# Logic Programming with Graded Modality

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# Motivation

- Express modal concepts: “at least as many as” and “at most as many as” naturally.

- Find natural and intuitive semantics for Epistemic Specification

Combine the basic idea in Graded Modal Logic and ASP!

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# Logic Programs with Graded Modality

- Syntax
  - Rule $l_1$ or … or $l_k$ :- $e_1,\ldots,e_m$, $s_1,\ldots,s_n$. $k\geq 0$, $m\geq 0$, $n\geq 0$
  - Literals
    - Extended literal $e$
      - » Objective literals: $l$
      - » Default literals: not $l$
    - Subject literals:
      - » With upper bound: $M_{[lb:ub]}\ e$
      - » Without upper bound: $M_{[lb:]}\ e$

$M_{[lb:ub]}e$ intuitively means: it is known that the number of belief sets where $e$ is true is between lb and ub.

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# Safe Rule:

A rule is SAFE if each variable in it appears in the positive body of the rule.

- The positive body of a rule r is composed of the extended literals containing no not in its body.

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# Semantics(ground program):

Let *W* be a non-empty collection of consistent sets of ground objective literals, $w \in W$,

- $< W, w > \models l$ if $l \in w$
- $< W, w > \models not\ l$ if $l \notin w$
- $< W, w > \models \mathrm{M}_{[lb:ub]}e$ if $lb \leq |\{w \in W| < W, w > \models e\}| \leq ub$
- $< W, w > \models \mathrm{M}_{[lb:]}e$ if $|\{w \in W| < W, w > \models e\}| \geq lb$

Then, for a rule $r$ in $\Pi$, $< W, w > \models r$ if

- $\exists l \in head(r)$: $< W, w > \models l$
- $\exists t \in body(r)$: $< W, w > \not\models t$.

*W* is a model of a program $\Pi$, if $\forall r \in \Pi$, and $\forall w \in W$,

$$< W, w > \models r$$

# A Special Model: World View I

- ASP Idea. [Gelfond,2014] give three principles for rational reasoning.
  - Believe in the head of a rule if you believe in its body (Satisfiability principle)
  - Do not believe in contractions (Consistency principle)
  - Believe nothing you are not forced to believe(Rationality principle)

**World View** will be defined on

- the principles (From ASP), and
- the above satisfaction notion for literals(From GML), and rules (Satisfiability principle in ASP).

# A Special Model: World View II

- Part I. For a disjunctive logic program, its world view is the non-empty set of all its answer sets.

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# A Special Model: World View III

- Part II. For an arbitrary *LPGM* program $\prod$, a non-empty set $W$ is its world view if $W$ is the world view of a DLP $\prod^W$ which is a reduct of $\prod$ by the following laws.

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# A Special Model: World View IV

- $\prod^{W}$

1. Removing from all rules containing subjective literals not satisfied by *W*.
2. Replacing all other occurrences of subjective literals of the form $M_{[1b:\ ub]}\ l$ or $M_{[1b:]}\ l$ where lb=|*W*| by *l*.
3. Removing all other occurrences of subjective literals of the form $M_{[1b:\ ub]}$ *not l* or $M_{[1b:]}$ *not l* where lb=|*W*|.
4. Replacing all other occurrences of subjective literals of the form $K_{[0:\ 0]}\ e$ by $e^{not}$. ($e^{not}$ is *not l* if *e* is *l*, and $e^{not}$ is *l* if e is *not l*)
5. Replacing other occurrences of subjective literals of the form Mω *e* by *e* and $e^{not}$ respectively, that is, two rules should be created, one in which Mω *e* is replaced by *e* and one in which Mω *l* is replaced by $e^{not}$.

# A Special Model: World View V

- Example $\prod$:

$$l \text{ :- } M_{[1:]} l.$$

For $W$={{}}, $\prod^W$ is empty and has a world view {{}}, therefore, {{}} is a world view of $\prod$.

For $W$={{$l$}}, $\prod^W$ has one rule: $l$:- $l$., and thus has a world view{{}}. Hence, {{$l$}} is not a world view of $\prod$.

# A Special Model: World View VI

**The reduct is natural and intuitive.**

- Law 1: Removing from all rules containing subjective literals not satisfied by *W*.
- **Interpretation**: The law directly comes from the notion of Rule Satisfiability and Rationality Principle in answer set programming which means if a rules body cannot be satisfied (believed in), the rule will contribute nothing;

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# A Special Model: World View VII

**The reduct is natural and intuitive.**

- Law 2: Replacing all other occurrences of subjective literals of the form $M_{[1b:\ ub]}\ l$ or $M_{[1b:]}\ l$ where lb=|*W*| by *l*.
- **Interpretation**: if it is known that there are at least lb number of belief sets where *l* is true and there are totally lb belief sets in *W*, then, by the meaning of the gradation based on counting, *l* must be believed with regard to each belief set in *W*. Then, by the Rationality Principle, you are forced to believe *l* with regard to each belief set. Hence, $M_{[1b:\ ub]}\ l$ or $M_{[1b:]}\ l$ should be replaced by *l* (instead of being removed) to avoid self-support;

# A Special Model: World View VII

**The reduct is natural and intuitive.**

- Law 3: Removing all other occurrences of subjective literals of the form $M_{[1b:\,ub]}$ *not l* or $M_{[1b:]}$ *not l* where lb=|*W*|.
- **Interpretation**: if it is known that there are at least lb number of belief sets where *l* is not true and there are totally lb belief sets in *W*, then, *not l* must be believed with regard to each belief set in *W*. Then, removing $M_{[1b:\,ub]}$ *not l* or $M_{[1b:]}$ *not l* in a rule will not effect the satisfiability of the rule.

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# A Special Model: World View VIII

**The reduct is natural and intuitive.**

- Law 4: Replacing all other occurrences of subjective literals of the form $M_{[0:\,0]}$ *e* by $e^{not}$. .
- **Interpretation**: if it is known that *e* is not believed with regard to each belief set in *W*, then we are forced to believe $e^{not}$ with regard to each belief set in *W*;

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# A Special Model: World View IX

**The reduct is natural and intuitive.**

- Law 5: Replacing other occurrences of subjective literals of the form Mω *e* by *e* and $e^{not}$ respectively.
- **Interpretation**: The last law states that, if it is known that there are at least lb number of belief sets where *e* is believed, and the number of belief sets in *W* is strict greater than lb, then *e* may be believed or may not be believed with regard to a belief set in *W*.

- Example using Law 5.

$$l \text{ :- } M_{[0:]}l.$$

For $W$={{}}($|W|=1$), $\prod^W$ is:

$$l\text{:- } l.$$

$$l\text{:- } not\ l.$$

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# Relation to Epistemic Specification

- Latest *Epistemic Specification* (2014 version): $ASP^{KM}$

where K, M, *not* K, *not* M are used to extend ASP.

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**Theorem**. An ASP$^{KM}$ Program can be represented as a LPGM program by

K$l \Rightarrow$ M$_{[0:0]}$*not l*

M$l \Rightarrow$ M$_{[1:]}l$ and *not not l* respectively.

*not* K$l \Rightarrow$ M$_{[1:]}$*not l* and *not l* respectively

*not* M$l \Rightarrow$ M$_{[0:0]}l$

# A Sound and Complete Algorithm for Computing World Views

**Algorithm 1** LPGMSolver.

**Input:**

$\Pi$: A *LPGM*;

**Output:**

All world views of $\Pi$;

```
n = max{lb | M_[lb:ub] e or M_[lb:] e in Π} {computes the maximal lb of subjective literals in Π}
WV = ∅
for every natural number 1 ≤ k ≤ n do
    WV_k = WViSolver(Π, k) {computes all world views of size k for Π}
    WV = WV ∪ WV_k
end for
WV_{>n} = WVgiSolver(Π, n) {computes all world views of size strict greater than n for Π}
WV = WV ∪ WV_{>n}
output WV
```

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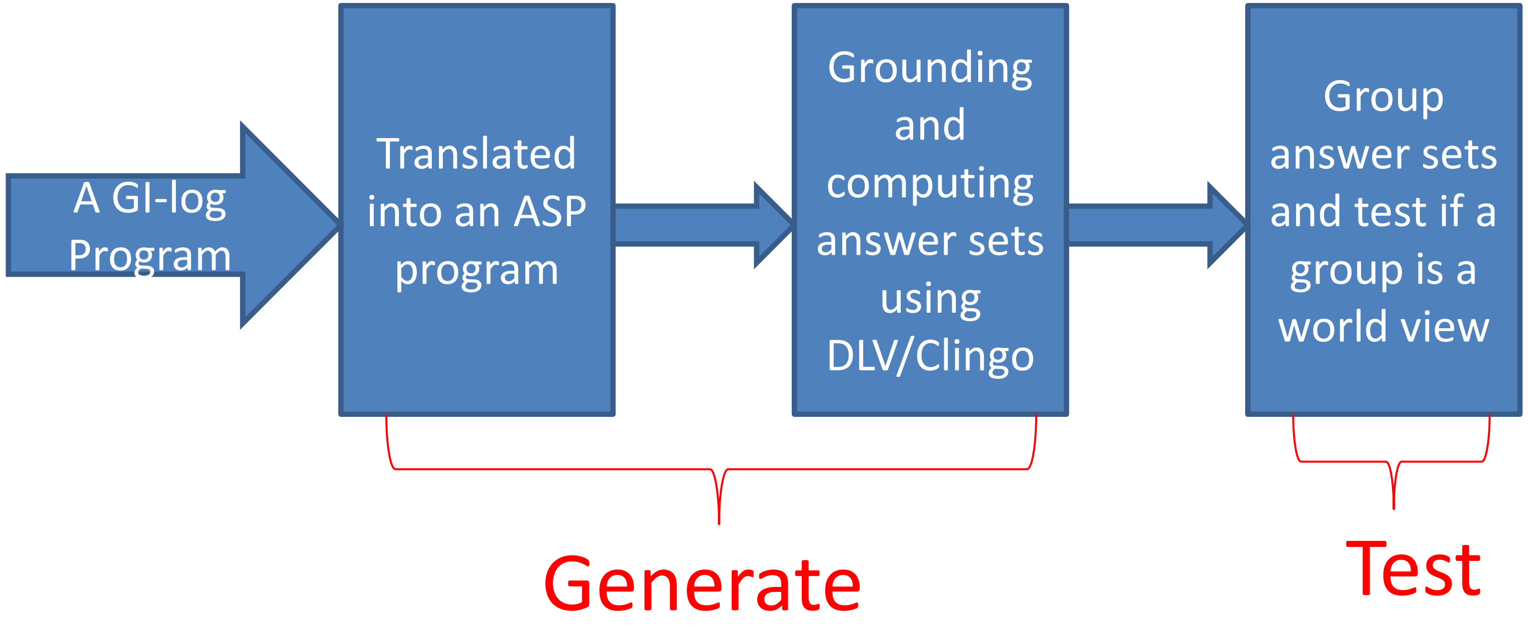
# An Algorithm for Computing World Views

## WViSolver and WVgiSolver



# Complexity of the Algorithm

- The algorithm is in PSPACE and $O(2^{3|\mathcal{L}|})$.
- A LPGM solver will be issued in http://cse.seu.edu.cn/people/seu_zzz/

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# Applications: A Case Study I

- N-critical edges problem. Given a directed graph $G = (V, E)$ where $V$ is the set of vertices of $G$ and $E$ is the set of edges of $G$, find the set of all edges that belong to $n$ or more hamiltonian cycles in $G$.

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# Formalizing Hamiltonian cycles

$$inhc(X,Y) \; or \; \neg inhc(X,Y) \leftarrow edge(X,Y).$$
$$\leftarrow inhc(X1,Y1), inhc(X2,Y1), X1 \neq X2.$$
$$\leftarrow inhc(X1,Y1), inhc(X1,Y2), Y1 \neq Y2.$$
$$reachable(X,X) \leftarrow vertex(X).$$
$$reachable(X,Y) \leftarrow inhc(X,Z), reachable(Z,Y).$$
$$\leftarrow vertex(X), vertex(Y), not \; reachable(X,Y).$$

# Defining N-critical edges

$$ncritical(X,Y) \leftarrow M_{[n:]} inhc(X,Y), edge(X,Y).$$

# Applications: A Case Study II

- N-exclusive paths problem. Given a directed graph $G = (V, E)$ where $V$ is the set of vertices of $G$ and $E$ is the set of edges of $G$, decide whether there are $n$ number of $m$-exclusive paths from a vertex $a$ to another vertex $b$, that is, decide whether there are n or more paths between a and b, and there are no edge belonging to m or more of the paths.

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# Formalizing the definition of *Path(a, b)*

$$inpath(X,Y)\ or\ \neg inpath(X,Y) \leftarrow edge(X,Y).$$
$$\leftarrow inpath(X1,Y1), inpath(X2,Y1), X1 \neq X2.$$
$$\leftarrow inpath(X1,Y1), inpath(X1,Y2), Y1 \neq Y2.$$
$$reachable(X,X) \leftarrow vertex(X).$$
$$reachable(X,Y) \leftarrow inpath(X,Z), reachable(Z,Y).$$
$$path \leftarrow reachable(a,b).$$
$$\leftarrow not\ path.$$

# Formalizing *n or more paths*

$$npath \leftarrow \mathrm{M}_{[n:]} path.$$
$$\leftarrow not\ npath.$$

# Formalizing *m-exclusive*

$$\leftarrow \mathrm{M}_{[m:]} inpath(X,Y), edge(X,Y).$$

# Conclusion

- LPGM language is a new way of reasoning with Negation as Failure and Modality together.
- LPGM semantics/reduct is intuitive and based on the principles of ASP.
- The application of LPGM seems potential.

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# Future Work

- Mathematical Properties.

- Methodologies for modeling with LPGM.

E.g. Contextual Reasoning, information fusion etc.

- Other graded modalities

A new result can be found in ASPOCP 2015.
"*logic programming with graded introspection*"

# Thank You!