j\in N_i}a_{ij}$. Define $L=D-A$ as the Laplacian matrix of the graph G . A path from node v_i to v_j is a sequence of edges $\{(v_i,v_k),(v_k,v_l),\dots,(v_m,v_j)\}$. An undirected graph G is called connected if there is at least one node having at least a path to every other nodes. Furthermore, suppose that there is a virtual leader denoted as node v_0 , then we have an augmented graph \bar{G} whose node set is $\bar{V}=\{v_0,V\}$. The connected weight from the i th UAV to the virtual leader is denoted by b_i , and $b_i>0$ if and only if there exists an edge between the i th UAV and the leader. The union graph \bar{G}_{1-m} of a collection of undirected graphs $\bar{G}_1,\bar{G}_2,\dots,\bar{G}_m$, $m\geq 1$ with the same node set \bar{V} is a graph whose node set is \bar{V} and the edge set is the union of the edge sets of graphs $\bar{G}_1,\bar{G}_2,\dots,\bar{G}_m$, and the connected weight between node v_i and v_j is the sum weight of $\bar{G}_1,\bar{G}_2,\dots,\bar{G}_m$. The collection of the graphs $\bar{G}_1,\bar{G}_2,\dots,\bar{G}_m$ is called jointly connected if \bar{G}_{1-m} is connected.

Note that the interaction topology may dynamically change in reality, so it is necessary to consider all of the possible topologies. Define $\sigma:[0,\infty)\rightarrow P=\{1,2,\dots,p\}$ as the piecewise constant switching function, where p denotes the number of the possible graphs. Denotes $\bar{G}_{\sigma(t)}$, $a_{ij}(t)$ and $b_i(t)$ as the topology graph at time t , the time varying versions of a_{ij} and b_i , respectively. Consider a finite sequence of non-overlapping bounded and contiguous time intervals $[t_k,t_{k+1})$, $k=0,1,2,\dots$ with $t_0=0$, $0<t_{k+1}-t_k\leq T_1$, $T_1>0$. Suppose that in each time interval $[t_k,t_{k+1})$ there exists a sequence of non-overlapping subintervals $[t_k^0,t_k^1),\dots,[t_k^s,t_k^{s+1}),\dots,[t_k^{m_k-1},t_k^{m_k})$ with $t_k^0=t_k$, $t_k^{m_k}=t_{k+1}$ satisfying $t_k^{s+1}-t_k^s\geq T_2>0$, $0\leq s\leq m_k-1$ for some integer $m_k>0$. Meanwhile, the interaction graph $\bar{G}_{\sigma(t)}$ is supposed to be invariant in each time subinterval $[t_k^s,t_k^{s+1})$, and to switch at time t_k^s .

2.2. Problem description

The dynamics of an UAV can be classified into the trajectory dynamics and the attitude dynamics. Since the dynamics of attitude are much faster than those of trajectory, the formation control can be decoupled into an inner-loop control and an outer-loop control,

where the inner-loop controller stabilizes the attitude while the outer-loop controller drives the UAV towards the desired position (Bayezit and Fidan, 2013; Dong *et al.*, 2016b). The two-loop control structure schematic diagram for a UAV is depicted in Fig. 1 (Dong *et al.*, 2016a). In this paper, the formation control is mainly concerned with the position and velocity in the outer-loop where the UAV can be regarded as a point-mass system, while the inner-loop can be controlled by a PD controller (Dong *et al.*, 2016b; Wang and Xin, 2013; Seo *et al.*, 2012). The outer-loop dynamics of UAV $i \in I$ can be approximately described as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t), \end{cases} \quad (1)$$

where $x_i(t) \in R^m$, $v_i(t) \in R^m$ and $u_i(t) \in R^m$ represent the position, velocity and control input vectors of UAV i , respectively.

Figure 1 here

In this paper, the expected time-varying formation is specified by the vector $h(t) = [h_1^T(t), h_2^T(t), \dots, h_n^T(t)]^T$, where $h_i(t) = [h_{ix}^T(t), h_{iv}^T(t)]^T \in R^{2m}$ is a piece-wise continuously differentiable function vector with $h_{ix}(t) \in R^m$, $h_{iv}(t) \in R^m$ and $h_{iv}(t) = \dot{h}_{ix}(t)$. Denote $\xi_0(t) = [x_0^T(t), v_0^T(t)]^T$ as the state of virtual leader with $x_0(t) \in R^m$, $v_0(t) \in R^m$ and $v_0(t) = \dot{x}_0(t)$, and $\xi_i(t) = [x_i^T(t), v_i^T(t)]^T$. For simplicity, let $m=1$ without loss of generality, since the analyses and the results can be easily extended to higher dimensional case by using the Kronecker product.

Definition 2.1 The multi-UAV system (1) is said to achieve time-varying formation if for any given bounded initial states $\xi_i(0)$, the state ξ_i satisfies

$$\lim_{t \rightarrow \infty} (\xi_i(t) - \xi_0(t) - h_i(t)) = 0_{2 \times 1}, \quad i = 1, 2, \dots, n, \quad (2)$$

Remark 2.1 The assumption of the existence of the virtual leader is reasonable, because the trajectory of the virtual leader we need to track can be regarded as that of a specified real UAV with no external control inputs whose dynamics are described by

$$\begin{cases} \dot{x}_0(t) = v_0(t) \\ \dot{v}_0(t) = g(t), \end{cases} \quad (3)$$

where $g(t)$ is a continuously differentiable function. Therefore, the time-varying formation control problem is transformed into time-varying formation tracking problem.

To make the definition more comprehensible, we take a time-invariant formation of square as an example, which is shown in Fig.2. Consider a multi-UAV system containing four UAVs, which means $I = \{1, 2, 3, 4\}$. Since the formation is invariant, $h_i(t)$ is equal to h_i . Choose the formation center as the virtual leader. If equation (2) in Definition 2.1 is satisfied, and $\|h_i\|_2 = \|h_j\|_2, \forall i, j \in I$ as showed in Fig.2, where $\|\cdot\|$ denotes the Euclidian 2-norm, the square formation is achieved. It is worth noting that $h_i(t)$ represents the relative offset vector of $\xi_i(t)$ with respect to $\xi_0(t)$ if the condition in

Definition 2.1 is satisfied. Therefore, we can achieve any desired formation by choosing the appropriate $h_i(t)$.

Figure 2 here

Next, the paper will focus on the time-varying formation tracking control protocol design and finite-time stability analysis under switching topologies.

3. Protocol Design and Stability Analysis of Time-varying Formation Control

In this section, the time-varying formation tracking control protocol to achieve time-varying formation finite-time tracking will be firstly proposed, and then sufficient conditions and stability analysis under switching topologies for system (1) are presented.

Lemma 3.1.(Sun *et al.*, 2012) Let $\xi=[\xi_i] \in R^n$, $\zeta=[\zeta_i] \in R^n$, and suppose that matrix $C=[c_{ij}] \in R^{n \times n}$ is symmetric. If $f: R \rightarrow R$ is an odd function, then

$$\sum_{i=1}^n \sum_{j=1}^n c_{ij} \xi_i f(\zeta_i - \zeta_j) = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n c_{ij} (\xi_i - \xi_j) f(\zeta_i - \zeta_j). \quad (4)$$

Next, the theory of homogeneity with dilation is given for the finite-time convergence analysis.

For the following system,

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}), \quad \mathbf{x} = (x_1, x_2, \dots, x_n)^T \in R^n \quad (5)$$

a continuous vector field $f(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_n(\mathbf{x}))^T$ is homogeneous of degree $\kappa \in R$ with the dilation $\mathbf{r} = (r_1, r_2, \dots, r_n)$, if for any $\forall \varepsilon > 0$,

$$f_i(\varepsilon^{r_1} x_1, \varepsilon^{r_2} x_2, \dots, \varepsilon^{r_n} x_n) = \varepsilon^{\kappa+r_i} f_i(\mathbf{x}), \quad i=1, 2, \dots, n. \quad (6)$$

System (5) is called homogeneous if its vector field is homogeneous.

Lemma 3.2. (Wang and Hong, 2010) Suppose that system (5) is homogeneous of degree $\kappa \in R$ with the dilation $\mathbf{r} = (r_1, r_2, \dots, r_n)$, the function $f(\mathbf{x})$ is continuous and $\mathbf{x} = \mathbf{0}$ is asymptotically stable. If the homogeneous degree $\kappa < 0$, then the equilibrium of system (5) is finite-time stable.

Assumption 3.1. The interaction graph $\bar{G}_{\sigma(t)}$ is invariant in each time subinterval $[t_k^s, t_k^{s+1})$, and switches at time t_k^s . Moreover, the collection of graphs in each interval $[t_k, t_{k+1})$ is jointly connected.

In Dong *et al.*, (2016a), the proposed time-varying formation control cannot guarantee convergence in finite time and demands the switching topologies to keep connected all the time. Aiming at the above weakness of the control protocol in Dong *et al.*, (2016a), we propose the following protocol to solve the time-varying formation in finite-time problem under the jointly-connected topologies.

$$\begin{aligned}
u_i(t) = & \sum_{v_j \in N_i} a_{ij}(t) \left[\text{sig} \left[x_j(t) - x_i(t) - h_{jix}(t) \right]^{\alpha_1} + \text{sig} \left[v_j(t) - v_i(t) - h_{jiv}(t) \right]^{\alpha_2} \right] \\
& - b_i(t) \left[\text{sig} \left(x_i(t) - x_0(t) - h_{ix}(t) \right)^{\alpha_1} + \left(\text{sig} \left(v_i(t) - v_0(t) - h_{iv}(t) \right)^{\alpha_2} \right) \right] + \dot{v}_o(t) + \dot{h}_{iv}(t)
\end{aligned} \tag{7}$$

where $h_{jix}(t) = h_{jx}(t) - h_{ix}(t)$, $h_{jiv}(t) = h_{jv}(t) - h_{iv}(t)$, $\text{sig}(\cdot)^\alpha = |\cdot|^\alpha \text{sgn}(\cdot)$, $\text{sgn}(\cdot)$ is the sign function, and $\alpha_1, \alpha_2 > 0$.

Remark 3.1 In the right side of (7), the first two items represent the tracking errors of relative positions and velocity among neighbor UAVs, while the third and fourth items denote the ones between the following UAVs and the virtual leader. Such a control protocol can guarantee a satisfactory formation during both transient and steady processes. Besides, $\dot{v}_o(t)$ and $\dot{h}_{iv}(t)$ can be considered as two feedforward control items, thus fast and stable formation can be achieved by the whole feedforward-feedback control structure.

Remark 3.2 By applying the nonlinear function $\text{sig}(\cdot)^\alpha$ to the control law, the finite-time stability of the close-loop system is realized. At the same time, the traditional linear control is turned into nonlinear control which leads to a better control performance.

Remark 3.3 In order to adjust the settling time of the close-loop system, we can add the control gains $k_1, k_2, k_3, k_4 > 0$ to the control law (7), such that the convergence process can be faster by choosing the appropriate k_1, k_2, k_3, k_4 . Thus, the control law can be further designed as

$$\begin{aligned}
u_i(t) = & \sum_{v_j \in N_i} a_{ij}(t) \left[k_1 \text{sig} \left[x_j(t) - x_i(t) - h_{jix}(t) \right]^{\alpha_1} + k_2 \text{sig} \left[v_j(t) - v_i(t) - h_{jiv}(t) \right]^{\alpha_2} \right] + \dot{h}_{iv}(t) \\
& - b_i(t) \left[k_3 \text{sig} \left(x_i(t) - x_0(t) - h_{ix}(t) \right)^{\alpha_1} + \left(k_4 \text{sig} \left(v_i(t) - v_0(t) - h_{iv}(t) \right)^{\alpha_2} \right) \right] + \dot{v}_o(t).
\end{aligned} \tag{8}$$

It should be pointed out that k_1, k_2, k_3, k_4 are dispensable for system (1) to achieve some time-varying formations.

Remark 3.4 The values of a_{ij} and b_i represent the confident degree on the formation received from the neighbors or the leader. For simplicity, choose $a_{ij} = 1$ or 0 and $b_i = 1$ or 0 in this paper. In addition, the following theorem will give an approach to choose the values of α_1 and α_2 .

Next, we will explore the stability analysis together with the method to design the values of α_1 and α_2 .

Theorem 3.1. Suppose Assumption 3.1 holds. Then under the control of protocol (7) with $0 < \alpha_1 < 1$, $\alpha_2 = 2\alpha_1/(\alpha_1 + 1)$, time-varying formation in finite time for multi-UAV system (1) with switching topologies can be achieved.

Proof. Define $\bar{x}_i(t) = x_i(t) - x_0(t) - h_{ix}(t)$ and $\bar{v}_i(t) = v_i(t) - v_0(t) - h_{iv}(t)$ as formation tracking position and velocity errors, respectively, then system (1) can be written as

$$\begin{cases} \dot{\bar{x}}_i(t) = \bar{v}_i(t) \\ \dot{\bar{v}}_i(t) = u_i - \dot{v}_o(t) - \dot{h}_{iv}(t). \end{cases} \quad (9)$$

Substituting protocol (7) into (9) leads to

$$\begin{cases} \dot{\bar{x}}_i(t) = \bar{v}_i(t) \\ \dot{\bar{v}}_i(t) = \sum_{v_j \in N_i} a_{ij}(t) \left[\text{sig}(\bar{x}_j - \bar{x}_i)^{\alpha_1} + \text{sig}(\bar{v}_j - \bar{v}_i)^{\alpha_2} \right] - b_i(t) \left[\text{sig}(\bar{x}_i)^{\alpha_1} + \text{sig}(\bar{v}_i)^{\alpha_2} \right]. \end{cases} \quad (10)$$

In the following, two steps are taken to prove the finite-time stability of the closed-loop system (10).

Firstly, we prove that the close-loop system (10) is globally asymptotically stable under control law (7).

Obviously, in each time interval $[t_k, t_{k+1})$, $k = 0, 1, 2, \dots$, there are at most $N = T_1/T_2$ subintervals during which the topology graph $\bar{G}_{\sigma(t)}$ maintains invariant. In each time interval $[t_k, t_{k+1})$, take a Lyapunov function candidate

$$V = \frac{1}{2} \sum_{i=1}^n \bar{v}_i^2 + \sum_{i=1}^n \int_0^{\bar{x}_i} b_i(t) \text{sig}(s)^{\alpha_1} ds + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \int_0^{\bar{x}_i - \bar{x}_j} a_{ij}(t) \text{sig}(s)^{\alpha_1} ds \quad (11)$$

Since $a_{ij}(t) = a_{ji}(t)$ stays invariant in time subinterval $[t_k^s, t_k^{s+1})$ and $\text{sig}(\cdot)^\alpha$ is an odd function, one gets that the time derivative of the Lyapunov function candidate V along the trajectory of system (10) is

$$\begin{aligned} \dot{V} &= \sum_{i=1}^n \bar{v}_i \left[\sum_{v_j \in N_i} a_{ij}(t) \left[\text{sig}(\bar{x}_j - \bar{x}_i)^{\alpha_1} + \text{sig}(\bar{v}_j - \bar{v}_i)^{\alpha_2} \right] - b_i(t) \left[\text{sig}(\bar{x}_i)^{\alpha_1} + \text{sig}(\bar{v}_i)^{\alpha_2} \right] \right] \\ &\quad + b_i(t) \sum_{i=1}^n \text{sig}(\bar{x}_i)^{\alpha_1} \bar{v}_i + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) \text{sig}(\bar{x}_i - \bar{x}_j)^{\alpha_1} (\bar{v}_i - \bar{v}_j) \\ &= \sum_{i=1}^n \bar{v}_i \left[\sum_{v_j \in N_i} a_{ij}(t) \text{sig}(\bar{v}_j - \bar{v}_i)^{\alpha_2} - b_i(t) \text{sig}(\bar{v}_i)^{\alpha_2} \right] \\ &= \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) (\bar{v}_i - \bar{v}_j) \text{sig}(\bar{v}_j - \bar{v}_i)^{\alpha_2} - b_i(t) \sum_{i=1}^n \bar{v}_i \text{sig}(\bar{v}_i)^{\alpha_2} \\ &\leq 0, \end{aligned} \quad (12)$$

where the second and third equations are obtained by applying Lemma 3.1. From the fact that collection of the graphs in time interval $[t_k, t_{k+1})$ is jointly connected, one gets that $\dot{V} \equiv 0$ if and only if $\bar{v}_i \equiv \bar{v}_j \equiv 0$, $\forall i, j \in I$ and $\dot{\bar{v}}_i = 0$. Then one can further get

$$\begin{aligned} \dot{\bar{v}}_i(t) &= \sum_{v_j \in N_i} a_{ij}(t) \left[\text{sig}(\bar{x}_j - \bar{x}_i)^{\alpha_1} + \text{sig}(\bar{v}_j - \bar{v}_i)^{\alpha_2} \right] - b_i(t) \left[\text{sig}(\bar{x}_i)^{\alpha_1} + \text{sig}(\bar{v}_i)^{\alpha_2} \right] \\ &= \sum_{v_j \in N_i} a_{ij}(t) \left[\text{sig}(\bar{x}_j - \bar{x}_i)^{\alpha_1} \right] - b_i(t) \text{sig}(\bar{x}_i)^{\alpha_1} \\ &= 0, \end{aligned} \quad (13)$$

which implies that

$$\begin{aligned}
& \sum_{i=1}^n \bar{x}_i \left[\sum_{\nu_j \in N_i} a_{ij}(t) \left[\text{sig}(\bar{x}_j - \bar{x}_i)^{\alpha_1} \right] - b_i(t) \text{sig}(\bar{x}_i)^{\alpha_1} \right] \\
&= \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) \text{sig}(\bar{x}_j - \bar{x}_i)^{\alpha_1} (\bar{x}_i - \bar{x}_j) - \sum_{i=1}^n \bar{x}_i b_i(t) \text{sig}(\bar{x}_i)^{\alpha_1} \\
&= 0.
\end{aligned} \tag{14}$$

Since the graphs in each time interval $[t_k, t_{k+1})$ are jointly connected, $\bar{x}_i = \bar{x}_j = 0, \forall i, j \in I$ can be obtained from equation (14). According to LaSalle's invariance principle, $\bar{x}_i \rightarrow 0, \bar{v}_i \rightarrow 0$, as $t \rightarrow \infty$, that is, system (10) is globally asymptotically stable.

Secondly, we prove that system (10) is homogeneous of degree $\kappa = \alpha_1 - 1$ with respect to the dilation $\mathbf{r} = (\underbrace{2, 2, \dots, 2}_n, \underbrace{\alpha_1 + 1, \dots, \alpha_1 + 1}_n)$. Let

$$\begin{aligned}
\boldsymbol{\psi}(t) &= (\bar{x}_1(t), \bar{x}_2(t), \dots, \bar{x}_n(t), \bar{v}_1(t), \bar{v}_2(t), \dots, \bar{v}_n(t))^T \\
&= (\psi_1(t), \psi_2(t), \dots, \psi_n(t), \psi_{n+1}(t), \psi_{n+2}(t), \dots, \psi_{2n}(t))^T,
\end{aligned} \tag{15}$$

$f_i(\boldsymbol{\psi}) = \psi_{n+i}$, and

$$f_{n+i}(\boldsymbol{\psi}) = \sum_{\nu_j \in N_i} a_{ij}(t) \left[\text{sig}(\bar{x}_j - \bar{x}_i)^{\alpha_1} + \text{sig}(\bar{v}_j - \bar{v}_i)^{\alpha_2} \right] - b_i(t) \left[\text{sig}(\bar{x}_i)^{\alpha_1} + \text{sig}(\bar{v}_i)^{\alpha_2} \right]. \tag{16}$$

Then system (10) can be rewritten as

$$\begin{cases} \dot{\psi}_i = f_i(\boldsymbol{\psi}) \\ \dot{\psi}_{n+i} = f_{n+i}(\boldsymbol{\psi}), \end{cases} \tag{17}$$

When $r_1 = r_2 = \dots = r_n = R_1$, $r_{n+1} = r_{n+2} = \dots = r_{2n} = R_2$, $R_2 = R_1 + \kappa$, and $R_1 \alpha_1 = R_2 \alpha_2 = R_2 + \kappa$ which implies $\kappa = \frac{1}{2}(\alpha_1 - 1)R_1$, $R_2 = \frac{1}{2}(\alpha_1 + 1)R_1$, and $\alpha_2 = 2\alpha_1 / (\alpha_1 + 1)$, it leads to

$$\begin{aligned}
& f_i(\varepsilon^{r_1} \psi_1, \dots, \varepsilon^{r_i} \psi_i, \dots, \varepsilon^{r_n} \psi_n, \varepsilon^{r_{n+1}} \psi_{n+1}, \dots, \varepsilon^{r_{n+i}} \psi_{n+i}, \dots, \varepsilon^{r_{2n}} \psi_{2n}) \\
&= \dot{\psi}_i(\varepsilon^{r_1} \psi_1, \dots, \varepsilon^{r_i} \psi_i, \dots, \varepsilon^{r_n} \psi_n, \varepsilon^{r_{n+1}} \psi_{n+1}, \dots, \varepsilon^{r_{n+i}} \psi_{n+i}, \dots, \varepsilon^{r_{2n}} \psi_{2n}) \\
&= \varepsilon^{r_{n+i}} \psi_{n+i} = \varepsilon^{R_2} f_i(\boldsymbol{\psi}) = \varepsilon^{R_1 + \kappa} f_i(\boldsymbol{\psi}) \\
&= \varepsilon^{r_i + \kappa} f_i(\boldsymbol{\psi}), \forall i \in I,
\end{aligned} \tag{18}$$

that is,

$$f_i(\varepsilon^{r_1} \psi_1, \dots, \varepsilon^{r_i} \psi_i, \dots, \varepsilon^{r_n} \psi_n, \varepsilon^{r_{n+1}} \psi_{n+1}, \dots, \varepsilon^{r_{n+i}} \psi_{n+i}, \dots, \varepsilon^{r_{2n}} \psi_{2n}) = \varepsilon^{r_i + \kappa} f_i(\boldsymbol{\psi}), \forall i \in I. \tag{19}$$

On the other hand, we can get

$$\begin{aligned}
& f_{n+i}(\varepsilon^{r_1} \psi_1, \dots, \varepsilon^{r_i} \psi_i, \dots, \varepsilon^{r_n} \psi_n, \varepsilon^{r_{n+1}} \psi_{n+1}, \dots, \varepsilon^{r_{n+i}} \psi_{n+i}, \dots, \varepsilon^{r_{2n}} \psi_{2n}) \\
&= \sum_{\nu_j \in N_i} a_{ij} \left[\text{sig}(\varepsilon^{r_j} \psi_j - \varepsilon^{r_i} \psi_i)^{\alpha_1} + \text{sig}(\varepsilon^{r_{n+j}} \psi_{n+j} - \varepsilon^{r_{n+i}} \psi_{n+i})^{\alpha_2} \right] \\
&\quad - b_i(t) \left[\text{sig}(\varepsilon^{r_i} \psi_i)^{\alpha_1} + \text{sig}(\varepsilon^{r_{n+i}} \psi_{n+i})^{\alpha_2} \right] \\
&= \sum_{\nu_j \in N_i} a_{ij} \left[\varepsilon^{\alpha_1 R_1} \text{sig}(\psi_j - \psi_i)^{\alpha_1} + \varepsilon^{\alpha_2 R_2} \text{sig}(\psi_{n+j} - \psi_{n+i})^{\alpha_2} \right] \\
&\quad - b_i(t) \left[\varepsilon^{\alpha_1 R_1} \text{sig}(\psi_i)^{\alpha_1} + \varepsilon^{\alpha_2 R_2} \text{sig}(\psi_{n+i})^{\alpha_2} \right] \tag{20} \\
&= \varepsilon^{\alpha_1 R_1} \sum_{\nu_j \in N_i} a_{ij} \left[\text{sig}(\psi_j - \psi_i)^{\alpha_1} + \text{sig}(\psi_{n+j} - \psi_{n+i})^{\alpha_2} \right] \\
&\quad - \varepsilon^{\alpha_1 R_1} b_i(t) \left[\text{sig}(\psi_i)^{\alpha_1} + \text{sig}(\psi_{n+i})^{\alpha_2} \right] \\
&= \varepsilon^{R_2 + \kappa} f_{n+i}(\boldsymbol{\psi}) \\
&= \varepsilon^{r_{n+i} + \kappa} f_{n+i}(\boldsymbol{\psi}), \quad \forall i \in I,
\end{aligned}$$

that is,

$$f_{n+i}(\varepsilon^{r_1} \psi_1, \dots, \varepsilon^{r_i} \psi_i, \dots, \varepsilon^{r_n} \psi_n, \varepsilon^{r_{n+1}} \psi_{n+1}, \dots, \varepsilon^{r_{n+i}} \psi_{n+i}, \dots, \varepsilon^{r_{2n}} \psi_{2n}) = \varepsilon^{r_i + \kappa} f_{n+i}(\boldsymbol{\psi}), \quad \forall i \in I. \tag{21}$$

Combining the above two steps of analyses, the vector field

$$f(\boldsymbol{\psi}) = (f_1(\boldsymbol{\psi}), f_2(\boldsymbol{\psi}), \dots, f_n(\boldsymbol{\psi}), f_{n+1}(\boldsymbol{\psi}), f_{n+2}(\boldsymbol{\psi}), \dots, f_{2n}(\boldsymbol{\psi}))^T$$

is homogeneous of degree κ with respect to the dilation $\mathbf{r} = (\underbrace{R_1, R_1, \dots, R_1}_n, \underbrace{R_2, R_2, \dots, R_2}_n)$.

Specially, when $R_1=2$, $0 < \alpha_1 < 1$, one can get $\alpha_2 = 2\alpha_1 / (\alpha_1 + 1)$, $R_2 = \alpha_1 + 1$, and $\kappa = \alpha_1 - 1 < 0$. According to Lemma 3.2, the equilibrium of system (10) is finite-time stable, that is to say, $\bar{x}_i \rightarrow 0$, $\bar{v}_i \rightarrow 0$ in finite time. Thus, time-varying formation in finite time for multi-UAV system (1) with switching topologies can be achieved under control protocol (7). \square

4. Multi-UAV Control System Simulations

In this section, numerical simulations are given to illustrate the effectiveness of theoretical results obtained in the previous sections.

Consider a multi-UAV system with six UAVs where the dynamics of each UAV are described by (1). In the case where $m=2$, $x_i(t)$, $v_i(t)$, $x_0(t)$, $v_0(t)$, $u_i(t)$, $h_i(t)$ can be respectively written as $x_i(t) = [x_{ix}(t), x_{iy}(t)]^T$, $v_i(t) = [v_{ix}(t), v_{iy}(t)]^T$, $x_0(t) = [x_{0x}(t), x_{0y}(t)]^T$, $v_0(t) = [v_{0x}(t), v_{0y}(t)]^T$, $u_i(t) = [u_{ix}(t), u_{iy}(t)]^T$, and $h_i(t) = [h_{ix}(t), h_{iy}(t), h_{ix}(t), h_{iy}(t)]^T$, $i = 1, 2, \dots, 6$.

Choose the interaction graphs as $\{\bar{G}_1, \bar{G}_2, \bar{G}_3, \bar{G}_4, \bar{G}_5, \bar{G}_6\}$ which are shown in Fig. 3(a) to verify the effectiveness of the proposed control law. The corresponding switching signal is depicted in Fig. 3(b) which means that the communication graphs switch in the following order: $\bar{G}_1 \rightarrow \bar{G}_2 \rightarrow \bar{G}_3 \rightarrow \bar{G}_4 \rightarrow \bar{G}_5 \rightarrow \bar{G}_6 \rightarrow \bar{G}_1 \dots$, and each graph stays active for 0.1s which conforms to reality. Such a scenario is to some extent really challenging since there are only a few communication links available at any time, which subsequently

extends the control period of a complete loop. Moreover, it can be seen from Fig. 3(a) that the collection $\bar{G}_1, \bar{G}_2, \bar{G}_3$ and $\bar{G}_4, \bar{G}_5, \bar{G}_6$ are jointly connected, respectively, which implies that Assumption 3.1 is satisfied. Thus, we have $t_k = k$, $t_{k+1} = k + 0.3s$, and $t_k^0 = k$, $t_k^1 = k + 0.1s$, $t_k^2 = k + 0.2s$ with $k = 0, 1, 2, \dots$

Figure 3 here

The six UAVs are supposed to keep a parallel hexagon formation with time-varying edges and at the same time to keep rotation around the virtual leader $x_0(t) = [5t, 5t]^T$ which lies in the formation center. Thus, the time-varying formation is specified by

$$h_i(t) = \begin{bmatrix} 0.2t\cos(0.1t + (i-1)\pi/3) \\ 0.2[\cos(0.1t + (i-1)\pi/3) - 0.1t\sin(0.1t + (i-1)\pi/3)] \\ 0.2t\sin(0.1t + (i-1)\pi/3) \\ 0.2[\sin(0.1t + (i-1)\pi/3) + 0.1t\cos(0.1t + (i-1)\pi/3)] \end{bmatrix}, \quad i = 1, 2, \dots, 6.$$

Choose $\alpha_1 = 1/3$, $\alpha_2 = 2\alpha_1/(\alpha_1 + 1) = 1/2$ in protocol (7). The simulation results are presented by Figs. 4-5. Fig. 4 (a) shows the initial positions of the six UAVs and the virtual leader, while Fig. 4 (b) displays the positions of the six UAVs and the virtual leader at $t = 50, 60, 70, 80, 90, 100s$. Fig. 5 depicts the formation position tracking errors $\bar{x}_i(t)$ on X-axis and Y-axis. From Figs. 4 and 5, the following phenomena can be observed: (i) the six UAVs successfully keep a parallel hexagon formation; (ii) the edges of the parallel hexagon are getting longer as time goes on; (iii) the parallel hexagon keeps rotation around the virtual leader; (iv) the virtual leader moves along a straight line, and keeps lying in the center of the formation, so that the formation moves along the same straight line; (v) the formation position tracking errors keep 0 after about $t=20s$ although the interaction topology is still switching, and the response time is a bit long which is reasonable as the initial positions are too far and the topology does not keep connected all the time. Therefore, the desired time-varying formation can be achieved in finite time by the control law (7) under jointly connected topologies.

Figure 4 here

Figure 5 here

5. Conclusion

For multi-UAV system formation control problem with variable topology, time-varying formation tracking control is more practical in many applications. Besides, jointly connection is a more general condition because it does not require the connection all the time, and finite time stability is more meaningful due to its higher performance of response rates, accuracy and robustness. Therefore, the time-varying formation finite-time tracking control under jointly connected topologies is studied and the control law is formulated in this paper. Based on graph theory, the finite time stability of the close-loop system with the proposed control protocol is explored through LaSalle's invariance principle and the theory of homogeneity with dilation. Numerical simulation results are presented to illustrate the effectiveness of the obtained theoretical results. A future research direction is to extend the results in this paper to the case where there exist time delays in communication.

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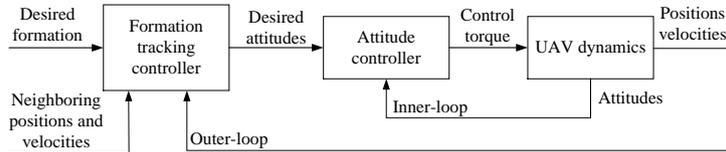


Fig. 1. The two-loop control structure scheme for a UAV.

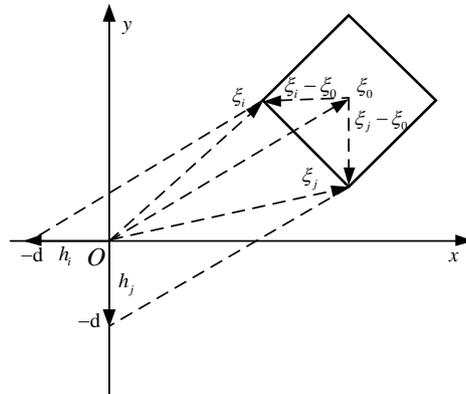
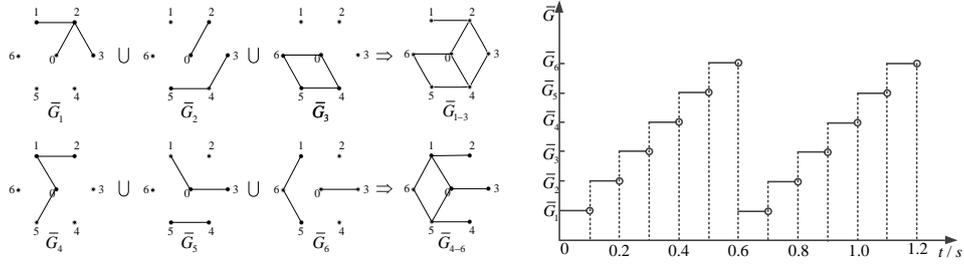
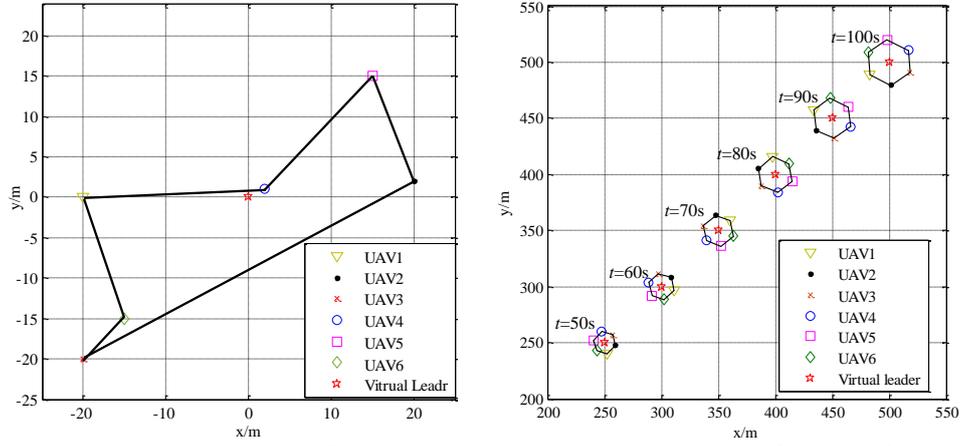


Fig. 2. Illustration example of square formation.



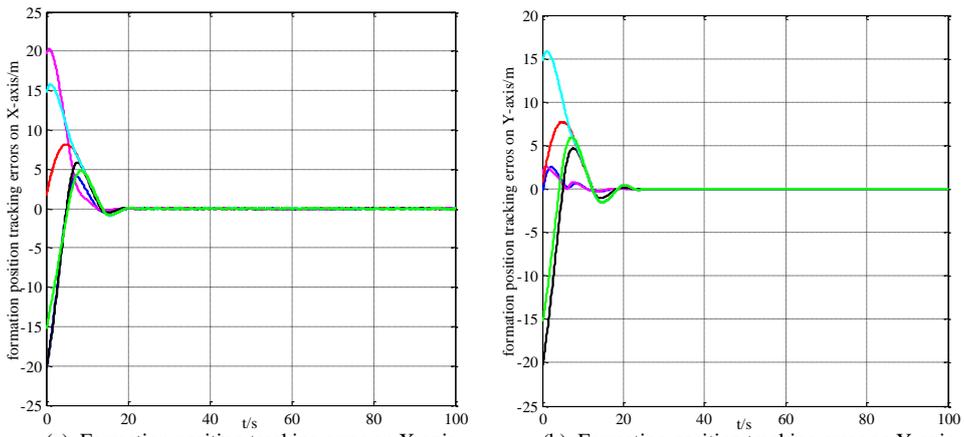
(a) Possible jointly-connected interactions graphs (b) A switching signal describing the graphs

Fig. 3. Jointly-connected graphs and a switching signal describing the graphs.



(a) Positions at $t = 0s$ (b) Positions at $t = 50, 60, 70, 80, 90, 100s$

Fig. 4. Position snapshots of the six UAVs and the virtual leader.



(a) Formation position tracking errors on X-axis (b) Formation position tracking error on Y-axis

Fig. 5. Formation position tracking errors on X-axis and Y-axis.